

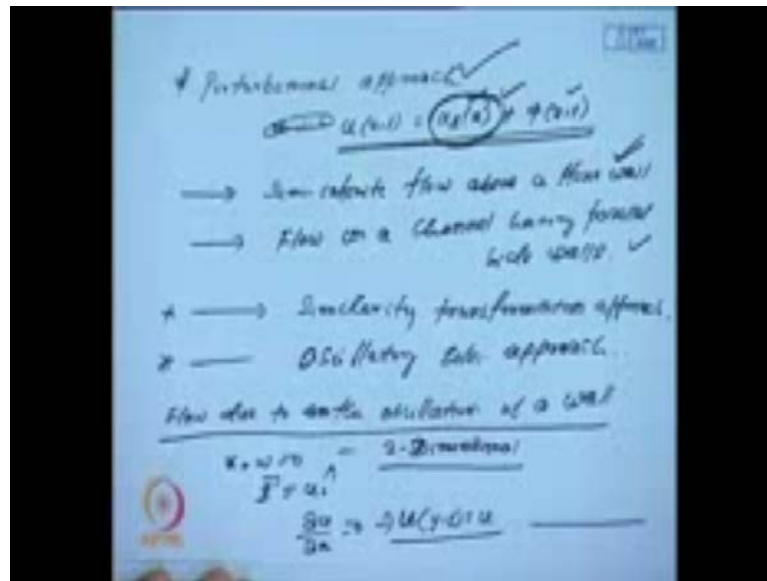
Marine Hydrodynamics
Prof. Trilochan Sahoo
Department of Ocean Engineering and Naval Architecture
Indian Institute of Technology, Kharagpur

Lecture - 36
Unsteady Unidirectional Flows

Welcome you to this series of lectures and marine hydrodynamics. In the last lecture we are talking about some of the basic flow problems as respected with Navier Stokes equation. The problems we have discussed in the last 2 lectures that are on study flow. And today, we will discuss about unsteady flow. One of the unique advantage or features of this unsteady flow particularly when it is unidirectional is, that we can, we can get back the solution of associated with the unsteady motion from the unsteady solution motion of the unsteady solution. That is one of the beauties. Another thing is in case of unsteady motion.

We will have sometimes there are 2 types of problem comes into. Sometimes the ocean is set by due to a sudden impulsive pressure gradient. Initial changes in the pressure gradient and sometimes it is due to the initial transient. The motion becomes suddenly start accelerating and that is a transient motion one is oscillatory motion. One is the transient motion and another type is what are at the, what the prude is set in motion due to the change in the pressure gradient. And that also can be oscillated in nature and these kinds of problems when you look into a solution approach again. We follow 3 approaches one is the perturbational approach.

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Perturbational approach in the perturbation approach, what we do we always approach u . The solution then u we said is, if it is u is a function of x and t if it is a solution, then you always say as it is $u_s(x) + \phi(x,t)$. So, divide the solution under 2 parts. That means one part is the steady-state solution. The other part is contributing parts of the unsteady characteristics of the solution, characteristic due to unsteady motion. But together it gives the solution and the steady. So, if you say that in a simple case is that this solution the steady-state solution can be obtained from the full solution as I mentioned that it is one of the input and feature of this as a problem. Then here we have 2 types of problems.

We will discuss in this one is semi infinite flow, semi infinite flow above a plane wall. And the other is a flow in a channel having parallel side walls and as a emulsion the flow can be driven by the change in the unsteady pressure gradient change in due to the change in pressure gradient. And also it can also be driven by the unsteady boundary condition like may be one of the boundary condition which will be suddenly set into motion. And that will be and the flow can be transient in it is nature, and as I say also the flow can be oscillated in nature.

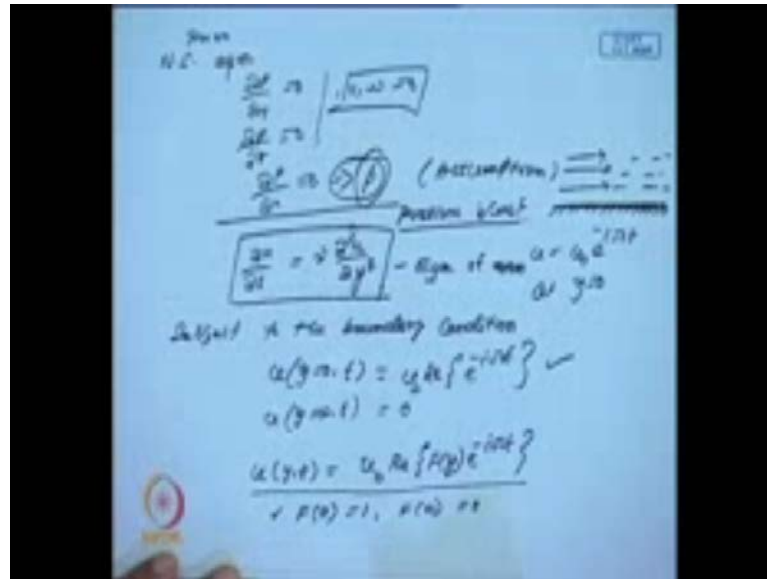
So, then when you come to solution approach of this kind of problem, as I say one of the salient feature is the potential approach is that you can always get the solution associated with the study motion, that what we say the perturbational approach. The other major

approach is which are used basically is the simulated transformational approach either it is simulated transformation, there is a perturbational approach. This is what I said the simulated transformation approach. And the oscillatory solution approach. That is called the oscillatory solution approach.

So, now we will discuss the case of problem associated. Semi infinite flow above a plane wall, once we have a clear idea about this we will discuss in few cases. And then we will go to the second case that is flow in a channel having parallel side walls to do. So, the first problem semi infinite flow above a plane wall, I will let us look into the problem like flow due to an oscillate. The oscillation of the wall flow due to the oscillation of a wall, this is comes under the semi infinite flow above a plane wall. That means initially a wall was there, I see that I will still looking into the unidirectional flow basically flow.

Here what happens there is a only one wall, the wall is set in it starts oscillating basically. I will have u and the component velocity v and w will be 0 and v by q is equal to u . And then by because flow is time dependent, so I can have because u here w . So, from this continuity equation, you know that $\frac{\partial u}{\partial x}$ is equal to 0, that gives me $\frac{\partial u}{\partial y} + \frac{\partial w}{\partial z} = 0$. $\frac{\partial w}{\partial z} = 0$ means I will get q is $\frac{\partial u}{\partial x} = 0$. So, u can be as a function of y and z . And if I have seen the one dimensional, you have seen that two-dimensional flow closed two-dimensional. Then, I will have u can be because it is independent of x . So, u can be a function of y and t that is my u . So u can be as a function of y and t and I have seen that the pressure gradient is 0. Because I have seen that there is no change in the pressure, that means again if I look at the Navier Stoke equation.

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From Navier Stokes equation, we have $\frac{\partial p}{\partial y}$ is 0 and we have also $\frac{\partial p}{\partial z}$ is 0. And then here we have we assume that the right side is the constants. So, I assume that $\frac{\partial p}{\partial x}$ no change in the 0 because this is coming. And that this means p I call this, once I say this comes from the criteria that u here v w are 0, on the other hand since this is coming as sum this is coming by assumption. I assume this, and then under this assumption the equation of motion gives me, if I assume the pressure is same throughout the fluid. Because I have a plate it is a large infinite plate, is a fluid which is flowing. It is the direction the fluid is flowing, and then I have also my from the Navier Stokes equation. For its component of the Navier Stokes equation, gives me $\frac{\partial u}{\partial t}$ is equal to $\nu \frac{\partial^2 u}{\partial y^2}$. And since u I have taken $\frac{\partial p}{\partial x}$, this is my assumption that is p is equal to constant. So, this is constant

So, the pressure is constant here. Now what I will say that? I will assume that as if I am looking at the wall is oscillating. And it is a in the direction of the oscillation is the wall, if I say that u is equal to $u_0 \cos(\omega t)$ and u_0 is the amplitude. So, this is oscillation and this oscillation is where it is happening. It has y is equal to 0 at y is equal to 0. u becomes this is this is the way the plate is oscillating along this axis and the plate is infinitely large. So, this becomes as one of the boundary conditions. That means I am basically I have to solve this equation this is my governing equation, and I am solving this subject to the boundary condition. The boundary conditions will be u at y is equal to 0, u at $y \rightarrow \infty$ is 0. And that is called to u real part of $e^{-i\omega t}$, just take it minus.

And then that is the wall is oscillating but what is happening to the fluid when the wall is oscillating away from the wall. For distance u at y is equal to infinity fluid is extended infinitely, extended fluid y is equal to infinity t and this is 0. So, it basically here we are looking at a, it is basically the flow generated by a flat plate which is oscillating in its plane along the x -axis with angular frequency ω . And otherwise away from this is oscillating. But what happen away from this structure away from the plate, the flow speed is 0. Now this characteristic because I have 2 my governing equation is this, this is the equation of motion this is my equation of motion. And then I the known fact is that about t I know that initially the fluid is oscillating with a making a simple harmonic oscillation.

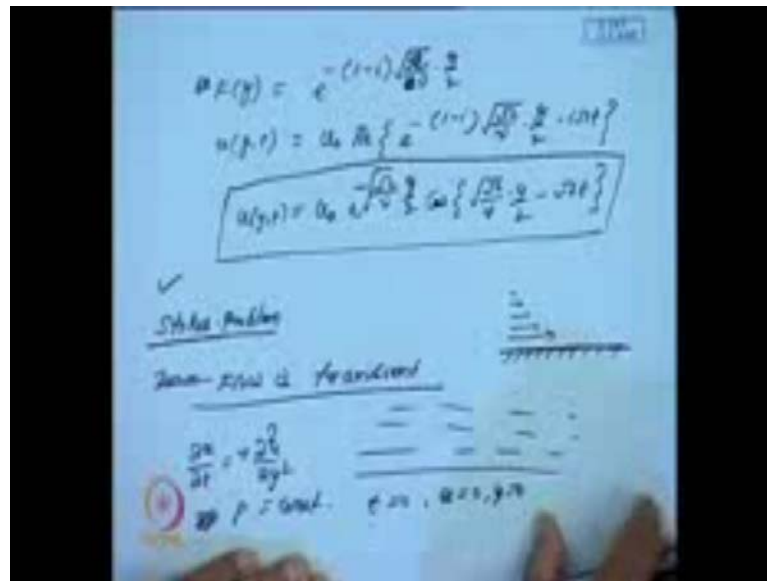
So, that gives me an idea that if I look for a solution that is $u(y,t)$, if I say that is u because u is the initial u into real part of that, I call this as u naught real part of $F(y)e^{-i\omega t}$. Suppose I assume, because I know I assume that the time dependent at the flow is oscillatory in nature, because of the plate is a boundary condition is oscillatory in nature, see if I assume I have a flow of this type, the solution is of this type. And if I substitute for this in the governing equation first of all I will see what happened to my boundary condition if u is equal to u naught u naught $F(y)e^{-i\omega t}$. What happen at y is equal to 0 at y is equal to 0, this is u naught. So, if I substitute y is equal to 0 here then what I will get that will give me $f(0)$ that means $f(0)$ will give me u naught is there. So, $f(0)$ will give me one from this condition this is sum of this. Now what will happen at u y infinity t that means it is 0. So, that gives me f of infinity and 0 so this becomes that boundary condition. Now, if I look at suppose I substitute for here in this equation what will be my f will satisfy the f will satisfy because you can easily see that.

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$F''(y) + \frac{i\omega}{\nu} F(y) = 0$ ✓
 subject to the BC:
 $F(0) = 1, F(\infty) = 0$
 $k^2 = -\frac{i\omega}{\nu}$
 $\Rightarrow F''(y) - k^2 F(y) = 0$
 $F(y) = A e^{ky} + B e^{-ky}$
 $k = \sqrt{-\frac{i\omega}{\nu}} = \frac{(1-i)}{2} \sqrt{\frac{\omega}{\nu}}$
 $F(y) = A e^{\frac{(1-i)}{2} \sqrt{\frac{\omega}{\nu}} y} + B e^{-\frac{(1-i)}{2} \sqrt{\frac{\omega}{\nu}} y}$
 $F(0) = 1 \Rightarrow A + B = 1$
 $F(\infty) = 0 \Rightarrow B = 1$

Capital f will satisfy $f''(y) + \frac{i\omega}{\nu} f(y) = 0$ this becomes. So, basically I have to solve this differential equation subject to the boundary condition. That is $f(0) = 1$ and $f(\infty) = 0$ if I look at the solution of this differential equation, then I will get if I simply take $\frac{i\omega}{\nu}$ as p into. So, what will happen to this and then my solution will be if I say k is equal to k^2 is $\frac{i\omega}{\nu}$ then I will have $f''(y) - k^2 f(y) = 0$. And this will give me $f''(y) - k^2 f(y) = 0$ if I take this minus then it will be $f''(y) - k^2 f(y) = 0$. And if this becomes this, then what will happen to my $F(y)$. $F(y)$ will be $A e^{ky} + B e^{-ky}$. Now what will be my k here? And then my k will be $\sqrt{-\frac{i\omega}{\nu}}$. If I look at this then this will give me $\frac{1-i}{2} \sqrt{\frac{\omega}{\nu}}$, and if this is the case and this one is a square $\frac{d^2 u}{dt^2} - \frac{i\omega}{\nu} u = 0$. So, this becomes this. So, then what will happen to my $F(y)$. My $F(y)$ will be $A e^{\frac{1-i}{2} \sqrt{\frac{\omega}{\nu}} y} + B e^{-\frac{1-i}{2} \sqrt{\frac{\omega}{\nu}} y}$. So, this will be wise in the all form this and this one and if I substitute because one of this, this e will be 0, because if infinity is 0 if $f(\infty) = 0$ gives me $a = 0$. And on the other hand $f(0) = 1$ gives me $f(0) = 1$ gives me $y = 0$ is $1 = b$. So, which one gives me b is equal to 1. So, that is what I got the final form. So, if I add to this so my form of the solution of $u(y, t)$.

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Now so I get my F_y equal to e to the power minus 1 minus I root over ω by 2ν . It will not be there is a little correction here. This is by ω by 2 into y by 2 this is y by 2 not 2 here, because this is 1 minus I by 2 . So, this becomes ω by ν into y by 2 . So, that way we got $u_y t$ is u naught real part of e to the power minus 1 minus I into ω by ν y by 2 minus I, i ω t . So, this is my form of the solution. And if I just put it I will keep it like this u naught. If I take it to the power minus ω by ν root over into y , and then this should give me \cos . Because I am looking at the real part \cos . This will give me root over of ω by ν y by 2 minus ω t . this is minus minus plus and this is \cos ω by ν into y by 2 . So, this becomes the my total solution.

And there is a y by 2 , here this is my $u_y t$. If you look at the solution, if you look at the solution because the plate was oscillatory and that as y tends to be $p t$ this tends to 0 . In fact the one can easily feel that if this plate which was oscillating, and then because of this term it will be minus ω by ν y by 2 it decreases exponentially. So, the speed will be here slowly, slowly the speed will decrease. And after certain time there is no, no fluids remain intact. So, the plate is oscillating here. But as you go up away from the plate the fluid goes and becomes stagnant. And there is no flow in fact often you can always relate this factor is like it is a u to this the fluids remain, otherwise the flow remains as a steady flow. So, it is like you can call this as a u to this unsteady motion will be effective for the and which the flow remains steady. And again we can see that this part if you look at the flow is oscillatory in nature, and there is a phase shift of this

factor ω by ν . That is a phase shift ωy , that is a y shift and the phase that is ωy and that is a . So, this is what when you look at the oscillatory motion of the flow as you know that the flow is oscillatory. And in fact this kind of flow it was a first I have discussed by Stokes then another sometimes you call it Stokes; Stokes problem. Then there is another flow of this nature here the flow we have taken this is as oscillatory in nature that means we assume that the plate is oscillatory. Now, if I look at another problem of the same, because just on a plate that means when I say that is sometimes we call this the second problem. I will consider assume that as if the flow is transient. Basically here transient flow here you look into that. I will again look into the same problem that means I have a plate infinite plate and the plate starts to usually the plate is at rest.

So, I will consider a plate, so there is a fluid you have a fluid and the plate initially is at rest. That means I will assume that again you have under the same assumption of unidirectional flow. You will have $\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2}$ and again p is constant, p will come. So, $\frac{\partial p}{\partial y} = 0$. So, you have to assume that your $\text{grad } p$ is equal to constant. There is, but what happen here, I am looking to the case where initially at t is equal to 0 u as 0. But suddenly it starts to oscillate that means t is equal to 0 u is equal to 0. On the other hand at the same point what y is equal to 0? But again when t greater than 0. I will put this condition like this when t is greater than 0.

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$(y=0, t>0), u = u_0$ ✓ / Boundary condition
 $(y=0, t=0), u = 0$ ✓ / Boundary condition
 $u = 0$ at $t = 0$ ✓ / Initial condition

Similarity transformation
 $\eta = \frac{y}{2\sqrt{\nu t}}, \frac{u}{u_0} = f(\eta)$
 $\frac{\partial u}{\partial t} = \frac{\partial u}{\partial \eta} \frac{\partial \eta}{\partial t} = u_0 f'(\eta) \left(-\frac{1}{2} \frac{y}{\nu t^{3/2}}\right)$
 $\frac{\partial u}{\partial y} = \frac{u_0 f'}{2\sqrt{\nu t}}, \frac{\partial^2 u}{\partial y^2} = \frac{u_0 f''}{4\sqrt{\nu t}}$

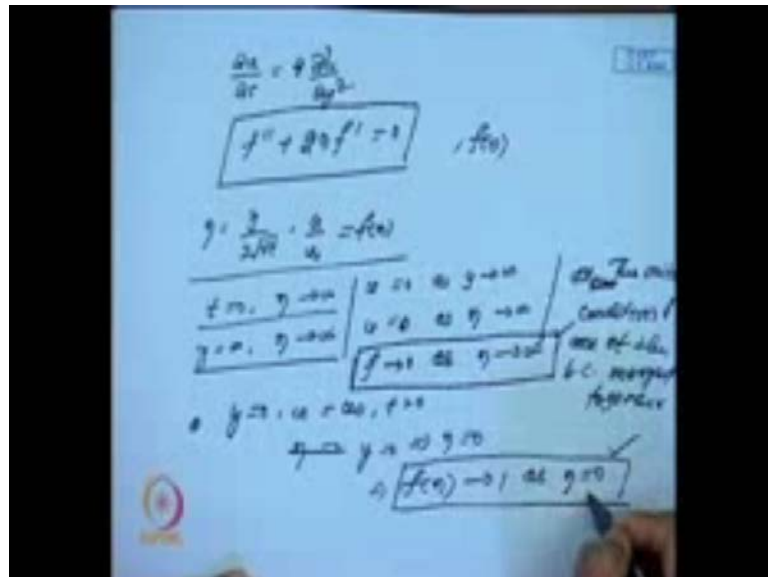
U is 0 as y is equal to 0 t is greater than 0 the moment from t 0 to t is greater than 0. What happens when the u becomes u naught. Suddenly it starts the motion is generated on the plate and fluid starts. The plate starts oscillating and that y is equal to infinity t is greater than 0. This u tends to 0 it is clear now. So initially the plate was the initial condition was that u was 0 at t is equal to 0. This was the initial condition suddenly it starts to oscillate as I was telling that there was a one type of flow is oscillated. The other type which is a fluid even flow velocity driven flow that is here suddenly, the velocity u becomes u naught as t greater than 0. Again that means the plate starts oscillating with a speed in the x direction u naught and it is a unidirectional flow. So, and here in the, in both the cases here it is t greater than 0.

Here it is, so this becomes boundary conditions and the same problem and this becomes the initial condition. So, to solve this, what I will do? I will follow what is called similarity transformation approach. The objective of any transformation is to reduce a complex problem to a similarity transformation. In the similarity transformation approach, what I will do as I mentioned that the objective of any transformation is to reduce a complex problem to a simpler problem. So, by we will use in this case, the similarity transformation by putting eta is equal to y by 2 into root over nu t. If I substitute for this further I will say that u by u naught. I call it as f of eta, this using this transformation what I will do. Then what will happen to my del u by del t del u by del t

will be equal to del u by del eta into del eta by del t and del u by del eta del u by del eta gives me u naught f dash eta..

Again del by del t del eta by del t that gives me minus half times 1 by root nu into t to the power del eta by del t. That is your y to the power minus 3 by 2 into y. And that simplify to minus u naught f dash eta into eta by 2 t. Because again y by nu t will give me eta and gives me u naught f dash eta into eta by 2 t del u by del t. Then what will happen to del u by del y? You can see that it is u naught f dash eta u by del by 2 root over nu t. And if you look at del square u by del y square that gives me u naught f double dash divided by 4 nu t, it can be checked and if you substitute for this in the governing equations that is.

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We have the governing equation del u by del t is equal to nu nu times del squared u by del y square if I substitute for this equation. Then what I get in terms of f I will get f double dash plus 2 eta f dash is equal to 0. So, initially it was a partial differential equation in with a derivative in t and y. Now it becomes a ordinary differential equation for an eta f is a function of eta and what will happen to my boundary condition. Because I have the initial boundary conditions I have since my eta is equal to y by 2 root over of nu t and u by u naught is f of eta. And my boundary conditions are when t is who will equal to we have t is equal to 0 t is equal to 0. So, here it is t is equal to 0 means eta times infinity again y tends to infinity this gives me eta tends to infinity. So, these are the 2 things which are based from here. And then if I go back to my plate boundary condition

that means what will happen to my u . So, my u is 0 my u as 0 as y tends to infinity it was my given boundary condition. So, that means in this case y tends to infinity same as η tends to infinity.

So, u will be 0 as η tends to infinity that is one thing and again what happen to f in the process as η tends to infinity because u is 0 as η tends to infinity. So, that means, so my u is 0 as η tends to infinity. So, that this means my f because u by u naught is $f \eta$. So, my f will tend to 0 as η will tend to infinity that is one condition. And the other condition I have y is equal to 0, u is equal to u naught. That was another condition. But t is greater than 0. And then these 2 y is equal to 0 u is equal to u naught then what will happen to then η u is equal to u naught t greater than 0 y is equal to 0. So, that means η will be 0, y is equal to 0, u is equal to u naught. So, when my η will be 0, so y is equal to 0. Same as in place η is equal to 0. So, in that case what will happen u by u naught is $f \eta$. So, that means my $f \eta$ will tend to one as η equal to 0. So, this is one more condition and this is a second condition.

So, in this case we see that 2 things happen from this t is equal to 0 and y is equal to infinity. t is equal to 0 is associated with an initial condition, y is equal to infinity associated with boundary condition. But these 2 together gives me only one condition that f is 0. So, this is that means the here in this case the initial the initial one of the initial the initial condition on one of the boundary condition merge one of the boundary condition merge together. So, I had initially one initial boundary problem now with this substitution one of the initial condition and the boundary condition merge together to give me only one condition. So, it becomes a one boundary condition and another boundary condition is this. So, this has reduced to one boundary problem where f double dash plus 2 η f dash is 0 and satisfy the 2 boundary function $f \eta$ is tend to 1 as η terms to 0.

Whereas, and the second one is f tends to 0 as η tends to infinity. So, the initial boundary will provide by using this similarity transformation the initial boundary value problem as reduced to one of the boundary value problem. And this is one of the characteristics of the similarity transformation. It is one of the very rather I will say important characteristics of similarity transformation. Now if I look at the solution

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$$f'' + 2\eta f' = 0$$

$$\frac{f''}{f'} = -2\eta$$

$$f(\eta) = c_1 \int_0^{\eta} e^{-\eta^2} d\eta + c_2, \quad f(0) = 1$$

$$f(\infty) = 0$$

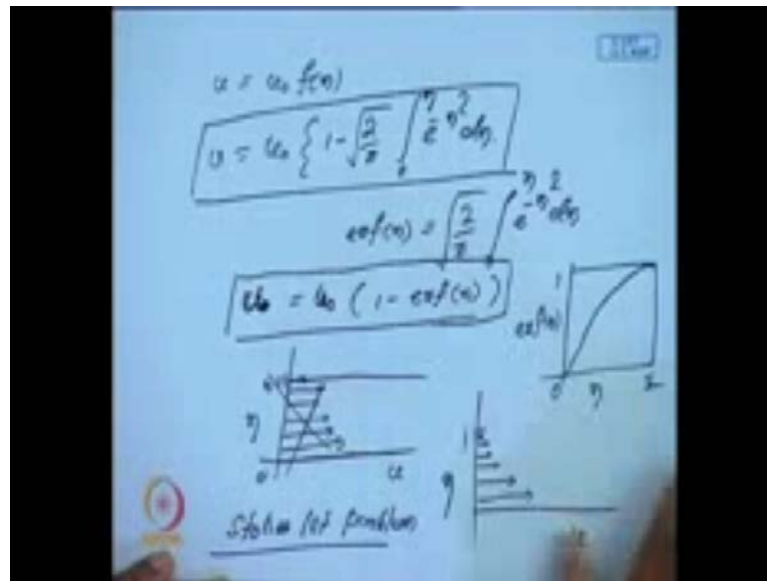
$$c_1 = -\frac{2}{\sqrt{\pi}}$$

$$f(\eta) = 1 - \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-\eta^2} d\eta$$

$$\int_0^{\infty} e^{-\eta^2} d\eta = \frac{\sqrt{\pi}}{2}$$

What will happen my $f'' + 2\eta f' = 0$ if I look into the solution of this. Then what will happen? $f''/f' = -2\eta$. And if I integrate it twice what will happen? I will get my $f(\eta)$ that is some $c_1 \int_0^{\eta} e^{-\eta^2} d\eta + c_2$. This will be the general form of solution and here c_1 and c_2 are arbitrary constraint. And I have 2 conditions that is my $f(0) = 1$ and $f(\infty) = 0$. If I substitute for this boundary function $f(0) = 1$ means $c_2 = 1$, because this will go. So, if $f(0)$ in this c_2 is equal to 1 and $f(\infty) = 0$, that means $c_1 \int_0^{\infty} e^{-\eta^2} d\eta + c_2 = 0$. So, $1 = 0$ that is $f(\infty) = 0$ implies. This is equal to this. And once this is what we all know that this is nothing but $\int_0^{\infty} e^{-\eta^2} d\eta = \frac{\sqrt{\pi}}{2}$. This is equal to $\frac{\sqrt{\pi}}{2}$. This is a standard integral exist. So, that gives me my c_1 is equal to $-\frac{2}{\sqrt{\pi}}$. This is and if this will occur, I think it is root over of π by 2. It can be verified so this is root over π . So c_1 is this. c_2 is this. So, my $f(\eta)$ becomes $1 - \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-\eta^2} d\eta$. This is my $f(\eta)$ and once I know $f(\eta)$ and $f'(\eta)$ is then.

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My u comes u naught into f η or else it is u naught f η is $1 - \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-\eta'^2} d\eta'$. This is the u that is the full the flow distortion is like this. u is given by this and sometimes we call this erf η is also that is called the error function 0 to η into e minus η square d η . So, in that case we call this u is equal to u naught $1 - erf$ η and the characteristics, because this is called the this function erf η is called the error function. If you look at the characteristics of the error function, it looks like this at 0 to 2 if it is η and if it is erf η then in fact this is a 0.2 . I call 0 this is 1 . Then in fact this result is like this erf η tend to near η is equal to 2 , if this afterward it is 2.1 . So, that is why this is u ; this is y u is equal to u naught into this. So, this is the total solution and erf η is this. Then I if I look at the plot u this is my u and this is my η . Then I will have a c and this is η is equal to 1 η is equal to 0 . Then and this is the plate moving. Then, I will have the flow speed initially it will be reducing and then reducing. So, finally, there in fact there is no flow as we raise η is equal to 1 , that means the flow reduces beyond which beyond this rather, I will say I will put it this way.

So, initially the speed will be there then the speed decreases, decreases, and decreases. And finally, there is so, this is one this is u . So, this is your η . So, as η goes to beyond 1 , then the fluid speed, because it is 1 , so $1 - 1$ is 0 . So, the fluid speed u becomes 0 beyond this. That means you can say the motion beyond one beyond η is equal to 1 . The motion becomes unsteady, and because the oscillatory behavior will not be affected the initial impulse. That is provided basically the, to the velocity that is become effective

up to η is equal to 1 beyond which again the flow will be like a steady motion. Unsteady characteristics will not again be effective beyond η is equal to 1. That is what it talks about and this problem is sometimes called the Stokes first problem.

Now we will just go into another problem. Basically, there in both the problems we have considered that the both are in the pressure gradient is 0. And another problem I will just say that, suppose I say that my pressure is pressure is if I just say that my pressure gradient is nonzero.

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Flow due to an oscillatory pressure gradient

$$\rho \frac{du}{dt} = -\frac{dp}{dx} + \mu \frac{d^2u}{dy^2}$$

$$-\frac{dp}{dx} = A \sin(\omega t)$$

$$\rho \frac{du}{dt} = A \sin(\omega t) + \mu \frac{d^2u}{dy^2}$$

$$u(y=0, t) = u_0 \cos(\omega t)$$

$$u(y \rightarrow \infty, t) = 0$$

$$u(y, t) = u_0 \operatorname{Re} \left\{ \frac{A}{\gamma} e^{-\gamma y} e^{i(\omega t)} \right\}$$

$$P(0) = 1, P(\infty) = 0$$

I will just look at the flow due to an oscillatory pressure gradient in the previous 2 cases we have considered the flow on where the pressure gradient is taken as 0. But if I just look at on the same thing with pressure gradient nonzero I will have my Navier Stokes equation, same unidirectional flow $\rho \frac{du}{dt}$ is equal to minus grad p plus $\mu \frac{d^2u}{dy^2}$, one of the same assumption And if I say that my grad p is equal to a sin ωt the pressure gradient is just oscillated in. So, this is also induce a I have a plate infinite plate which will it will also induce a flow and set the fluid to move. Also and then in this case what will happen?

So, if I substitute for here my governing equation $\rho \frac{du}{dt}$ equal to a sin ωt plus $\mu \frac{d^2u}{dy^2}$, and again I have the assumptions the flow is oscillatory, I will assume y is equal to 0 and time t this is cos ωt as. And that this is my initial flow and the pressure gradient of this and at infinity where say y is equal to

infinity t is a 0. So, here what we are adding that we are adding a pressure gradient which is oscillatory in nature. And as usual our flow is oscillatory; initially the plate is moving oscillating in the speed like the previous case. Like the first problem, we discuss which is and then at infinity what time t is equal to 0, this is 0. So, then what will happen in this case also if you look at a solution of this oscillatory type, we start a solution of this type. Then we can easily see that our $f(0)$ will be one and f at infinity will be 0. So, like the first problem we have discussed, if you proceed in the same way because here what is happening the additional term is this term. The term this is then we can easily see that like we do for a simple boundary value problem or in a differential equation we can easily see.

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$$u(y,t) = - \frac{\rho}{\rho \nu} (a \cos(\omega t) + \phi(y,t))$$

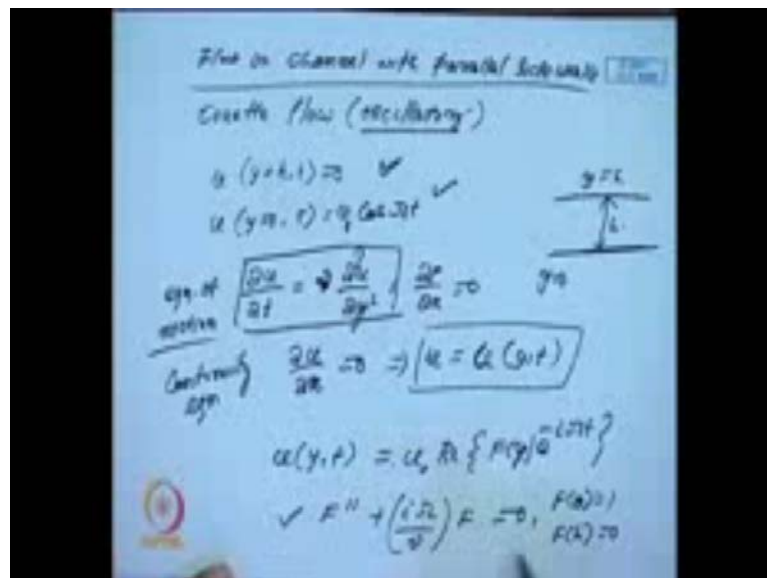
$$\phi(y,t) = \left(\frac{\rho}{\rho \nu} \right) \exp\left\{ -\left(\frac{\nu}{2\omega} \right)^{\frac{1}{2}} y \right\} \times \cos\left\{ \omega t - \left(\frac{\nu}{2\omega} \right)^{\frac{1}{2}} y \right\}$$

That my $u(y,t)$ minus a by $\rho \omega \cos \omega t$ plus sum when we call it some $\phi(y,t)$. This is and this ϕ can be given as if a substitute for this. Then I can easily say my ϕ will be $\phi(y,t)$ will be of this type on a by $\rho \omega$ into exponential of minus ω by 2ν to the power half into y multiplied by \cos of ωt minus ω by 2ν to the power of half into y . So, here to flow characteristics will be this, and again here we see that this term the ϕ will be this. Once ϕ is this, my u is already, so it has a one of the part. This is because there are 2 parts of the solution. This is this part is contributing change in the pressure gradient. And this is coming directly because of the oscillatory behavior of the flow and this is our total solution is depending. So, the solution full solution is becoming. this $u(y,t)$ and here we can see that even if the flow remain

oscillatory, because there is a if pressure gradient is there this flow remains oscillatory even if at far this term will go to 0. So, what will happen so this phi by t this phi by t 0 has?

So, if this term will be contribute to this, so if you look at the whole solution that the whole solution $u(y,t)$ will be of this part, and it is you can say it is a generalization of the solution which we obtained initially, where we did not have the pressure gradient. And so in the 3 problems, what we have discussed today we have come across there is a plate. And the plate is oscillatory in 2 cases, there is a low pressure gradient in the third case there is a pressure gradient. Whereas, this plate is in one case the 2 case is the plate was oscillatory in the other case plate was oscillatory. In the other case the plate was suddenly there was a velocity which was provided. And that oscillated that oscillated the plate which created the motion in the fluid. And so these 3 problems are all problems where only one plate is there. And infinitely extended fluid is there the plate is to start to vibrate or by certain motion whether it is by difference. Or now if you look into another class of problem particularly flows in a channel.

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Now I said that I have 2 walls with parallel sidewalls is and in this category the first thing comes that comes to mind is of coquette flow that means I am looking at a oscillatory coquette flow. So, in this case as usual we have 2 plates this is suppose y is equal to 0.

Another plate is at y is equal to h and in this case you have u at y is equal to $h t 0$ and u at wy is equal to $0 t$. Suppose this plate that moves at the speed $u \cos \omega t$ in the earlier case we used to say it moves with a constant speed u_{naught} . Now I say that it is a this is a is a oscillatory flow time dependent bias in this. And then if I go back to my couette flow particularly when the motion is unsteady. That will give me $\frac{\partial u}{\partial t}$ is equal to $\frac{\nu}{\Delta y^2} \frac{\partial^2 u}{\partial y^2}$. And here I in couette flow we have $\frac{\partial p}{\partial x}$ is 0 there is no change in the pressure gradient this is a fully developed flow. So, this becomes my governing equation these are the 2 boundary conditions. And if I say and as usual u because of from continuity equation this is from the equation of motion and then my continuity equation u becomes $\frac{\partial u}{\partial y}$ is 0 $\frac{\partial u}{\partial x}$ is 0 . This is continuity equation and that gives me u is equal to $u y t$ and these are the 2 boundary conditions. Because the plates start to initially this plate is where kept fixed.

And this plate is moving at a speed $u_{naught} \cos \omega t$ at y is equal to 0 and this becomes equation. So, if I look at the solution of this I will start with $u y t$ again I will go for a because oscillatory solution u_{naught} real part of $F y$ same approach I will use $\cos \omega t$. Then I will easily get if I substitute for this in this equation then I will get $\frac{\partial^2 f}{\partial y^2} + i \omega \frac{\partial f}{\partial y} = 0$ this becomes. And I have 2 boundary conditions if I substitute in terms of this will have 2 boundary condition that is $f(0) = 1$ and $f(h) = 0$. Then this is and this is at going at the same speed, because ωt then these are the 2 boundary conditions than if I solve this, I solve it substituting these 2 boundary condition then.

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$$F(y) = u_0 \operatorname{Re} \left\{ \frac{f\left(\frac{\sqrt{2}}{\nu}\right)^{\frac{1}{2}}}{1 - \exp\left[\frac{1}{2}(-i\omega t)^{\frac{1}{2}}\right]}\right\}$$

$$u(y,t) = u_0 \operatorname{Re} \left\{ F(y) e^{-i\omega t} \right\}$$

Transient Couette flow

I will get my flow as $f(y)$, I will get it u_0 naught where you can apply the boundary condition and solve it. I will directly get it u_0 naught into either I will write the full solution $F(y)$ is u_0 naught into real part of e to the power minus $i\omega t$ by ν to the power of half into $2h$ minus y . This is divided by 1 minus exponential of 2 minus $i\omega t$ by a new to the power half into h and into β the power minus $i\omega t$ in fact this is a general form of $F(y)$ ones I know $F(y)$. Then it can be easily checked that this satisfies that is 0 is 0 and if 0 is 0 and f h s itself to 0 is 1 and f h s 0 . And again once I know $F(y)$. I will know you of the $u(y,t)$ by u_0 not, real part of $F(y)$ into e minus $i\omega t$. And we can always see that the steady-state solution that particularly when it stands to infinity, then I will get back the solution that is the oscillatory bottom plate when it is oscillating, I will get when there is a constant pressure in the pressure gradient that was initially discussed and so this is one. So, that means flow where the flow is oscillatory in nature again I can in a similar manner I can go to a Couette flow by assuming transient. I can also discuss this buyers instead of Arab resume can set always sector boundary condition that initially a fluid as it set to motion u_0 . And then suddenly set to motion and there are 2 plates so by noon doing.

So, I can also get the solution of the transient Couette flow solution. And again the equation of motion will remain as we have discussed in case of oscillatory motion where only boundary condition will change. And that insist instead of oscillatory motion that motion will be a transient motion that means initially flowed will be addressed suddenly

to begin be given a speed at which it will be moving. And then we can get back the solution, in that case this is also straight for solution and it can be tried in the coming election I will leave it as a homework. So, this will stop here today.

Thank you very much.