Elements of Ocean Engineering
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Lecture - 27
Mooring Systems (Contd.)

So, we will continue with the mooring systems in this lecture. So, as I have last class as I have told you there are three types of analysis that we have to do.

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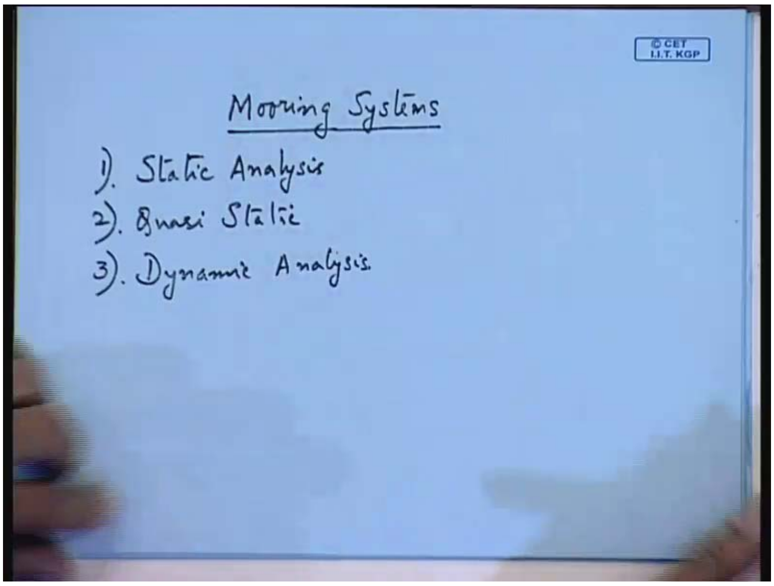

So, in the mooring systems which are very important for offshore structures the reason is they are stationary at one place and they have to encounter the motions from the waves. So, moving systems there are 3 types of analysis. First what you will do that we will be doing in this class is static analysis, then you have quasi static, and the last one is your dynamic, so all these has to be done. Now, coming to the static analysis part that is your mooring chain actually assumes the configuration of a catenary wire. So, it is actually a catenary wire and some of you have done the mechanics also, but let us see a similar mechanics out here.

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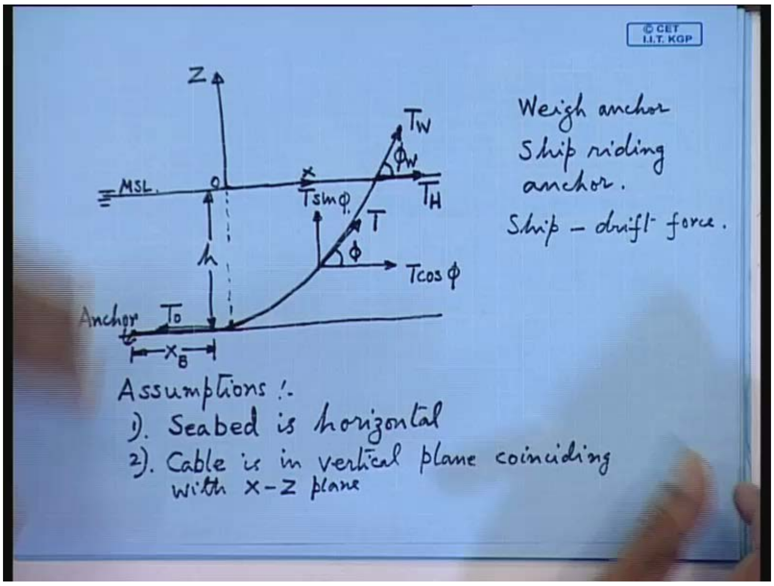

So, here is our water line. So, the, you take a reference on the means of the mean sea level. So, this is your water plane or you can call it this mean sea level whatever it is now we will make some assumptions in order to simplify matters. So, this is your x positive direction and x positive direction on z axis. And you assume your boat or platform to be out here let us say that the deck is more or less at the mean sea level or little bit away from it. So, this will be the horizontal part of your cable retention. So, this is T H now your mooring cable actually is wound in a windlass or in semi submersibles you call it a mooring winch. So, the mooring winch will exert a force in this direction you call this as T W or T winch and this make an angle of say phi w with the water plane.

So, now let us see your see bed actually is the height of the sea bed say let us take here now you remember the sign of the the z axis. So, this x z and y is on this. So, h you take this as h now your mooring cable actually assumes the shape of a catanery. So, you draw a catanery wire rope now, the static analysis that I will be doing here is quite simple, but in the actual case it is not. So, now, what happens is that the mooring line will lie to some extent on the sea bed like this. Now, it will take a certain angle from this point and this is your anchor point say this is your anchor point now this distance you write as X B. So, this is the shape of your mooring line now mooring line it assumes a shape of a catanery it is never steel this is a steel cable.

So, it will never be taut or in one straight line, it will assume the shape of a catanery and a large portion of the line will be lying flat on the sea bed as you can see. Now, what happens if we exert a force of T W on the boat? In this direction you will exert a horizontal force in this direction. So, anchor line tension is always horizontal as you can see from the diagram anchor line tension is always horizontal, it is never in one particular direction away from the sea bed. So, now, when you want to lift this anchor so this your semi submersible must what is called ride the anchor that is it is going to wind the chain cable. And then it has to come over the, this h and then again it has to come over anchor then you lift it up you can never lift a anchor in this mode.

So, that is what it is called ship riding anchor and where y ou drop a anchor say these are common ship term they say weighing your anchor when you drop the anchor. They say you weight your anchor weigh anchor there is this term and other terms is when you lift the anchor it is called ship riding anchor. So, these are called marine a sort of marine engineering terms the captain of the ship will say weigh your anchor; that means, you drop your anchor and if you want to say. So, when the anchor is dropped at this point ship will not come stop at this point. So, it will continue to go in the horizontal direction. So, so that is your the point that you will come here. So, it is acted upon by what is called ship is always acted upon by a drift force.

So, this drift force actually comes on the anchor. I do not know drift force can calculation ship motions actually you will be doing so anyway. So, now, you take a portion of the anchor line say this is your tension is let us say this is the tangent to the catenary. So, here you will find that this is your tension is tangent and this makes an angle of right this juncture let us this makes an angle of phi with the horizontal. So, so that means, the wire rope is supported by T H in this direction and T 0 in the opposite direction and the part T W will be in the vertical direction which is supporting the weight.

So, this will be T cos phi and; obviously, the other one will be T sin phi. Now, there are certain assumptions which you have to make before we I draw the free body diagram. Assumption number 1 is sea bed is horizontal otherwise this T 0 will not be horizontal that is then you have to take a component of T 0. So, here you write this is your anchor or now number 2 is cable to make our problem simple you assume cable is in vertical plane coinciding with x z plane anyway. So, this actually we are assuming otherwise you have to take components on the exit plane which let us for the present let us ignore.

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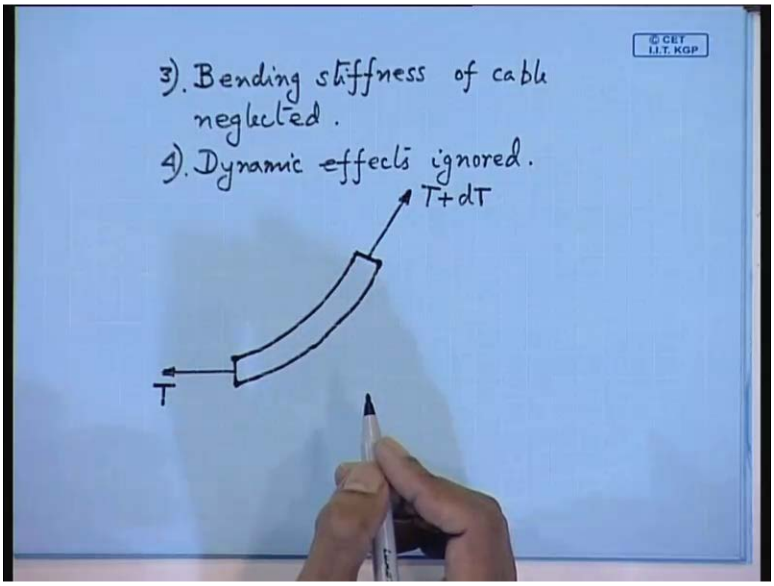

Now, number 3 is bending stiffness of cable line now we are making static analysis. So, right at the top you write static analysis. So, these are not to be confused with quasi static or dynamic analysis. So, you do not make any dynamic effects dynamic effects ignored. So, before we proceed in the calculation that is with the free body diagram. So, next I will try to draw the free body diagram. So, you make these assumptions. Now, let us take a portion of the chain cable say at this point.

So, in your magnified font you write you draw the free body diagram of the portion of the cable. So, your cable is having a curvature like this or if it is having a curvature; that means, the horizontal component at the top of the element let us take an element of a cable. So, in rush take it like this then what are the forces acting on this now this is your cross sectional area of your cable. So, your tension out here will be how much. So, this will be T plus d T if your tension out here is T. So, there is a change in the tension T plus d T not only in the magnitude, but also there is a change in the direction as well. Now, direction actually I have drawn this horizontal and your phi will become 0 in this case I can let us make it more you keep a certain angle out here for your problem.

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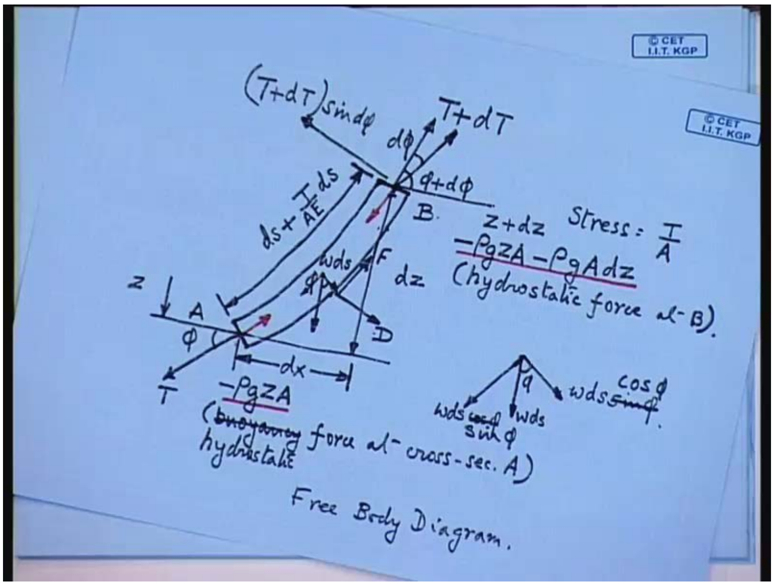

Just modify the diagram or you just draw another diagram, because in this diagram the T is becoming horizontal. Now in this case let us assume a direction of T now the direction of T is normal to your cross sectional area. So, this is your T and this will be your this T plus d t. So, there will also be a change in the angle because of the shape of the catenary now angle out here this you take the horizontal line. So, angle you write phi plus d phi this is phi plus d phi and this one you write as phi.

So, at the top end of the cable the tension is T plus d T angle making with the horizontal is phi plus d phi down below is phi and T phi now there are in the free body diagram there are other forces. So, this is your free body diagram now what are the other forces now, because of the action of your tension what is happening to the length of the cable because this is a wire you know. So, you are stretching it so; that means, if your original length is d s then this has become as a result of the application of T this will change. So, this will become how much that is your strain in the line. So, you find out the strain. So, your stress is say T divided stress in the cable line is how much stress is equal to T by A. So, now, you find out with from your modulus of elasticity.

So, this will be T over A E multiplied by d s if your d is the original length. So, this is the amount of elongation. So, your length has become T s plus T over A E multiplied by d s. So, that is the length of your cable now the horizontal direction the this one you take this as d x and this one you take as d y, not d y what have said the direction is z. So, this will be d z and this one will be d x. So, what we have to find out is the from the equation or static equation we find out T and the length of the cable line you have to find out d s. So, this is the situation now what are the other forces acting at the end? Now you write before we other forces are the hydro dynamic forces. So, the tangential component of the hydro dynamic force you write this as f and normal to it you have another force which is called the drag force or D. So, these are the hydro dynamic forces. So, before we analyze these you write.

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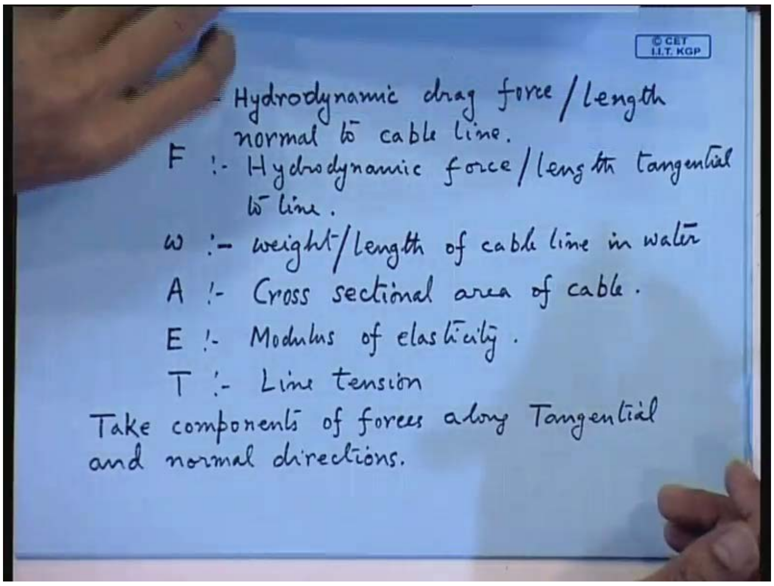

D is hydro dynamic drag force. Actually in our analysis you will find that since our equations have become more complicated. We have to neglect the hydro dynamic forces, but hydro dynamic forces are quite substantial. So, hydro dynamic drag force now this force is denoted per unit length. So, this is per unit length. So, total force will be d multiplied by the length of the cable line. Now, f is called sorry d you right drag force this is normal to cable line the other one is say hydro dynamic drag force or rather simply this one is tangent this is also per unit length tangential to line.

Now, weight the other portion is w is the chain weight. So, this is weight per unit length of cable line in water now this you per your note is not in air so; that means, we are taking the buoyancy also into consideration. So, that is the net weight actually weight in air minus the buoyancy. So, this force is actually acting vertically downwards if you take out here. So, the force that will be acting downwards at this point is w multiplied by d s followed extension you for this weight you leave it as it is. So, now, how much you get. So, A is your cross sectional area of the cable. Now, you take the components of these forces in the horizontal and also in the vertical direction. E is your modulus of elasticity now what other T is what this is called line tension. So, this is the very simplest analysis that we are doing actual analysis is not all that simple.

So, now this force is acting now if you take the, this line to read same out here. So, what is the change in the angle at this point? If you take component of force you do it either in around this line and normal to this line you take the component so; obviously, this angle is d phi. So, if this was your phi angle this angle. So, the change is d phi now you take the component of T plus d T along the line of T now about around this cross section what are the other forces acting. So, your tension is acting on both the faces say on the downward face this is you can say at A and this is your B. So, what are the other forces acting?

Now, for the convenience of your calculation you take. So, you refer to the diagram which we have started with. So, this is the diagram So, you are doing say you have taken an element out here. So, we are doing our analysis say at this particular point followed. So, now, you tell me what are the other forces which are acting now here you write this z from mean sea level and there is a certain and this will be. And this point is actually there is a correction the, you write this as z plus T z this point which is above the point A at higher elevation? The reason will come to in our analysis to make it simple you write this as z plus T z.

So, now you tell me what are the buoyancy forces? So, buoyancy forces at A will be this is z is negative you write this as minus row g Z A. So, this is buoyancy force. Buoyancy force at cross section; this is at cross section A. Now, you calculate the buoyancy force at cross section B now, buoyancy force at cross section B, you write like this; this is minus row g Z A and you have to actually this is at a higher elevation. So, then we you have to subtract another term. So, row g A d z now here actually you remember the sign of d z is opposite to that of a otherwise you will get a larger hydro static pressure at B. So, the for convenience of your calculations later on you will find that you take the hydro static force occurring at B in this form.

So, this is your buoyancy force at B buoyancy or hydro static force sorry you do not write buoyancy you write hydro static force. So, this is your hydro static force at B now, you know what will be the direction of the hydro static force at end a your hydro static force will at in this direction. This one I am writing and here what will happen hydro static force is nothing but the pressure of water act acting at this point. What is the pressure of water in which direction I would act? So, it will act in this direction. So, this is your this force. Now, you do the get the components of this forces in the x and y direction. Now, you take say you take T plus d component around d tangential direction. So, after this take components of forces along tangential and normal direction. So, what is the component around the tangential direction? First let us try to do our tangential direction or tangential component.

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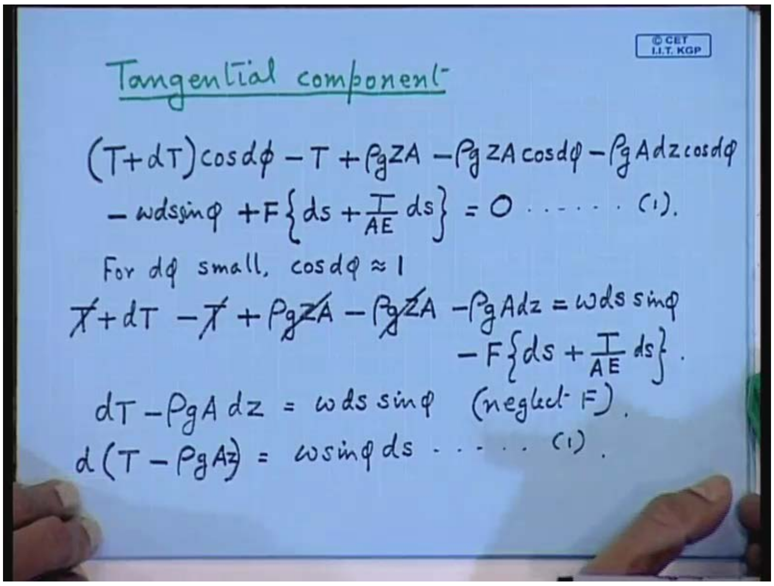

So, tangential is along the cable line. So, what about T plus d T? So, you will look at this diagram. So, if you take the positive direction along the upward line upward direction. So, the, this is T plus d T cos, how much T plus d T cos d phi? So, that is I am taking component along this T direction. So, you take only T plus d T cos of d phi, but do not take cos of d phi out here. So, this is acting in the upward direction and upward direction you take as positive. And if your take the opposite T here actually you tension is occurring in the reverse direction. So, this will be minus T now you will take the components of the buoyancy forces.

So, buoyancy forces out here is along this minus rho g Z A. So, this is acting in the opposite direction to T. So, here you add. So, minus of minus, minus sign is coming because of these Z. So, this will be plus rho g Z A now, you do this the buoyancy in this two forces how much in what direction will happen? So, this is actually acting in the same direction of T is not it or your or in which direction is it acting? This will act in the normal, is not it? This is your normal line, so obviously, you have to take a component on this line that is you have to take cos of d phi. But you do not reverse the sign it will be acting in the same direction of T. So, here so you remember this sign otherwise there will be a problem.

So, this will be minus rho g Z A this you simply multiply this by cos d phi and also the other term this is minus rho g a d z cos T phi. So, here actually I have changed the direction of this minus rho g is that, because it is acting in the opposite to T. Here it is along the line towards the direction of T only thing is you have to take the component that is you multiply it by cos d phi. Because this is acting on the direction of T plus d T your T plus d T is the normal to this section of area followed. Now, what are the other forces? The other forces are your this F d j and weight. Now, weigh T W is your net weight. So, this angle is how much this angle if you write this is phi now you take the components.

So, if you take the components in which direction this is going to act. So, w d s there will be 2 forces w d s cos phi and w d s sin phi you take components along normal that is along the direction of your drag force. So, that will be w d s sin phi and along this your w d s is acting vertically downwards. And your angle is how much is your phi angle? Now, here along the line of T this will be how much? So, we are resolving our forces along the line of T and perpendicular to T. So, this will be w d s cos phi and this one will be w d s sin phi now you tell me the directions. So, w d s cos phi this is w d s. So, this will be cos phi if this is your phi angle and this one will be sin phi now what tell me the direction. So, w d s sin phi will be this is the same direction of this t. So, I am just writing your arrows like this, but T we have taken this as negative. So, this will be minus w d s sin phi and tell me the other force.

So, this is coming from the weight of the chain cable now other force is the of the hydro dynamic component now hydro dynamic component which one you will take d or F. So, f is you assume F to be acting in the direction of T to make matters simple and d is normal. So; obviously, co component of d along the tangent will be 0 is not it cos 90 degree is 0. So, the only force you are acting is F. So, what is now f is acting in which direction? So, these sign of the forces you be careful. So, F is acting in the direction of T plus d t. So, this will be plus in our equation; this will be since we have taken the T plus d T to be positive it is acting in the same direction of T plus d t. So, this has to be taken as positive, but if F we have denoted F as what this hydro dynamic force per unit length? It is not the total force.

So, it is force per unit in length. So, you multiply by the total length of the chain cable for that element. So, what is that length? That is d s plus which one where do we have taken along the line of T that is why I have multiplied by d s this d s. So, anyway so so F multi multiplied by d s plus the extension of the length. So, this is T over A E. So, later on actually we neglect this F. So, all this trouble that we are taking will go waste.

So, this is your actual equation. So, this is equal to how much angle of the force is what. So, this is your first equation is your tangential component. Now, this d phi angle for d phi very small d phi is there is suppose change in the angle around this elemental length d phi small what is the value of cos d phi? Cos d phi is very near to 1. So, now, w d s here actually this will be another s out here sin phi.

So, now, what will be this equation? So, this will be simply T plus d T minus T. So, your T cancels out and how what will be the equation for this? This will be plus rho g Z A and this one will be minus rho g Z A. So, your rho g Z 1, it will go out and you are left with only minus rho g A d z on the other side you will get w d s sin phi and the minus sign will come out here so minus F multiplied by d s plus T A d s. So, from this equation your T and T will go and this term will also go. So, we are left with d T minus rho g A d z. So, this will be equal to w d s sin phi you neglect F.

Now, this equation you can write in differential form; you write in differential form this should be d of T minus rho g A; this will be w sin phi d s. So, this is one equation we can get the, we are making lot of assumptions [fl] this is z oh this is a z a z I have left out or what we are taking d. So, that is all right we are take the differential it will be d of d T and this is now you take the normal component. Now, here actually what you are saying that d s I have not multiplied by d s plus T A E d s. These are weight per unit that is quite small that is why this term I have neglected for this case if you are taking d s plus this all this trouble. But for w we are not taking, it is not it that is what you are asking anyway. So, now, you get another equation from the normal component the…

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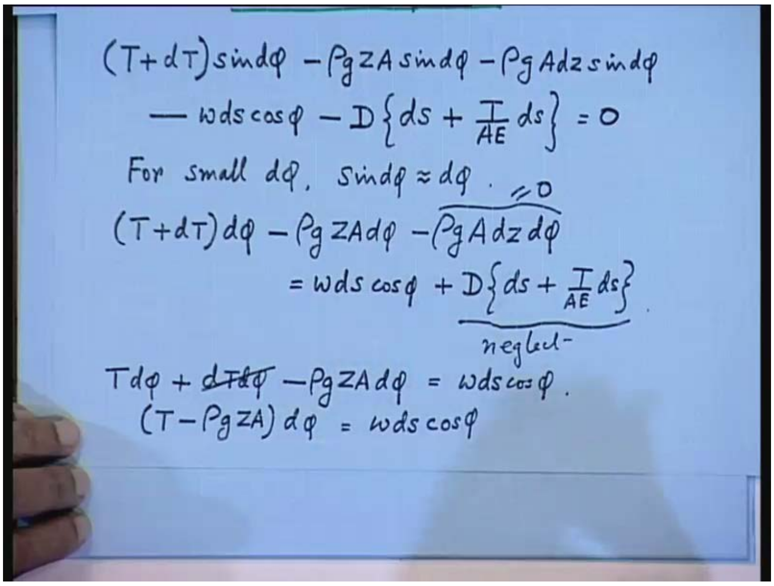

That means, you are taking normal to T. So, you find out what is your T plus d T go. So, T plus d T cos phi or sin phi sin d phi now what are the other forces. So, you look at your free body diagram. So, T plus d T cos phi will be around the direction of T T plus d T sin phi will be on the up this direction upward direction you take as positive. So, what about T t is you are taking component around t. So, T around the normal side would be 0. So, that is cos 90 degree. So, fortunately we are not having any T component now, we just take the components of your the hydro static equation.

So, hydro static pressure will you take only this one at the top, because your rho g Z A is around the direction of t. So, this red arrow so if you take component of the perpendicular direction it will be 0. But here this is acting as around T plus d T this rho g minus rho g Z A and this minus rho g A d z which is acting along T plus d t. So, there is not along the direction also. So, component along the tangential direction will be cos of this and the other one will be you will simply take sin of this thing. Sin of this thing will be acting in which direction upward or downwards in the direction of d is not it. So, minus rho g you take the component this will be sign d phi and minus rho g A d z sin d phi and this will be you take the weight component, weight component will be in the direction of d.

So, this will be minus w d s cos phi and this also will be the drag component will be minus d this will be how much you multiply by the total length of the cable. So, this is d s plus T A E multiplied by d s. So, this will be equal to 0 and now you simplify that is all. Now, here you take d phi for small d phi you take sin d phi almost same as d phi with that you make the simplification w and no this T plus d T sign d phi you are taking as positive is not it.

So that means, the in this your sin component in this direction, it will be this is your direction of what T plus d T sin d phi if this direction you take as positive. Then this hydro static it will be in the opposite direction is not it? But then why he has taken this as negative the hydro static no sure I think this will be in the upward direction is not it the upward direction. Then it will be the same as T plus d T sin d phi. So, then the, this thing remains the same.

So, now if we make the simplification what is happening? So, if you make simplification. So, this will be T plus d t. So, instead of sin phi you write d phi. So, here this will be minus rho g Z A d phi and this will be minus rho g A d z d z sin instead of sin d phi you write d phi around the other side how much we are getting. So, this will be w d s cos phi plus d of d s plus T over A E and d s or now this you neglect this term we are neglecting the hydro dynamic forces. Because we have neglected that other 1 F then this one also you take this as equal to 0, because d z multiplied by d phi are too small quantities. So, they will vanish.

So, we are left with T d phi this will be T multiplied by d phi coz again we are getting small quantity as d T d phi. So, all these small quantities you ignore. So, minus rho g Z A d phi this will be equal to w d s cos phi. So, this one again you may ignore. So, this is we are getting T minus rho g Z A multiplied by d phi this is equals to w d s cos phi. So, this we are getting from equation two the normal direction. Now, you can simplify this further.

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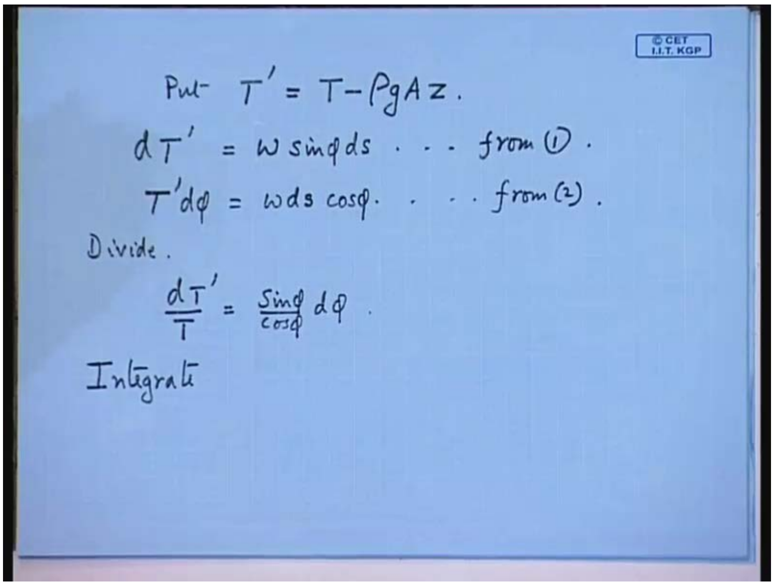

Now, you put this T prime is equals to T minus rho g A z. So, then we have d of T prime is equals to w multiplied by sin phi d s and the other one this is from 1 and from 2 how much you are getting simply T prime d phi and this is equals to w d s cos phi. So, this we are getting from 2, now you divide now if you divide you will get d T prime over T prime and what is coming out here? So, w d s will go. So, we are getting sin phi of over cos phi that is tan phi this is multiplied by d phi.

So, we are getting this equation now you integrate. So, you after you integrate you put the boundary conditions then you get another equation. Now, from that equation you can calculate what is going to be your d s or length of chain cable. So, next class, we will do this calculate length of chain cable. So, these are equation of a catenary the only problem is the sign conventions.