Marine Hydrodynamics
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Lecture - 11
Worked Examples on Two Dimensional Flows

Welcome to this lecture on marine hydrodynamics. Today, we will talk about several worked examples, basically we will try to in this lecture, we will try to work out several examples and show the application of blasius theorem. And also see, in some, some of the examples, we will try to calculate the velocity of the pressure using the Bernoulli's equation or rather I will say the equation of motion. Because in the last few classes, we have seen that how this various flow problems are simply analyzed by using the complex function theory in two dimensions. So, in many situations like in the last class I have mentioned, just same function with a little change in the functional characteristics, how the flow description changes. Here we will talk some of the complex flows and see how they are analyzed.

As I have already mentioned that in the when the potential theory was being developed large class of problems were handled, and flow descriptions were made by using this analytic characteristic of the complex function theory and some of the characteristics of the complex function theory. Like we have seen in the blasius theorem, how just by calculating the 2 integrals, we are able to calculate the force and the drag force, the lift force, the movements. And also we have seen how even if just the description of various flows described by using by choosing a proper complex velocity potential. So, we will continue today to show few more examples, that how the, this complex potential gives nice description of flows, so this we will start today. Now, let us start our first example.

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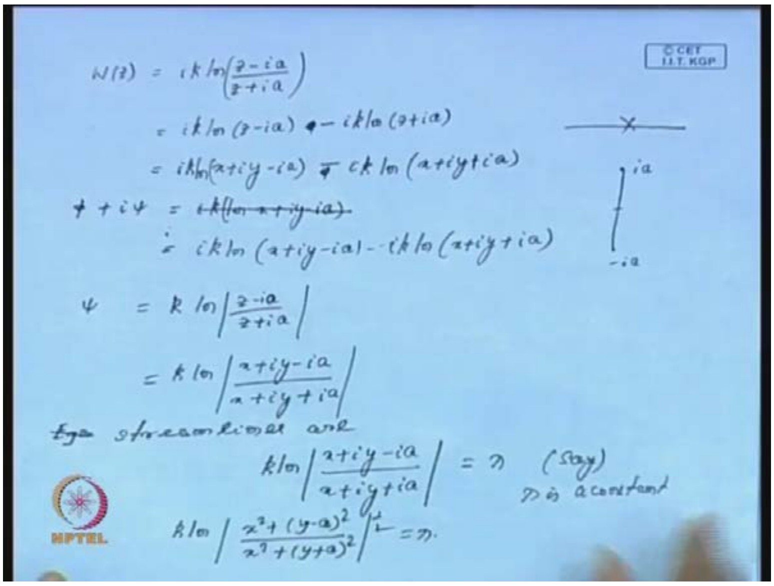

Suppose, if I take W z i k, now z minus i a by z plus i a. Now, this example, if you look at let us have simplify, this example this; this is i k log z minus i a plus rather, we will say this is minus i k log z minus i a z plus i a. And this shows that this is a vertex of strength k and this is also, what is of opposite strength. And the 2 vertexes there located at a distance not in the x axis rather than the y axis from the origin one is at i a, the other is at minus i a. So, if I say that if i a is the vertex here, we can always say that as if this is the mirror image of that. Like, we have seen in case of a source that, we have seen that how a mirror image of a source has given us, in the wall source acting forces acting in a wall due to a source. Similarly, let us see whether we get a similar result that if I have this one as a vertex of strength k, and whether it acts behaves like a mirror image of that. To look into that let us see that, what happen here.

So, this one I can always write it, this I can always write it this will give me i k z a x pus i y minus i a, log of plus or rather minus, i k log of x plus i y minus i a. Now, this is w here so what will happen to 5 plus i psi, if i look at this this is i k log x plus i y minus, this is plus minus, this is plus i this is plus i a. And if we say that from here what will happen to psi, psi will be, we can easily say that k, because I have the, I have to look at the imaginary part of this. So, I will get it k log modulus of we can see that z minus i a by z plus i a, and if you look at the modulus of this, this is same as from here. We can get that is k log x plus i y minus i a divided by x plus i y plus i a and this is the modulus of this.

Now, if I say psi is equal to, suppose I text, so if I say this is equal to for this, to find the equation of the stream line. For the stream lines rather I will say the stream lines are the psi is equal to constant, will give me the stream lines are the k log x plus i y minus i a divided by x plus i y plus i a. The modulus of this and that should be a constant, say that is a constant lambda is a constant. Now, this is same as k log of this is x square plus, this is y minus a square, on the other hand this is x square plus y plus a square. So, modulus of this I did power half and that is equal to lambda. Now, from this I can easily get this will give me that suppose I say.

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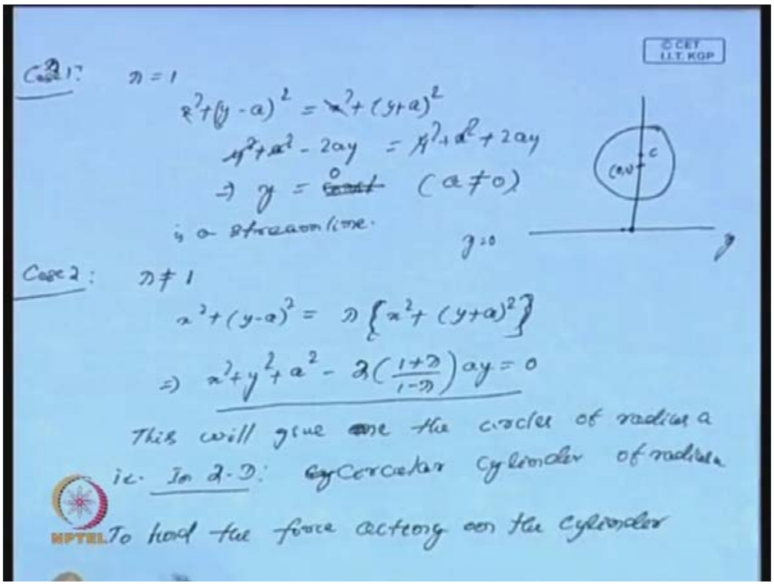

If I say my lambda, if I say lambda is equal to 1 if lambda is equal to 1, then I will have x square plus y minus a square is equal to x square plus y minus y plus a square. And that gives me, this will get cancelled and that will give me cannot be simplify plus a square minus 2 a y is equal to y square plus a square plus to a y. So, this goes this goes, so is gives me y is equal to constant, means if a is not equal to 0. Since y is equal to constant, what does it gives me, this is I will come to this now, the line if it is y is equal to constant. This is along x axis or any line which is parallel to x axis. So, if I take that constant as 0, then y can be 0, if that constant I take it as 0 sorry here y is equal to 0 to a y this is y is equal to 0. So, y is equal to 0, the line y is equal to 0 or the x axis represents and there will not be any fluid, which will pass through this cross through this line because this is a, is extreme line.

Now, the second case I will say, this is my case 1, now if I go to case 2, the case 2, I suppose lambda is not equal to 1 if lambda is not equal to 1. Then what I will get and I will get x square plus y minus a square is equal to lambda times x square plus y plus a square. And once this is, this then I will get x square plus y square plus a square minus 2 times 1 plus lambda by 1 minus lambda 2 a 1 plus lambda by 1 minus lambda into a y is equal to 0. So, for each value for lambda is not equal to 1, for each value of lambda I will get, and then I can easily get, from this I can easily get that as if I will get a circle. And here the region will be the point x is equal to 0 will be 1 of the, if I say this point is let me think of a circle. And if this is the axis then this point is 0 a, so because of this factor center somewhere can be here, that is the center.

So, it will be circle which will be on the so this line is 0 this line is one of the line. And again it gives me this is, so we get a circular cylinder of again I will say we have a circular cylinder. So, these are non-intersecting circles this will give me circles of radius a, that means in 2 d. Since I am dealing with the 2 d, so I have a circular cylinder of radius a, of radius a, and whereas, so this is also looks like a stream line. And also I have a other stream line the line y is equal to 0 is another stream line. Now if I want to say that, I want to know, what is the force? So, then I will to find what is the force acting on the cylinder. And the force acting on the cylinder but initially we have seen that we are starting with as if we are looking at a vertex. Because, our complex velocity potentially is basically one is a vertex and another is the mirror image of it where the strength of the two vertices are different.

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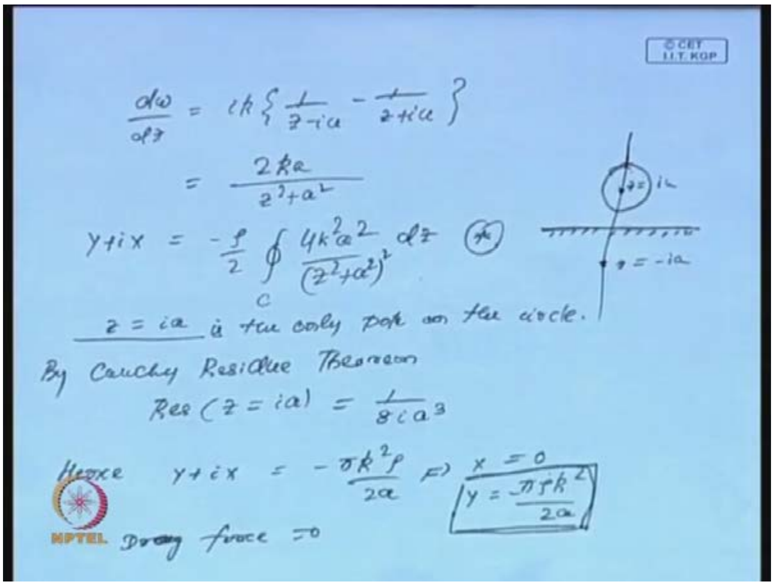

So, I will get it d w by d z, this will be i k 1 by z minus i a minus 1 by z plus i a and that gives me 2 k a 2 k a by z square plus a square. So, if this is this then I will get the my forces, I will get Y plus i x will be rho by 2 integral over the circular cylinder that is 4 k square a square by z square plus a square, square d z. And here it can be seen because the cylinder is on the proper plane and here I will say z is equal to i a, is the only similarity for of this, and in that case is the only pole on the circle. So, if I apply circle apply the Cauchy residue theorem, so by Cauchy residue theorem. So, my residue r z is equal to i a, and the circle that will give me 1 by 8 i a cube.

So, then hence if I put this as star because this is my blasius theorem, so from star my y plus i x will give me minus pi k square rho by 2 a. I am not going to you can always substitute for these two pi i into some of the residue. Hence and then it will get we will get this, so here we have seen that which implies my x commonly 0. Whereas, my y force is Y rho k square by 2 a minus and this minus an, indicates as if the force is acting in the downward direction concern, on the downward direction it is not enough.So, so the liquid we show that the liquid acting in the downward direction, and in the cylinder. So, it is trying to push the cylinder to the down, and this is the however here also we have seen that there is no the drag is 0 drag force is 0 and only the lift force is acting on the cylinder. So, this a, this is one of the example which shows that how by considering just one vertex.

And it is just the, you can say that suppose vertex is here z is equal to a. If the vertex is here then a look at as, if there is a mirror image of it which is located the strength is same. But, opposite and the opposite direction in the negative strength minus k here, and here, if it is that a here it is a z is equal to i a, and this point is z is minus i a. So, it is, it this is a wall because we have seen this x behave like a cylinder, it can behave like a wall that is due to a source a vertex on the mirror image of this.

So, in the so what we have seen in this example, that just considering two vertex of same opposite strength located at a distance 2 a. We are able to get a flow past cylinder, which is at the flow direction is along the y axis. And there is a horizontal wall on which the fluid is towards the wall the fluid is flowing. Now, I will go to another example, what will happen if we think of, because we have seen that when combination of source, and sink we have seen the combination of source. And source and again we have seen the combination of vertexes vertices. Now, let us see what will happen if I have source and a vertices.

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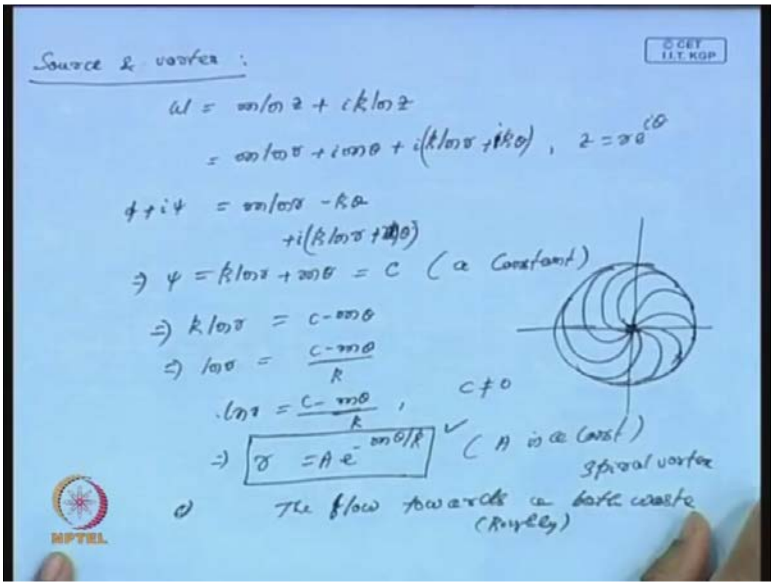

Source and vertex if I just said the both are together in a, suppose I said w is equal to this is will be m log z where m is the source of the strength and i k now, that where k is the strength of the vertices vertex. So, in this case this will give me m log r plus i m theta plus I will get from here i k log r minus k theta, where theta z is equal to r e to the i theta factor i k this is z is r e to the power i theta k log r plus i k theta i is there. So, plus k theta will there, will be I here, so that will m m log r m log r minus k theta plus k log r plus m theta. This is minus k plus k theta into i, so this implies if this becomes i plus i psi.

This gives me my psi will give me plus k log r minus plus m theta, this is m k m plus m theta. And if I say this is equal to constant, and let me call this constant c a constant. And that means log r log r will be, we can also call this as c minus m theta. This implies k log r is this c minus m theta which implies my log r and be c minus m theta by k are also I can call it minus m theta by k so taking c and 0. Then that will be log r which implies my r is e to the power, e to the power of minus m theta by k.

So, so also if I take c is non-zero, then I will say this is c minus m theta by k and I call it some A into e to the power minus m theta where A is a constant A is a constant. And that is so which implies I am getting, so if I look at that what happen suppose we have plot this along the axis. Then I will have a source if I plot this curve, I mean in take a few example. In fact this will give me, gives me this is called as spiral vertex, this gives me spiral vertex.

And then this shows that the flow towards A if I look at this this is sometimes roughly we will call this look at the fluid is flowing in this way. And then I call this the outer directions this one I call this a flow, we have waste in a water particularly in a bath bathroom particularly. The flow towards a sometimes bath waste roughly it, it can say roughly. This is we can related to other examples, another one, because here it is another example, I will give you, if I just take this, instead of if I look at the same flow in a different manner.

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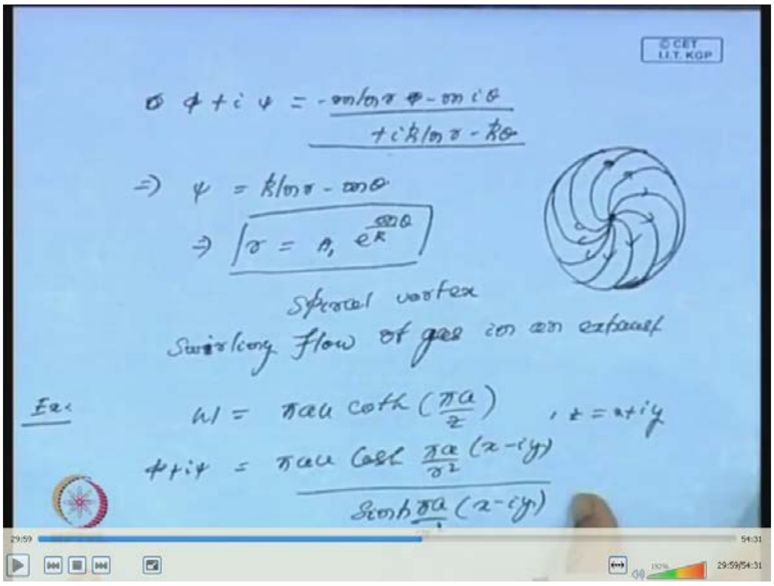

If I call this, I will take my r if psi minus, psi plus i psi is minus m log r, if my psi will be minus m log r plus minus m log r plus minus m i theta plus i k log r minus i theta. So, that means if instead of a source if I constant this as a sink, then what will happen the in the corresponding case I will get psi is equal to k log r minus m theta. And in that case what it will give me, it will give me r as some constant a A 1 e to the power m by k into theta. Now, other case again it will also give me a spiral, that only just take circle that only, so here the flow is always in the outer direction. In the previous case the flow was it was tending towards the waste, but here the flow is always in the outer direction. Because this is going r is in the radial outward direction, because you can see that this, will expressional goes.

And then this example I will call this again it is a spiral vertex, here this is in this case we call it the sink plus vertex. And in the other case we have a source plus a vertex it is again a spiral vertex and roughly I can say that is the swirling flow of a gas, of gas in exists here. The flow will always in the, because as if like when the water goes out of a chimney. So, this two examples gives us very good understanding, that how the flow the help of source and sink how nicely we can describe very complex flow phenomena.

Now I will go to another example, the third example let me consider a third example. In this example I will take that my w is equal to y a u, this is cot hyperbolic y a y z. And again it it is neither a, let us see how will where if it z is equal to where z is equal to x plus i y. And if you look at phi plus i psi that will give us, y a u cos hyperbolic, y a by r square into x minus i y divided by sin hyperbolic, y a by r square into x minus i y, same can be rewritten as, we can write the same as.

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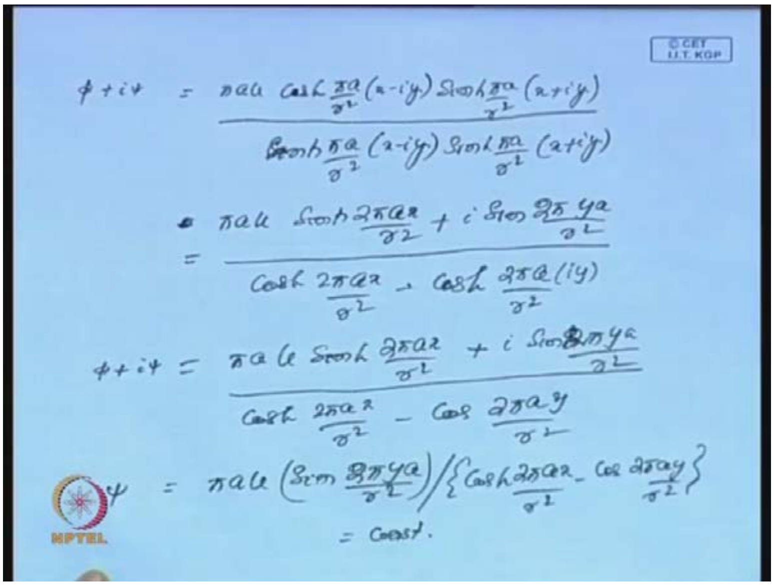

Again, we can write phi plus i psi we can write it in the form of i a u, we had cos hyperbolic y a by r square into x minus i y. Just I will put it multiply with sin hyperbolic pi a by r square x plus i y. So, I will divide by the same, if I divide by the same then I had sin hyperbolic sin hyperbolic pi a by r square into x minus i y. And if we such then I will apply cos into cos hyperbolic, a into sin hyperbolic b, and here sin hyperbolic, in the sin hyperbolic a plus b minus b a b rather I will put it if I utilize this formula. I will get pi a u that will give me this will give me sin hyperbolic 2 pi a x by r plus i sin 2 pi y a by r square.

And this divided by, divided by this will give me, cos hyperbolic 2 pi a x by r square minus, this will give me cos hyperbolic 2 pi a by r square rather into i y. This will give me let me put it in other line, sin hyperbolic 2 pi a x by r square plus i sin hyperbolic. Rather i sin hyperbolic sin 2 pi y a y r square divided by cos hyperbolic 2 pi a x by r square minus cos 2 pi or there is a y a y by r square and then if I look at psi. Because, this is y phi plus i psi, if I separate the real and imaginary parts then my psi will give me pi a u sin 2 pi y a y r square divided by cos hyperbolic 2 pi a x by r square cos a y by r square. And this is my psi if I say and this will give, constant means I get the stream lines. Now, if I choose the constant if psi is equal to 0 if I chose my psi is equal to 0.

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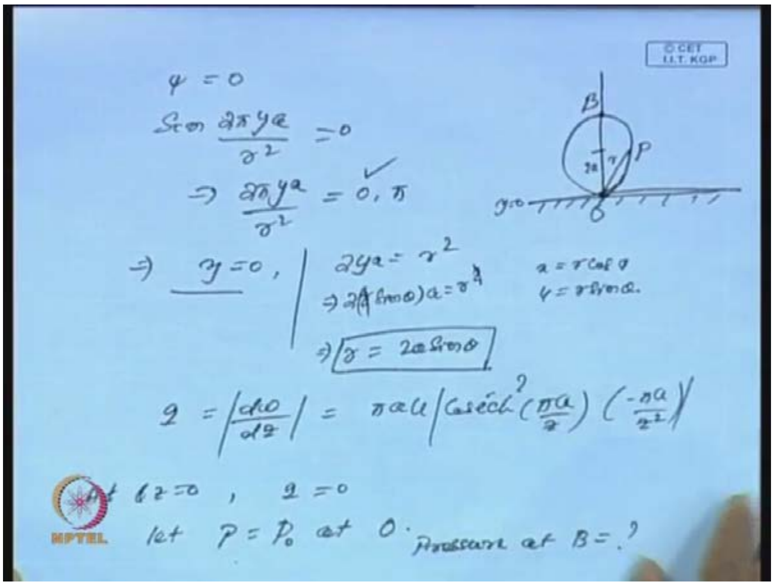

I get 2 things then putting psi is equal to 0, I will get sin, sin of 2 pi y a by r square is equal to 0. And it can be 0, because which gives me 0 or this is 0, and that gives me 2 pi y a by r square I can take it as 0, I can take it as pi. If this is 0, that means I will get my line, one of the line I can get y is equal to 0, that is for this. And another case I can have 2 y a is equal to r square, and which gives me which implies, there is a if I put x is y is x is r cos theta, y is r sin theta in the polar coordinate. Then I will can get y is r sin theta, if I put 2 r sin theta into a is r square, so r r will get cancel, this gives me r is equal to 2 a sin theta.

So, if I look at this, so this also, this will behave like a stream line, this is also behave like another stream line. I have the two situations so how this will look like so I also can get here y is equal to 0 streamline. This is the line y is equal to 0 this can behave like a stream line, and then another thing, we can have a r is 2 a sin theta. We can have circle here, and because this distance can be 2 a, and any point. If I take angle here, we can call this p, and this is my r this angle will be theta. So, we can again get a cylinder flow past cylinder of radius 2 a, again I am getting a cylinder flow past cylinder of radius 2 a. And where again this line also, and the line y is equal to 0 also behaving like a streamline. And as well as the stream, the cylinder is also as we have flow past cylinder.

Again, we fully look at this what will happen to q a q 2 d w by d z, and if I look at this d w by d z and this will give me pi a u my if I look at my w. This will give me pi a u cosic hyperbolic square, pi a by z into minus pi a by z square, that modulus of this, and if super at 0 0. If I calculate at z is equal to 0, then at z is equal to 0, I can easily see there is as a cosic hyperbolic square there is a z square, I can always find q is 0.

It can be easily form at z is equal to 0, this is the, suppose that means at if this point is o. And at this point the fluid speed is 0, and which is here obvious because you have a there is a no flow occurs this point and there is a cylinder. There is so let me say that, let p is equal to p naught is the pressure, at this point at 0. Then what will happen at any point on the cylinder, and one of the obvious question what will happen at this point, from the cylinder that only the pressure. If can I get the pressure here, if I will apply the bernoulli's equation knowing that p is equal to at z, is equal to 0 p is equal to p naught. If I call this point as a b what is the pressure at B, but I want to find out what is the pressure at B

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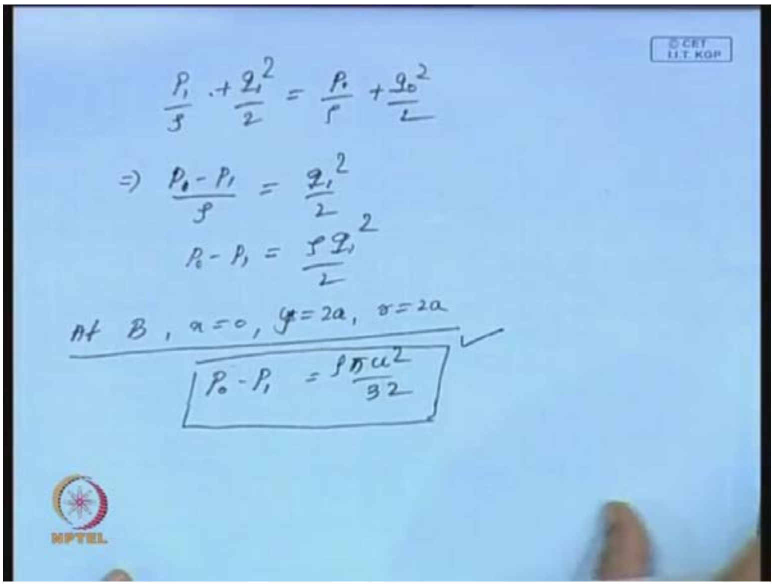

Then I can use this formula p 1 by rho plus q 1 square by 2 is equal to p 0 by rho plus q 0 square by 2. And already I had found q 0 is 0 at origin q 0 is 0, and I take p is equal to p naught. So, which gives me my p 1 so this will give me p 1 q 0 0, so my p 0 minus p 1 by rho is equal to q 1 square by 2. And we have seen that, again q 1 square by 2, so that will give me p 0 minus p 1 is equal to rho into q 1 square by 2.

And that can be easily seen that, this value we can easily see that if I substitute for q, because we know q is equal to we have already known q. So, if I say that at at B my x is 0 and y is 2 a, and r is 2 a. If I substitute this in a relation for q, and I will get my p naught minus p that, will give you pi u square by 32. I think there will be a rho, rho times, rho by u square by 32 and this is what the pressure I am interested to calculate the pressure at B. So, c it become a very simple function by considering a hyperbolic function, quite hyperbolic function.

We could get that there is a flow, there is a cylinder, there is, a, the base is the plane one of the plane is y is equal to 0. And then we have a cylinder on their where is a laying on the top of it. And then we can easily calculate, what is the force that is acting on the cylinder? With this understanding, I will now go to one more example, and that is the one which I will, what happen suppose I gave you for example, is…

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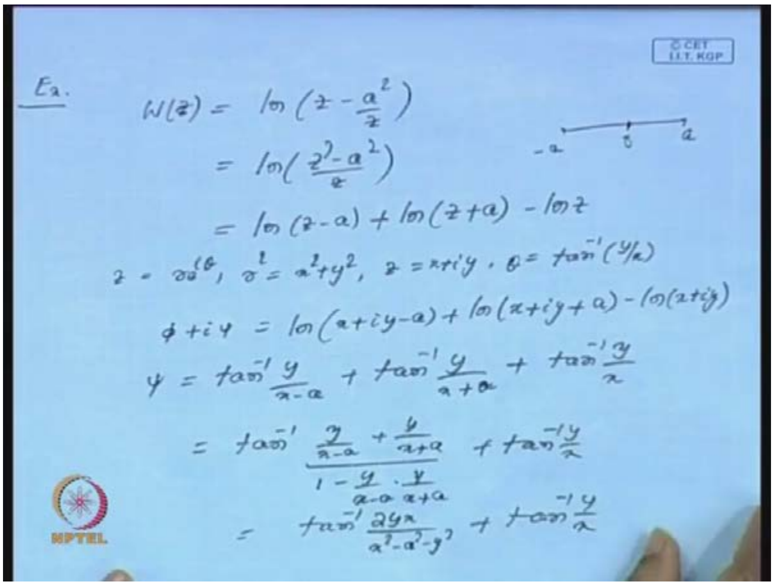

For the example, I will just consider in this example, I will just say let me say the w z equal to log of z minus a square by z. Let us first interpret this function how this is interpreted, this can be interpreted as log of z square minus a square by z. And which can be written as log of z minus a plus log of z plus a minus log of z. So, this is a like, we have unit source that are two units source located at distance 2 a. One is at the sources at one sources at here, there is a source minus a, this is a source of a, and I have another sink which is located at 0.

We, have already seen the presence of two sources, and presence of looked at two ends, by now we are taking two source plus a sink. Let us see how this flow is not looks like, if this arrangement if I look at phi plus i psi z is a r e 2 the power i theta. If I 2 to the power r is x square plus y square r square is this and z is equal to x plus i by theta is equal to tan inverse y y x. Then we can always get, from this it will give from this w z, we have got five plus i psi, that will give me log of x plus i y minus a plus log of x plus i y plus a minus log of x plus i y. If I say this then my psi will be it is basically I look at the argument in each of this case, that will may give me tan inverse y by a x minus a plus tan inverse y by x plus a. Then plus tan inverse y by x, if this as the flow, and if we I just take these two terms. I will get tan inverse y by x minus a, plus y by x plus a divided by 1 minus y by x minus a into y by x plus a plus tan inverse y by x, which again can be rewritten as, tan inverse 2 y x by x square minus a square minus y square plus tan inverse y by x.

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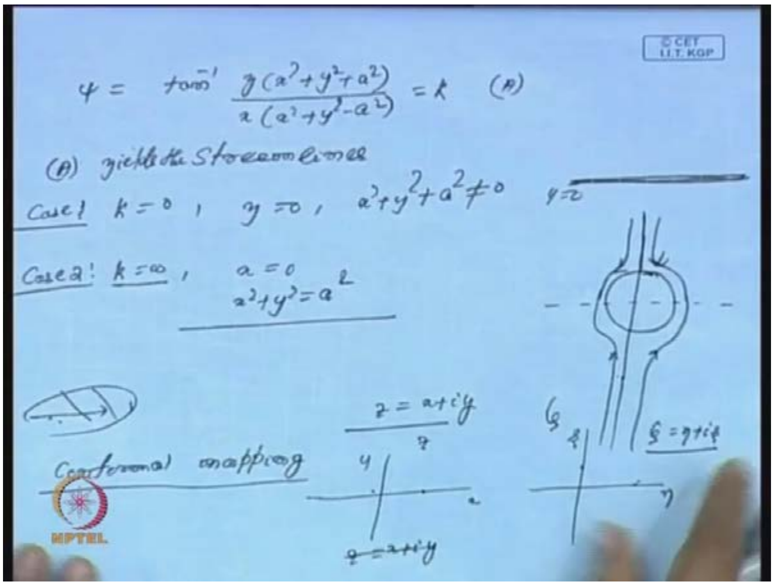

And again if I add the last two terms, last two terms, gives me tan inverse y times x square plus y square plus a square divided by x x into x square plus y square minus a square and this is my sin. So, if the, I say this is the constant, if I put this as a constant k, then that will give me the stream lines. A will give me, y is stream lines where there is values of k, now I will say take a typical, case suppose I say k is 1 that is I will say k is equal to 0. If k is equal to 0 then my y can be 0 or y has to be 0, because x square plus y square plus a square cannot be 0. So, line y is equal to 0, this line y is equal to 0 can after this stream line here

Now, I will go to the second case, it can behave like this stream line, they will not be any flow, which flow cross this line. Now another 2 this is interesting case 2, in this case if I just say k is equal to infinity. There are several cases which can we work out but if I say k is equal to infinity. If I say k is equal to infinity, I have x can be 0, that will also give me a stream line and also x square plus y square equal to a square. I have to also will give x square. So, if I look at the, that means, in this case, my, this line x is equal to 0, this line this will be a stream line. Again I have a x square plus y square is equal to, if I say this is my line which is along the x axis.

So, if I say all these are stream line that means there is a fluid which will be flowing which fluid is flowing, from this side. Then this will flow in this way, and then what will happen this is a cylinder, a fluid flow can flow in this direction. Again there is a no flow along this line, so along this axis there is no flow. And there is a cylinder, the flow can be in this direction also because in additional fluid also that can flow, because this is symmetric. The fluid will flow in this direction it can also flow in this direction.

So, we have seen this is like a flow past cylinder whereas, the flow direction is very different. So, this is what, we get and here, we have obtained these result by just considering two sources, which are located of unitary unit strength, which are located at difference at a distance of a 2 a. And there is another source which is located at rather another source, which is a sink, which is located at origin. So, through this, we can get flow pattern like this.

Now, with this I will this for example, now we will think of another thing, that, this for examples now I will go to but will happen, if I will give a brief introduction to conformal mapping. In case of conformal mapping, what we are doing because we have now worked out several examples on simple flows and also, me of the complex flows. This can be expressed in terms of very simple flows simple complex velocity potential. Now what happen there are I just give a introduction to conformal mapping today, here I considered two planes actually. I have 2 planes, that is I have a z plane, and I will think of another plane which is my zeta plane, both are 2 complex planes. Often, we find that problems, are if I just say that a problem is going to be very difficult in the z plane.

So, what I will do, I will convert it to a zeta plane, the same problem from z plane I will go to zeta plane work out the problem. In the zeta plane, and then again, if z is equal to x plus i y or i put it here z is equal to x plus i y, this is the z plane and zeta plane here a zeta is equal to eta plus i zeta. So, this is another plane, so in this plane this is eta, this is eta or this one is x, this you call it as y. So, in this conformal mapping what we do, we always transfer map from that means, if we can find a function in the zeta plane. If every point here I can find a corresponding hill point here.

And in the both the cases like, we will see later that how from a single simple geometry from here. How, we can find out, how we can find out another point in this plane, and for each point that is a one to one correspondence from one point here to another point here. Then always we can transfer map from the z plane to the zeta plane so that is what we will do here. And this is only possible in fact by using the conformal mapping, several complex problems, particularly when we look at the problems of flow. In a, allow a hydrofoil or an aerofoil or even if like, we are looking at if I think of a stream flow, in a z axis which or we can always related it to a parabolic flow across a paraboloid.

We can always related, it to a flows just, along a stream line along a uniform stream by suitable choosing. Again I will say that suppose I have an ellipse, and I have a circle in the z plane, again I relate with them ellipse in the zeta plane. Similarly, as I have mentioned that aerofoil's cambered airfoils, symmetric aerofoil's can be by using suitable conformal mapping, suitable transformation. We can always relate from z plane to zeta plane and like the geometry like cambered airfoil, the geometry is very complex in nature. But, such a geometry we can always bring it back by considering a suitable transformation. In fact it is the joukowski transformation, which will transform a circle to a cambered aerofoil, a hydrofoil for a symmetric hydrofoil or or an aerofoil.

These aerofoil's, we can work out to then suppose you want to calculate the drag. And drag force, or the lift force an aerofoil through using this uniform flow or there is a when a aircraft is moving particularly look at the lift and drag forces by using this concept of conformal mapping. Although, the geometric is very complex, we can easily find out the flow characteristics particularly, the forces moments that, is acting on aerofoil. In a air field particularly when the flow, is assume that the flow, is two dimensional in nature A similar concept is used for hydrofoil in fact in many situations hydrofoil aerofoil concepts are over road, because there are certain analogy between the two.

And in our next class, we will come to conformal mapping. And there we will talk in detail about how from one plane, we can relate to another plane. And simplify the problems, may be the problem is very complex in the z plane. We can go to zeta plane, and solve a simpler problem by using a suitable transformation. And then we can come back to z plane, by considering the corresponding inversion and then we can simplify whole problem. So, the concepts of choosing a suitable mapping or suitable transformation helps in a reducing the concepts of many problems. And which will in the next couple of classes, we will concentrate on this conformal mapping, and its application in hydrodynamics or aerodynamics problems.

Thank you all, we will stop here.