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# Lecture - 12
# Waves – III

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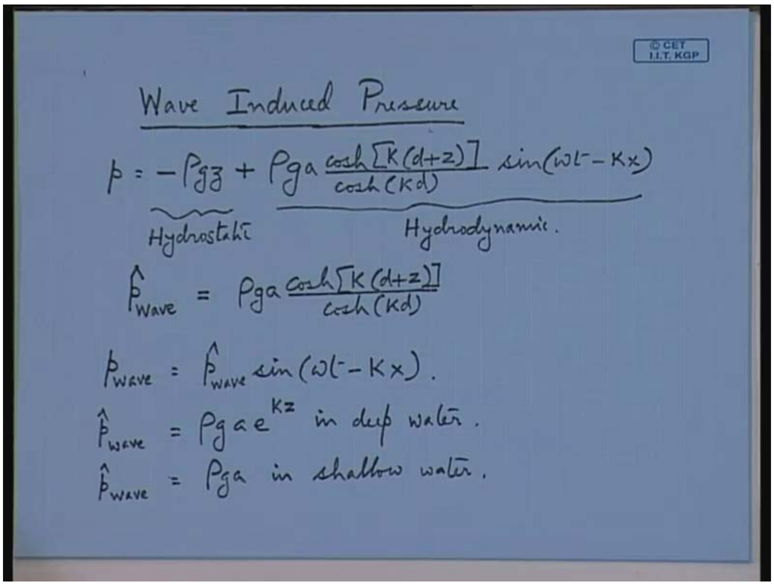


Today, we will continue with Waves, we have already calculated the pressure term. So, waves induced pressure after this we will go to non-linear waves. So, this we have calculated from your linear or I U theorem, so wave induced pressure we have got the expression as p. So, there are 2 components; one is minus rho g z, and the other is the hydrodynamic component, so this is given by rho g a cos hyperbolic of, k multiplied by d plus z, in cos hyperbolic k.

Now, if you want to analyze this equation of pressure. So then the first term is quite simple that is the hydrostatic term, so this is your hydrostatic pressure, now the one which is of interest to us the hydrostatic this all of you know there is this rho g h the other one is the hydrodynamic component, which is the more complex part. Now, this hydrodynamic component you write this as p wave. The other part that is the hydrostatic we are not much bothered, but what about this.

So, this is equals to rho g a cos hyperbolic k multiplied by, d plus z cos hyperbolic k d. Now, in shallow water or you can write, like this then the whole equation, that is the hydrodynamic component you can write as p wave, is equals to p wave with a cap and then your sin term will come, sin of omega t minus k x. Now this term that this which p wave, becomes rho g e the power k z that is in deep water.

So, this is of interest to us on the other in shallow water, this p wave, so this will become only rho g a, in shallow water. So, these actually you can find out but substituting this hyperbolic equation. it is erase to k z k m I, it is your minus k z you will come out like this. So, these are 2 important inferences that we get from the hydrodynamic part. Now if you look at the diagram, then you will find that, the pressure behavior is little bit different.

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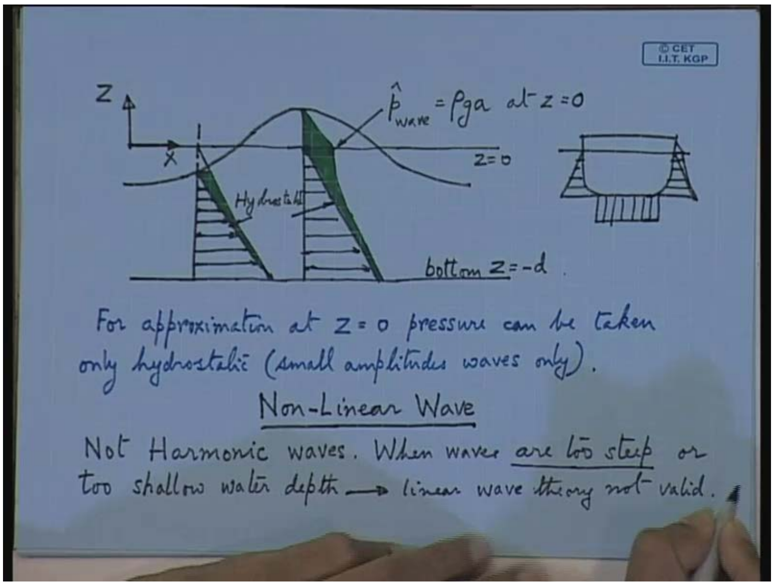


Say this is your, surface at z equal to 0 and we are taking the linear wave, so right now I am not going to the non-linear wave, so more complicated, so this is your x and z direction. So, if you take a linear wave then it will have a sin or a cosine profile, so this will come like this. Now if you want to calculate, the pressure with which we are interested, this are all main purpose is to calculate the force. So, this is the bottom is at z equals to minus d, now this is important.

So, this 1 let us take as certain, let us certain position on the wave there you hydrostatic pressure if you calculate, with very linearly. So, what is your hydrostatic pressure at this point, so this is your hydrostatic part, so this is the hydrostatic component, that is the rho g z. Now on top of that, you have to superimpose, this 1 that is the p wave, the hydrodynamic part. So, that is also varying, so you can calculate this in this is your, hydrodynamic part.

So, this is affect your calculation for your immersed object for special ships, now in the there is near to the draft and near to the crust, your variation will be like this. So, this is again your hydrostatic part, but the hydrodynamic component will have a variation. Something like this, so this is your p wave equation. Now there is a tendency to neglect the hydrodynamic part.

So, this is a quite small that you can see from this sphere, but never this is important. So, hydrostatic is your this 1, and this is your, what is the value of p cap wave, see at z equal to 0. So, simply rho g a, so that means, at z equal to 0 that is a hydrodynamic component, but no hydrostatic component in the p wave we substitute z equal to 0 you will get this, your cos hyperbolic k d z equal to 0 you cancel lout you will be getting rho g a.

So, this much is your hydrodynamic component even at that equal to 0, now if you take ships, sometimes when you are not break even this hydrodynamic component, you can take a hydrostatic part. So, for approximations at z equal to 0 pressure can be taken only hydrostatic, but there is a big.

If. So, this is when, was small and this is called small amplitude waves, otherwise you take this rho g a, but hydrostatic pressure will be 0 at this point at z equal to 0. Now this actually this is more in the important case of your ships and offshore structure calculation. So, the ships you will find a large part of the vessel is even still below z equal to 0.

So, now you have been told to calculate the pressure, so if there is no wave we are on the safe side, but if there is a wave you have to take account the hydrodynamic component, so that means there we some pressure in this linear. So, this has to be added, so this has to be taken care of, now let us go to the other part, that is the non-linear wave. Now why we go for more complicated waves, when we have the linear wave theory, which is the harmonic wave. So, this non-linear waves they are not, harmonic that is they do not have the sin or cosine term.

So, this is to be noted now this you have to apply, when waves are too steep then you should not apply your linear wave theory, or too shallow water depth, especially near the coast that is where you have very shallow water. Then the linear wave theory is not to be applied. So, this you specially note that linear wave theory is no longer vary, so more complicated stuff, now actually in the sea is a very confused state of measure. Now, what you will get if you take there is large number of wavelengths, even either get completely linear waves non completely non-linear waves.

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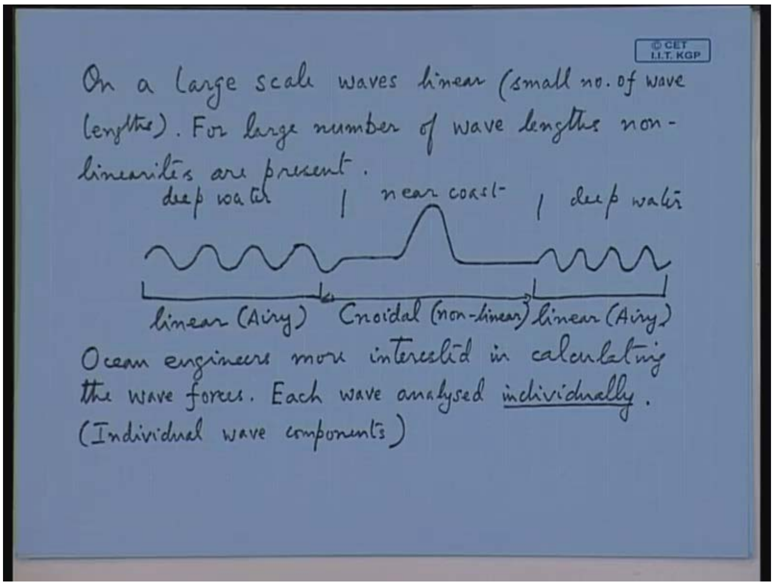


See on an large scale, so this is very interesting on a large scale waves linear to some extent linear say you write for small number of wave lengths. For large number of wave lengths non-linear it is will be present. Now if you draw a diagram the picture will look like this, so this is your theory wave on linear wave theory.

So, this is find, but after if you evidence you will find it is not like this. So, you will find the draft to be more flats and suddenly you will get a pith and come like this and then again it will go on legendry find linear wave. So, this part you can write linear or sometimes this is called airy wave.

So, this part also is linear that is called airy wave, but this part is non-linear. So, the diagram that we have drawn this is called a sinusoidal wave. So, this is the actual lecture in the sea now if you want to in all case since we are ocean engineers naval updates we are more interested in calculating the forces. So, ocean engineers more interested we are not interested in the profile of the wave as such, but we are more interested in calculating the wave forces calculating the wave forces.

Now, we are getting say this kind of waves, so how you are going to another is more interested in calculating the wave forces. Now how you are getting say this kind of waves. So, how you are going to another is more interested in calculating the wave forces. Now here actually what is done see each waves, so this is 1 waves, so linear wave this is another wave and this is another wave.

So, each wave has to be analyzed individually, so this is more important and then average or sometimes averaging is not done you find out the maximum waves force that be you are getting on the structure from the individual waves or individual wave components, this phenomena that is this sinusoidal wave we are more predominant near the coast and the linear you will get in deep water. So, you can in the diagram itself you can write near coast and this is important, so this is our measure interest now coming from the non-linear waves. Now there are various non-linear waves better not to go too much deep into this.

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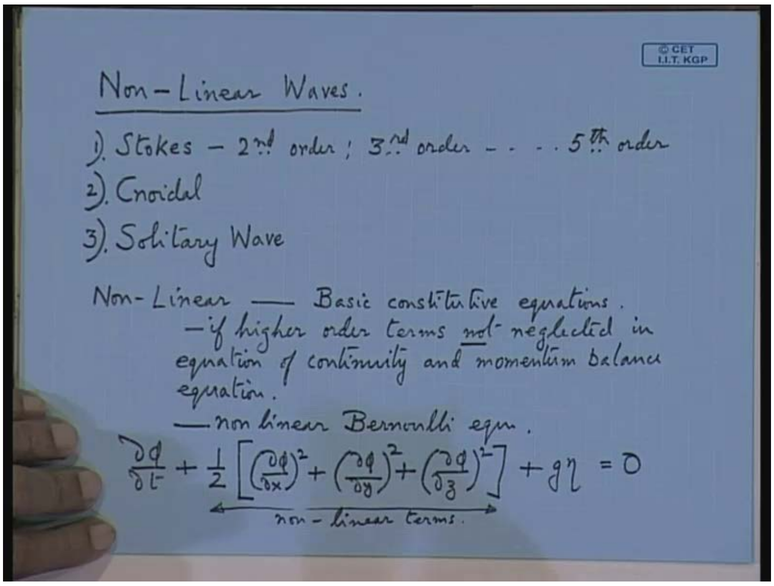


There are various types only 2 types I will tell you, say non-linear waves the 2 types that we will analyze number 1 is stokes. Now, stokes also there are various categories. So, you have second-order, then you can have third order, right and the of our interest we can go up to the fifth order, where this is more complicated. So, the stokes we have this theory the number the other one you have sinusoidal wave.

The last category you have this is actually a variation of this sinusoidal wave you have solitary wave. So, I will give you the diagram of this sinusoidal and solitary wave you can see where it varies you know solitary waves is an extension of this sinusoidal wave very large wave length there is a one single crust you will find. So, these are the 3 type which are very convenient more frequent this studied in the non-linear wave category. Now, here actually what you will find the term from where this non-linear term has come.

So, can you guess this now if you want to study this non-linearity. So, then we have to go back this non-linear term, I do not know how much of mechanics you have studied non-linear term actually comes from the basic constitutive equations So, there is it is coming if you neglect the higher-order terms in the equation of continuity and the momentum equation.

So, this will come if higher-order terms not neglected in equation of continuity in all of you know what is the equation of continuity. So, from this if you do not neglect equation of continuity and momentum balance equations neither 2 equations and another equation is there the Bernoulli equation other equation is non-linear Bernoulli equation remember in the airy wave we have taken the Bernoulli equation linear

Now if you take non-linear Bernoulli equation you will get an expression then this is dell phi over dell t velocity potential. So, this will be plus half this term you have neglected that is dell 5 over dell x square plus dell phi over dell y square plus. So, this term is the creation of your nonlinearity, now this is quite complex in the last term is plus g multiplied by this surface is not see eta equal to 0.

So, this you can this is your non-linear term, now if you want to go get this stokes expression you have to substitute the values of delta phi dell x dell phi dell y and dell phi dell z. So, this is your non-linear now what you do we have got the expression for the surface elevation.

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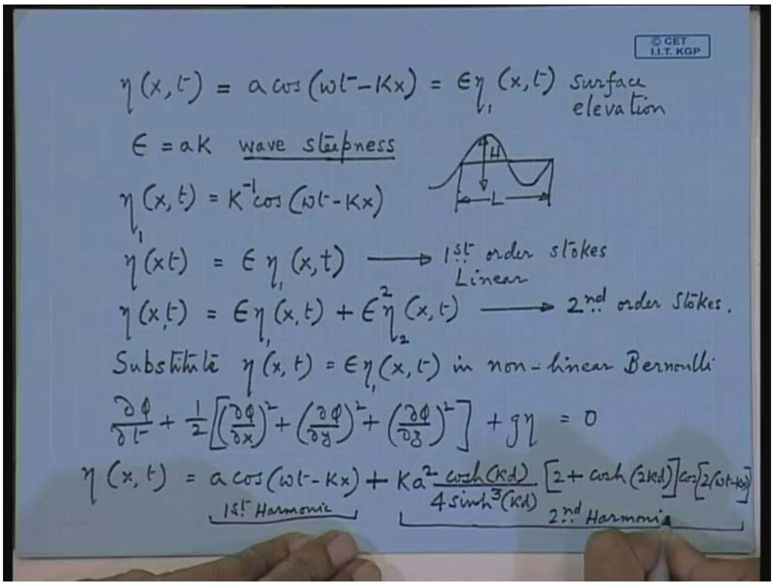


This expression for surface elevation is eta x t that is there are 2 terms there is 1 is this facial and other is the time. So, this is given as a simple cosine term you can take sin also, but for simplicity I think let us take omega t minus k x now this you can write this as epsilon eta 1 and this is x t now what is this eta sorry. In this epsilon is equal to a multiplied by k is called wave steepness that is if you take a wave like this what is that tissue of the wave height over the length of the wave, see this is your h and l is of course,, this is half wavelength.

So, l will be like this one, so h over l is a measure of wave steepness. So, later on we will see that this wave steepness is a factor in deciding what is called wave breaking or white capping. So, anyway, so those are the much more complex you can discuss later on now let us have a look at this stoke starry. Now, you write your expression like this that is the expression for what is this expression eta x t this is your surface elevation your surface elevation equation is written in this form in terms of wave steepness.

Now if you’re written epsilon as a k then of course, you to modify your eta as eta 1 x t. So, this you can express as k to the power minus 1 k is your wave number, so this will be cos of omega t minus k x now you are getting your expression for x or for eta getting it on down. So, what is the mention for eta.

So, this is simply as we have formally than before, so this is your first term in the stoke regression. So, this is called past order stokes for some time you can call this as linear theory. Now if you write the surface elevation as with second-order term of epsilon, so second-order you can write like this as epsilon eta on x t plus a higher-order term. So, epsilon square and this will be eta to x t.

So, this will give you second-order stokes now the second-order stokes. Actually this equation you can obtain by you substitute eta x to be equal to epsilon x t in the Bernoulli equation. So, your Bernoulli's equation that we have written you substitute this, and you can you find an expression with the second-order term.

So, which 1 you substitute we will substitute the linear one. So, this is x t equal to epsilon now Bernoulli's equation you take the non-linear Bernoulli's equation, not the linear 1 in non-linear Bernoulli expression, and then you simplify. So, your non-linear Bernoulli's this dell phi by delta t plus half of dell phi dell x term will come, then this will dell phi or dell y square when you substitute this plus g eta equal to 0.

That means, you can see this eta term eta term you take as this expression epsilon eta on x t, now here you substitute this and then at the we have already got the expression for velocity potential there are given in the earlier class. So, that expression you find out the derivatives square. So, this quite a long exercise and after that after doing all that you get an expression like this to eta x t will be equal to the first term will be your linear aggression a cos omega t minus k x.

Now your second term will be the quite complicated you can work this out in your hall. So, this is cos hyperbolic k d. So, this is what you write as the second-order Stoke expression. Now from this expression you can calculate the see or wave speed yes tell me how much is the wave speed.

So, this is called first harmonic why because it contains your cos omega t minus k s which is a harmonic expression now if you look at the second term this is the cos of nonlinearity you will also find another harmonic term that is cos twice of oh omega t minus k x. So, this is called second harmonic what is your c value or this is called the pre-speed.

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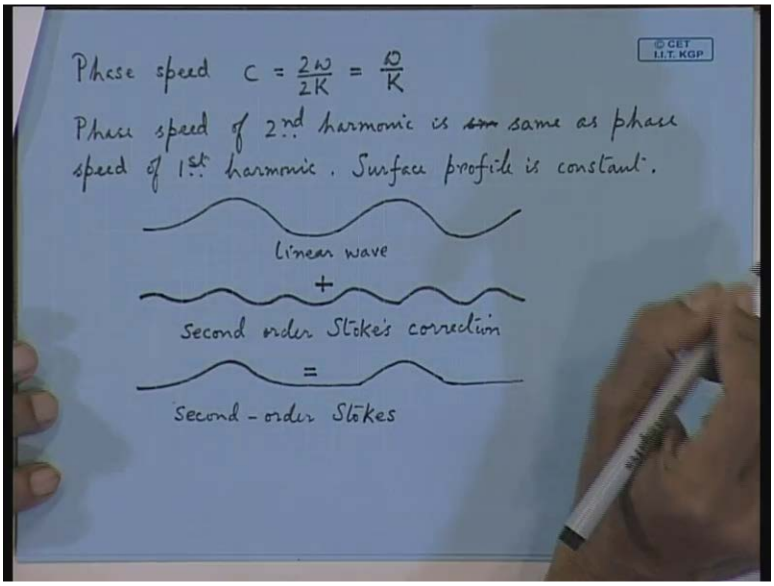


So, phase speed c is equal to, so you look at the expression it is that omega by k, so here it will be twice omega twice k ((Refer Time: 35:43)). So, what we are seeing is this phrase speed is same as the first harmonic. So, this is to be noted. So, that phase speed second harmonic. So, now, what will happen to this wave phase speed of second harmonic the same as first harmonic or rather you write same as a speed first harmonic, so now, what you know what is going to be your weight profile can you guess.

So, you actually we are getting 2 waves is it not, so your surface profile will be summation of the 2 surface profile. So, this is called surface profile is constant. So, now, we are getting 2 waves, draw the linear waves stokes first-order, so this is linear wave profile. Now, you had the surface elevation from second-order or second harmonic, so second harmonic you get some a smaller wavelength see something like this.

So, I am not wave go like this, it is a now you had this and see what happens. So, this is called second-order stokes correction to actually you are getting two waves from stokes second-order theory. Now you add these two you will get this is wave which will look like this now in this wave you find that the trough is more flat. So, this is the actual nature of see wave you know will go like this, so this is second-order, so this plus this is giving this. So, this is called second-order stoke.

So, this stoke theory is frequently used in offshore structure calculation post calculations is simply second-order now what you should know is where to apply this. So, there is a diagram is called a range of applicability.

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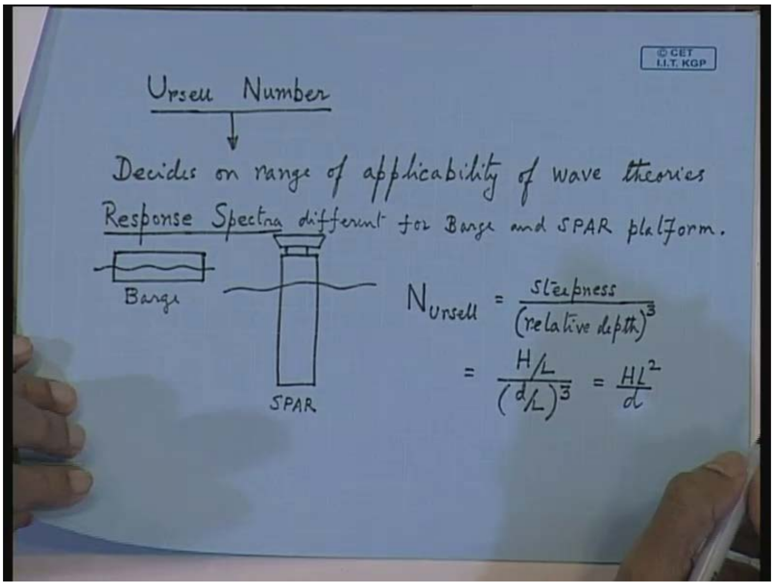


Now, if you want to want to understand this diagram, before that you have to come across a number which is called an Ursell number this person has devised this number. So, this decides on the range of applicability of various wave theories. Now here actually why this is important later on you will find that you see you are the whenever talk about there is another spectra which is called the response spectra.

The spectra I will tell you in a in a subsequent cross you will find this response spectra is different for various structure in the ocean see a barging shallow water will be excited by now your stokes is having stokes second-order there are two waves. So, which wave is going to excite your bars the most followed and that same excitation frequency will be different or for say is spar you know what is the spar?

A barge having is a, what is the shallow where that is a draft is pretty small now this is a spar platform is having a deep draft. So, which way frequencies going to affect a barge and is spar that will be decided by the applicability of stokes theory now this is you have the ocean engineers how to be careful about.

So, response spectra different for barge and spar flat form. So, after studying all this weight mechanics then actually we have to find out this response spectra there is what is emotionally barge in this spar and also the pressure term and which wave theory is your going to apply. So, this depends on the numbers which is called the Ursell number now. Ursell number if you write it is given as n Ursell.

So, this is a ratio of steepness over l depth cube, so this is the expression. So, steepness is the wave I divided by the wavelength and relative depth, you will get by the depth of water and divided by the wavelength and c b cube this. So, this reduces to h l square over d. So, this is called a Ursell number and if you want to look at the applicability diagram you can still reduce your expression to this form.

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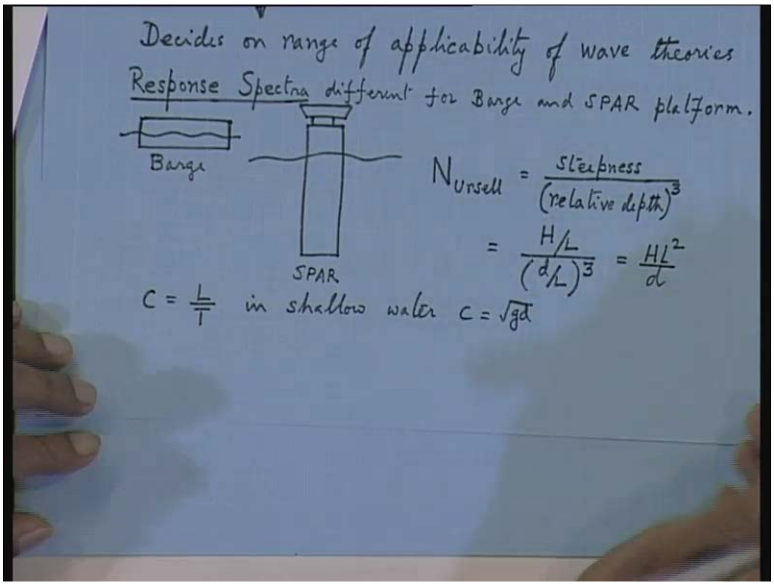


Now, the velocity of propagation is nothing but l by t, c is equal to l by t and in shallow water, you will get c is equal to root over g d.

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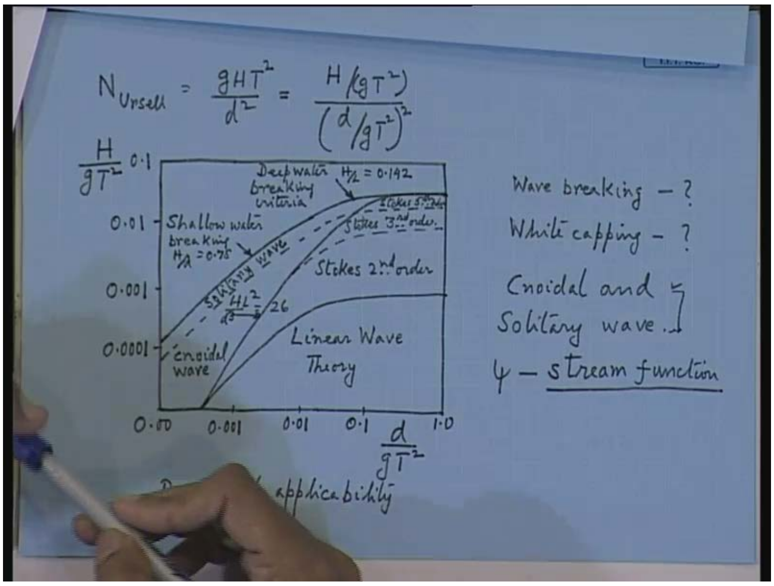


So, now this n Ursell you can write in this form g h t square over d square, so that ultimately if you substitute you will become h over g t square divided by. Now, this is what you have to plot d over g t-square wholes square, are you getting it, now in this actually l square you can write c square t-square. So, this becomes how much h t square will come and c square will be g d.

So, ultimately you can write divide both the numerator and denominator by g t square. So, this come especially Substitute this 2 value. So, now, here we can plot the graph and this graph is very important, for ((Refer Time: 47:09)) structure force calculation. Now on the horizontal axis you plot d over g t square, and the vertical axis you plot h over g t square now you have to refer to this graph, so this start from 0.00, now there are actually 4 1 2 4 intervals. So, this you can divide as 1 2 3 4 this is 0.001, 0.01, 0.1. The other is 1.0, now here also you make 4 divisions, last 1 in 0.1. So, your vertical axis is your horizontal axis is actually 10 times now here you get bands you will get a band like this.

Now this region is the applicable range for your linear wave theory, be careful now do not apply linear wave theory in all cases, and this 1 you will find sinusoidal wave, and here you will find another wave which is called is solitary waves; solitary waves is another transformation of sinusoidal wave, is called a solitary waves, and you have number of stokes theory. So, linear the linear 1 is the first order, then you have second-order. So, second-order will come from wave like this.

So, this region is applicability of stokes second-order. So, you remember like this linear wave theory is first-order, this 1 is second-order, about that you will get stokes third order, and this at the top you will get stokes fifth order, fourth order is not shown. Now this region is beyond this you write h by l that is 0 point 1 4 2 this is called deep water wave breaking.

So, there are other things we studied about wave breaking and white capping, this of course I will tell you not in this class will sufficient what is this now what is white capping, so this is your breaking criteria and this is deepwater other 1 you will get is shallow water breaking, now this happens at H by d. So, this is equal to 0.78 and this line is h by h l square d cube. This value is 26, that is the stokes limit of beyond there do not apply stokes you apply sinusoidal or solitary waves. So, this is called a range of applicability, so next class we will see this is the nature of this sinusoidal wave.

Sinusoidal and solitary waves, if you want to analyze this 2 waves, you have to take recourse to what is called is phi the is called a stream function. This stokes we have called by from the nonlinearity of the velocity potential, but we introduce another term which is called a stream function in order to get this 2 wave theories. The solitary wave is actually designed form of sinusoidal wave. So, this waves some more present near to the shores there affective coastal structure. So, tomorrow actually I will give you some description of offshore structure, and then we will finish this and go to wave script.