Marine Hydrodynamics

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Lecture - 18
Motion of a Cylinder

Welcome to this series of lectures and marine hydrodynamics. Today is the 18 th lecture in the series and will talk today about motion of cylinders, in a fluid. Particularly when the fluid is infinity extended. So, in this the cylinder will emphasize, as the cylinder as a, is a circular cylinder of radius and flowed is in infinity extended. So, two aspect of the (( )) when the cylinder is fixed, the fluid is moving flowed is moving. The second part is suppose the cylinder is moving where as the flowed at rest of a infinity.

So, these two cases will consider then will consider the other it is a kind of repetition of what I done in come flow fast a cylinder, and also income flow fast cylinder in the presents of circulation. So, this will be a very good idea about how the flow, there is a flow it flowing, and there is an object and object is moving what happens and flowed is moving what happens? So, that will gives as a clear idea where this a kind of part of flow pattern is come only observe in (( )) varies, problems of marine hydrodynamics because a it is a kind of a give as inside to the flow characteristic.

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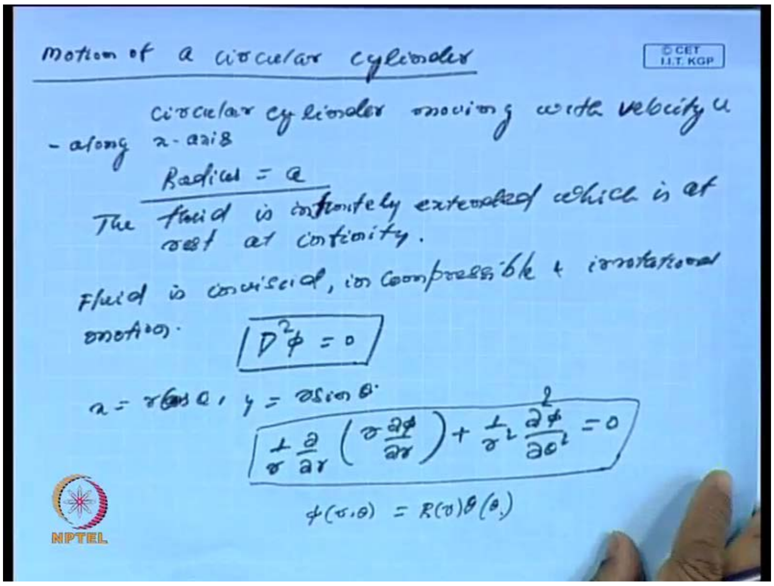

So, first will start the motion of a cylinder in a fluid, in a series motion of a circular cylinder, this approach take very general approach. So, here what you want to studies suppose I have a circular cylinder radius and consider a circular cylinder, its moving with velocity a with velocity u that is along x access and radius cylinder is a and fluid is infinity extended. The fluid each an infinity extended is infinity extended and which is red, which is a rest at infinity. So, since I have a cylinder inside the fluid what will happen to me. So, I am considering the fluid is in (( )) and compressible fluid is (( )) compressible and the rotational motion, a motion is a rotational.

Then we know that we have del square pi 0, this can be assumed into a way I stop approach in a standard organ all way del square pi 0 in the fluid is 0. In the cylindrical polar coordinate in 2 dimensional like x is equal to r cos theta and y is equal to r sine theta. What will happen to my last last equation? It is 1 by r del by del r this is r del pi by del r plus 1 by r square del square pi by del theta square 0. Once this is 0 what will happen. So, as a result what I will do because I want solve this equation. If I have to solve this equation, my simple approach is suppose I take pi r theta is R into r r is a function is a r keep it theta of theta.

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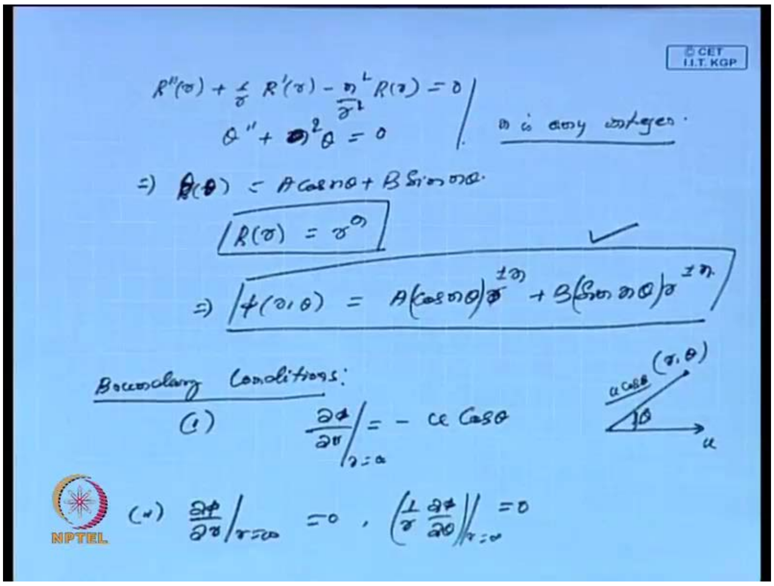

If I do this, then I can easily get it. What I will get? I will get easily that R 12 r into plus 1 by r. r per naught minus n square by r square r per r is 0 and theta 12 pi plus n square theta is equal to 0. So, hear put I have done this option individually equation by substation there can be n is an integer. So, that gives me my R of r is A cos n theta plus B sine n theta, sorry theta of theta is theta of theta is A cos n theta plus this equation solution of this. If look at the solution this that give me, this will be is easily see that R of r will be is a polar type of equation. So, will get it R of r r per n and consider this a certified. So, n is a integer if n is not equal to 0 then it can be positive negative.

So, if can get pi of r theta is a this perm A cos n theta plus r to the per plus minus n plus B sine n theta r to the per plus minus n. So, this side put it. So, this is let we are an orbit try constants. So, various wall of n I can chose any n and. So, according depending n I get a one of solution, but this is the most general solution. In the caution coordinate in to dimensional. Now, what are the bound? Suppose, I say that I have a cylinder which is moving with the speed u in the extraction, then what will happen to my boundary condition?

If I have a cylinder, the cylinder is moving speed is extraction, then I will have del pi by del r at r equal to A minus u cos theta, as that is this is any point on the surface of the cylinder r theta. Then theta is angle, this is the u component, then del pi by del r will give r equal to a minus u cos theta and that is… So, share it will have the speed will be u cos theta at this point a, this point and then… So, sine is I am talking of a fluid is which is as rest at infinity.

The velocity component at del pi by del r this is 1. So, again this is the, because cylinder is moving with u in the organ detraction that gives this is when I go to the second conduction. That means, fluid is at rest at infinity and gives d el pi del r at r equal to infinity is 0 and which also we can put interims of 1 by r del pi by del theta because the two components of velocity at r equal to infinity is 0. So, I have to solve this a, this velocity for a pi subject would this two condition to get the corresponding the set potential and once we get velocity potential. Then I will able to find at what is w because I can find out what is w, then can find what is compress potential. So, now, prime and B.

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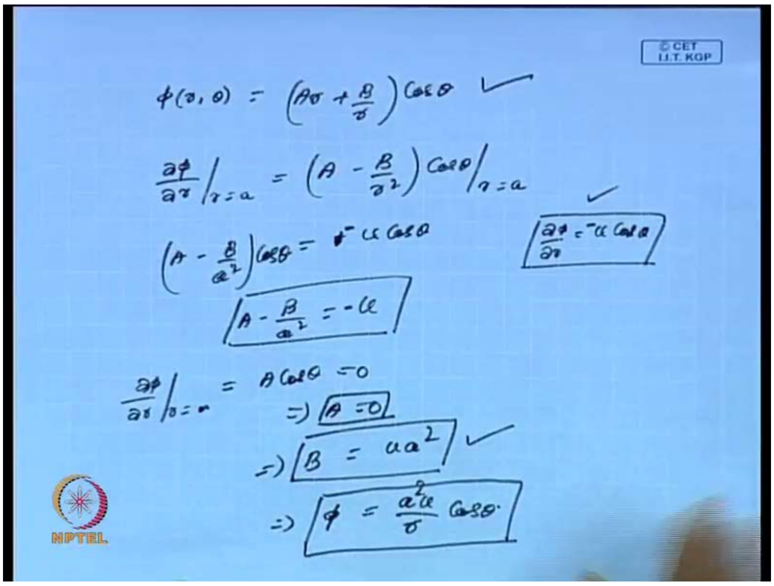

So, nature of the solution of the boundary solution, I can always find chooses by pi r theta because I have given it general solution pi r theta if I chooses A r plus B by r, cos theta this can be one of the solution out of the many solution. First when n is equal to I have taken the past part gin one that as the second part of solution I have taken n is minus one and one of the, because out of reshow that depending on n h. It will be advise approvers that why I choosing this from of the solution because then what will happen to del pi by del r I have to up riser to boundary condition del pi by del rat r equal to A will be A minus B by r square into cos theta at is r equal to A.

And its self give me minus u cos theta and what is the website gives me, that means, A minus B by a square into cos theta. If this is u cos theta that gives me a minus b by a square is equal to u sorry that is minus A minus B square by r del pi by del r a minus will I have, just a minute. So, this u cos theta purposely sorry because I have taken del pi by del r del pi by del r del pi by del r I clarify that is u cos theta what minus this is it will be clear. So, then what will happen?

Then the second condition gives me that is first condition gives B by A square is equal to u that is approach all of the condition and the other condition I got I have del pi by del r at r equal to infinity is equal to A cos theta del pi by del r A cos theta. That is 0, which infuse a 0 and if a is 0 then that gives me. So, this is acutely minus B cos theta this is minus this is fine. So, this minus my A is 0. So, which implies my B is equal to u a square and this is a point is clear. So, then what will happen my pi will be a square u by r. Because this is pi pi theta, so my pi will be from this 2 will pi equal to A square because A is 0, so u a square by r into cos theta.

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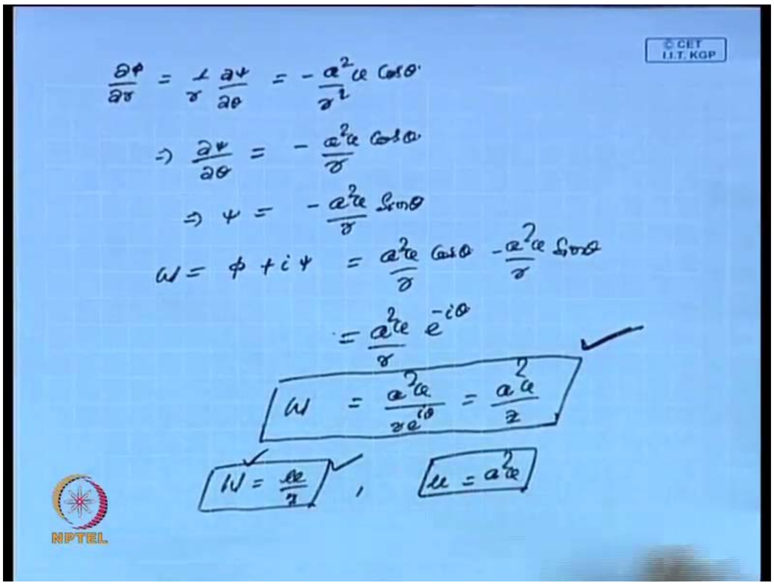

This is the velocity positional once a pi obtain then what will be my side? Then what will be my del pi by del r equal to del pi del r is nothing but 1 by r del psi del theta and that itself gives you minus a square by r square u cos theta. That gives me del psi by del theta minus a square u by r cos theta, which gives psi equal to minus a square u by r sine theta 1 sine, this then W will be this is the complex potential pi plus I psi and that gives me because pi is a square u by r cos theta minus a square u by r sine theta, which nothing but a square u by r e to the minus i theta, which I can call it gas a square u r e to the per i theta a square u by z. So, this is by W, so the complex potential in have uniform clue.

Here what I consider my clue it infinity is 0 is no clue at infinity the fluid is not clued at rest infinity and the boundary the cylinder moving speed u in the extraction. In that case the complex velocity potential W becomes this. Now, if we look at this, we have we know that in case of the dab let W is equal to mu by z W is equal to mu by z. This is the complex potential for dab let. So, the flop when there is a, so in this case what is happening in the case of cylinder, which is moving at a speed u if mu becomes a square u then the dab let is reducing to this flow pattern.

So, that is very come on thing be to dab let and a cylinder motion of a cylinder area fluid, which is at rest infinity. So, this is the most general case or the complex potential now with this let me analysis what is what is happing to the energy associated with the fluid.

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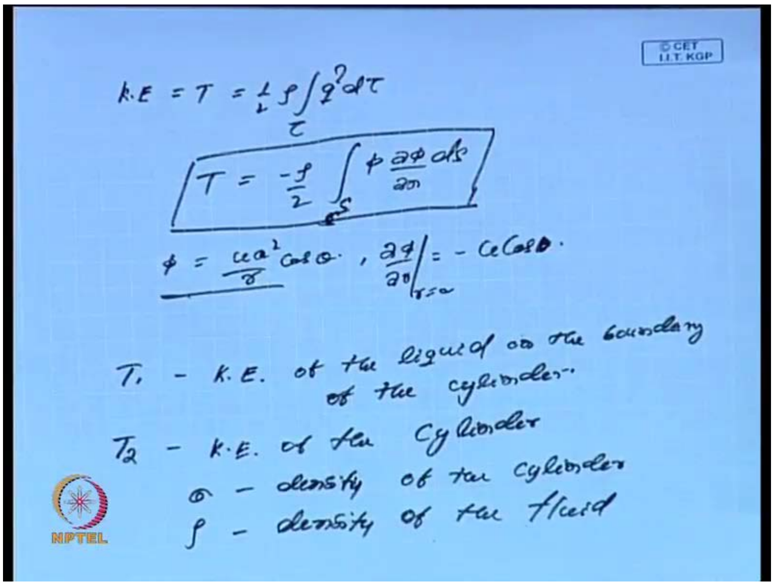

So, basically if look at the kinetic energy, because this will gives interesting result the kinetic energy is called it T. That will be 1 by 2 row integral for this, I have done it earlier, what just reputed q square d 2 and this can be written as minus row by 2. I have done this in one of the earlier lecture is unseeing the theorem this can be written pi del pi by del n d s this is the kinetic energy in a particular in confer reign in the problem to will surface area is s now we know that pi is equal to in aver case. In case of cylinder pi is equal to u a square by r cos theta. So, we have and also we know what is del pi by del r del pi by del r at r is equal to a is minus u cos theta then what will happen if I assume this is all deflation of t kinetic energy and in our case pi if pi is equal to this then we know that del pi by del r. R is equal to a is this then what will happen spouse assume that let T 1 we the kinetic energy of the liquid on boundary of the cylinder.

Let T 2 be the because cylinder booking is the kinetic energy of the cylinder because the cylinder is moving, this is the kinetic energy of the cylinder and T 1 is the kinetic energy of the liquid in the boundary of the cylinder if I say sigma is the density of the cylinder and I say row is the density of the fluid is the density of the fluid.

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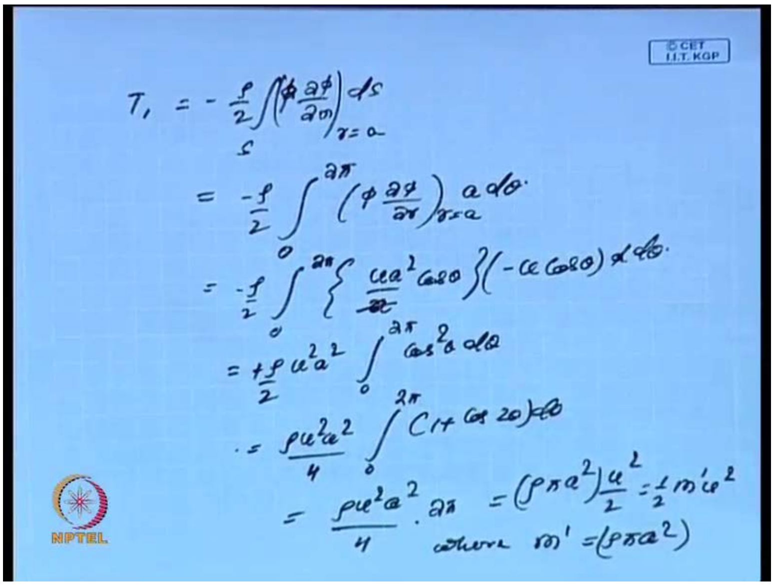

Then what will happen to T 1 then my T 1 would be minus row by 2 T the lower is pi del pi by del n d s and since this is the on the boundary of the cylinder. So, boundary of the cylinder what will this calculated it as r is equal to a. So, this I can write it minus row by 2 take all 0 2. 2 pi because I have a cylinder of radius a and r is equal to s will be. So, basically this will give me pi del pi by del r, r is equal to a into a d theta because on the boundary of the cylinder I am calculating. On this will give minus row by 2 0 to 2 pi this is u a square by r r is a. So, this is cos theta 2 minus u cos theta a d theta if I do this calculation then its gives me the simple, I will get it. If is simplify this I can easily get it minus row by 2 other minus plus row by 2 this is a is there and other is a has canceled that will give me u square a square u square a square and then 0 to 2 pi. I have cos square theta d theta that will give me.

Row a square a square by four if a put it 0 to 2 pi 1 plus cos 2 theta d theta then I will have row u square a square by 4 into 2 pi. That will give me row pi a square into u square by 2 and I can call it its half m twin u square. (( )) my put where m twin equal to row pi a square. What is this? If I look at row pi a square this is nothing but I by two dimensional cylinder.

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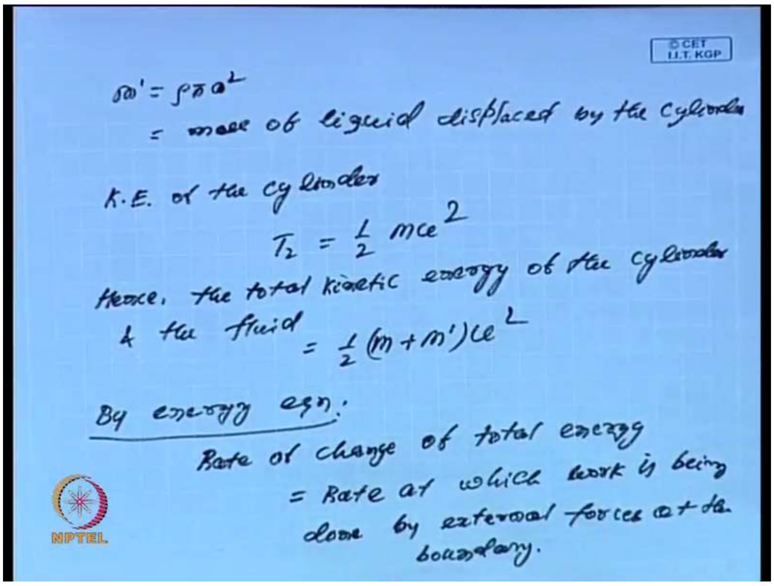

So, this m prime is row pi a square is nothing but the mass of the liquid displaced by the cylinder I can call this. So, this is by T pi T 1 now if look at the kinetic energy of the of the cylinder we are the cylinder is moving with speed u and they call image mass of the cylinder and it will give me T 2 half m u square because speed of a cylinder and Image mass of the cylinder then. So, hence the total kinetic energy of the cylinder on the fluid and the liquid fluid to a half m plus m twin into u square. This because the total kinetic energy of cylinder of fluid now by energy equation by equation of energy the rate of change of total energy in gives me rate of change of energy. In were to, but change of total energy is same as the rate which work done, rate at which work is being done by external forces external forces at the boundary at the boundary.

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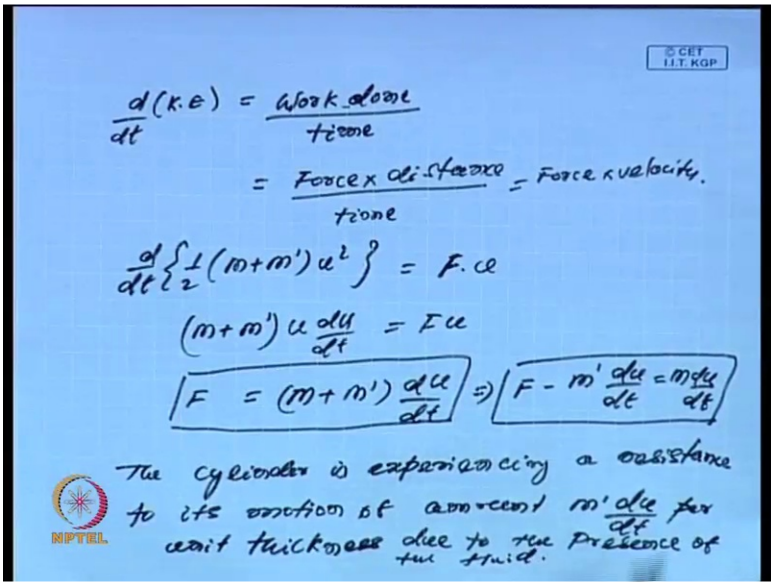

Then if I apply this to the equation this equation I will apply to this cylinder, then I will have d by d t of the kinetic energy is equal to basically work done for time for unit time and this is work for unit time and work done is nothing but force into distance what is wary standard deflation by time 1 is can I can write it has force in to velocity and if that is the case what is d by d t (( )) total kinetic energy is I have d by d t of half half of m plus m twin into u square and that is force is (( )) the force as f total external force f and then u is the velocity. So, that gives me a simplify this is me m plus m twine into d u by d t.

This is equal to f into u and that gives me my f as m plus m prime into d u by d t, so which I can also write f minus m prime d u by d t equal to m d u by d t. So, one of this from can write, so what is happening here. So, f minus m prime d u by d t m d u by d t m d u by d t m d u by d t is kinetic energy the total energy that is acting on the force that is acting on the for unit time this is force acetic on a cylinder and this is what happening heir in the total external force minus this m prime is mass displace because.

This is the associated with the displace mass. So, what it says of the cylinder is experiencing experiencing a resistance to its motions and what is the amount of the resistance. The amount m prime d u by d t per unit thickens, for unit thickens due to the presence of the fluid. So, this is the resistance it is experiencing due to the presents of the fluid and in this case. So, in the process and this is what this let me say…

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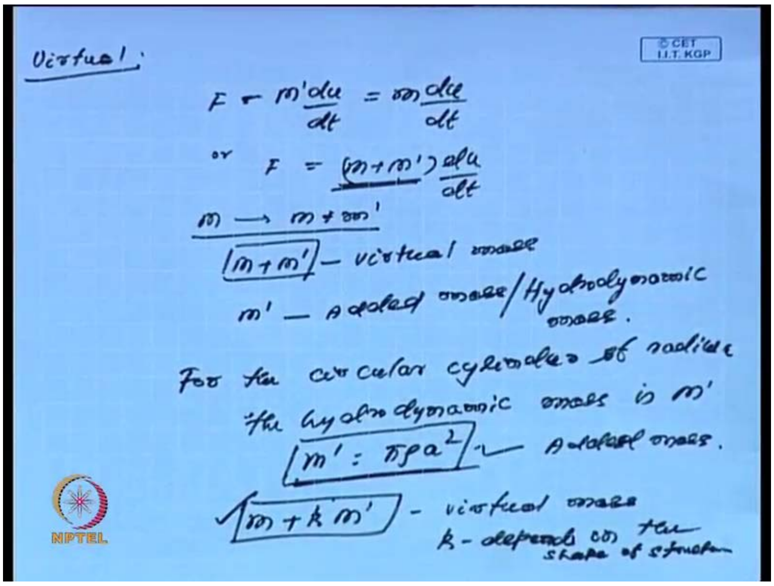

So, in the process the mass of the or define another term what is virtual mass. In this contest so the equation I will say F is equal to F minus m prime d u by d t equal to m d u by d t I will jest write it a. So, what is the happening the r I call it F equal to m plus m prime.

D a by d t this is, that means, the mass m of the cylinder m is mass of the cylinder mass of the cylinder and that is in case to m plus m prime when the fluid when the cylinder inside the fluid and this mass and prime is nothing but the mass of the equal is plus and this m plus m prime m prime this m plus m prime together this is call the virtual mass. Because mass of the cylinder is this, but one the produce the cylinder is inside the provide the mass is increasing which is moving the provide. So, m plus m prime is virtual mass and this is the often by increasing the mass of the cylinder by the added mass and the added mass that is the m prime and this m prime I call the this is the added mass.

Or we some call is the hydrodynamic mass. So, in the case of the circular cylinder of radios say when this is. So, for the circular cylinder of radios the added mass the hydrodynamic mass is m prime and m prime is equal to pi row a square this is. So, that is clear that is total is call the virtual mass m plus m prime this is call the virtual mass and the m prime is call the added mass. So, in fact this is a very important the contest of problems of a marine hydrodynamics.

Because all most all marine hydrodynamics which one body is moving in a fluid I body is some must in a fluid and force is the acting in the it there are register mute the fluid it and in the process particularly for all moving body it is very important because fluid is well the body will move. Amount of fluid in all ways associates it will that is the amount fluid that is the associates the basically the amount of fluid in that is a displace by the body and then this amount of fluid always this is provide this resistance and then this plays significant role.

Control in the all most calculates in of force of force calculation on a body and also this is the resistance calculation this is always taken into account and this added mass calculation the hydrodynamics mass the added mass is very significant control infect the this added mass. The virtual mass for all most all merino structure the virtual mass this is the this form m plus K into a m prime this is the general nature of the virtual mass, in all most all hydrodynamics fluid in, and this m prime is the added mass where K is a constantan which depends on the nature arch depends on this structure the shape of the structure the shape of the structure and this is does not well.

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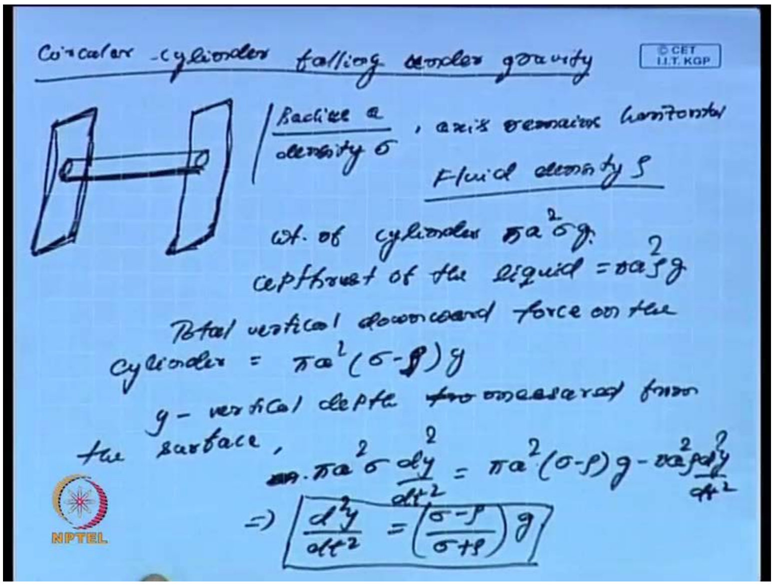

Some of the virtual mass now we this understanding I will just calculate what happen in a cylinder is falling under gravity, because when it is falling under gravity. And me just to say falling under the gravity, from this a general from the equation we can easily find out suppose they will cylinder of radius a, sorry. Suppose, I have a cylinder a cylinder radius a, circular cylinder of radius and density sigma about the cylinder is radius a. This is sigma and is falling which access is a horizontal which is axis of the cylinder axis remains horizontal. It is in fluid of density row fluid is density row.

And we consider and unit length of the cylinder which is a limited to smooth verticals basically these of the two walls. So, this cylinder is whole in and then what is the weight of the cylinder the weight of the cylinder. It will be pi a square sigma into g then what is the fluid of up thrust of up thrust of the liquid equal pi a square 2 g. So, if this is the up thrust and this is the weight of the cylinder because this is coming it is always trying to bring the cylinder downward high of the up thrust is to build.

It this is top to then what is the about downward of force total vertical downward force vertical downward force on the cylinder, it will be pi a square to sigma minus b g because sigma minus row into g if I consider y is the vertical displacement vertical depth all sometimes it is called vertical depth is up from the surface, I then is insurant of time then I will have to equation wholes of pi a square, sigma this is the because sigma is the depth consider cylinder y a square sigma into d y by d t this square y by d t square easy equal to pi a square this is the down not for pi a square sigma minus row into g minus pi. This is just by the horizontally a square row a square y by d t square this is coming exactly from the recently is the equation well the body is moving.

Because a fluid, so this will be the equation just we have derived the and this will give us if simplify this the y by d t square to give us sigma minus row a sigma plus row into g. This is the equation of the cylinder when the cylinder is calling and the direction of the gravity. Here we have seen that in a fluid particularly and we have seen this equation that the cylinder depends on this, if y is large enough sorry the other.

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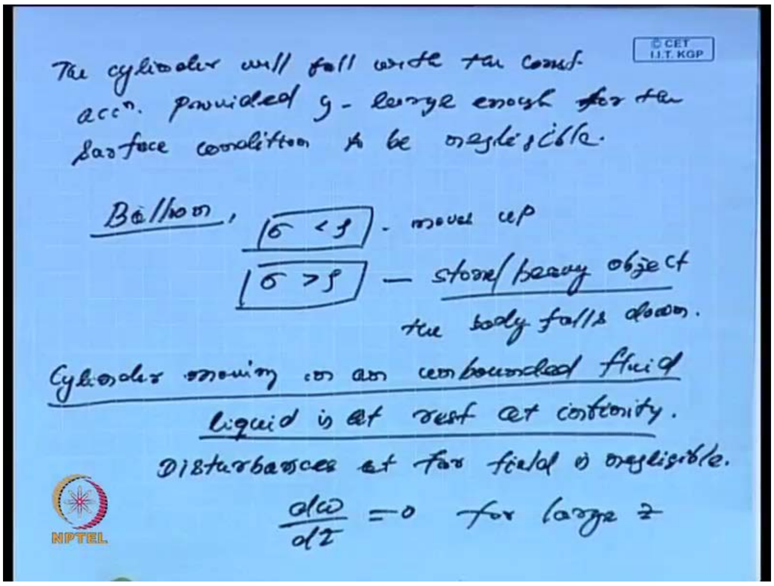

What will happen the, so what will happen if the cylinder will always will fall if the constraint with this constraint as lesson provided by g large provided by g large enough what the surface condition to way to be negligible. So, in that case because; that means, we have know condition of this cylinder because way is solar (( )) the surface on this always can be neglected the condition that is (( )) and what happens of the cylinder.

Can be easily neglected if we neglected surface condition then the should be the equation this square y by d t square is sigma minus row minus is by plus row into g. Now, what will happen in the case balloon, in case of balloon which is moving in the upper the direction and that is the sigma is always that then the row and that is why if sigma is less then row then this square by this there is up thrust and will the process the cylinder moves in the upper direction, and other hand in case in case of any other object sigma where the sigma is greater than row then this will move this will be move up.

On the other hand in case of the stone or heavy object the body falls down falls down even if we have a fluid now this concept what we have learn that when we have a cylinder body is moving in the fluid. So, we have always and double is experiences external force and always experiences a registrants and again we have seen that the cylinder fall and the direction of gravity always there is a up thrust.

Always in process this up thrust then it will be move in the upper direction because gravity is trying to pull into downer direction, but the up thrust will be it is always acting in the upper direction. So, in the process it is will be move on up in the case of balloon and other hand will case of a heavy object it always move the in the downer direction now I just give a general condition or a cylinder moving on a in an bounded fluid. If a cylinder each moving in an unbounded fluid liquid and suppose I seen the liquid is a rest at infinity at infinity.

So, if you will I say the liquid is a rest at infinity I can always say the velocity motion of the cylinder a disturbances on the motion of the cylinder in liquid is a rest at infinity that is store disturbances far field at for full is negligible for field in said always mean a of will abject is for field negligible. So, in that case d w by d z will be 0 at far field if d w by d z is 0 or large at is the z for large z where cylinder is that is field it is… because of this.

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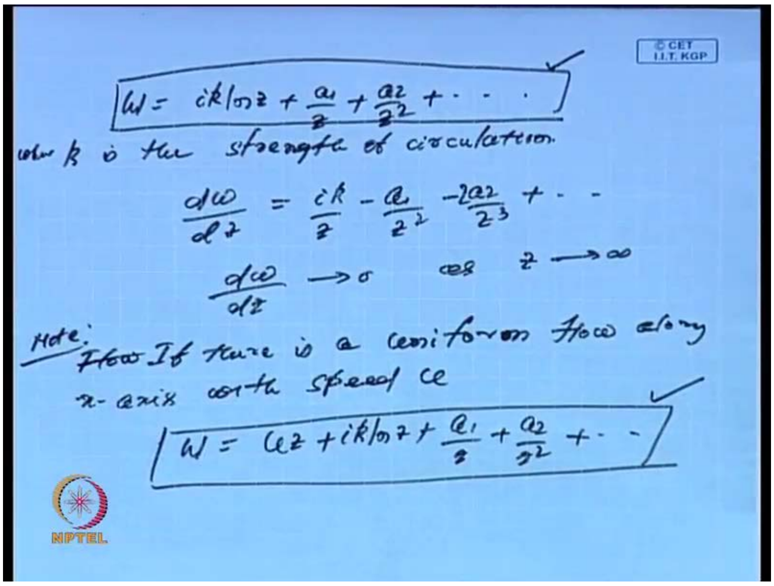

What we can, the most general former of the, in that case, what will happen I can if a write in w easy equal to i k large z plus a 1 by z plus a 2 by z square plus this. If I just check it is the k is the strength of circulation there is no source and sink there is the in case of this will be gives me that will be give me d w by d z I k by z minus a 1 by z square minus a 2 by z cube 2 by 2 z cube that is what and it can be see the motion if clue velocity is 0 as d w by d z into 0 as z infinity.

So, in because of this fall the most general from and of the of a cylinder of begin in the on bounded fluid it which is rest at infinity. So, I always can say if suppose in that is in face the flow is uniform suppose I say there is uniform flow when the cylinder is moving there is a if there is uniform flow this is another case not if there is uniform flow along x axis with speed u then the general motion can be w can be u z plus i k not z plus a 1 by z plus a 2 by z square plus so on.

When there is no uniform flow, these case will in the motion when there is no flow at infinity a flow it is a trust that infinity and other hand there is uniform flow along the x axis which force of the speed u along the x axis this will the most general of the from most the complex potential.

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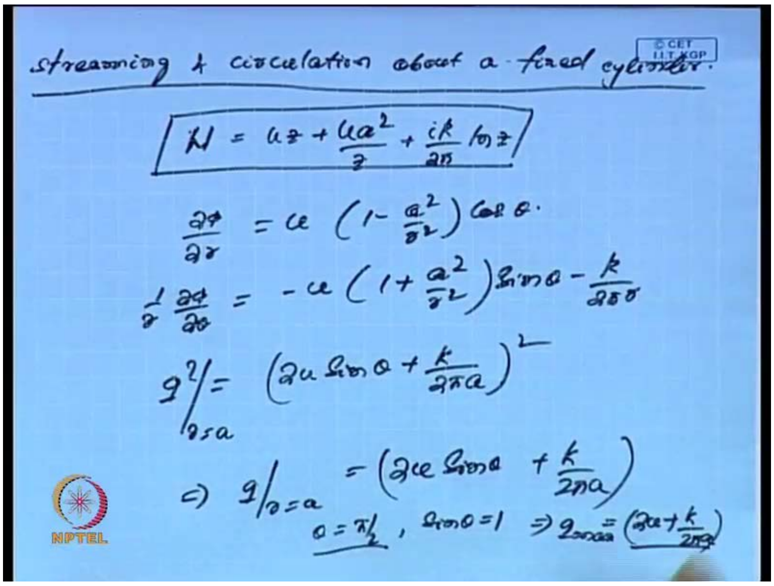

Now I will just give a detail streams about if have. So, in from this I can easily get streaming and circulation about to fixed cylinder. When I have I already know that flow for rest is cylinder when there is uniform flow which motion the will speed each is u or infinity in the horizontal direction then may w easy u z plus u a square by z and if I have introduce a circulation of strength case then may about self other term divided by i k by 2 pi knot z. So, this will be the complex potential for the uniform flow a stream in motion as well as circulation about of a cylinder and from here you can easily see that if may you can easily find out.

2 pi by 2 r in this case each u into 1 minus a square by r square cos theta on may 1 by r 2 pi by 2 theta each minus u into 1 plus a square by r square into sine theta minus k y 2 by r and in that. So, we can always get q square is equal to 2 u sine theta. So, other I will say

that q square r easy equal a I will get it 2 u sin theta plus k by 2 pi a k by 2 pi a square and hence.

What will happen which in place q on the bonded of cylinder will be 2 u sine theta plus k by 2 pi a. So, far theta is equal to, if I get a theta is equal to pi by 2, then may sine theta will be 1, which will be in place may q all is get 2 u plus k by 2 pi a. This only give me the maximum speed on this of surface of the cylinder and that will absorb only theta is equal to pi by 2 in the presence of circulation.

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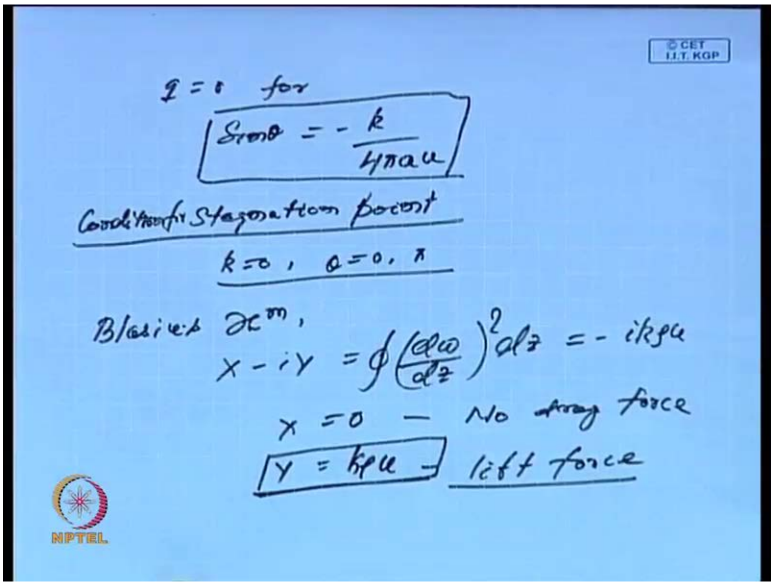

On the other hand what will happen by on the other hand we will see q easy equal to 0 for sine theta easy equal to minus k by four pi a u on this is give is the condition for in fact, is 0. So, this gives the condition first stagnation this is condition first stagnation point. So, what will see that then k easy equal to 0 if case there is no circulation; that means, theta easy equal to 0 or pi and in that case we have a stagnation point. Earlier what we are seen in case of here only from flow past is a cylinder again if will go by apply Blasius theorem because we have w. If we apply the Blasius theorem will can get x minus i y integral.

D w by d z at that the cylinder and if can easily see that this will be minus i k row u and that will give us x easy equal to 0 and y is k row u. So, what it is say that the in the presents of the circulation there is no drag for this each in place no drag force what there is a resistance and this resistance is the k row u. That is the other that is the lift force on this becomes the this is k row u. So, the cylinder (( )) it force of magnitude of k row u where are there is no drag force that is x proven by cylinder in the presents of circulation and here. So, this concept is very good to analyses the lift force; however, as.

We have already talked to that will not talk. So, not talk any detail about drag force because it is as a have explain it which is clear because for a in which at flow it will be difficult to drag force by using the because the drag force is will not be there that is the upload in is sent and that already explain in the well I will be talking about the well talk to about the paradox. On the other hand this will be give a almost have relation about the lift force that is acting on the cylinder and in fact this concept on the well used when we have talk to very well about the a lift calculation in case of a aerofoil. This is all I want to tell about the motion of a circular cylinder today.

Thank you very much.