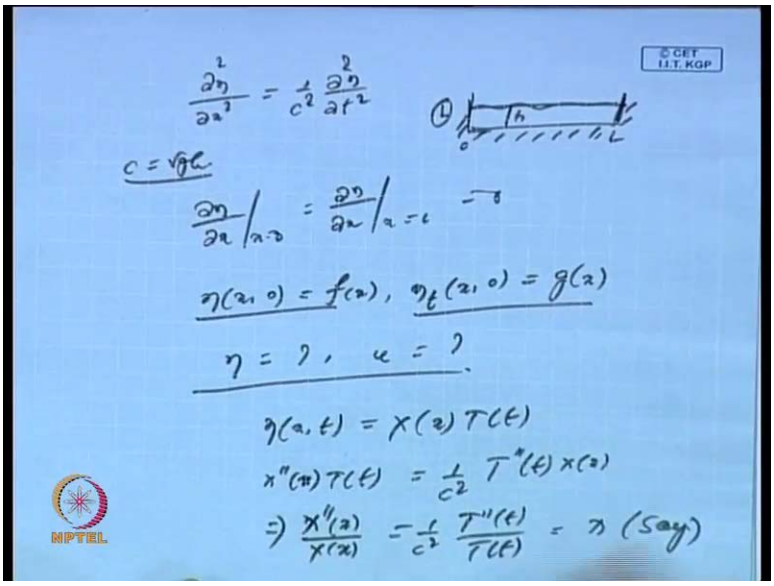
Marine Hydrodynamics
Prof. Trilochan Sahoo
Department of Ocean Engineering and Naval Architecture
Indian Institute of Technology Kharagpur

Lecture - 26
Linearised Long Wave Equation (Contd.)

Welcome you, to the this lecture on marine hydrodynamics. In the last class, we have talked about the basic equations of associated with linearised long wave, and here we have two equations; one is the equation of continuity, and other is the equation of motion. We have discussed this in case of a, both in case of one dimensional, one dimension, and in case of two dimension, and we have also looked in to the case of valuable width and breadth. So today let us workout one example to understand the solution procedure associated along with wave equations.

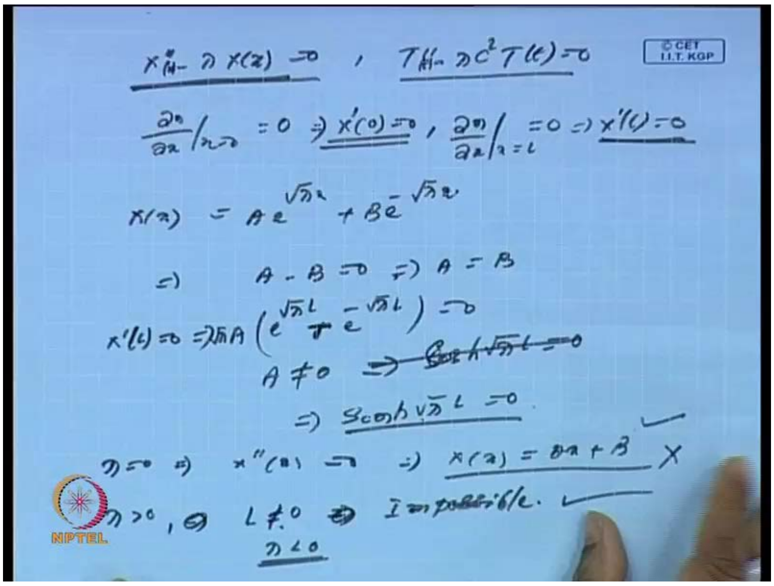
(Refer Slide Time: 01:03)



Suppose I consider the... Let us me let me say I consider the one dimensional hour, and I say suppose I have a channel of length, rather I will say, and I have length. This is the depth from the main order depth, this is h, and then the length is l, this is 0, this is l; this is the total length l. See if have a channel of length l, and the depth of the channel is h, then here we have c is equal to root g h, and for that since I have the two walls, and the two walls what will happen? I will have del eta by del x, because the velocity will be 0 along this wall, so that will give me del eta by del x at x is equal to 0 to del eta by del x at x is equal to l this is 0. I again take two initial conditions, suppose I have been given that suppose eta x 0 the displacement is f of x at t is equal to 0, and eta t x 0 is equal to g of x. Suppose these are the initial data, at time t is equal to 0.

So, in that case what will happen, how I will get, and we are looking into what will happen to eta, and then what will happen to u. So because here once we know eta you can know u or vice versa. So let us concentrate on, know what exactly eta is. So to know eta, so first will do, we all know that you can always write eta, because this conditions are homogeneous condition, this is a homogeneous equation, with the true homogeneous conditions boundary conditions, so you can always apply the method of expression as very good. So I can write it x t as eta x x into t t, and then this will be, if I substitute for eta this one, then I will get x double dash x double dash x into t t is equal to 1 by c square t double dash t into x x which gives me x double dash x by x of x is equal to 1 by c square t double dash t by t, and because this is function of x, this is function t, so I can call this as constant. Let me say lambda is a constant, so I am constanting a lambda, and in thus.

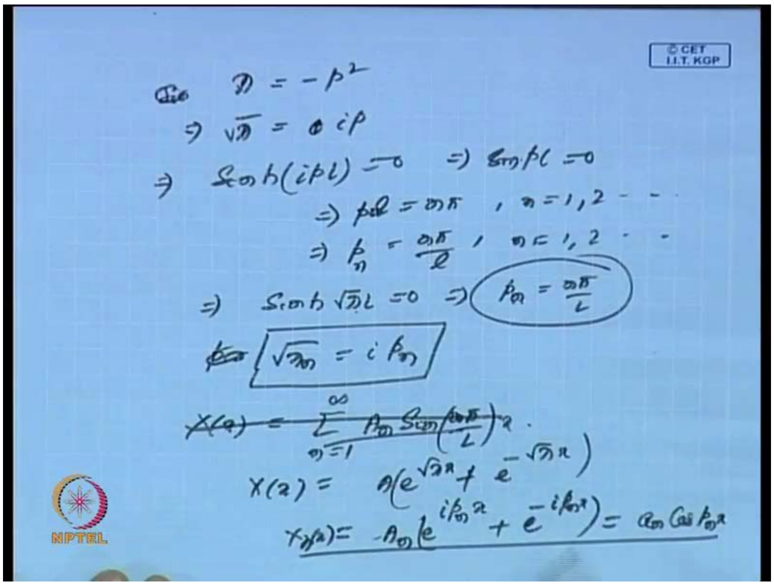
(Refer Slide Time: 04:15)



So because if, once we say this is lambda, so we will get two sets of equations from this, that will give us; x double dash minus lambda x x is 0, and the other equation is t double dash minus lambda c square t t is 0. Of course, this is a function of t, this is also function of x. So then, if I look at the solution of this I have three faces here, and in case of. So what will happen, we also have been giving, we have been giving del eta by del x and the x is equal to 0, and that gives us 0, which implies x x x 0 is 0 and again del eta by del x at x is equal to l is 0, and that gives x dash 0 is 0, and this also gives me x dash l equal to 0, so these are the two. So in x I have this equation I have to solve, subject to these two conditions. So this is a (( )) type of boundary problem becomes in a x be as two n(()) condition, two point boundary condition problem, because we have two points, boundary conditions have prescribed, and this is a two dimensional, second order differential equation. And then, so now what will happen to x, so x x will sat this way. My solution of x will be a 2 to the power root lambda x plus b e to power of minus root lambda x. So if I substitute for, one of these boundary condition, del eta by del x at x is equal to 0 is 0, so that gives me a minus b is equal to 0, because of this condition, and again it implies a is equal to b. Other that hand if I put x dash l is 0, so that gives me, extra cell means, this will be e to the power root lambda l, then a is b plus e to the minus root lambda l into a equal to 0.

And since, if a is not 0, implies cos hyperbolic root lambda l is 0. This is minus b is 0, so this will extra cell, or there is a minus sign, so it is minus, because this is extra cell is 0. So I will rather say x dash l is 0 implies, this is equal to root lambda is 0, so a is not equal to 0, so will have, which implies sin hyperbolic l is 0. Now question comes, if lambda is 0, then is non 0, so if lambda is 0, then I have implies. So hyperbolic lambda l is 0, so implies sorry I say lambda is 0, then I do not have any solution, implies x double dash x is 0, and that gives x x is a x plus b, and again that will not be any help to me, because I am looking for a solution of in x and t, it should behave like a wave, so it will not helpful, this solution is ruled out, this cannot be a solution, because I am looking forward type solution. On the other hand if I look, lambda is greater than 0. If lambda is greater than 0, that means sin hyperbolic root lambda l is 0. And so if lambda is greater than 0, that means l has to be 0, l is non 0, lambda is greater than 0, l is non 0, implies so which implies, this term cannot be 0; impossible. This is impossible solution, because I have sin hyperbolic root lambda is 0, and l is not equal to 0, lambda is greater than 0, which is impossible, so this also another case. So the only choice remains, that lambda has to be less than 0.

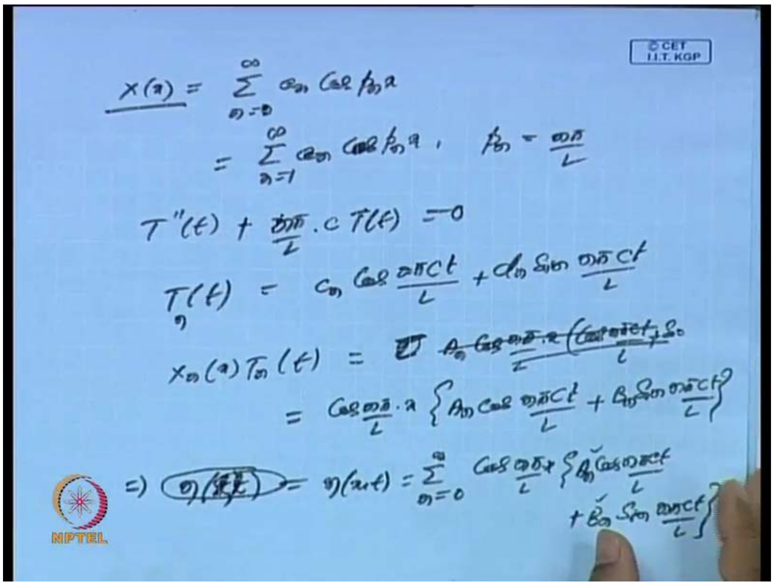
(Refer Slide Time: 10:03)



If lambda is less than 0 that means. If lambda is less than 0 then I will have sin, let me say lambda equal to minus p, and if lambda is minus p, so then I can always say, rather I say lambda is minus p square, which implies root lambda is equal to minus, or that I will say i p, and then that will give me sin hyperbolic root lambda l, root lambda is i p l 0 which implies sin p l is 0, and if sin p l is 0 which implies p l is equal to n pi implies p is equal to n pi by l, thus implies. So that means, my l is equal to 1 2, this is n is equal to 1 2, when I call this as p n. So what happens I got. So sin hyperbolic root lambda l is equal to 0 implies p n is equal to n pi by l, and where as that gives me. And again we have p n is. We have seen that root lambda, and then in this case I have, because I have for each p n each p n I will get lambda, so I call this as corresponding as lambda n, so root lambda n is i p n. I can call this, this is. That means for each lambda n I can get, and this call, so in the process what I will get.

I will get my x x as sigma n is equal to one to infinity A n sin hyperbolic sin; that is n pi by l in to x, this is by x part. And then this becomes a i n volume problem, and this is call the x x. So I have infinitely many such i n values, and these i n value is l root lambda at the i n value p n s are the i n values, and corresponding i n functions are the sin p n l, sin n pi by l into x, these are the corresponding i n values. So if these are the i n values, now sorry x x is not this, because I have started with lambda, these are main lambda n, but, by x x is equal to a e to the power a is b, so it root lambda x plus into the minus root lambda x, on that will be. Again rather I will say A n into the power root lambda, root lambda is i p n, this is i p n x plus this becomes a x and x and into the minus i p n x which I can call it as some A n cos p n x. And so in the process I will have, this is one of Eigen functions, so this is wrong, this is one if the Eigen functions.

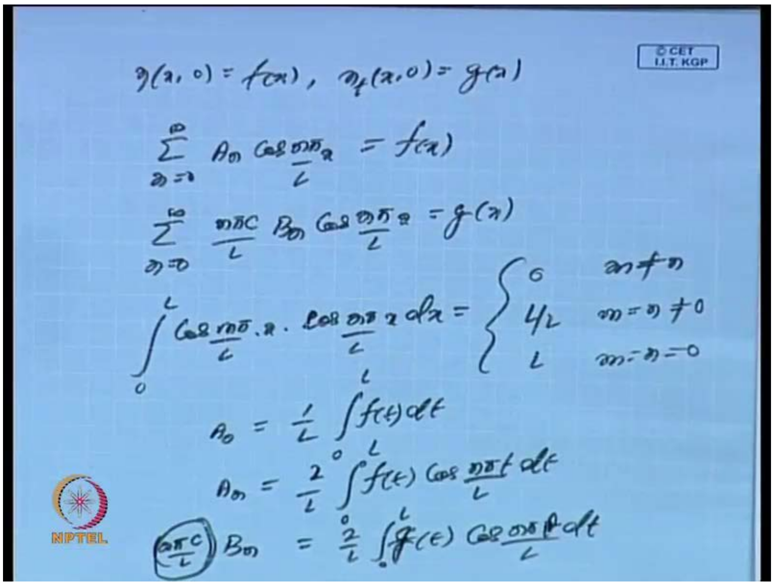
(Refer Slide Time: 14:34)



And in the process we have the super position of all the Eigen functions, will give me the full solution, so that will x x is sigma n is equal to one to infinity A n cos p n x, and p n is n pi by l, so that implies sigma n equal to one to infinity A n cos. In fact I can also take a 0, because that will give me, n is equal to 0 I can take, because that will give me a constant also, when n is equal to 0 then p to A n cos p n x or p n is n pi by l, and what will happen to the t t, we have t double dash t minus lambda, lambda is A n minus p n square, lambda is minus p n square, and that will give me plus lambda c square t minus lambda plus lambda is n pi by p n n pi by l into c into t t is equal to 0 and that gives me that t t are the solution is, I can also call it cos n pi by n pi c t by l plus d n sin n pi c t by l.

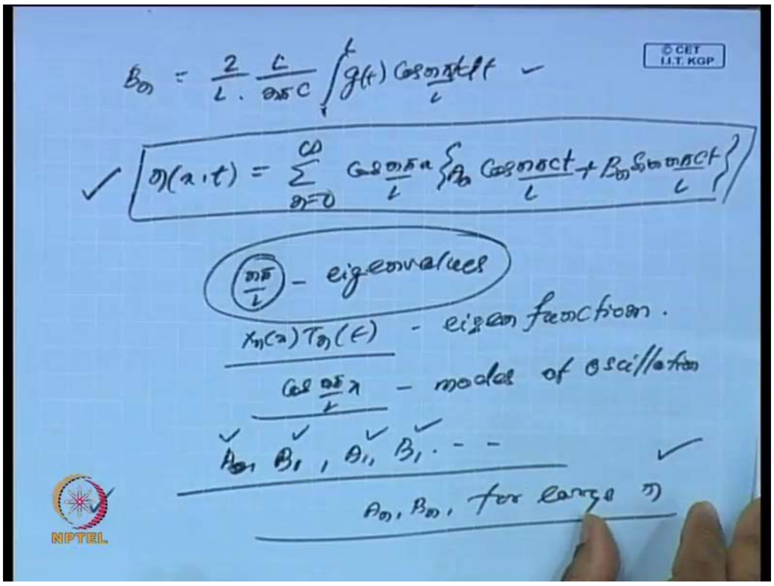
So then this I can call it s t n, so if I combine this with x x then what will happen, because this if I look at this, multiply look at the individual x n d n x n x n to t n t, also will be solution that will give me sigma, sorry that will give me my x n x is A n into c n so I can call it sum b n, or call it A n cos n pi by l into x into cos n pi by n pi c t by l plus sorry s is, let us put it this way. This is I can always call this cos n pi by l into x into I can call this as A n cos n pi c t by l plus b n sin n pi c t by l. This is my x n x so which implies my eta. So I have infinitely many such n, so eta is x t my eta x t that will give me sorry sigma n is equal to 0 to infinity cos n pi by l into x. I can call it A n n pi c t by l plus b n sin n pi c t by l, so this is my eta x t. Now I have two constants, two sets of unknowns A n s and b n s, and to obtain this two sets of unknown, I have again two more condition, because I have only utilize the condition at eta is equal to, that at the two boundary conditions.

(Refer Slide Time: 18:18)



Now I have two initial condition; the two initial condition gives me; that means my eta x 0 I have, eta x 0 is equal to f of x and eta t x 0 is equal to g of x. Then to utilize this, so if I substitute for this in eta, then what it will give me sigma n is equal to 0 to infinity A n, because it will give me cos n pi by l into x is equal to f of x, and if I look at this one that will give me sigma n is equal to 0 to infinity n pi c by l into b n cos n pi by l into x is equal to g of x. So this gives me the two sets of. This will give me the two sets of equations for A n and B n, and if I substitute, because I will utilizing these two results 0 to l this is one integral as a; that is of the arithmetic criteria m pi by l into x into cos n pi l into x d x and that is 0; form is not equal to n, this is a l by two, if I say m is equal to n is not equal to 0 and this m if m is equal to n equal to 0. So if I apply this condition, then what will happen, then utilize this and this then that will give me my A 0, A 0 will give me 1 by l m n is 0 so 1 by l 0 to l f t to t and if just multiply by now it. If I using this again, what will happen to my A n, this will be 1 by, 2 by l 0 to l this will be f t cos n pi by l into t d t, and in a similar manner if I utilize this one, this is like the (()) coefficient, this will give me 2 by n 0 l this is n pi c n pi c by l into b this gives me f t g t g t into cos n pi x by l d t and this is called d t.

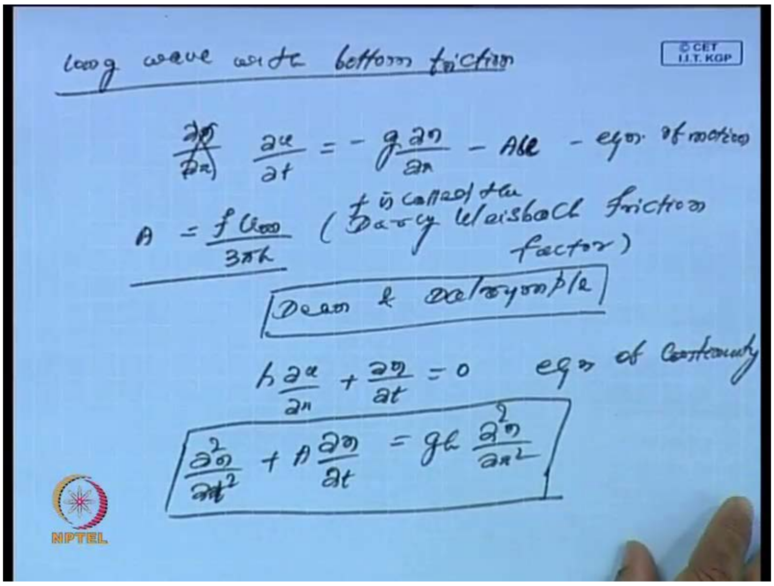
(Refer Slide Time: 21:28)



So if I have this factor this m pi c by l I can always blame into the other side, and that will give me by B n is 2 by l into l by n pi c into 0 to l g t cos n pi by l into t d t. So these are the A n and B n, and once I have this n means. So I will get as I said the main solution is; sigma eta x t sigma n is equal to 0 to infinity cos n pi by l into x; that is an cos n pi by n pi c t by l plus b n sin n pi c t by l so this is my full solution, where all the a ends and ends are known in terms of f t and g t and this is what represents, but, this each of them, or each n I get them, out I call then as Eigen functions, this n pi by l this is call a Eigen values, and then the individual functions x n x t n t there call the Eigen functions. And sometimes we call them ends, but what will happens, this cos n pi by l into x, these are called the modes of oscillation, this is the mode of oscillation.

So this is what, and then, and this Eigen values, in fact any computation we will do, as you seen in case of a series ,we have seen earlier in case of a wave oscillation in a tank, we have seen that, how the modes of oscillation, plays significant role, and we have seen that the primary modes of the initial few modes of the primary secondary modes, so that fast few modes in (( )) significant role in a similar manner, we can easily see that only, it can easily prove the, soon that these terms will contribute significantly. And again we know that this past few modes, particularly the A 0 A 0 B 0 A 1 B 1 in the series, fast modes of oscillations are the, this constants, initial constants will play a significant role, compare to the unknowns, the constants A n B n for a large n, in fact we can see that, I will say rather the series is, of course the series is converse this point and in that case A n B n they will tend to 0 for large values of n. In fact that is what also will find out, from the theory of analysis that the A n B n will tend to 0, as intends to infinity, that is sometimes known as the remain (( )) in theory of analysis. So this is what I want to say, this is one of the simplest way, to find the solution of a wave professional problem, in the conditions are very simple, provided f x and g x unknown, we can always find that what exactly form of eta. And once we know eta, then easily we can obtain what is u, because eta is a related to with u, by the equation of, long wave equation of continuity, or the long equation of motion. So this is one of the ebriel, one of the best example, to illustrate the solution procedure.

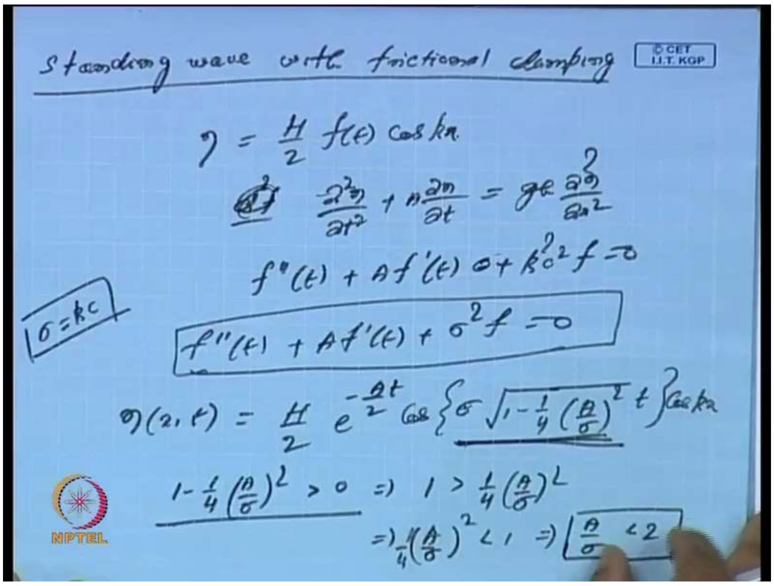
(Refer Slide Time: 25:44)



To this now let us go to talk about long wave with bottom friction. So in case of loge wave with bottom friction what will talk about. here rather in this case, the equation of continuity will change del eta by del x, because when there is bottom del e by dell t sorry is equal to minus g del eta be del x, this use to be the in case of a the equation of motion for the l loge wave, but, if there is bottom friction, then there is another term which is added to this is a u ,and that what is that a u term where u is the velocity, and a is a constant that constant is again f of u m by three pi h, and this is called a small number Wisthan unity, and this is called Darcy Weisbach friction. Take some factor so the details are, this one kind effort to the book of dean and dalrymple, her detailed reference, what we have mechanize for scientist and by dean and dalrymple, which I have already prescribed.

So this is a constant, by Eoily Pennan, so we have this becomes the equation of motion, and inside this constant a is small quantity, it will less than unity, but, that will contribute, because of the friction factor .Then again we have continuity equation, we have the continuity equation; that is k del u by del x plus del eta by del t 0, this is our equation of continuity, in case of long wave. If I combine this two equation or if I eliminate eta, that will give me del square eta by del x square plus a del eta by sorry del eta by del t square was a del eta by del t, is equal to g h del square by del x square, this becomes my equation of motion, in case of when there is about on friction, and where is given by this number, and depends on the u m becomes the maximum speed, and f is a f is factor which depends on the bottom friction particularly, f is called a f is called the Darcy Waisbach of friction factor. So this becomes the equation of motion, for the long wave, and if I say a is equal to 0 this is my long wave equation, in case of one dimensional.

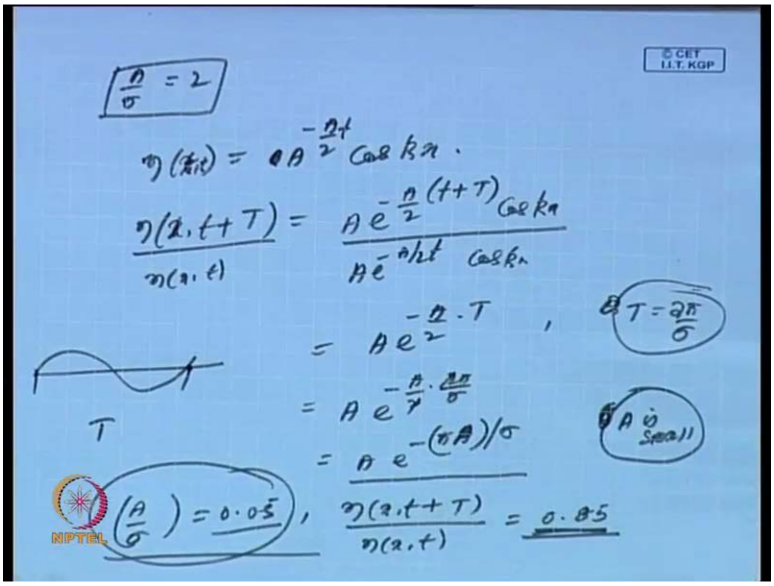
(Refer Slide Time: 29:23)



Now, look at analyze this, to analyze this problem, let us consider two separate cases. Suppose I am looking at a standing wave, with friction damping, then let me say that suppose eta is equal to h by 2 into f of t into cos k x, then what will happen, if I substitute for this in the equation of motion with bottom friction, then that I will get. I will get d square f by d t square, because my original equation is, my original equation is del square eta by del t square plus a del eta by del t is equal to g h into del square eta by del x square. If I substitute for eta is this, then I will get f double dash t plus a f dash t, and that is g h is nothing but, the g h is c square, and this will be del square eta by del x that will k square and this will give you plus k square c square into f, and the this nothing, but, c square k square is nothing but, omega square or sigma square. So for that will give it f double dash t plus a f dash t plus sigma square plus c is equal to sigma square f is 0. And if I look at the solution of this equation, that solution will give me eta x t, because this is my f t rather, the f t and eta if I look at it, I will easily say this h by 2 and then into a minus a by 2 t into cos of sigma 1 minus 1 by 4 a by sigma square into t into cos k x, if has this will be the solution.

Basically you can solve this, as a algebraic. Look at the characteristic equation, and the root of the equation give us this, and there then we can easily say that this is the volume of the solution. And here we have taken my sigma is k c plus c is sigma is by k, and that itself so sigma is basically frequency of the wave. So now what will we do, what will happen to u. So if this is my eta x t, then. Again now let us let my analyze this term, if analyze this term. Suppose when this term is cos sigma into 1 minus, suppose what will happen, if sigma, if this term is greater than 0; that means 1 minus 1 by 4 a by sigma square, if this is greater than 0, then only I have this form of solution, otherwise this will be hyperbolic term, and it gives me hyperbolic term, then this will not represent, it will not satisfy the web type solution, and then my purpose will not be solved. So when this will dominate if it is, this has to be greater than 0. If this has to be greater than 0; that means 1 is greater than 1 by 4 a by sigma square, which implies a by sigma square 1 by 4 less than one, which implies I should have a by sigma, so will less than 2, so this is one of the criteria that I should have a basic mode, it should be less than 2.

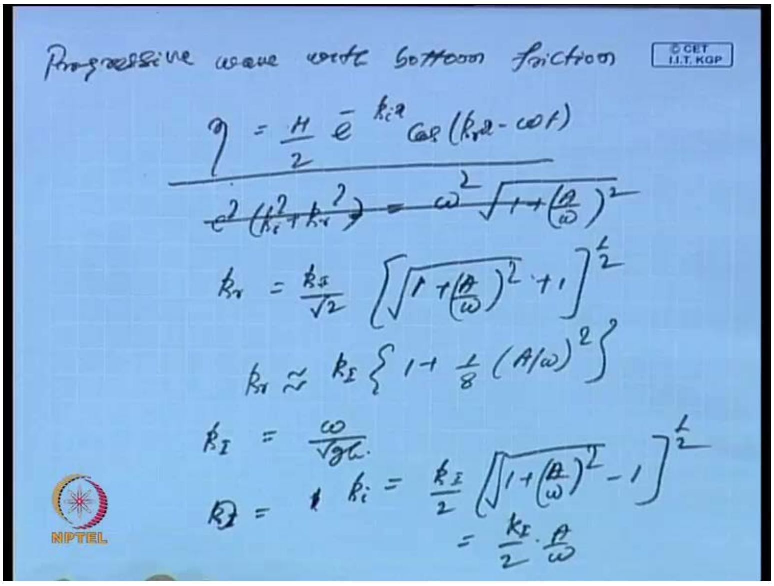
(Refer Slide Time: 34:51)



And if sigma r is 0; that means if I say, that means a by sigma, if it is a by sigma is 2, then in that case the term will not contribute in that case, I will have eta t is a to the power minus a by 2 into t into cos k x, this will my eta x t. And in that case, now what will happen if I say eta t x t plus t, where capital t is the period then this will give me a e to the power minus a by 2, t will be replace by plus t into cos k x divided, if I divided by eta x t, this also I will divided eta x t, and that gives me a e to the power minus a by 2 into t, and a is a by 2 ,t and t is nothing but, when t is 2 pi by sigma, this is the period, so then it will give me a e to the minus a by 2 into t is 2 pi by sigma, on that gives me, so that will give me a e to the power of minus pi pi a by sigma, and that is what, so I will get, so this is the. Now it can be easily seen that, if a by sigma is 0.05, then I can see that eta x t plus t by eta x t that will give me 0.85.

so what it shows that over one period of, over one period eta that, means when this was my elevation, and after one period t is the period of this, then when it reaches here, the elevation only reduce by 15 percent, that is what is so that means if and when, I have taken a by c sigma is just point 0 point 0 5; that means the sigma is how small is how, whatever small it is it is very small quantity for this small values. Again under this condition that, this a by 2, this is what I am getting; that means when; that means sigma is a small, a is a small quantity, even if a is small that contribute significantly to reduction of the amplitude of the wave, when there is bottom friction, because over one period that is reducing by 15 percent, it is what it is in fact it is, because of this reason in many situations, particularly during storm adjustment, when wave propagate to the land area, large amount of, we always suggested to leave part of open spaces, because the open space will reduce the energy, because of friction and that energy dissipation. Energy dissipation will be very fast, when there is bottom friction, so because of that it is always suggested, to leave lot of open space, along the coast line, so that it will reduce rate of energy, and due to by dissipation.

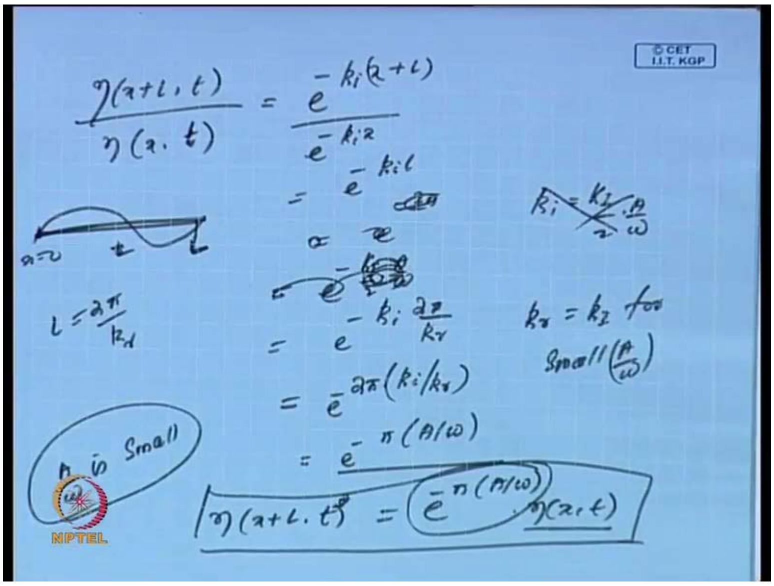
(Refer Slide Time: 39:25)



Now I will will look at the corresponding case, in case of a progress wave. This is the case if a standing wave, how the bottom friction takes place in case of progressive wave, with bottom friction. In this case, I will look at a solution of this form ,because from the previous experience I always can say, that let us me look at a solution of the form, h by 2 into minus k i x into cos k r x minus omega t. And if I substitute this form of solution in the long wave equation, with the bottom friction, then one only come across. One can easily see that, this k i k r satisfy the relation c square into k i square plus k r square is equal to omega square 1 plus A by omega square. So since which we can always write it as. So this will give us two equations, so here what will sorry if this is the form of the solution, then if I substitute, because here I have to know, what is my k i, what is my k r let me just I come to this.

I have to know what is my k i, and what is my k r, and then if I substitute for this in the original equation, what it will give me, that it will give me my k r is, in terms of k i, is k i by root 2 into 1 plus a by omega square root of plus one to the power half. And again if you simplify this, then you will get k i because a by omega as seem it is small, so you will get it one k i into 1 plus 1 by 8 1 by 8 into a by omega square, this is what one will get, and this is only, this result is valid only when a by omega is a small, and here again what we have writing here k i also will be omega by root g h. In addition we can say that my k i will be. So k r is in terms of k i, so again further if I look at this, substitute for this k i in terms of. Then another relation I have k i, small k i is k i is omega by g h. On the other hand I have a small I, so my small k i will be k i by 2, k i is omega by root g h, this is k i by 2 into 1 plus a by omega square root of minus 1 to the power of half, and this is nothing but, equal to k i by 2 a by omega, so this is my. So I have one is k r and k r 1 is k i, so my k r is given in terms of these, this is my k r, and I can get my k i as this where, this k i is given by this. So once I know k i and k r, so what will happen to eta x plus l by eta x minus l.

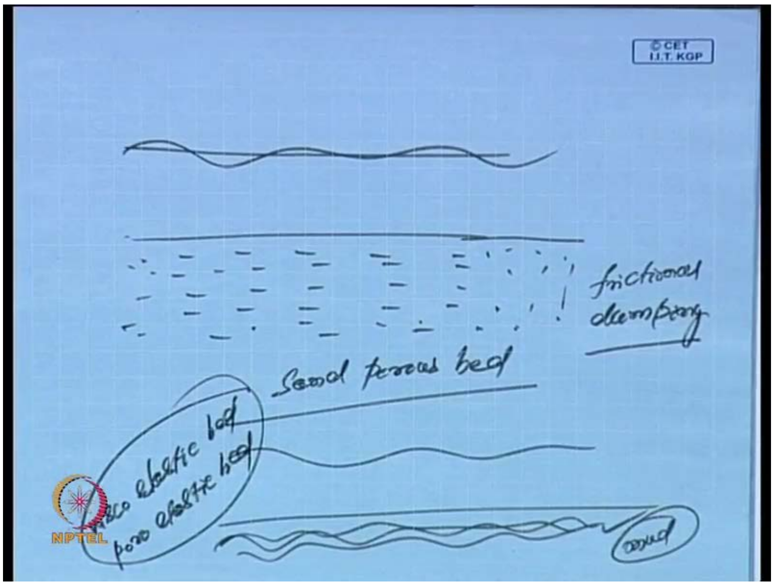
(Refer Slide Time: 44:22)



I just look at here what happen to here, over one wavelength, l is the wavelength, so then what will happen. This will give me, if I substitute for h by 2 e to the power h by 2 will get cancel, that will cos k r x, because k r x is 1 plus a omega, that will not change, because over one period, so only factor will change it the minus k i x by x plus l in to divided by e to the minus k i x. So you can term contribute to one that will not contribute, so this will give me e to the power minus k i l, and l is 2 pi by, so that again that will give me e to the power minus 2 pi my k i by k r. Say again small k i, is given by, I have already given, my k i is k capital I by 2 into a by omega, so if I substitute for, this will give me e to the minus k k i is k i by 2 into a by omega k i by k r into sorry k i l minus k l, oh sorry lambda is 2 pi by k, I am sorry so we have l is 2 pi by k, l is two pi by k and that k i call it as k r, so this will be e to the power minus k i into 2 pi by k r into k i k r is equal to k i, because k r is k i in terms of, because this term a of k r I equal to k i, for small a by omega, because there is a term. Then it will be 2 pi by k i k r, and that will give the e to the power minus two pi into k i by k r this will what.

And again k i by k r k i by k r, we have k i by k r, so it can be easily seen that, this is equal to into the minus pi into A by omega, because of the other relation k i have k r relation. Once this is there then, what will do here, because again in a similar manner we will see that by A by omega is small. This quantity will be a very small quantity. So then what will happen to eta x plus l t; that will be the e to the minus pi into A by omega into eta x t. So that means over one wavelength, again we can see, that it covers from x is equal to, if it is x is equal to 0, this is l, this is l is the, this is l is one wavelength, so when it, the wavelength, the wave passes from o, just crosses one wavelength, then immediately there is a factor. This factor will reduce the amplification of the waves. Basically it will reduce the amplitude of the wave exponentially. So however small it is A by omega, if A by omega how small it is, at this factor will be, again bring it, because again there is factor eta minus 0.5 by omega, exponentially to lift this. So the wave can be dissipate, very fast. So whether in the last example we have seen, that when there is wave ,which is propagating ,with time, because we have taken timing here, but, we have seen that eta x plus t eta t plus t by t eta t there we have seen that with the time indicates, and also we have seen in this example, that will the space also the wave nature, ditch a particularly amplitude of the wave, will decay, that means continuity of friction damping, so this is in case of a long wave.

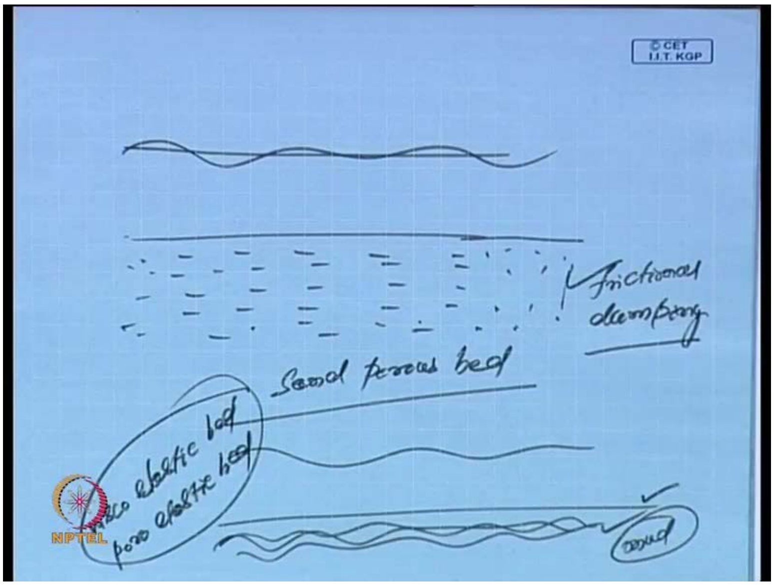
(Refer Slide Time: 50:30)



There are other ways what also, we can show, suppose I say that if there is a bottom friction instead of bottom friction, suppose I have bottom porosity. I have bottom porosity, particularly (( )), if I have bottom porosity, then because of, there will be frictional damping. If this is the wave, that is propagating, frictional damping. So in a Sandi bed also frictional damping will takes place, and then because of fine pores, that can be energy dissipation will takes place, and also will have frictional damping. Again this is case of a sand bed, sand porous bed, also you can say that same situation will happen, one of the typical example is that. Another is that, suppose we say that we have body bed, sometimes will call it visco elastic bed, or poro elastic bed, then also we see that due to the deformation, and energy dissipation will take place, deformation and dissipation energy porosity, whether is a poro elastic bed. In case also particularly sometimes in the theory of mud, we define this as a, the bottom bed, is often define as a poro elastic bed, or a visco elastic bed. So even if situation, where this situation, when we have a visco elastic or a poro elastic bed, then also wave energy dissipation take place.

This is a little complex, but, it can be easily soon, that energy dissipation takes place, on a visco elastic bed, particularly on a muddy bed or even if a sandi bed very fast, but, sand bed will be, energy dissipation will be, if it is sandi porous bed, then energy dissipation will be very fast, because it is a friction takes place, due to the porosity, in the bottom bed, and that decays. So it is, because of these reasons, in the coastal area, particularly the wetland area, or even if the beaches, often large amount of wave energy get dissipated. In fact it is one of the reason why, it is always add bite in the coastal area, not to destroy the natural beaches, because beach provides us a very wave observing, it has every wave, dissipating, it acts like a natural wave dissipating device, so it is always measures are taken, to protect the existing natural beaches, and often garment, and even if come it takes various states, to protect the beaches, not only for to retraction, or human activities, or particularly corresponds and other things are recessional passionities, but, it indirectly help us in saving our coast line, because of the large amount of sand, which as porosity, it helps in a dissipating lot of lot of energy, wave energy.

(Refer Slide Time: 50:30)



So that is why this is one of the simplest example I have shown but, this is a little complex, and I think, if time formats, I may go for this sand porous bed, but, these I will not go, perhaps this will be little more complex, and it will on the syllabus, what we are planning. So this will stop here.

Thank you very much.