Aircraft Propulsion

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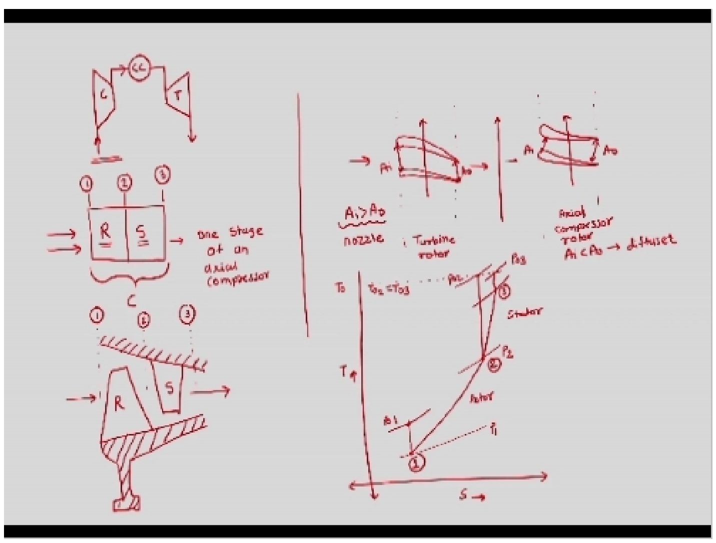
Lecture-29
Axial Flow Compressor Velocity diagrams, Workdone and Degree of Reaction

Welcome to the class, today we will see and talk about the axial compressors. Axial compressor is one more option for us as a first entity in a gas turbine power plant.



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(refer time: 00:45). We know that in our gas turbine power plant we have air entering into the compressor and then it goes to the combustion chamber and then onwards it travels to the turbine. So this gas turbine power plant needs a compressor and then there are multiple options like centrifugal compressor and then the axial compressors. So among that we are going to see an axial compressor. So as it is shown over here this is a compressor.

The schematic of the axial compressor would be simple and first the flow of air would experience a rotor, this is air which is entering into the compressor, this is a complete compressor first it will come into the rotor and then there will be a stator. So rotor entry conditions are 1, exit conditions are 2 and stator exit conditions are 3. So one compressor is composed of 2 entities, one is rotor another is stator.

Rotating entities rotor stator is a stationary component of the compressor. So in that we will see how it can be sketched, so first we will have rotor, this part is the frame which is attached to the shaft and then this is part which is stationary and then we will have rotor blades and then we have stator blades. So this is rotor, this is stator, then air enters into the compressor of one stage it experiences first rotor and then experiences the stator.

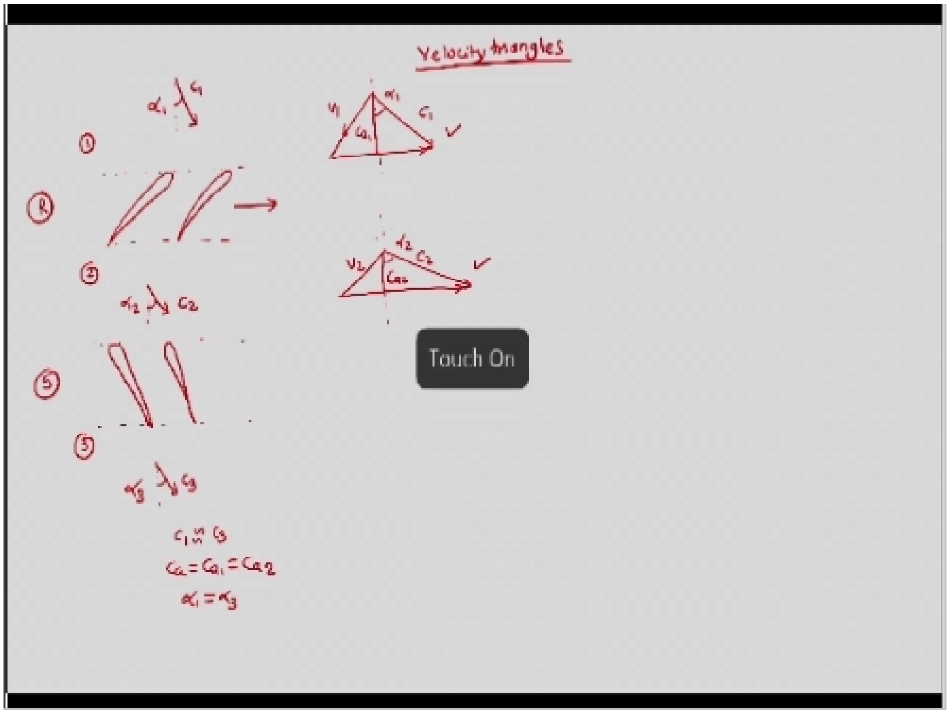
So that is as it is said we will have in that condition to the rotor as 1, outlet conditions are 2 and outlet conditions for stator are 3.So this is compressor but we will stay it as one stage of an axial compressor, it is possible that multiple rotor stator pairs would be put in series such that we will have a multi-stage compressor. Then we have actually axial compressor and axial turbine where state rotor would have different configurations.

So if I consider an axial turbine which will be separately discussed or which as a separate topic as axial turbine rotor would have, this is turbine rotor, air enters and air leaves. So this is the normal area which is A_i, this is the outlet area A_o, in general A_i is greater than A_o and the action is basically action of a nozzle. Complimentary separately in case of an axial compressor we have different issue where we have this is A_i and then this is A_o.

So this is axial compressor rotor and A_i is less than A_o and action is diffuser. So while flowing through the rotor blades there is a revaluation in the area. So this is the direction of rotation, so this is the direction for rotation for turbine and this is for the compressor and by flowing air experiences variation in area and it leads to the actions like nozzle or diffusion having said this we can plot the T-S diagram for the rotor and stator and hence one stage of the axial compressor.

So if we say T as y-axis S as x-axis, then this is state 1 which is P_1 so we are at state P_1 so we will have compression from 1 to 2, so we have P_2, this is point 2, this is point 1, this is in the rotor and in the stator we have further compression and then we will reach state 3, then this is in the stator, further here if we would have tried to calculate the total conditions at 1, then this would be P_{01} and then at 2 we would be P_{02} here and P_{03} as well will be here.

But such a way that we will have same total temperature as T_0 such that $T_{02} = T_{03}$. So practically we have gone from in the rotor we are grown from 1 P_{01} to P_{02} and in the stator and rotor combination when you have gone from P_{01}, we actually would change the total pressure from 1 state 2 to state in the rotor and 1 state to 3 state in rotor + stator combination. So this is how we would have thermodynamic variation of properties in the rotor of the axial compressor and rotor + stator of the axial compressor.



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(refer time: 08:30). Now we will try to plot velocity triangle, while plotting the velocity triangle let us plot again, sketch again the stator rotor combination which would help us to plot the velocity triangle. So this would be first rotor R, so these are the rotor blades, air is entering from here where this is the direction of absolute velocity C_1 with respect to the axis of the shaft and this is the direction of rotation. So this angle is said to be α_1 and then here this is state 1, this is state 2.

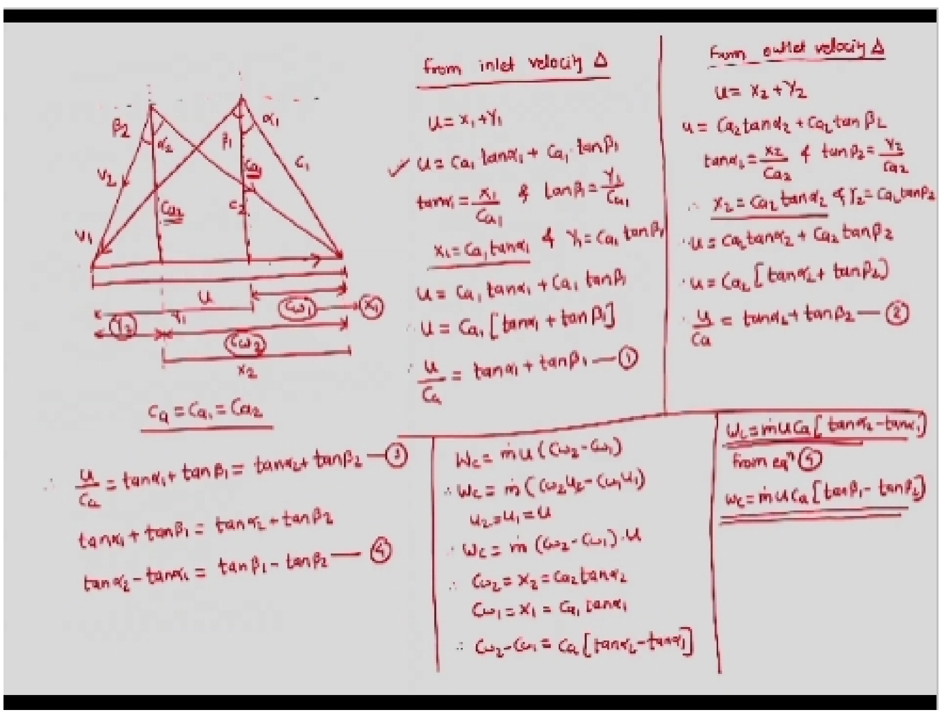
So before the rotor state is 1 after the rotor thermodynamic state is 2 and then air enters into the diffuser, the diffuser should again be the blades which are other stator blades. So these are the stator blades, so here air enters with velocity C_2 and angle with the axis as α_2 and then this is state 3 where again air leaves with velocity C_3 and angle α_3 and thence this fluid or air would again forget past if required to the next stage of the compressor.

Hence practically we would have C_1 is almost equal to C_3 such that $C_a = C_{a1} = C_{a2}$ and then we have $\alpha_1 = \alpha_3$, axial velocity should remain constant while flowing. So if we have tried to plot the velocity triangle at the inlet of the rotor then as we plotted this, so this is velocity C_1 which is making an angle α_1 with the axis of rotation and then as we showed this direction of rotation.

So this would be the direction of rotation, so this is u velocity and hence the relative velocity is V_1. So we have C_{a1} as the vertical velocity. Then at the outlet we have again as plotted here the C_2 as the velocity. So this is C_2, this is C_{a2}, this is α_2, this is V_2. So since if we go back and see this is the rotor and we are trying to plot the velocity triangle at the station 1 and at station 2 but at the midplane height of the rotor.

The velocity triangle would be different at different heights of the rotor, since we know that the velocity which is tangential velocity u is going to vary in the direction which is R since it is πDN/60 where D is diameter of the rotor. So since the diameter would vary a different location you would vary and hence we would have different velocity triangles at different heights. But here we are trying to plot all the velocity triangles at the mid plane height.

So we have inlet velocity triangle and outlet velocity triangles both at the mid plane height. Now having said this if we go ahead and plot the combined velocity triangle for our analysis then the combined velocity triangle would be this where we have first C_1 velocity here.



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(refer time: 13:42). And then we have u velocity which is complete , u velocity here, then we have this as V_1, this is Inlet velocity triangle where α_1 as the angle for absolute velocity and β_1 as the angle for relative velocity, having said this we can plot the outlet velocity triangle where due to the rotor we will get increase in absolute kinetic energy and then this is C_2 which is absolute velocity and then this is V_2 such that we have α_2 and β_2 are the angles for absolute and relative velocities respectively.

Since we are plotting at same mid plane the u velocity which is tangential velocity of the blade that is going to remain constant for inlet and outlet velocity triangles. So if we continue this and plot the normal we will have this vertical as C_{a1} and then this vertical as C_{a2}, further this C_1 and C_{a1} would constitute this as whirling velocity C_{w1} and then C_2 and C_{a2} would constitute the whirling velocity C_{w2}.

Let us say that this is X and this is Y required for us for further calculation from inlet velocity triangle we can write u, so this is X_1 for us so parallely for inlet velocity triangle we are decomposing u into 2 components. So here we are saying that we are decomposing u for inlet velocity triangle as $X_1 + Y_1$, where X_1 is basically C_{w1} and Y_1 is rest of the u. Similarly for outlet velocity triangle from outlet velocity triangle we have $u = X_2 + Y_2$ where X_2 is C_{w2} and Y_2 is remaining component of u.

So u is $X_1 + Y_1$, we can write down X_1 which is C_{w1} in terms of C_a, we are emphasizing to write in terms of $C_a = C_{a1} = C_{a2}$, height of the velocity triangle is constant. So we will try to represent X_1 and Y_1 in terms of C_a. So it will give $u = X_1 + Y_1$ but $X_1 = C_{a1} \tan \alpha_1, + C_{a1} \tan \beta_1$, we know that $\tan \alpha_1 = X_1 / C_a$ and $\tan \beta_1 = Y_1 / C_a$. So we have $X_1 = C_{a1} \tan \alpha_1$ and $Y_1 = C_{a1} \tan \beta_1$.

So this is what we have written $u = C_{a1} \tan \alpha_1 + C_{a1} \tan \beta_1$, we can take C_{a1} as common and then we have $\tan \alpha_1 + \tan \beta_1$. So further since a is constant we can mention it as C_a and u/C_a becomes $\tan \alpha_1 + \tan \beta_1$, having said this and proceed with the outlet velocity triangle and then again mention $u = X_2$ where X_2 is this distance and then this distance is $C_{a2} \tan \alpha_2 + C_{a2} \tan \beta_2$.

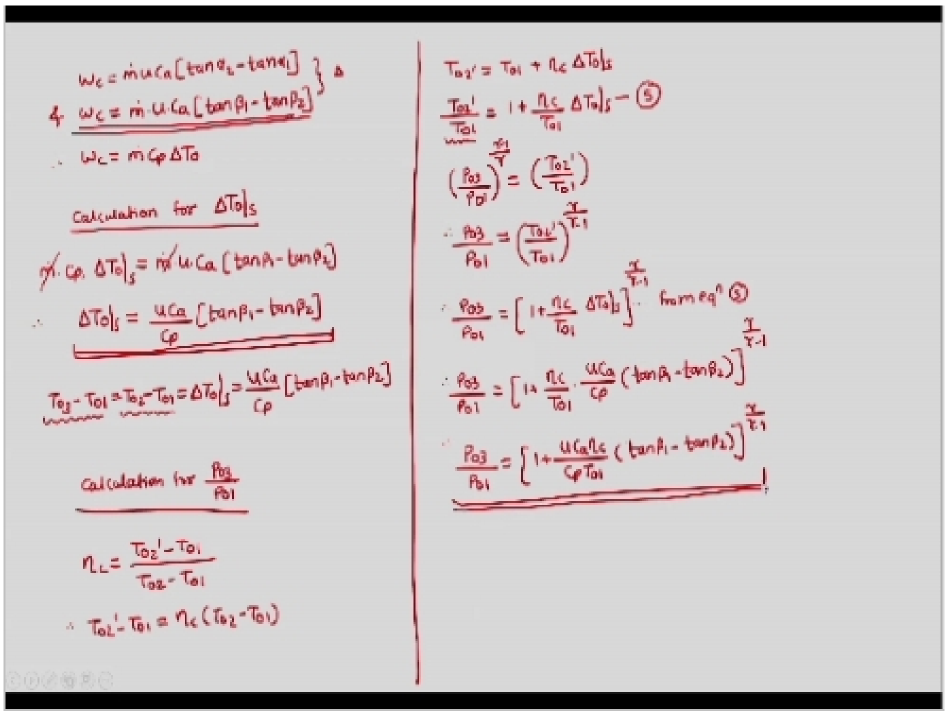
We can again mention $\tan \alpha_2 = X_2 / C_{a2}$ and $\tan \beta_2 = Y_2 / C_{a2}$, so $X_2 = C_{a2} \tan \alpha_2$ and $Y_2 = C_{a2} \tan \beta_2$. So we got u as $C_{a2} \tan \alpha_2 + C_{a2} \tan \beta_2$. So we get $u = C_{a2} (\tan \alpha_2 + \tan \beta_2)$. So we get $u / C_a = \tan \alpha_2 + \tan \beta_2$, if we name this equation from inlet velocity triangle as equation 1 and this equation as equation 2, which is obtained from outlet velocity triangle we can write down from equation 1.

We can write down from equation 1 and 2 as $u/C_a = \tan \alpha_1 + \tan \beta_1$ which is equal to $\tan \alpha_2 + \tan \beta_2$, this is what we can mention. Further now we will proceed and we can write down an expression for work input for the compressor from the velocity triangle before that we should write down one more expression from here which is equation number 3 that $\tan \alpha_1 + \tan \beta_1 = \tan \alpha_2 + \tan \beta_2$ that $\tan \alpha_2 - \tan \alpha_1 = \tan \beta_1 - \tan \beta_2$.

Now we will limit as equation number 4, having said this will proceed for the calculation of work input for the compressor and we know that work input = $\dot{m}\, u\, (C_{w2} - C_{w1})$, basically this expression was $\dot{m}(C_{w2}u_2 - C_w u_1)$ but we are working on the same mid plane height such that $u_2 = u_1 = u$ we are writing compressor work as $\dot{m}(C_{w2} - C_{w1})u$.

So we will write C_{w2} is basically X_2 and we know $X_2 = C_{a2} \tan \alpha_2$ and we know $C_{w1} = X_1$ and then that $= C_{a1} \tan \alpha_1$. So we have $C_{w2} - C_{w1} = C_a (\tan \alpha_2 - \tan \alpha_1)$. So we can use equation number 4 and before that we can write down expression for work input as $\dot{m}uC_a(\tan \alpha_2 - \tan \alpha_1)$, but from equation 4 we can also write $W_c = \dot{m}uC_a(\tan \beta_1 - \tan \beta_2)$.

So this is the expression for work input for an axial compressor. So we have seen how to plot velocity triangle and how to derive the expression for work input.



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(refer time: 24:42). But we will write down again $W_c = \dot{m}$ u C_a ($\tan \alpha_2 - \tan \alpha_1$) and $W_c = \dot{m}$ u C_a($\tan \beta_1 - \tan \beta_2$). These 2 expressions are from velocity triangle, but we know from thermodynamics that $W_c = \dot{m}\, C_p\, \Delta T_0$. So here we are starting toward the calculation for stage temperature rise, so we can equate that and then we can find out what is the stage temperature rise.

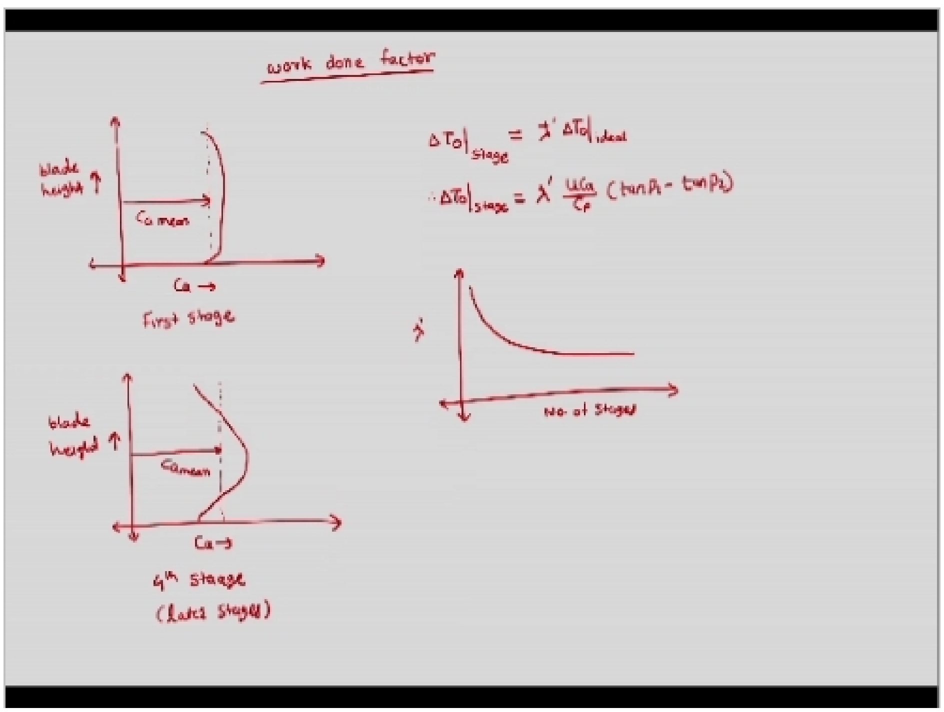
So $\dot{m}\, C_p\, \Delta T_0|_s = \dot{m}$ u C_a ($\tan \beta_1 - \tan \beta_2$), so $\dot{m}$ $\dot{m}$ will cancel and then we have $\Delta T_0|_s = \frac{uC_a}{C_p}(\tan \beta_1 - \tan \beta_2)$ and then this remains the expression for the interrogate and total temperature rise in the stage. So we have $T_{03} - T_{01}$ which is $T_{02} - T_{01} = \Delta T_0|_s = \frac{uC_a}{C_p}(\tan \beta_1 - \tan \beta_2)$. So having said this next we will try to find out what is the pressure rise in a stage.

So calculation for P_{03}/P_{01}, so for that we needed the temperature rise total temperature rise in a stage. So we will move and we will use the efficiency of compressor or stage efficiency which states that (T'_{02} - T_{01})/(T_{02} - T_{01} and then we can get T'_{02} - T_{01} = η_c (T_{02} - T_0 1). But T_{02} - T_{01} is also equal to T_{03} - T_{01} and hence T'_{02} = T_{01} + η_c $\Delta T_0|_s$.

So we have T'_{02}/T_{01} = 1 + (η_c/ T_{01})$\Delta T_0|_s$, but this is isentropic temperature ratio in a process which is isentropic and we know this is equal to $(P_{03}/P_{01})^{(\gamma-1)/\gamma}$ = T'_{02}/T_{01}, so P_{03} / P_{01} = $(T'_{02}/T_{01})^{\gamma/(\gamma-1)}$. So P_{03} / P_{01} = $(1 + (\eta_c/T_{01})\, \Delta T_0|_s)^{\gamma/(\gamma-1)}$ where we will continue the equation number and state this as equation number 5.

So this is from equation 5, so we can write down (P_{03}/P_{01}) = [1 + (η_c / T_{01})(u C_a/C_p)(tan β_1 - tan β_2)] $^{\gamma/(\gamma-1)}$. So we can write down in a simplified manner that P_{03}/P_{01} = [1 + (uC_a η_c/C_p T_{01})(tan β_1 - tan β_2)]$^{\gamma/(\gamma-1)}$. So this is how we can calculate the pressure rises in a centrifugal compressor from the velocity triangles.

Knowing the velocity triangle we can also calculate the total temperature rise, we can calculate the work input and so we can calculate the pressure rise. Now moving on to the next concept which is work done factor.



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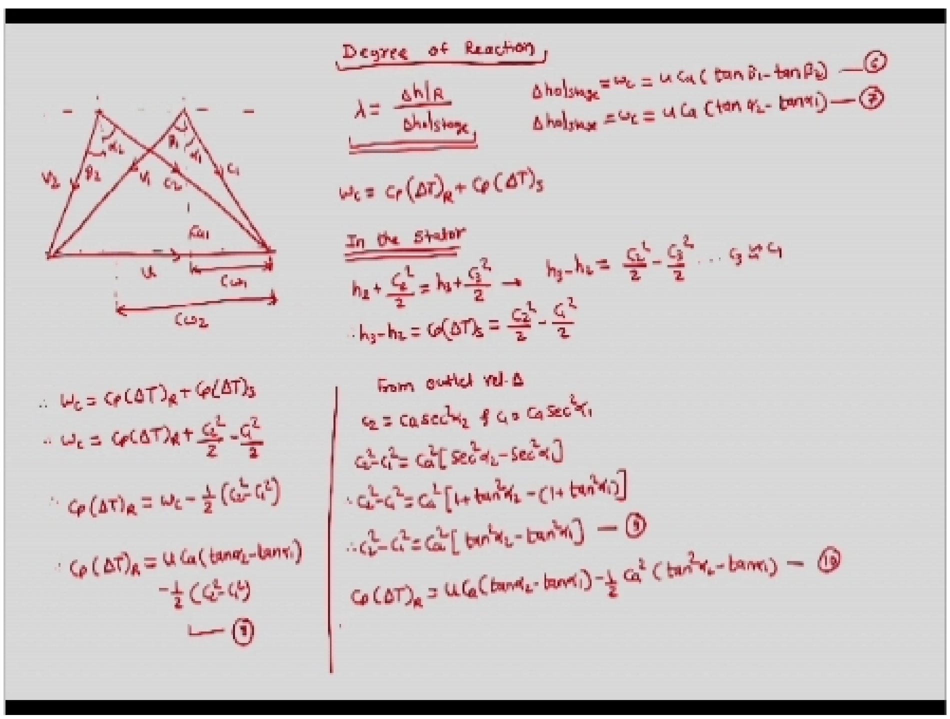
(refer time: 31:09). If we are in one stage and then if we see in y-axis if we have laid height and x-axis if we have C_a then we have fairly this variation as the variation for the C_a blade height is varying in this direction and we have C_a. So this is mean C_a and the blade height different blade heights the main C_a is varying but this is the mean value of C_a and this such variation can be seen for first stage or an early stage.

But if we go down a stage then again plot blade height versus C_a then the variation will be much cued and then this would remain the value of mean C_a and this is where C_a and this is for suppose nth stage and practically we are talking about suppose third or fourth stage and we will say latter stages, such variation of C_a would very different work inputs and different blade heights and hence the different total temperature rise.

So to account that there is a factor which is called as work done factor and then this give us $\Delta T_0|_s$ which is reality is equal to $\lambda' \Delta T_0$, what we have calculated from the velocity triangle. So this is we will say as ideal, so we have $\Delta T_0|_s = \lambda'(uC_a/C_p)(\tan\beta_1 - \tan\beta_2)$ and if we see the variation of λ' over number of stages would be like this.

So this accounts the variation of C_a along the different blade heights, we will move on for the next step and then we will try to calculate the degree of reaction.



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(refer time: 34:25). Degree of reaction for the axial flow compressor, again plot the velocity triangle here, this is outlet velocity triangle, this is inlet velocity triangle. So these are outlet and inlet velocity triangles, this is C_1, this is C_{a1}, this is α_1, this is β_1 this is C_2, this is V_1, this is α_2, this is β_2, this is V_2, this is u and then we have this as C_{w1} and then this as C_{w2}, having said this we will proceed for the degree of reaction we know degree of reaction $\lambda = \Delta h|_R/\Delta h_0|_s$.

This is our definition for degree of reaction, physically we mean that in the compressor we are having changing enthalpy, so among that how much enthalpy has got changed in the rotors what is the static enthalpy rise in the rotor divided by total enthalpy rise in the stage. So we know $\Delta h_0|_{stage} = W_c$ and that is you C_a (tan β_1 - tanβ_2) or $\Delta h_0|_{stage} = W_c$ = u C_a (tan α_2 - tan α_1).

So where we can also write W_c, this we will name as equation number 6 and this is maybe, so named this as 6 and this is 7. So W_c we know it is equal to $\Delta T_0|_R + C_p \Delta T_0|_S$, so if we go in the stator, stator is basically a diffuser which has 2 as inlet and 3 as outlet, so we have $h_2 + C_2^2/2 = h_3 + C_3^2/2$.

So we can write $h_3 - h_2 = C_2^2/2 - C_3^2/2$, but we know C_3 is almost equal to C_1 so we have $h_3 - h_2 = C_p \Delta T_0|_s = C_2^2/2 - C_1^2/2$, so now we can write $W_c = C_p \Delta T_0|_R + C_p \Delta T_0|_s$. So $W_c = C_p \Delta T_0|_R + C_2^2/2 - C_1^2/2$. So $C_p \Delta T_0|_R = W_c - 1/2(C_2^2 - C_1^2)$, hence we can write $C_p \Delta T_0|_R = uC_a$(tan α_2 - tan α_1) - 1/2 ($C_2^2 - C_1^2$.

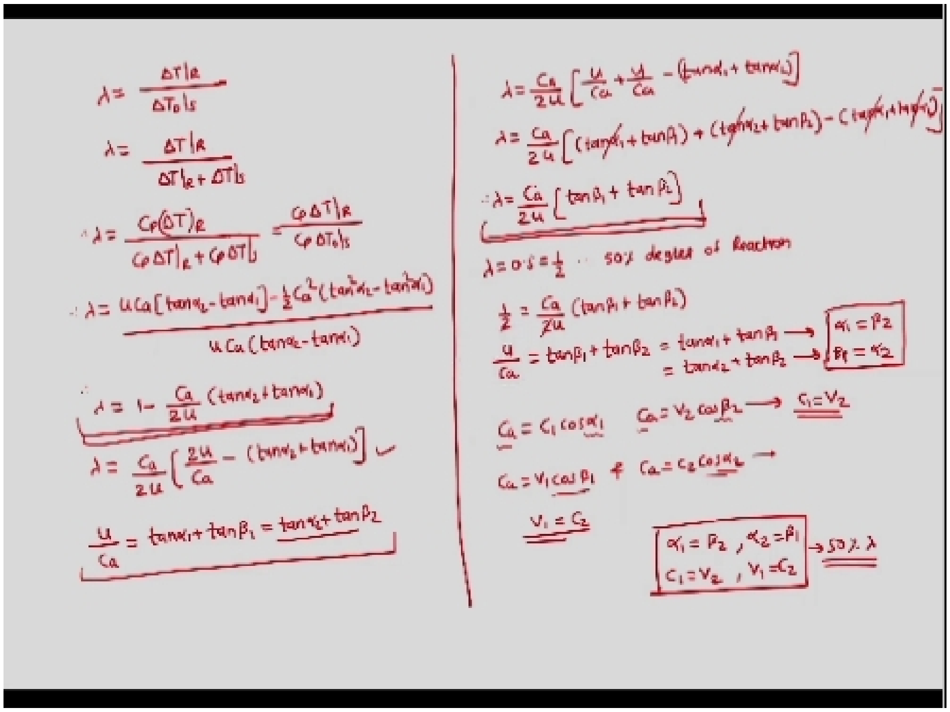
We are taking this from here where we have calculated $W_c = \dot{m}uC_a$(tan α_2 - tan α_1), but here we are taking specific work input so $\dot{m}$ is not considered, having said this we can further simplify C_2 and C_1 from the velocity triangle. So from outlet velocity triangle we have $C_2 = C_a$ sec α_2 and $C_1 = C_a$ sec α_1, having said this we can get $C_2^2 - C_1^2 = C_a^2$ (sec^2 α_2 - sec$_2$ α_1).

So $C_2^2 - C_1^2 = C_a^2$, we can write down sec^2 α_2 as 1 + tan ^{2} α_2, similarly here as well we can write down 1 + tan^2 α_1. So we can write down $C_2^2 - C_1^2 = C_a^2$(tan^2 α_2 - tan ^{2} α_1). So we can rearrange the equation number 8, we will name this as equation 8 and write down $C_p \Delta T_0|_R$ = u C_a (tan α_2 - tan α_1) - 1/2(C_2^2-C_1^2).

We can take from equation 9 and write down C_a^2 (tan ^{2} α_2 - tan ^{2} α_1). Now we can use this equation number 10, which gives us the $\Delta T_0|_R$.

(refer time: 42:24). So $\lambda = \Delta T_0|_R$ and we can use that $\Delta T_0|_R/\Delta T_0|_S$ or we can say $\lambda = \Delta T_0|_R/(\Delta T_0|_R + \Delta T_0|_S)$. So it is equal to λ = u we can multiply numerator and denominator by C_p. So this will be enthalpy change in the rotor divided by enthalpy change in the stator + enthalpy change in the rotor. So we have practically this can be written as u C_a (tan α_2 - tan α_1).

We are using equation number 10 for writing the numerator, so for to avoid the confusion we are saying that $C_p \Delta T_0|_R/(C_p \Delta T_0|_R + C_p \Delta T_0|_S)$, so numerator can be obtained from the



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equation number 10 which says that [uC_a(tan α_2 - tan α_1) 1/2 .C_a^2(tan^2 α_2 - tan ^{2} α_1)] divided by rotor + stator and then that is basically Δh_0 and which is equal to uC_a(tan α_2 - tan α_1).

So this is C_p $\Delta T_0|_R$ divided by C_p $\Delta T_0|_s$ and we have seen here that it is equal to uC_a(tan α_2 - tan α_1). So λ = 1 - (C_a/2u)(tan α_2 + tan α_1). So this we can take (C_a/2u) as common and then this will become 2u/C_a - (tan α_2 + tan α_1), we can go back and see that we have written that u/C_a = tan α_1 + tan β_1 and that is also equal to tan α_2 + tan β_2.

Having said this we can write down this expression λ = C_a/2u where u/C_a $\times$ 2 we can take (u/C_a) + (u/C_a) - (tan α_1 + tan α_2). So this λ = C_a/2u $\times$ u / C_a we can take it as tan α_1 + tan β_1 = tan α_2 + tan β_2 - (tan α_1 + tan α_2). So we have λ = C_a/2u and then this will get cancelled we will have tan β_1 + tan β_2.

So this is an expression for degree of reaction for axial compressor and this is also an expression for degree of reaction for axial compressor. Now let us take a special case where we have λ = 0.5 or 1/2 which is called as 50% degree of reaction, for this special case we can write down we will get 1/2 = (C_a/2u)(tan β_1 + tan β_2). So this 1/2 of will cancel u/C_a = tan β_1 + tan β_2, but we know here that u / C_a = tan α_1 + tan β_1.

So this expression gives us α_1 = β_2 or if we equate this with tan α_2 + tan β_2, this expression leaves tan β_2 and we get β_1 = α_2. So for the special case 50% reaction we have α_1 = β_2 and

$\beta_1 = \alpha_2$, further if we go back and see velocity triangle we know that $C_a = C_1 \cos \alpha_1$ and $C_a = V_2 \cos \beta_2$. We can see it from the velocity triangle.

So $C_1 \cos \alpha_1$ is the C_a, so $C_a = C_1 \cos \alpha_1$ similarly C_a is also equal to $V_2 \cos \beta_2$, but we know $\alpha_1 = \beta_2$, C_a is also same and then this gives us $C_1 = V_2$. Similarly we can also write $C_a = V_1 \cos \beta_1$ and $C_a = C_2 \cos \alpha_2$. So from inlet velocity triangle this is β_1, so we can write down this C_a in terms of β_1 as $V_1 \cos \beta_1$ same height outlet velocity triangle will be $C_2 \cos \alpha_2$.

So again we know $\alpha_2 = \beta_1$, so these terms will cancel and we get $V_1 = C_2$. So in all for the 50% degree of reaction we have $\alpha_1 = \beta_2$, $\alpha_2 = \beta_1$, $C_1 = V_2$ and $V_1 = C_2$, this is for a special case of 50%, 50% λ. This is how we start with the basics of axial flow compressor, further basics will be seen in the next class thank you.