Marine Hydrodynamic
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Lecture - 34
Analysis of Basic Flow Problems

Series of lectures on marine hydrodynamics, in the last class, we have talked about basic equation, basically the Navier Stoke equation of motion for incompressible and vicious fluid. And then we have talked about what happen when the Reynolds's number is very small, and then it leads to kinetic motion which is called clipping flow or stokes flow or slow motion. As a continuation of that, today we will talk about fluid which are, which are Reynold's number are small, but it is comparatively larger than that of the clipping flow motion.

Particularly, here only again concentrate on some of the analytic solution, as I have told in the last class that these analytic solutions plays a crucial or plays a significant role in analyzing generalized flow problems of real, which are of the physical importance and arises in real world problems. And these problems gives this simple flow problem gives a benchmarking to benchmark results to when we have we get approachment solution for generalized flow problems. And also it gives a bit, little bit understanding about the flow characteristics. So, today again as I have told that we will start with the some of the basic flow problems and analyze how the flow, analyze the flows associated with these problems.

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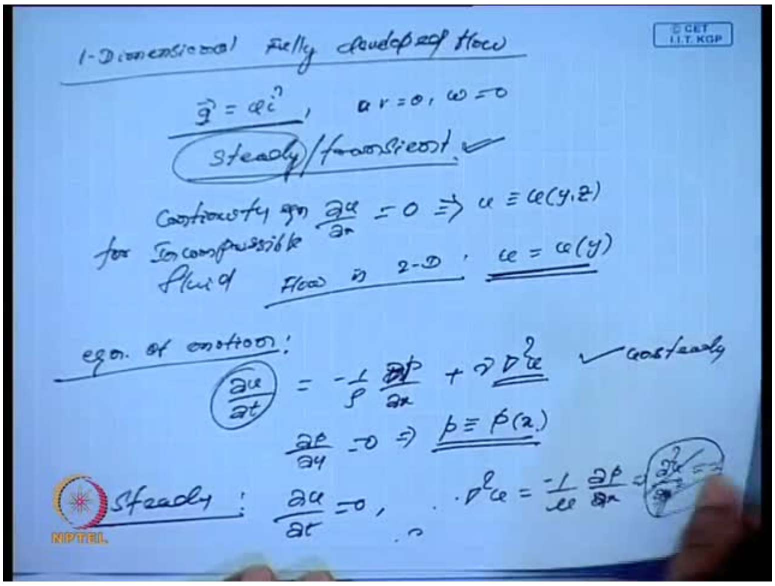

So, as simplest one we will think of that, we have, I have again talked about one is the one dimensional flow problem, fully developed flow, one dimensional fully developed flow, and what is a fully developed flow? In this case the inertia term is negligible, and one is the inertia term is negligible and the second is that the flow is unidirectional. Particularly, suppose q bar which has component of, so here we will only consider as if flow is in the x axis or it can be considered in one of the axis.

So, it is unidirectional, flow exist in one direction and the inner set terms are negligible and these are called fully developed flows. And there are other flows, some of the cases where we may again look into (( )) of the inertia flow, inertia terms may be written. So, this is, comes under this category of flow. And again when we look into this we have two class of problem one is steady, one is transient. In case of steady, it is a time independent where in case of transient flow, where the time independent flow with a inertia value being described.

On the case of a steady flow, we have no initial data because it is independent of time. So, only the wall boundary condition has to be described. On the other hand in case of transient flow we need to describe, describe the initial values, our time t is equal to 0, what happens to the flow in addition to the wall boundary conditions. So, with this let us look at the flow in case of a, when if I say that I have a fully developed flow I already have told you that q bar is equal to u, i hat where u is the x component of the velocity and here if this is, then you have a, the velocity component in the y direction is 0 and in the z direction is 0.

If this is the case then what will happen to the, for an in compressible fluid, the equation of continuity, it will gives us that del u by del x is 0, continuity equation for incompressible fluid, incompressible fluid. If this becomes del u by del x is 0 and once del u by del x is 0, which is same as telling u is a function of u y z. And if I again say that the flow is two dimensional, flow is 2 D, then what will it give me that u can be written as a function of u y, because it is independent of z. So, for a two dimensional flow, two dimensional fully developed flow, we have if the motion is a, we have for a incompressible fluid, we have u is equal to u y.

Now, what will happen to the equation of motion? If you look at the equation of motion, this was two things. Then again this is say, that this will give me inertia terms are negligible del u by del t that will give me minus 1 by rho grad p plus nu del square u and this grad p is nothing but del p by del x, because along the x axis this becomes the equation of motion. And again from y component you have a two dimensional flow from y component you have to give me del p by del y is 0 and once del p by del y is 0, that gives me p is a function of p x only.

So, here u becomes a function of y and p becomes the function of x only. Then what will happen? And this is the unsteady motion. On the other hand if the motion becomes steady, this is for unsteady motion. However, if the motion becomes steady then the equation of motion reduces to this term will be 0 that means del u by del t is 0 and in that case we have del square u is 1 by mu because it is mu by 1 by mu (( )) del p by del x. Here, we can see that u is a function of and del square u is what? That will be basically the, we are looking at the x component of the… So, del square u and here u is the function of the y only. If it is a two dimensional then that will be give me del square u by del x square or del u (( )) del square u by del y square, this becomes minus 1 by sorry I will come to this later, the next paragraph say. So, what it gives?

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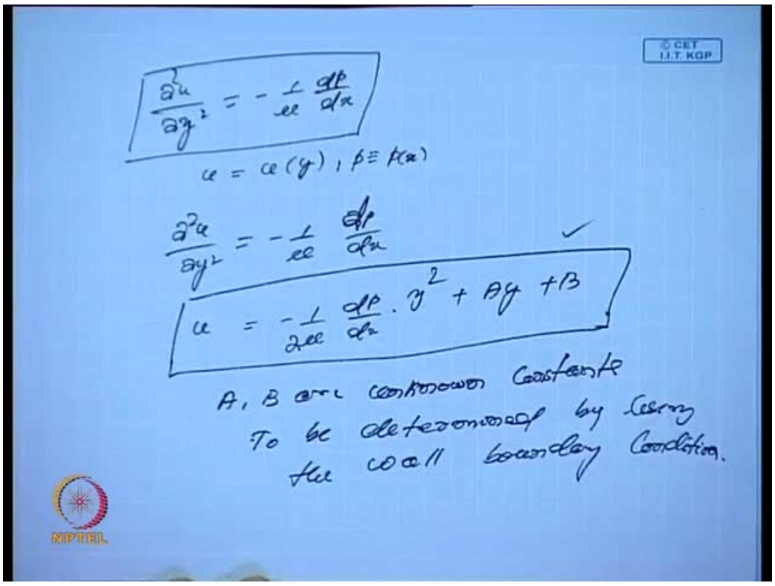

If the flow is two dimensional then it gives me del square u by del y square is equal to minus 1 by mu and d p del p by del x because p is a function of x and this becomes d p by d x and this becomes the flow. Now, this is for the two dimensional flow and here u is a function of u y and p is a function of p x. Now, if I have to solve this because this is a function of y and I assume that difference of pressure is known then I will have del square u by del y square is equal to minus 1 by mu if this is d p, d p by d x.

Since, this is a function of x, this is a function of y I can always integrate it twice to get it minus 1 by mu, it will be 2 mu d p by d x into y square plus A y plus B. So, this is (( )) one dimensional flow, this becomes the general, the flow (( )) is u along the x axis becomes of this form whereas A B are known constants which are arbitrary and to be determined, there to be determined by using the wall boundary condition, wall boundary condition. On the other hand, if I say the flow is a two dimensional, three dimensional in nature. Just let us see the general from, how it looks like then will come to specific cases.

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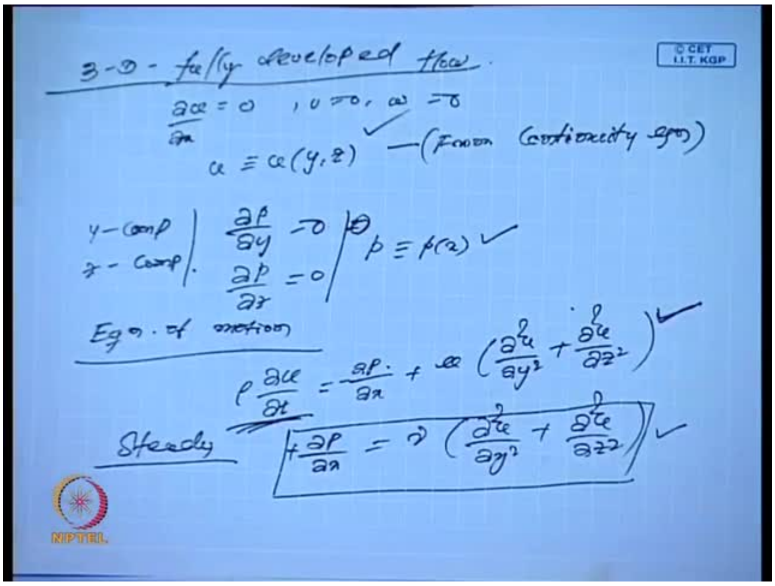

If the flow is a two dimensional in nature, three dimensional in nature for 3 D plus, the flow is 3 D then and it is again fully developed then I will have u is a, we have del u by del x is 0 means because it is 3 D flow whereas the flow direction here v is 0, w is 0. So, you have u can be a function of u is a constant and you can be dependent on y and z. This comes from the continuity equation, from continuity equation. On the other hand from the equation of motion we will have as usual we have v is equal to 0, so that will give us del p by del z del, p by del y is 0 and again we have w is equal to 0, that gives of del p by del z is 0.

In fact, in all these cases I consider either we observe the boundary condition, the pressure term or sometimes we neglect that because we are considering (( )). Basically, it is observed in a pressure term and if this is 0 that, these two things is 0 that means pressure is independent of y and z that means p can be written as a function of p x only. And in the process, what will happen to the x component of the equation of motion? This is from the y component of the equation of motion and this comes from the z component of the equation of motion and then equation of motion the x component will give up del u by del t because inertia terms are negligible, this is equal to minus rho into del u by del t is del p by del x minus del p by del x, the body (( )) is a involved with this plus into be mu into del square u by del y square plus del square u by del z square because u is a function of the new del square u that will show u is a function of y z.

So, x component does not come into picture. And then when the motion is steady then this becomes this term will be 0. So, that will give us del p by del x equal to mu by rho that will be nu del square u by del y square plus del square u by del z square. So, this becomes for steady case, this becomes the equation of motion and this (( )) for whereas, here p is a function of x and u is a function of y z. (( )) 3 D fully developed flow. So, this is the general form and here again we need to prescribe the wall boundary condition.

To solve this equation if we are solving this as a steady problem then only you need the wall boundary condition and if you want to solve this we need apart from their wall boundary condition one initial boundary, initial condition to take care of the transient part. Now, we will go to simple problems one by one. So, the simplest one in the series is the straight, parallel flow in a straight channel.

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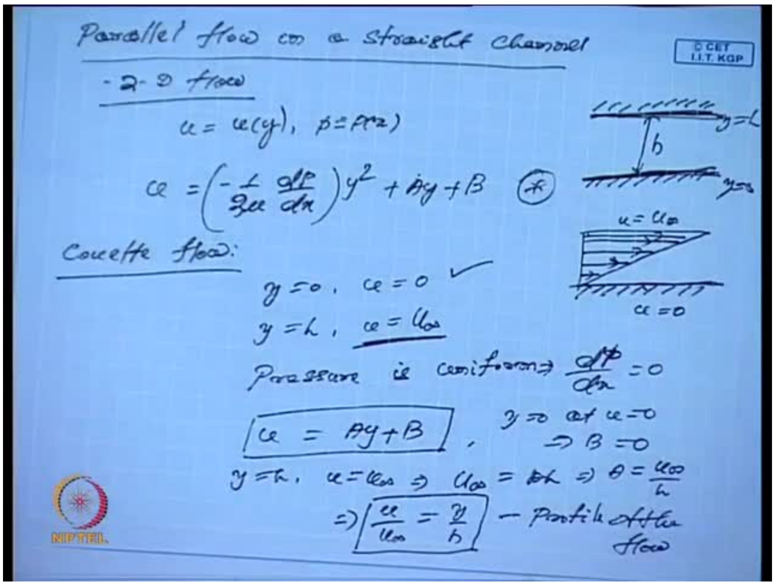

Let us consider the flow between two, two parallel plates. Consider the flow between two parallel plates, so and this distance between the two plates is, I consider this as h. Now, as I have seen, we have seen it becomes a one dimensional flow, I consider it as a two dimensional flow, 2 D flow and just we have seen that for a two dimensional flow the governing equation we have u is equal to u y, we have u is equal to u y and p is a function of p x only.

And then we have seen that the equation of motion that gives rise to u the flow (( )) is u because minus 1 by 2 mu d p by d x into y square plus A y plus B, this has another flow and here the wall boundary condition there are two channels, two plates. These are the side of the channel and then on this we have to describe the boundary condition which are once we know A and B then we can know what exactly the flow distribution. And now the simplest flow of this parallel flow is the Couette flow and in case of a Couette flow what we do that y is equal to 0, u is equal to 0, y is equal to h, u is equal to u infinite.

So, that means one of the plate y is equal to 0, one of the plate, this plate, if it is y is equal to 0 then this is called y is equal to h. This plate is fixed, kept fixed. So, no slip condition will be there. There is no, fluid will pass through this. So, horizontal direction the fluid velocity is 0, at y is equal to 0 and then y is equal to h, y is equal to h along this plate. Look at the plate boundary of this. Then in this plate, this plate is, we assume that plate is moving at a speed u infinity, u is equal to infinity, steady motion, whereas infinity is a constant uniform flow that means this plate is moving at a uniform speed at y is equal to h.

And again in this case we assume the pressure is uniform, pressure is uniform. Once, pressure is uniform then which implies d p by d x is 0. So, if I look at this original equation star that gives me my u is equal to A y plus B and this u is equal to A y plus B, if I substitute for y is equal to 0 u is equal to 0 that gives me from the equation one, so y is equal to 0 at u is equal to 0 which implies B is equal to 0 because u is 0, y is 0. So, that will give B is equal to 0.

Then the other condition y is equal to h, u is equal to infinity because already B is equal to 0 that gives me u infinity is equal to A h and which implies, if I substitute for A, A is u infinity by h and thus if I put B is equal to 0, A is u infinity by s that gives me u by u infinity is equal to y by h. In fact, this is the flow distribution and this gives the flow profile, profile of the flow. And if you look at this how the fluid will be moving that means u by u infinity is y by h that means if this is the, I consider this as a u by u infinity y by h.

So, in fact the, initial is the fluid flow is a, there is no fluid which is flowing across this, then slowly, slowly the speed increases. This is the way fluid will be flowing and then u becomes u naught, u infinity. Here, u is equal to… So, the speed goes on increasing from u is equal to 0 to u is equal to infinity as u by u infinity is y by h, as it, as you go away from the, this wall to this wall. So, this side the fluid will not be flowing at all whereas, this side the fluid will be flowing at the same speed, just adjacent to this wall the fluids will be same as the u infinity and this will be the rate at which the fluid will be flowing. The speed will be this. So, this is, this flow is called a Couette flow, one of the most simplest flow for the vicious fluid one dimensional flow.

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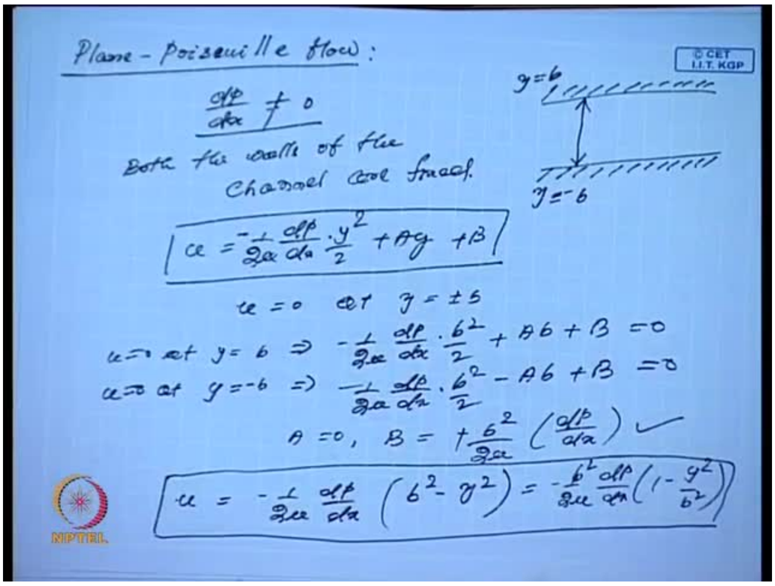

Now, I will go to the second example that is called a plane Poiseuille flow. And in case of a plane Poiseuille flow what happen? The pressure gradient d p by d x, it is again the flow between two channels and the pressure gradient in this case the d p by d x that is this is non zero, further both the walls are fixed, both the wall of these are fixed. That means none of them are moving. Then if you go to the general solution what we have just derived for a one dimensional, two dimensional flow between two plates then we have already seen u is equal to 1 by minus 1 by 2 mu d p by d x y square by 2 plus a y plus b, this is our general form of the solution for a two dimensional fully, one dimensional fully developed flow, if this becomes this then when d p by d x is non zero then we have both the walls of the channel are fixed.

So, what will happen? If I just say that one of the channel is y is equal to minus b and one is at y is equal to b then look at the distance to be, I can always use the same notation, but since there is no harm in always using different notation, the distance and, but we can always promote 0 to h. Here, h is in the previous Couette flow we have taken this distance as h here. We are taking this distance as two b, does not matter. So, u is equal to minus 1 by 2 mu d p by d x y square by 2 plus A y plus B, this becomes the general flow and in this we have d p by d x is not 0 and we have u is equal to 0 at because both the walls are fixed.

So, u is equal to 0 at y is equal to plus minus b. If u is equal to 0 at y is equal to plus minus b that gives us suppose I say u is equal to 0 at y is equal to b and that gives me u 0 at y (( )) that gives me y is equal to b, y is equal to b means minus 1 by 2 mu d p by d x into b square by 2 plus A b plus B and this equal to 0. And then u is equal to 0 at y is equal to minus b that gives me minus 1 by 2 mu d p by d x into b square by 2 then minus A b plus B equal to 0. And if you simplify this and that gives us A as 0 that gives me from this we will get A as 0 and B will get minus b square by 2 mu d p by d x.

This is how we have (( )) the (( )) eliminate B then you get A. Thus, these two terms eliminate B means you just subtract one from the other and then you will see that these term will get cancelled, these term will get cancelled because they are of same sign, they are of opposite sign, so that will give us A is 0 and when you, if you add these two things then A will be already A is 0, so you have 2 B is equal to this. So, B equal to plus b square by 2 mu into this is what we have got it.

Now, here if I substituted for this B then what will happen to my u? A B u will be minus 1 by 2 mu d p by d x into, this is b square minus 1 by 2 b square minus y square and this is same as this gives minus 1 by 2 mu, if I take this square by 2 mu into d p by d x 1 minus y square by b square. This is the general pattern of the distortion of the flow and here d p by d x is a non zero. So, this is the general pattern.

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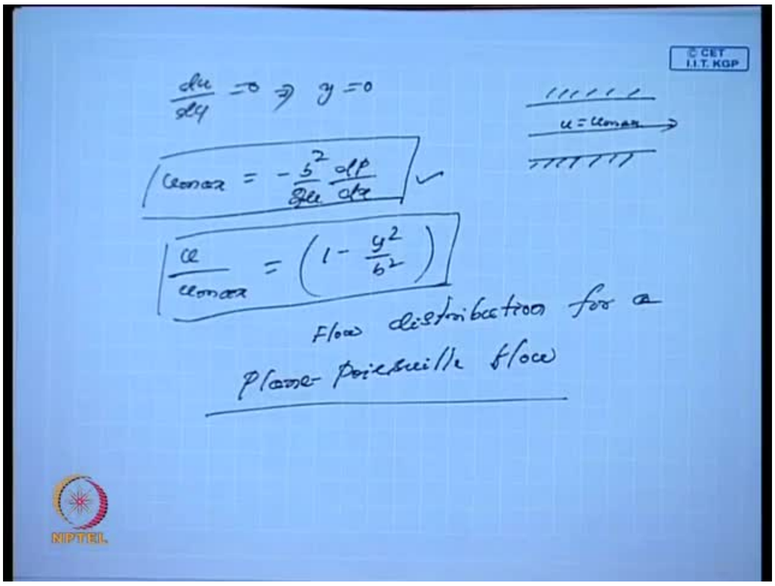

And if I look at the what is the, what will happen to d u by d y is 0, if d u by d y is 0, if d u by d y is 0 that means to know whether the flow is maximum it can be (( )) in that at y is equal to 0 the flow will be maximum which implies y is equal to 0. That means the channel there is no flow here. So, at this point the flow velocity is maximum, u is equal to u max. So, what will happen to my u max? What happened at y is equal to 0? So, u is equal to u max is minus b square by 2 mu into d p by d x.

Question comes, this becomes the maximum. The speed will be maximum; the positive value has to be taken if you are looking at the speed and then what will happen to u max? And that gives me u by u max, if I substitute because b square by 2 mu if I just take it from the original equation of u then I will get 1 minus y square by b square, that I get, this is what and this gives the velocity distribution for a plane Poiesuille flow, if the flow distribution for a plane Poiesuille flow. Now, whether these two simple problem, very, very simple problems. Now, we will go for a generalized plane Couette flow.

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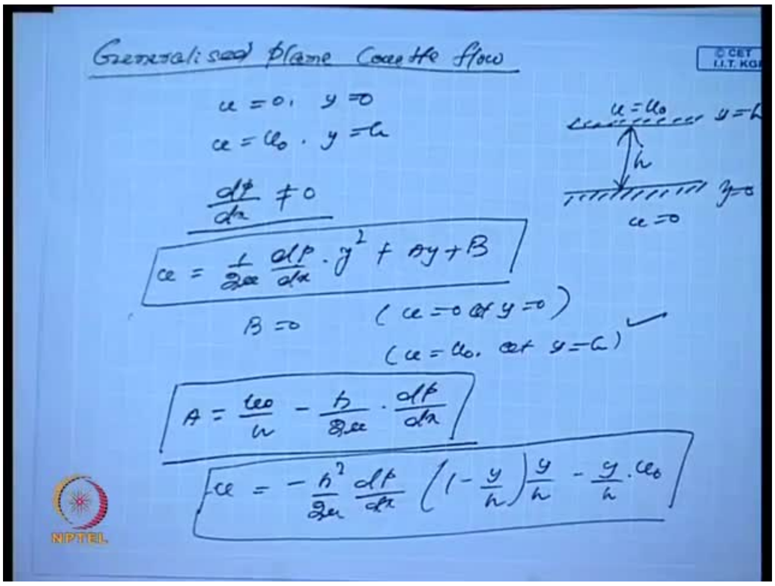

If I go to a generalized plane Couette flow this is a… In case of generalized plane Couette, we have seen that in case of a Couette flow, plane Couette flow you have taken, we have the two plates, there are distance sorry h, u is equal to u naught and that was moving at a uniform speed, this plate was moving at a uniform speed, rather this plate is kept fixed u is equal to 0, this is y is equal to 0, this is y is equal to h. And then the plane Couette flow, so we have seen that we have taken u is equal to 0, y is equal to 0 and u is equal to u naught, y is equal to h.

Unlike, the case of a plane Couette flow, in case of a generalized Couette flow would take d t by d x is non zero. If the pressure difference is not equal to 0 then as usual we will have u is equal to 1 by 2 mu d p by d x into y square plus A y plus B. And if this is the general flow, nature of the general flow then we have again this term will not be 0, but u is equal to 0 at y is equal to h and then it gives us if u is equal to 0 at y is equal to 0 that gives us B as 0, u is equal to 0, y is equal to 0.

That gives us B is equal to 0 and when, then if you substitute that u is equal to u naught at y is equal to h that gives us A as, that will give your A as u naught by h minus h by 2 mu into d p by d x. This can be obtained, the substitute for u is equal to u naught at y is equal to h and in this equation then you will get A is this whereas, B 0 and A is this and in the process you get u is equal to minus h square by 2 mu into d p by d x 1 minus y by h into y by h, y by h into u naught. This will give you… So, so this gives us the general flow and that is the general expression for u and if you substitute for this.

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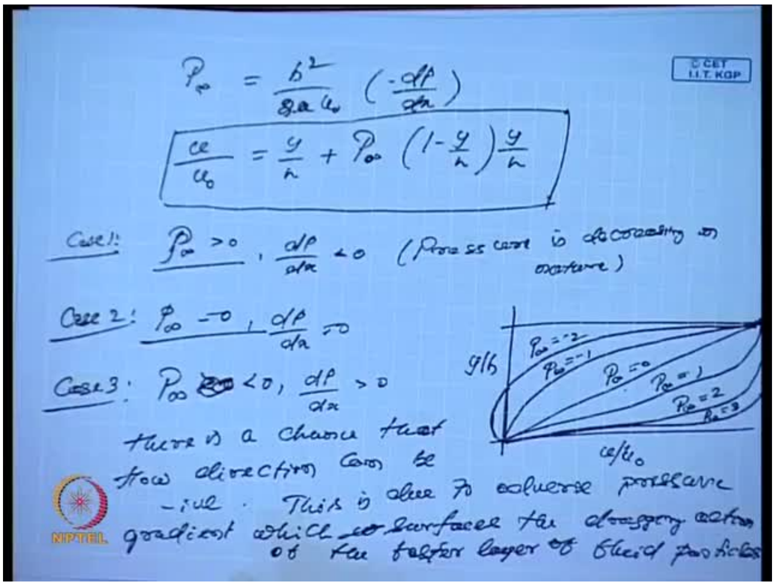

Now, if I, in this case if I take a little, define the pressure terms as p is equal to h square 2 mu u naught into minus d p by d x then the velocity distortion term gives me in the non dimensional form, then u by u naught is equal to y by h plus P infinity into 1 minus y by h into y by h from the previous expression you will get it. As I have taken a negative term (( )) and then once this is the general form where I have taken in P infinity the negative sign.

So, what it says that the velocity distortion depends on the pressure, the infinity and there are three cases that arise. In case one, suppose P infinity is greater than 0, P infinity is greater than 0. If P infinity is greater than 0 I have a d p by d x is, so that means this greater than 0 means d p by d x is less than 0. And once d p by d x is less than 0, that means the pressure is decreasing, is decreasing in nature which increase in the flow direction.

So, d p by d x is less than 0 gives us pressure is decreasing in nature and so what happened to the velocity distortion then? And if I just plot it u by u naught and this is y by h. In fact you will see just take this, then this is P infinity is 0, P infinity is 0 then u by u naught is (( )) P infinity is 0, I will call this P infinity as 0 that gives you u by u naught is y by h as I have seen in the previous case and if it is P infinity is 1. So, this is P infinity is 1, P infinity is 2, 3 on the other hand we have this is P infinity minus 1, this is the case P infinity is minus 2 and here what happen if P infinity is 0, P infinity is greater than 0, that means the flow is always in the, moving in the positive direction over the whole width of the channel. Flow is in the positive direction.

On the other hand in case two if P infinity is 0 that means you have a, the flow speed that means d p by d x is 0, the pressure (( )) end is 0 and this is similar to the plane of the flow and the flow distortion is in this case it is linear and it will go on increasing and we have seen in case of Couette flow, plane Couette flow and in case of case three when P infinity is greater than 0 or less than 0 sorry P infinity is less than 0. P infinity is less than 0 means d p by d x is greater than 0.

The pressure difference is increasing, there is a reason and which particularly in this part you see that the flow distribution is in the negative direction that means fluid flow in the opposite direction. There is a chance that, chance that flow direction can be negative, can be negative. The fluid flow is in the opposite direction and what does it means because that means the viscosity is dragging the drag the motion and there is a situation which makes the flow to be in the positive direction.

It is due to the basically, this is the due to the adverse pressure gradient, which is due to adverse pressure gradient which surfaces the dragging action for the faster layer of the fluid which surfaces adverse pressure gradient which surfaces the dragging action of the faster layer of the fluid particle. So, these are the three cases where in one case in this case particularly this is one of the interesting observation that the flow can be in the negative direction and this is the only possible when we have this particularly in the presence of a vicious drag with difference in pressure.

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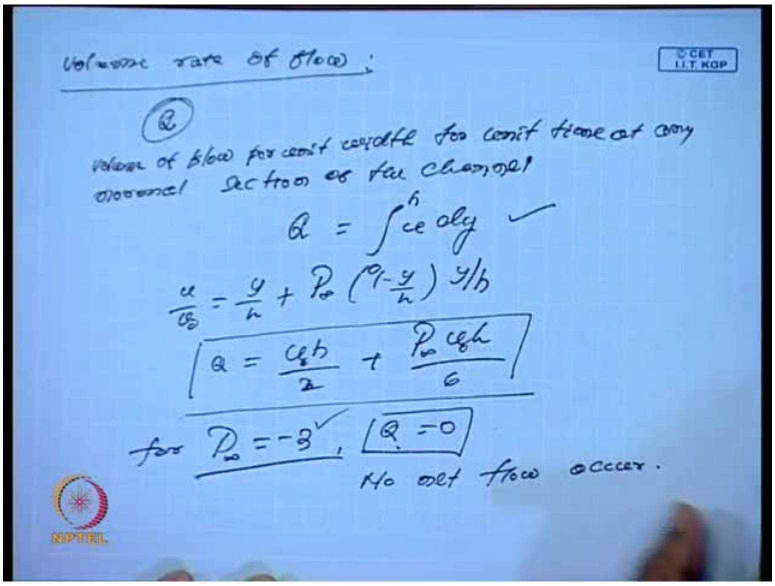

Now, this if I go to, what is the, let me calculate what is the volume rate of flow? Volume rate of flow and this is defined as if I say Q as the volume of flow per unit width per unit time; I call this volume of flow per unit width per unit time at any normal section of the channel. Then this Q is given by is equal to 0 to h, you just integrate it u d y. If I substitute for u, the general flow distribution in this expression that means I know that u by u naught is equal to y by h plus P infinity into 1 minus y by h into y by h and if I substitute for this here then I will get Q, you get u naught by h by 2 plus P infinity u h by 6, this becomes the volume rate of flow per unit cross section and it can be seen that because we can have, we have seen that three cases that P can be negative.

So, you can easily see that for P infinity is minus 3 then this u naught by h, P infinity is minus 3 and this is minus 1 by 2 or this is also u naught h. So, then that will give us Q as 0. So, that means when P infinity is minus 3, Q becomes 0, volume rate of flow is 0. That means there is no net flow along any section which is perpendicular to direction of motion when P infinity is minus 3, no net flow occur. Now, with this I will go for another quantity that is called the coefficient of skin friction.

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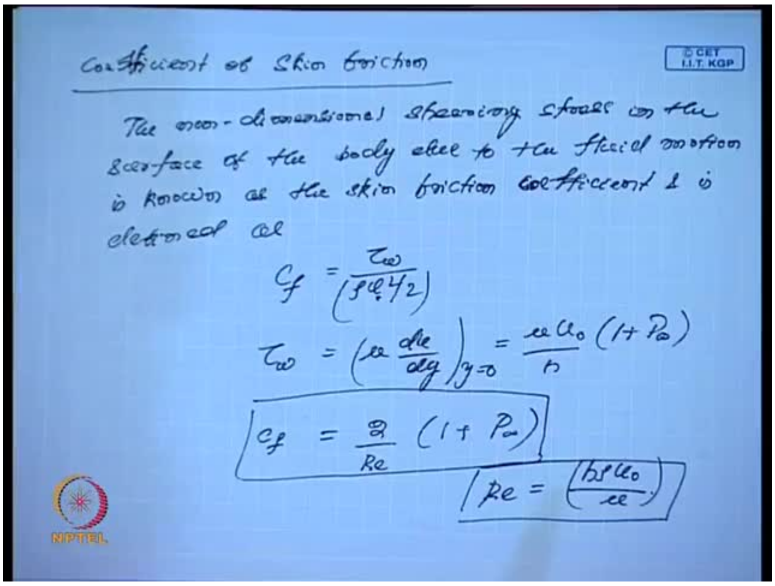

Coefficient of skin friction and this is defined as this is a non dimensional, a non dimensional shearing stress in the surface of the body due to the fluid motion is known as the skin friction coefficient, known as the skin friction coefficient, known as the skin friction coefficient and is defined as C f is equal to tau w by rho u square by 2 and in this case and tau w is given by that is a shear stress near the wall mu d u by d y and this is at y is equal to 0 because one of the wall is at y is equal to 0.

And in this case if substitute for you in tau w we will get, this will be mu u naught by h and this is 1 plus P infinity, this becomes tau w and then in the process my C f will be (( )) by tau w here then this should be 2 by R e into 1 plus P infinity. Here, my R e is h rho u naught by mu. So, this is kind of Reynold's number where h is the typical length parameter, u is the speed and mu by rho, this is the kind of … So, the skin friction coefficient is becomes this and it all depends on the Reynold's number of the fluid flow and P infinity, the pressure gradient and again if you see that the skin friction coefficient C f, if P infinity is minus 1.

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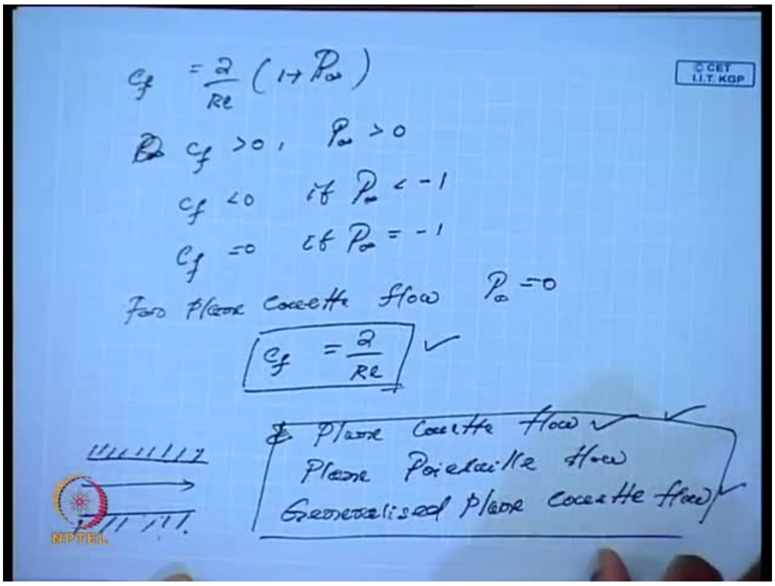

Our C f is equal to 2 by R e 1 plus P infinity and we can see that P infinity, this C f, the skin friction coefficient C f, this is greater than 0 for P infinity is greater than 0. Obviously, this is greater than 0 and C f is less than 0 if that means this quantity is less than, if P infinity is less than minus 1. And again C f equal to 0 if P infinity equal to minus 1. So, we also see that for the plane Couette flow, for plane Couette flow P infinity becomes 0 because d p by d x is 0 and in that case C f is 2 by R e. So, this is another important observation this case.

Now, with this understanding of, we discuss three types of flow, today. One is the plane Couette flow, plane Couette flow the other is the plane Poiseuille flow and the third type of flow I call this is the generalized plane Couette flow. In all three problem this is basically we consider the flow between two plates basically in a channel. In one of the case in plane Couette flow you consider the pressure gradient is 0 and one of the plate is fixed, the other plate is moving, the channel, the other plate is moving, the channel, the other plate.

That means here the fluid and we have seen that the fluid flow increases from 0 to till it reaches the speed, it reaches the highest speed along this other end of the channel where the fluid is moving at the speed on the other hand when you came to plane Poiseuille flow here we assumed as the both the plates are fixed in the channel both the walls are fixed and whereas d p by the pressure gradient is non zero and in such a situation we have seen that the maximum full speed occurs at this point.

On the other hand in the case of generalized plane Couette flow we have seen there are three cases all this depends on the P infinity that means the non dimensional pressure gradient. And also we have seen the volume rate of flow and volume rate of flow when this P infinity term because minus 3 there is no net flow which no net flow occurs the channel under this condition and the next, this simple three cases.

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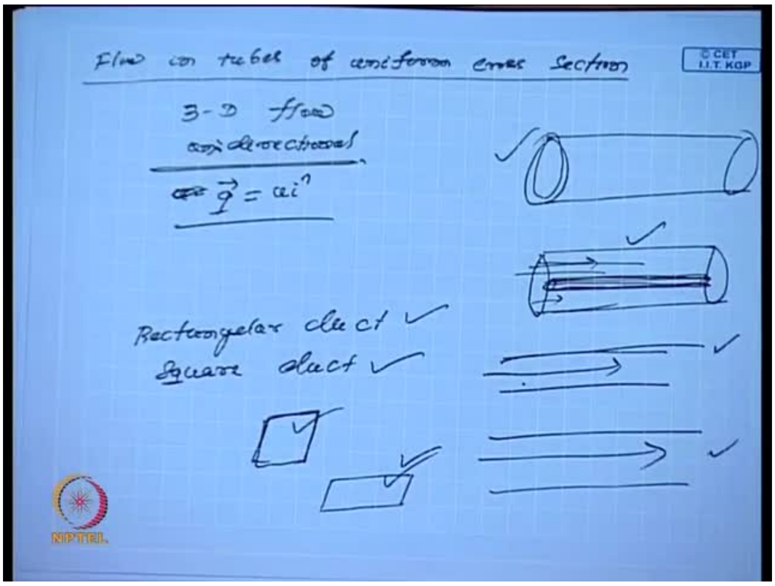
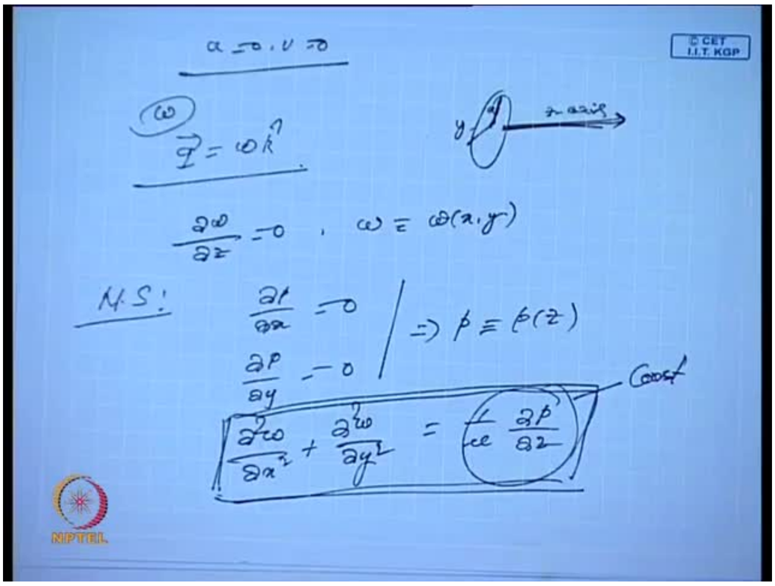


The next problem we will discuss, you want to flow, flow in tubes of uniform cross section, flow in tubes of uniform cross section. Again, here when I consider a tube of uniform cross section of course, I assume here as if the flow is in two dimensional in nature, three dimensional flow, this becomes a three dimensional flow. And in case of 3 D flow and again I have the flow is unidirectional, then when I say flow in tubes I can have several tubes, I can have a circular tube, I can have a co axial tube like suppose I may have tube, two tubes the inner cylinder is a rigid one and the flow across this and this between the cross section of this.

So, the fluid is flowing through this, but there is a closed, this is another cylinder and which may be fixed inside this. There is a flow between two co axial cylinders and there can be a situation, I may have a duct, rectangular duct or I have a square duct. It is a channel whose cross section is rectangular whose like its front face is square, its front face is rectangular one and it is flowing. It is a large channel having the front face is square here unlike it is a circular one, it is a cylinder.

So, the duct is like a, it is a channel whose front face is square in case of a rectangular duct, in case of a square duct and in case of a circular duct the front face is rectangular in nature. So, and it can be the fluid can flow in this direction, one directional. Let it be in the x direction, the fluid flow is there. So, this is, all this flow, these are all two dimensional, three dimensional fluid flow, but the fluid is unidirectional that means again u is equal to, q is equal to u I, only one and, in one direction the fluid is flowing. (Refer Slide Time: 50:56)



And if we look at the general form of the solution, this problem, again it is also considered another fully developed flow category and in this case also as I say my, if I assume that the, the flow is along the like I take x and y along the axis, along the plane of the cylinder, so only the, if I think of a cylinder I consider as if the z axis is along the cylinder, generator of the cylinder. So, if this is the z axis along the generator of the cylinder and this is the, this is x, this is y then if I do that then I will consider my u is equal to 0 as v is equal to 0.

And I will have only w component, I have only w component, z component that is w. So, my q bar I can call it w k hat. I just take this in this case, I can always do any, but for sake of simplicity. If I take the q bar is w k hat that means u v are 0 and then you will have, what will happen? This becomes my equation of motion becomes equation of continuity gives me del w by del z is 0 because u is 0, v is 0 and del w by del z is 0 gives me w as a function of w x y.

And in a similar way if I go back to the Navier Stokes equation of motion, I will have del p by del x is 0 because my u is 0 and del p by del y is 0 because my v is 0 whereas the third component of the velocity gives me del square w by del x square plus del square w by del y square and that gives me because this gives me, what? p is a function of z only and then the z component of velocity will give me del square w by del x square plus del square w by del y square because in the z direction there is no flow and that gives me 1 by mu del p by del z. This will be the and again p is a, because this itself is a giving a p is function of z and here we can see that w is a function of x y.

So, this part will be a function of x y, this part is a function of z, it cannot be possible. So, this has to be a constant. So, the pressure gradient is constant, del p by del z is a constant ideally and here w is… So, basically the governing equation will be of this nature and depending on the flow characteristics, the type of flow in the tube whether it is a rectangle cross section, whether it is a square duct, square cross section or whether it is a circular one only substitute the boundary condition and the wall boundary and will analyze this flow in our next class.

After this, we will go to some of the transient flow problems. So, let us see in the next class about the flow in the circular flow in the tubes of various cross section with this fundamental, basically we will be concentrating on this equation, with this I will stop here.

Thank you.