Marine Hydrodynamics
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Lecture - 10
Source, Sink and Doublet

Today is the tenth lecture in the series on marine hydrodynamics. In the last few 2 classes, we spent time on understanding several 2 dimensional flows. In fact, in between I talked to about how using the complex function theory, how can represent various flows in the process. We have talked about sources and sinks. We have seen that a sink is nothing but a negative source. In case of a source, the flow is always in the radial direction. On the other hand, we have seen that in case of the vertex, the flow is always in the tangential direction.

Now, with this understanding, again we have seen that when we have seen the additive characteristics of the various sources in a flow that is possible, the individual identities source characteristics would maintain. On the other hand, when we think of a boundary surface inside the flow; particularly we have seen. In taking a case of cylinder, in the presence of a source, we have seen that the source additive property will not on go when cylinder is introduced to the flow.

Now, this understanding today, we will spend some more time on various types approach, particularly after source are sink what happens? What is a double-t? Then, we will mention few theorems, results, relative source sink and doublets and also vertex. With this, we will before going further. I will keep one very important result that is circle theorem. Although this result I would have told much before, but let me mention this. What is a circle theorem?

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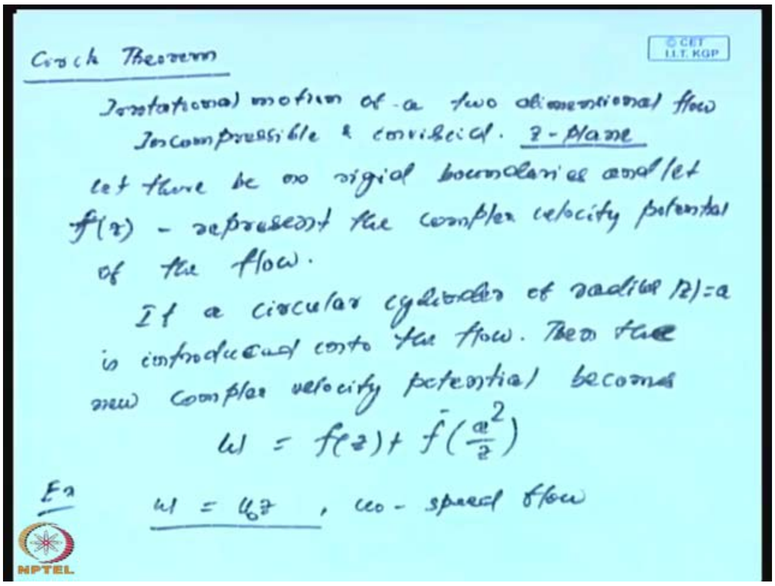

This theorem says that if there be a suppose, we have a irrotational motion, we have a irrotational motion of a 2 dimensional flow of the irrotational motion of a 2 dimensional flow. Of course, the fluid is an incompressible and in viscid. We are in the z plane, particular in the compressed plane. So, we are talking about a flow of irrotational flow of a fluid, where the motion is irrotational. It is 2 dimensional in nature. The fluid is assumed incompressible and in viscid. So, we are in the z plane basically in the incompressible plane.

If we introduce suppose, there are no region boundaries. We usually assume, let there be no rigid boundary. Let f z represent the complex velocity potential of the flow complex velocity potential potential of the flow. If f z represents the complex velocity potential of the flow, what we will do if you introduce a circular cylinder; if a circular cylinder is cylinder of radius where z is equal to a is introduced in to the flow? Then, what will happen to the new complex potential? Then, the new complex potential becomes w is equal to f of z plus f bar a square by z. So, this is what happens. This is the circle theorem.

This result we apply, we can apply to several several for the several applications. This result, the simplest application is suppose, I consider a flow w is equal to u z. That means I have a uniform flow, which is p d o. Then, if I apply for example, this is an example I will take. So, this is a uniform flow that has speeded as u naught. The speed is the flow.

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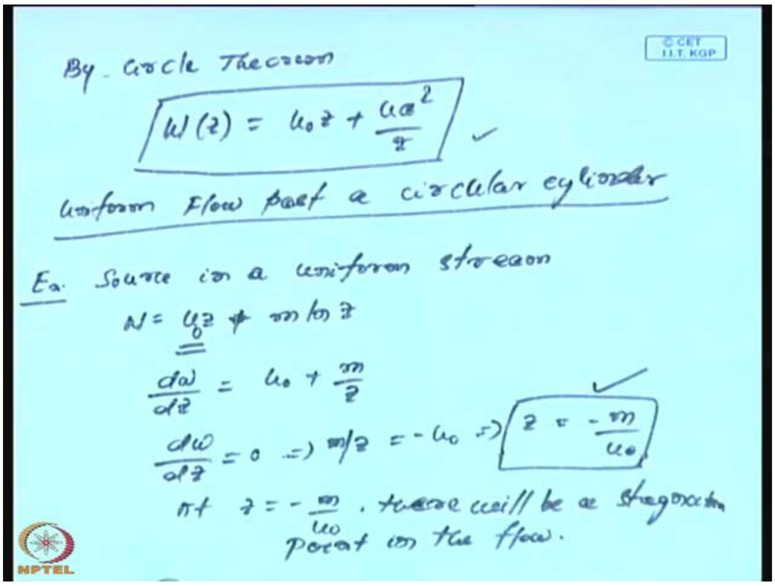

Then, if you apply circle theorem, so by circle theorem, if I introduce a circular cylinder the flow, then my new w z will be equal to u naught z plus u a square by z. This is the flow will come from past circular flow past circular cylinder. This is the uniform flow past of circular cylinder. Already, you we have derived several several results on this flow in one of our previous class. So, it is directly coming in this. This result is directly coming from the circle theorem.

With this now, I will go back to sources and sinks another example. We know about source. I will mark out another example of a source. Suppose, suppose, I just consider a source in a uniform stream. Seeing the source, let me consider N as w is equal to u z plus m n z. So, this is already called this as u naught.

So, initially I had a stream velocity uniform flow. I have added to that suppose, I have a source of the stream present in this. What will happen to this? If I from this say, I can get what will be divided d w by d z? This is u naught plus m by z. So, if d w by d z is 0, it gives me z is equal to my, rather I will say m by z equal to u naught. This in place my z will be minus m by u naught. So, that means when z is minus m by u naught, this will be this floor.

So, at z is equal to minus m by u naught, there will be a stagnation point. There will be a stagnation point in the flow. We can easily see that that this this point will lie on x axis.

Further, it can be as we seen that what is the compressible velocity potential particularly in the streamlines.

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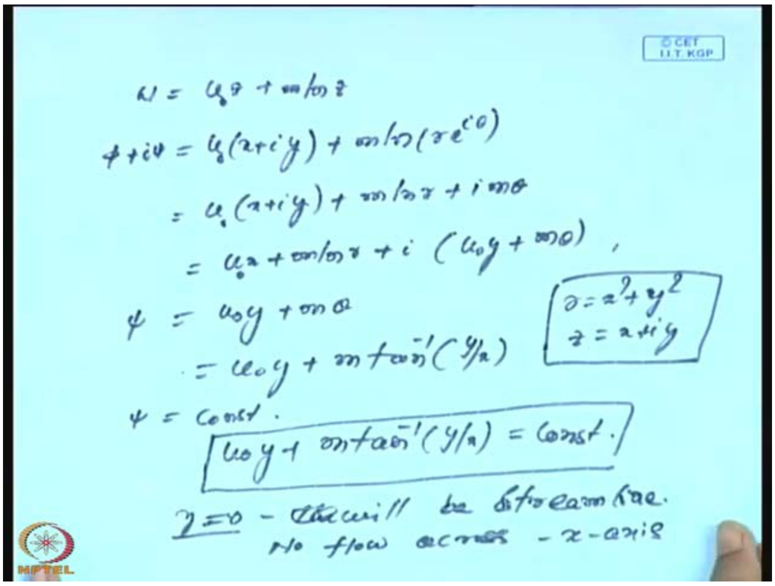

We have already seen w is equal to u z plus m bar z, If you look at the complexity potential, so from this, we can get phi plus i psi will be u x plus i y plus m bar r e to power i theta. If you put it u times x plus i y plus m bar n plus i m theta, which we can write as u x as started with u naught.

So, it is u naught x plus m bar n r plus i times u naught y plus m theta. See, if I look at the streamlines, psi will be u naught y plus m theta, which is u naught u naught y plus m tan inverse y by x where my r is equal to x square plus y square. I call it z is equal to x plus i y. So, if a psi equal to constant that means the streamlines will be tan inverse y by x that would be constant.

These are the streamlines. These will be the streamlines. We can see what will happen when we put y is equal to 0. If you take y is equal to 0, then that itself will give us a constant that means we can always say y is equal to 0 will be the line x is equal to y is equal to 0. It means it is the line x axis on that will be there would be no flow, which will be x axis means y is equal to 0 will be streamline.

All we can say no flow across the x axis. This is no flow across x axis. So, this is an example where we see that when there is a stream and the uniform flow, you can have a stagnation point in the flow. That stagnation point is on the x axis. They will not be any flow that is about there is no flow, which will be crossing the x axis. This is because x axis itself gives like a streamline. Now, I will take a different example. Suppose, I have a source, which is can we consider a source? You may consider a source.

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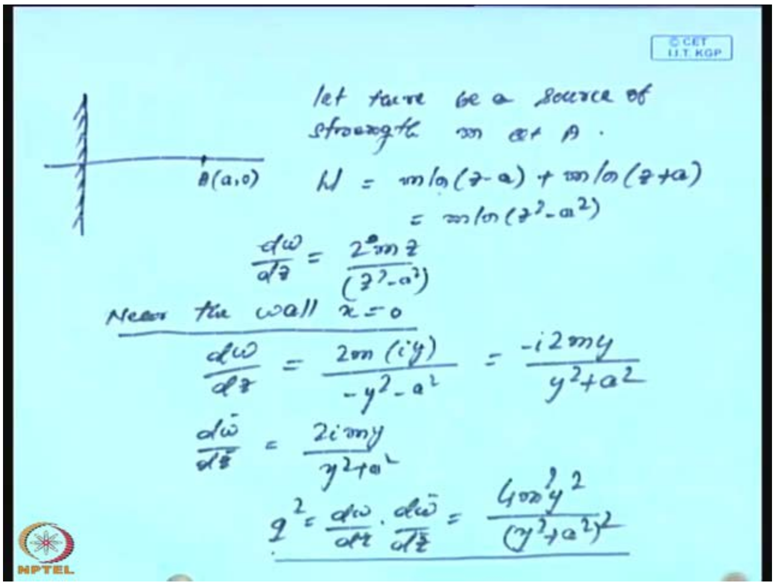

Now, they have wall and middle it is along the x axis. Let there be point A that is a coma 0. Now, let there be source of strength m at a. This is on the x axis. There is a wall. If that is a source, what will be the, with much will be at minus a 0 that will be on the other side.

Suppose that there is a mirror image. So, then what will be the complex potential? If I look at the complex potential m log z minus a plus m logs n z plus a, so that means the point z is equal to; if z is equal to point ratio r, I have a 1, then this z is equal to minus a. It is just on an opposite side of the 1. That means it is automated. The distance is not the same distance on the wall and appears as if it is an opposite side. Then, it is like a mirror image. So, what will happen to this? So, this also can be written.

So, from here what a happen d w by d here if d w by d z, I calculate that will give you m this can be from here. It will be m logs z square minus a square. So, when I said d w by d z that will give me 2 m by z square minus a square into; so this will d d w by d z. Then, only of wall what will happen near the wall? We have that the wall means x is equal to 0.

So, what will happen to d w by d z near the wall? d w by d z is this will be 2 m z is x is 0.

So, this will be i y my z is x is 0. So, this is minus y square minus a square. So, I can call it minus 2 i times 2 m y by y square plus a square. Again, if you look at d w bar by d z bar from here itself, we will get 2 i m y square plus a square. So, from this, I can get q square, which is d w by d z into d w bar by d z. That gives me 4 m square y square by y square plus a square, square.

Now, that means on the wall, x is equal to 0. This is my wall. Of course, the speed will direct towards the wall in this direction. This is the speed. At this speed, the fluid will be flowing from the source once we have q. But, since this wall is an also streamline, then we can have, we also can see that what will happen to the force pressure only 1.

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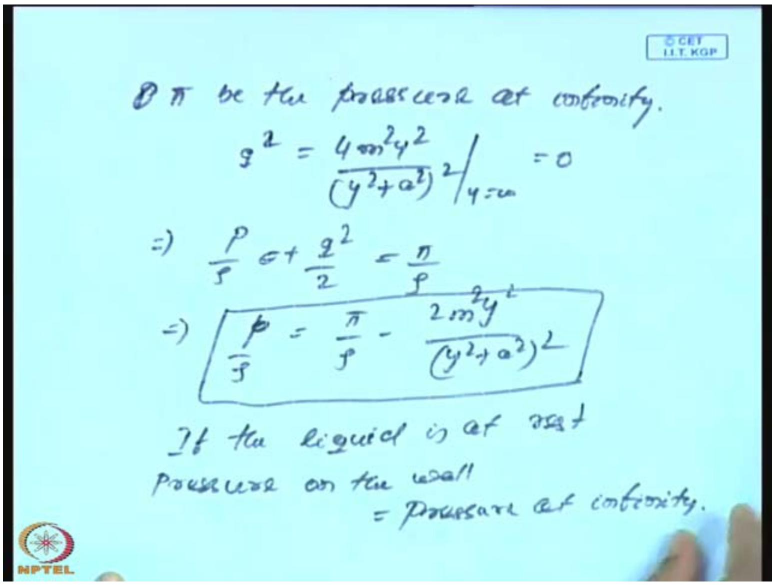

So, if I say while the pressure is the pressure at infinity, pi is the pressure at infinity. From other lesson, q we have seen, q square is equal to 4 m square y square by y square plus a square, square. So, that is q at infinity, this it will be 0. So, at any point on the wall, that will be 0 at infinity, pi is the pressure. q square is this. y is equal to infinity. It should be 0 because there is, if it is 0 that means I will have p by rho is equal to p by rho plus q square by 2. If I go to Bernoulli’s equation, I have pi by rho.

That means my q my p is equal to whether also p by rho equal to pi by rho minus q square by 2. This is 2 m y the q square by 2. So, q square is a 2 half is there. So, we will get 2 m square y square by y square plus a square, square. So, this will give me the pressure at any point the wall. If I know, if liquid is the rho is at rest, so if q is again I will say if q is 0 that means if the liquid is at rest that means these are at any point on a wall. It will be same as the pressure at infinity.

So, if the liquid is at rest, pressure on the wall is same as, same as the pressure at infinity. Now, with this understanding, if I want to calculate what is the force that is excited on the wall due to the source? Then, I can always do because I know p. Then, I can calculate the pressure. From the pressure, pressure from pressure, I can calculate the force.

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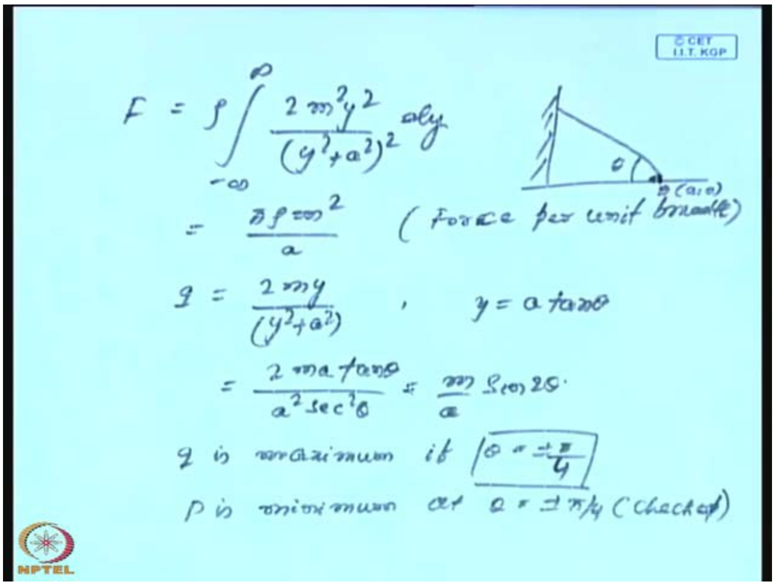

So, the force will be F is equal to rho. This will be you can easily see. This will be minus infinity to infinity that is 2 m square y square by y square plus a square, square d y. So, the source will if this one would and this if you will calculate this, this will give us by rho m square by a, because basically what we are doing here. We are substituting further pressure and integrating. This pressure is per unit breadth of the wall. This force this is force per unit breadth. This becomes the force.

Again, if I have the relation q, my q is on the wall. My q is 2 m y by y square plus a square. If this, if I substitute, if I take y is equal to a tan theta, then this will give me y is well tan theta. That will give me 2 m a tan theta by a square sec square theta. That will give me m by a sin 2 theta. That will give me the velocity. So, if I am, my wall is here, the point is here a, that is the point a, 0. Then, if this makes an angle is theta with this x axis, this point is here. This angle is theta. Then, q is m by a sin 2 theta. It can be seen theta will be maximum, and then q is maximum. q is maximum if theta is equal to phi by 4 plus minus pi by 4. This is pi by 4. It can be seen if q is maximum. Then, you can see that at that point, the speed is maximum.

We can see that p is minimum at that point. Theta is equal to plus minus pi by 4 because from Bernoulli's equation, once I have one form boundary, it has speed which is maximum. p has to be minimum. This can be checked from the relations. So, this is the way we calculate. Suppose, due to a source, how the force is calculated on a wall in a fluid flow? Now, this I will go to doublet because we have seen in case of a source or a sink, we have a source. It is radial in flow, is radial. Now, if we think about doublet where as if you are thinking of 2 sources, so let us look at what I mean by a doublet.

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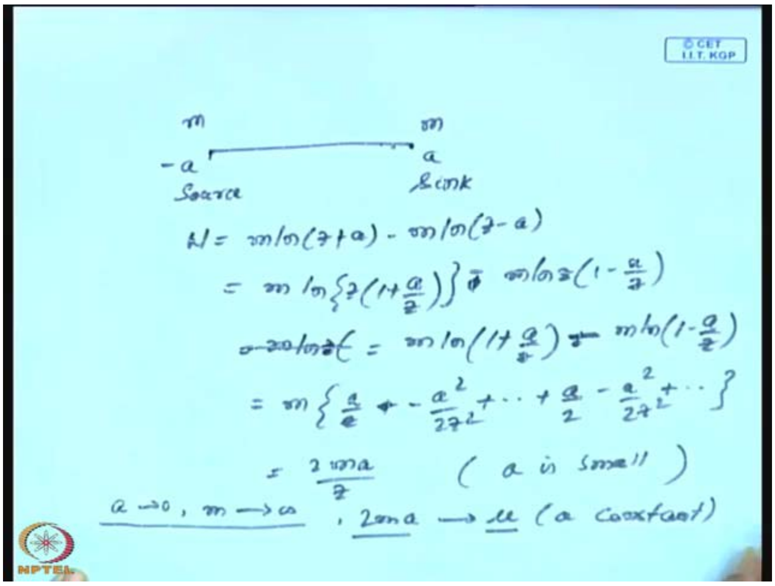

So, now let us look at two things. I have a source of strength. This is the point is minus a. This point is a. Here, I put a source and here, I will put a sink. So, what will be the complex velocity potential? The 2 are at a distance to way source and sink they are at distance to a. so, the complex velocity potential will be m logs z minus a z plus a minus m log z minus a. I can write it as m log of, I will put it z into 1 plus a by z plus m logs, minus m into logs z into 1 minus a by z. This gives me m logs first time.

It would b m logs z, either I will say m log 1 plus a by z plus or minus m log 1 minus a by z. This is because I have a m log z term here, m log z term here. They will cancel. So, what I will do? I like expand it. So, m, if I expand it, therefore this is so. Let us see this.

Then, I will get it a by z plus where they are at the second terminal minus a square by 2 z square plus higher order comes here. Then, plus a by z minus a square by 2 z square plus the higher times this and this coming that will give 2 m a by z plus 1 higher terms. This is because I am assuming a is small. If I assume a is small, this gives me. So, I look at 2 things.

If a is tending to 0, the strength of source is turning to infinity in such a manner, so that m a tends to a constraint, 2 m a turns to constant. What is called a constant? So, let me repeat. Well, there is there is a source; there is a sink, which is of strength m. Now I will just say that the source and sink, there are very, they become very close to each other, whereas they become close to each other. The strength m will tend to infinity. So, in such a manner that 2 m a will give m a constant and that y becomes a constant.

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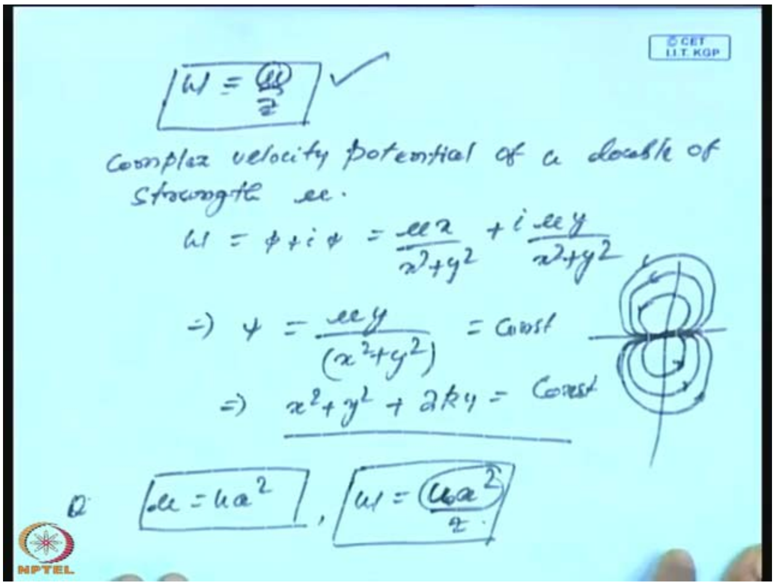

w will be w mu by z. This is called a doublet. So, this w represents the complex velocity potential of a doublet of strength mu. If you look calculate what are the streamlines, w can be written as phi plus i psi. This can be written as mu x by x square plus y square plus i times u y by x square plus y square. It gives me psi is equal to mu y by x square plus y square.

Then, this is constant. It gives me a constant that means this will give me the streamlines that is x square plus y square plus 2 k y is equal to constant. We have seen this with lot of examples in the last class. This is like this. It will touch the x axis. The flow factor would be like this. Of course, this is cylinder circular flow. So, again the flow will be the center will be, it will touch the x axis. Again, the circular flow and flow will be and this is the flow that will due to a doublet.

So, from a source and sink, when they rest together, particularly close to each other, we will get a doublet and a flow again will be in fact. In in our last class, if mu is equal to u a square, you have seen that when mu is equal to mu a square, we have seen a simple singular example. That means with mu is equal to u a square, then w is equal to u a square by z that we have seen in this example that a it is a like we have speed u naught test. There is a cylinder of radius a, other speed called strength speed u naught. This is the velocity potential.

At the same time, here we are seeing that the similar function complex velocity potential of a w is representing a doublet. Thus, it is sometimes this this example such as that this constant value changes the nature of the problem that can be different problem. By physical characterization, we define on how we look at the problem and what is the value associated, the constant associated with it. So, that is that is why, in many situations, we always have a similar problem that can be expressed, rather a similar function can be used to understand the various flow characteristics. All that will depend on that constant vector, the physical parameter that is involved in the flow.

So, that is one of the examples. This is one example. We showed that physical characteristics will change the nature of the problem. Often, even often the complex velocity potential is similar in nature. With this, I will go to another part of the problem that is I will go to the blasius theorem and the blasius theorem.

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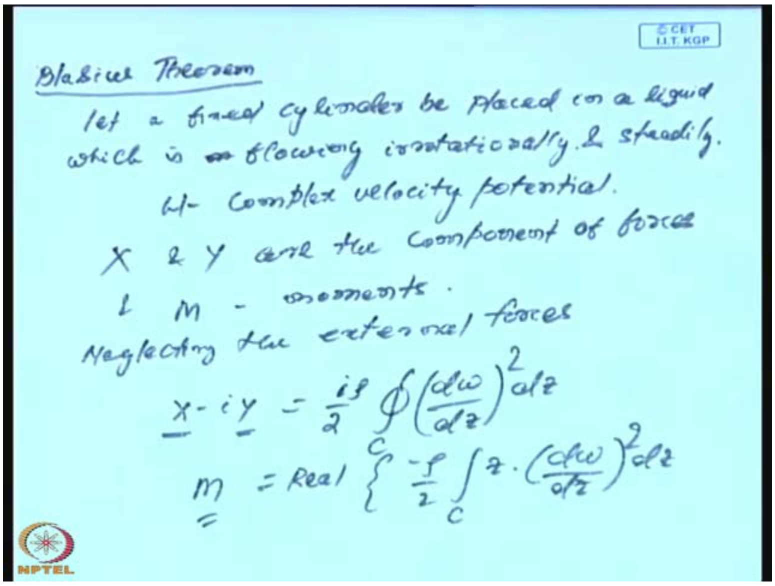

So, basically this theorem talks about the forces, component of forces that is acting on a body or on a boundary when there is a flow. Suppose, let us think of let a fixed cylinder be placed in a liquid, which is flowing. The fluid is flowing irrationally and steadily. Let w is the complex velocity potential associated to flow. A fixed cylinder is placed in a liquid, which is flowing irrationally and steadily. w is the complex velocity potential of the flow.

Let x and y be the component of forces. m is the moment on the cylinder. These are the force on moments in cylinder, moments acting on a cylinder. Then, neglecting, if you neglect the external forces due to the fluid, the forces acting on the cylinder will be x minus i y. that will be given by 1 by 2 i times 2 integral over c. I told you d w by d z square d z whether it is called moment, m is the real part real part of minus rho by 2 integral over c that is z d w by d z square d z.

So, this theorem, which we will have, which cylinder is include, which is moving irrotationally, if you want to calculate the force on moments. So, let us see. Here, I can easily say that x is the force and y is the left force m a, m is a moment that is acting on the cylinder. Now, I will illustrate through an example. We take the case of an example.

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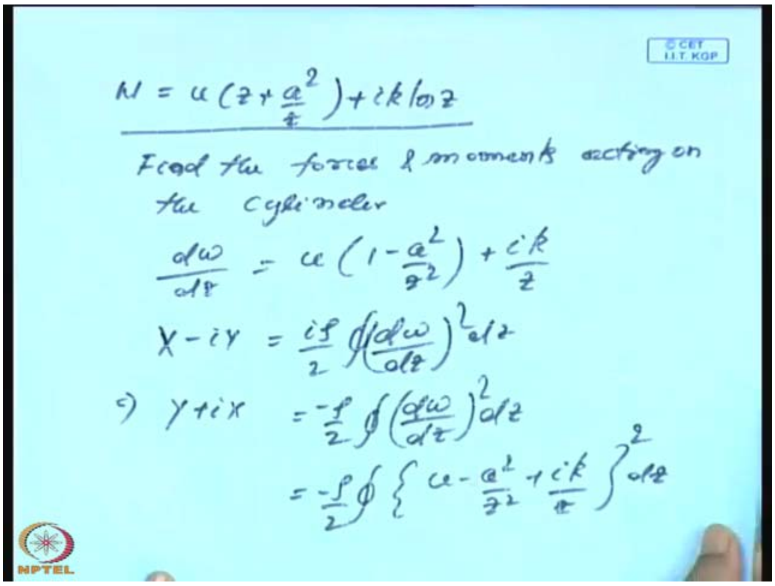

Suppose I say my w is u z plus a square by z plus i k log z. This will prevents this velocity potential. It prevents. I have a infinite cylinder placed in a uniform flow in the presence of a. Here, I have a vertex and the circulation k, the strength of the vertex. Then, if I have to find the vertical force, find the forces and moment that is acting on the cylinder.

If I have to do that, look at here, w is given. What will happen to my d w by d z? My d w by d z in which u 1 minus a square by z square plus i k by z, if this is, then what will happen to my x plus i y? If I go to this in other way, my x minus i y will be rho by 2 into rho by d w by d z square d z. Again, we check an answer at if i y plus i x as a rho by 2 per minus rho by 2 d w by d z square d z. So, if I explain this, d w by d z means minus rho by 2 d w by d z is u minus a square by z square plus i k by z square d z. So, this is a i by circle; is a closed circle.

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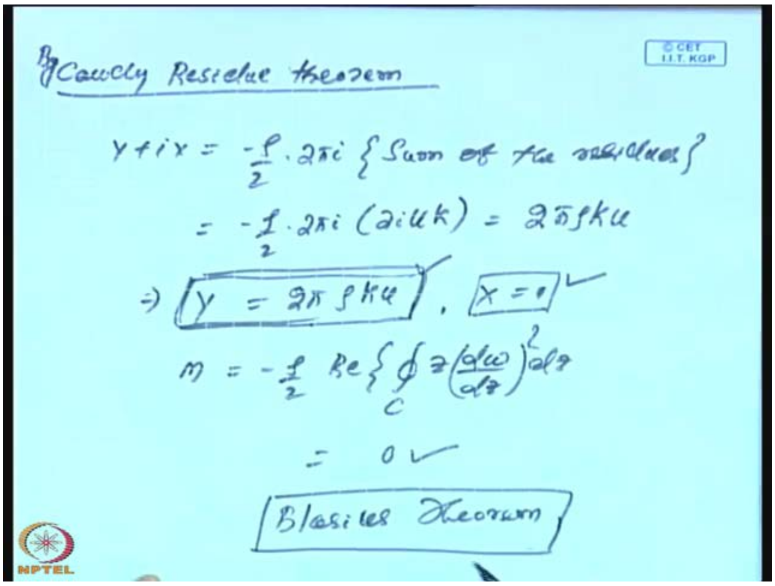

So, if I apply Cauchy residue theorem. It looks like Cauchy residue theorem. So, this will give me y plus i x will give me minus rho by 2, 2 pi i in to because I have at z is equal to 0. I have a is equal to 0. There are flows. So, you can say sum of the residuals and only since we have, we can see that in this problem the only residues and z is equal to 0 that is a residue. It can be calculated 2 pi into 2 i u k.

That will give me 2, 2 cancel i, i minus pi 2 pi rho a. From this, I can always say that my y is 2 pi rho k. Again, I do not have any here x what is 0. Further, if I calculate the moment, we are applying the same process minus rho by 2 ideal part of minus rho by 2 integral c z d w by d z square d z. I can easily find that this is 0 and not going to because this again can be applied by the Cauchy residue theorem rather says by Cauchy residue theorem, this will be 0.

So, here we have a; this y part with this del j. In the present surface, there is a; but if x components they represent able force vertical force. If the lift force and x component in fact the horizontal force that is a drag force that means there is no drag in this case, whereas the lift force is gained by this. On the other hand, we have to calculate the moment and that is given 0.

Now, this is a very classical example to calculate the course based on blasius theorem. We have the blasius theorem to calculate this. Now, I will take few more examples in this class on this source and sink. Now, suppose I have a source. We have seen that if I have source and sink, which are placed, we have taken 2 to 3 cases.

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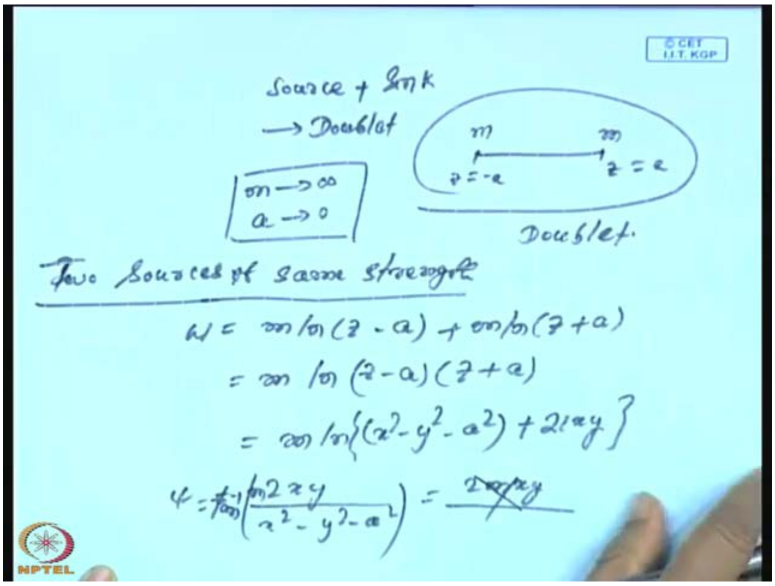

The first cases is if I say that I have a source placed at that main a have sink, a source placed at z is equal to minus a, we will have this sink which is placed at z is equal to a. Both are of strength m. Then, we have seen that that will relate to a doublet. Now, on the other hand provided m tends to infinity, a tends to 0, on the other hand if I suppose I say that I do not say that the situation. Suppose, there is a. What will happen if I have just a source or sink, both 2 equal sources? Here, it is a source and the sink source plus sink. It has giving a doublet.

What will happen if we have 2 sources? We think of 2 sources. 2 sources, both are of a same strength, 2 sources of same strength. Then, my w will be m log z minus m plus m log z plus a. That gives me m log z minus a into z plus a. That gives me m block. That gives me z square minus a square. That is x square minus y square minus a square plus 2 i x y z square. That gives me this from which I can get my psi I will get my psi as because same. So, this will give me 2 x y by x square m times 2 x y by x square minus y square minus a square. So, that means 2 m x y by rather it will give it tan inverse this.

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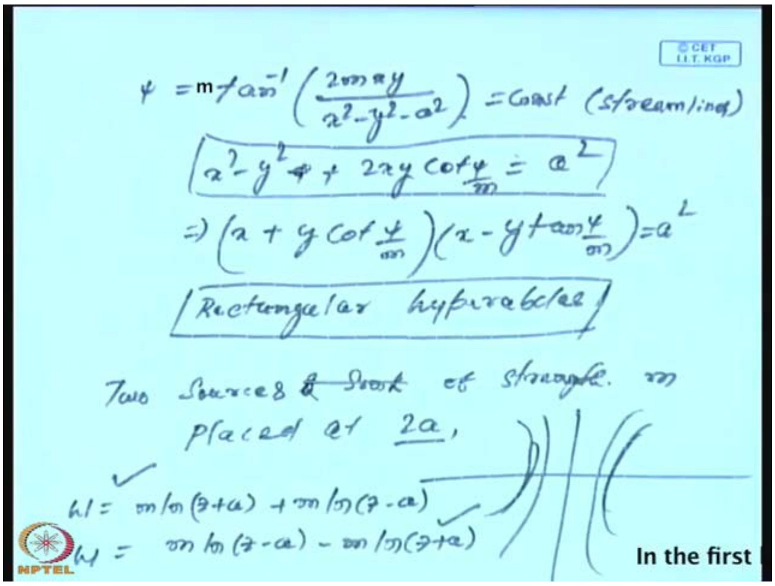

So, my psi is tan inverse 2 m x y divided by x square minus y square minus a square. If psi is equal to constant are the equations of the streamlines that will give me the streamlines. If I further simplify this, then I will get x square minus y square plus this will give me plus 2 x y. We would have taken as 1, then I will get 2 x y cot psi, but other 2, 2 x y psi by m, I do not have to take this. That will be equal to a square. If I simplify this, I can get this is same as x plus y cot psi by m into x minus y tan psi by m. This is equal to a square.

So, if you look at this, they are nothing but we will assume that these again give me rectangular hyperboles. So, in the rectangular hyperboles, so what we have seen in the 2 examples? We have a where you have a source and sink. We have a source and sink of strength, rather 2 sources 2 sources of strength m placed at this tends 2 a. Then, we are getting hyperboles, rectangular hyperboles.

On the other hand, we have seen that we get a doublet if you take a source and the sink placed against same distance 2 a, whereas the distance is small. So, that is a. So, what here I mean to say again that just look, if you look at the authentically compress potential w is equal to m log z plus a plus m log z minus a, at the same time you are looking at the the compress potential w is equal to m log z minus a minus m log z plus a.

So, if these 2 potentials, this to compress potential, there difference is mathematically looking, there is only a change of sign in both the cases. Only one psi it has, but that leads to a flow, which is quite different in both the cases. So, what we have here? So, that always it suggest that simple representation of the flow like complex number gives us very simple change in the flow complex number pattern. It particularly gives us very different types of flow. Physically, it is quite; it is very interesting physically to understand such kind of flows.

This has in fact because of this in understanding of 2 dimensional flows. The role of complex velocity potential of the role of complex function theory cannot be ignored. This is because without much of the difficulty, we are able to analyze several complex flows. That is possible because of not only we are able to analyze the flow pattern, we are also able to get the streamlines. Even if we have seen that using the blasius theorem, we are able to analyze various flows. Particularly, we are able to calculate the force and moments on the boundary of the flow. This is what blasius theorem has given.

So, in the next class again, we will look into a detail about complex flow, various types of complex flow. We will come to that particularly afterwards. We will come to little more about conform and mapping where how can we represent a simple flow. We can change them simple flow, a complex flow to have in one plane to very simple plane in other plane. In the process, we will be able to analyze more complex flow patterns by using simple complex velocity potential.

This is because we can transfer from one plane, a complex flow in one plane can be transferred, transform to another plane where we can analyze the flow in a very simpler manner. Again, we can go back to our new plane where the flow is well complex. That is known as confirm and mapping. In the next class, we will talk a little about confirm and mapping, its application to several flows.

Thank you all. Today I will stop here.