Marine Hydrodynamics
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Lecture - 40
Solution Methods for Boundary Layer Equations (Contd.)

Welcome you to this series of lectures on marine hydrodynamics and in the last lecture we are talking about various solution methods to deal with the boundary layer problems. In fact two methods we are emphasizing out of the three, one is the final difference method, but our emphasis we have even emphasis just two methods that is self-similarity method and then the momentum integral method.

So, we have derived the momentum integral method, the general form of the momentum integral method for two dimensional boundary layer flows in the motion steady and the equations. Let us relate the stationary wave pattern in case of when a wave propagates near a wall.

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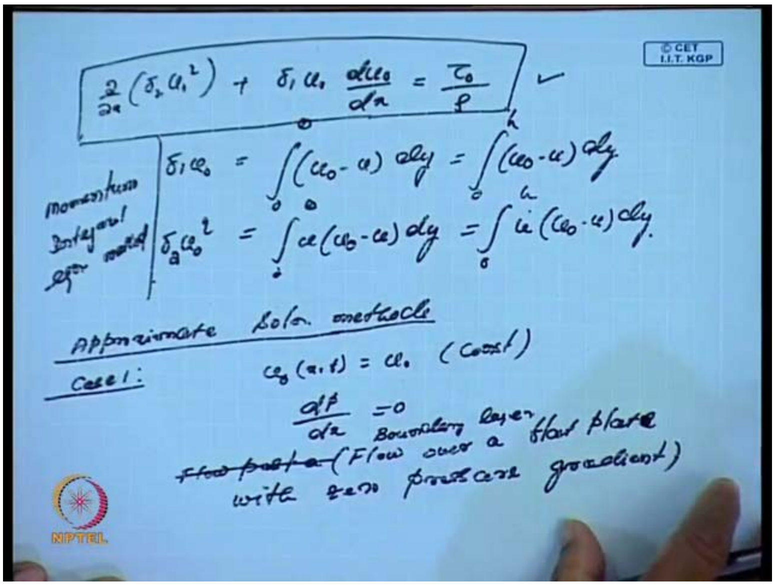

Equation says that is del by del x delta 2 u naught square plus delta 1 u naught d u naught by d x this is equal to tau naught by rho this is what we have derived in the last class. Delta 1 u naught is equal to 0 to infinity, u naught minus u d y and that is nothing but 0 to h u naught minus u d y. Similarly, delta 2 u naught square is nothing but 0 to infinity u into u naught minus u d y this is same as because they are in the boundary layer the stream velocity is same as the velocity, this is 0 to h.

So, this is u 1 to u naught minus u d y, so this was basically the momentum integral method and one of the advantage of this method is that this method is suitable to not only to deal with boundary layer flow equation, laminar flow laminar boundary layer poorness. But also it can be dealt with pulpiness understanding or getting the solution for turbulent flow problems this is the same what general and one of the robust method and which is called the momentum mating momentum integral equation method.

This method can be further generalized to deal with when the motion is unsteady the same method with a unsteady contribution will also be there and also it will be one of the most general approach to deal with this both laminar and turbulent flow problems. Now, today we will discuss only from these only we will go to look into approximate solution, this is the general form of the solution of the equation, but how to get a solution. So, let us look into approximate solution for the momentum of integral equation approximate solution methods.

So, there are many ways we can do that, so one of the things that let us write assume on the case one and this case we will assume that suppose u naught x t is u naught. So, basically we are going to look into the plus a solution the same problem semi infinite flat plate when there is a gradient is 0. So, this is constant you can say this is constant you say same as d p by d x is 0 because u naught to d 1 naught by d x will be constant it will be 0. So, d p by d x will be 0 and sometimes it is we call this problem as problem as a flow past flow past a rather we call it flow over a flat plate flat plate with 0 plus variant.

So, basically here the provision mean the boundary layer flow, I will call it boundary layer flow over the flat plate with 0 plus gradient, so this is the case then what will happen to our the boundary conditions. So, in this case because the d p by d x is 0 u naught is 0, so what will happen here u naught will be 0 here and d u naught by d x will be 0.

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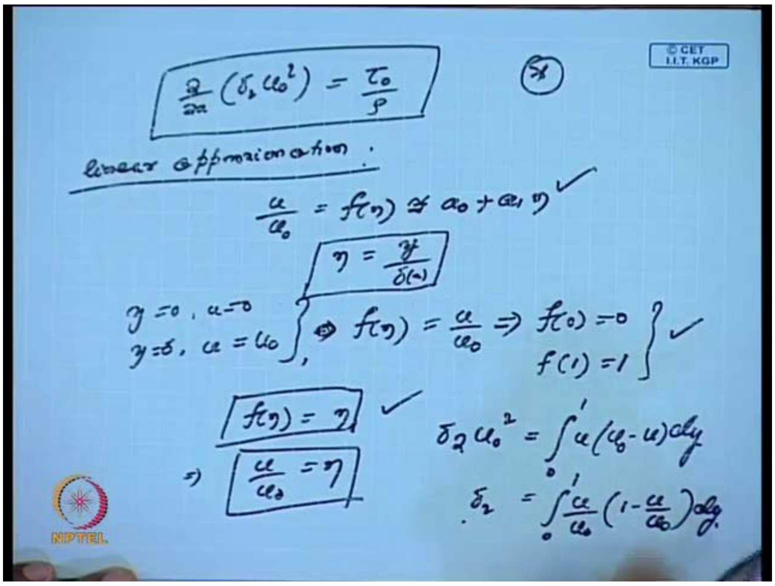

So, the momentum integral method equation will reduce to thereby del x it will further simplify it will be concentrate by del x delta 2 u naught square is equal to turn out by rho this becomes my equation of momentum integral equation and what is the boundary conditions. The boundary condition will be now to deal with what I will do; I will go for early linear approximation best on linear approximation the solution of this if I look into the linear approximation then what will happen. I will just calculate my u by u u naught u by u naught, I will say that this is f of eta and that is a 0 because a 1 eta where eta is nothing but y by delta, delta is a boundary layer it can be a function of h.

So, if this is the case then what will happen because if you look into the y is equal to 0 u is equal to 0 and y is equal to delta, u is equal to u naught because beyond y is greater than equal to delta u is equal to u naught. So, if I take this into account then that will give me which in place f of eta and further I say f of eta is u by u naught already, so if f of eta is u by u naught then f of eta is my u by u naught. So, f of eta this is 1 and f of eta is a by u naught which in place my y is 0 u is 0 at y is equal to 0 u is equal to 0, so that gives me f of 0 is 0. Further, y is equal to delta u is equal to u naught that gives me f of 1 is equal to 1 because beyond y is equal to delta that is y is equal to delta y is equal to delta means eta will be y, y delta eta is y by delta is 1.

So, in the process when eta is equal to 1 then u by u naught becomes 1, so this case means a 0 this becomes the two boundary conditions and my f eta is a 0, so I have two unknowns and two boundary conditions. So, if I substitute for f 0 and f 1 in this expression approximate solution, I call this as an approximate solution then I will get my f eta as eta f eta as eta. So, this should be just from this using this and this, so this becomes f eta is eta if f eta is eta then what will happen my u by so that means u by u naught becomes eta where eta is equal to y by delta.

Now, I will substitute for this u by u naught eta in the original equation that is my star equation, so if I do that then that will give me del 2. So, if I substitute for this del 2 u infinity, so that will give me del 2 u infinity or del 2 u naught square and it will be nothing but 0 to 1 u into u naught minus u d y this is what delta u infinity square if this is the then which implies. Now, I will substitute for I have a del by del x del 2 u and del 2 u infinities this becomes 0 to 1 if I call it del 2 when this becomes u by u naught into 1 minus u by u naught into d y now del 2 becomes this and if del 2 is this, so which gives me in terms of delta.

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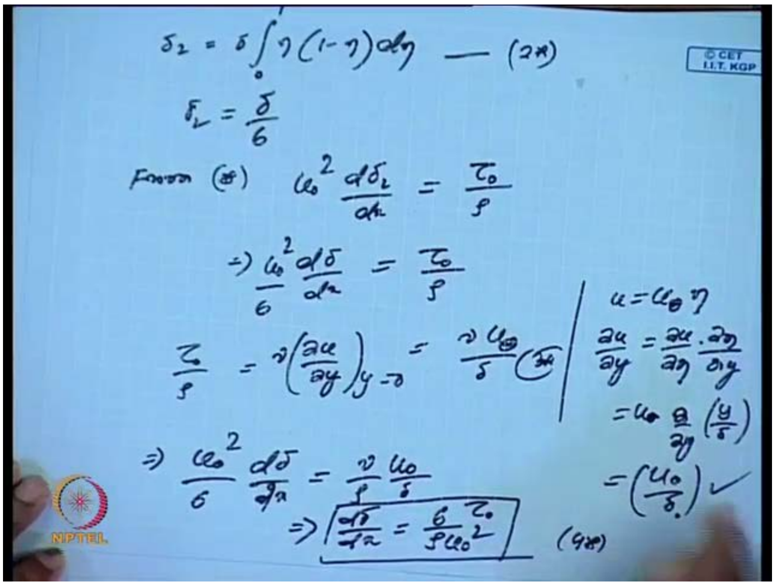

So, del 2 will give me delta times 0 to 1 eta into 1 minus eta d eta and which is nothing but delta by 6, so del 2 is delta by 6. Now, I substitute this same into the equation of integral equation in a similar manner, what I will get if from the from star, you say call it two star from star and double star I will get u naught square d del 2 by d x equal to tau naught by rho and which gives me d del thus del 2 is del by 6. So, I will have u naught square by 6 into d del by d x is equal to tau naught by rho, but what is tau naught by rho and if I look at tau naught by rho this is mu into del u by del y at y is equal to 0 and that is nothing but mu into u infinity by delta because I have u is equal to u infinity eta.

That means what will happen del u by del u del y, that will give me del u by del eta into del eta by del y and del u by del eta this is u infinity or u naught I am taking at u naught del eta by del y del eta by del y is eta is y by delta. So, that will give me del by del y y by delta and that gives me u naught by delta. So, this becomes, so that is why nu into u naught by u infinity, sorry that I am taking as u naught, so this becomes u naught by delta.

That is what I got tau naught by rho, so I substitute for this, so that will give me u naught square by 6 into d delta by del x d x is tau naught is mu by rho into u naught by delta. So, that brings gives me which in place d delta by d x is equal to 6 by rho this is u naught square and tau naught because tau naught tau naught tau naught is u naught delta. So, now if I substitute for tau naught in terms of nu u naught by delta then I will get it first substitute for this I call it three star from four star, so from three star and four star.

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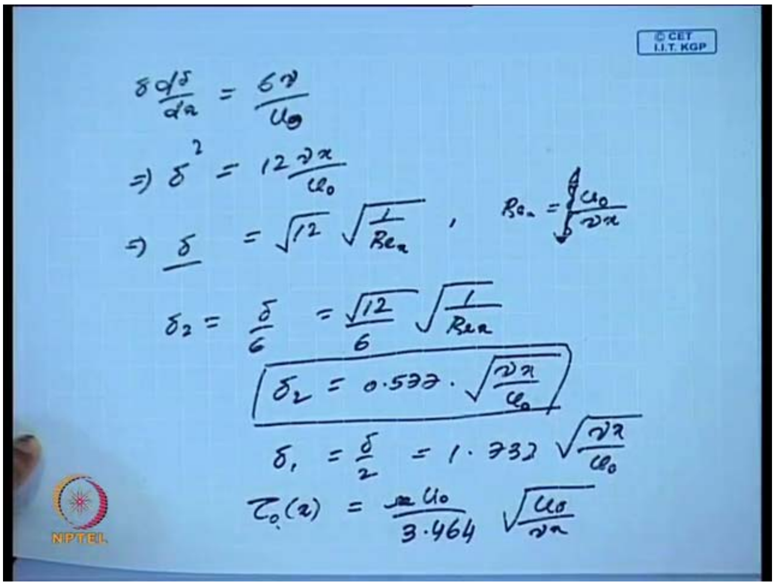

I will get delta into d delta by d x that will give my 6 nu by u naught and that gives me delta is equal to 12 delta square is equal to the square into 12 mu x by u naught and that means when delta becomes 12 root of 1 by r e x, because where r e x I am taking u naught by nu x. So, this becomes and then delta is this then delta 2 is delta by 6 that is 12 by root of r by 6 1 by r e x and that is root of a rho that becomes 0.577 into square rout what we called mu x by u naught this is my delta 2.

In the similar manner if I look at delta 1 I will see easily that it is nothing but delta by 2 and that will be 1.732 into mu x by u naught and in the process tau naught x that is the void shear stress settlers one can easily find that this is mu u naught by 3.464 into u naught by mu x this should be shear stress. So, this is what we have what we have got delta we have got del 2, we have got del 1 and we have got tau naught x.

So, you can always find the fixing coefficient and other factors associated with basically, if you want to find the drag coefficient then we can easily find it because we know tau x and we know all these parameters because from there it is placed. So, this is a way this is a 1 of the way what we have got that how we can solve the momentum integral equation.

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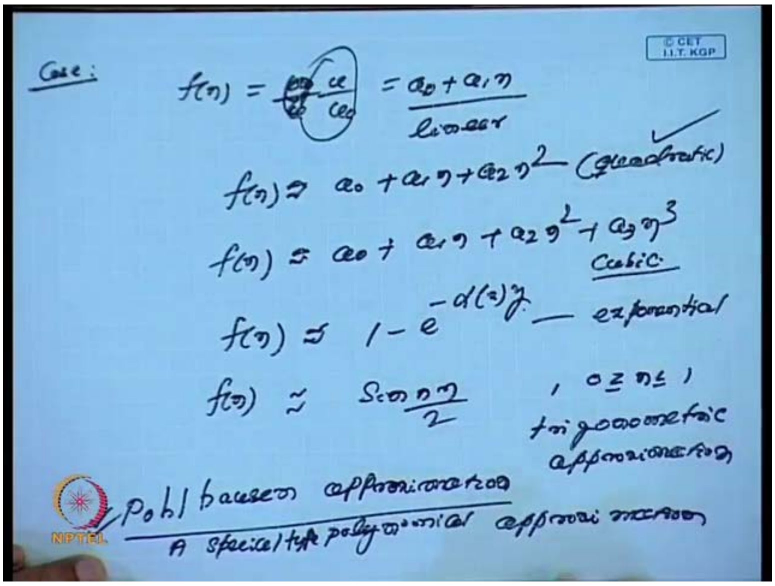

We can start with the several other cases, we can also consider to find the final solution of the momentum integral equation like instead of a considering f of eta as u naught by u, sorry u by u naught and that we have taken a 0 plus a 1 eta, that is a linear approximation. We can also think of approximation like f of eta I can always take as a 0 plus a 1 eta plus a 2 eta square or I can think of another approximation like it is a quadratic approximation and you can think of also a cubic approximation. I can also think of further approximations like exponential approximation 1 minus e to the power minus alpha x into y this is a expansion of this is a cubic approximation.

This is a exponential then we can also go for a to approximation the various approximation of course, all depends on there will be, but that depends on the nature of the problem. These approximations were the details and I will not go to that, but in case of a and other sinusoidal approximation that is sin pi eta by 2 0 less than equal to eta is less than equal to 1 this is called a trigonometric approximation and many more approximation many such approximation.

There is another approximation called Pohlhausen approximation, this is a have not, it is a kind of you can it is a kind of polynomial approximation it is a kind of polynomial approximation and rather called a special type a special type polynomial approximation this can be found in various text program. So, out of this approximation we can also find what eta is and once we know f of eta that means we know u by u naught the will velocity profile and then we can find the shear stress and other physical quantities of interest. So, I will just give him update for suppose I have a cubic, sorry I have a quadratic one then what I am doing what additional thing because my problem is how I am going for this, so in the case of a quadratic approximation.

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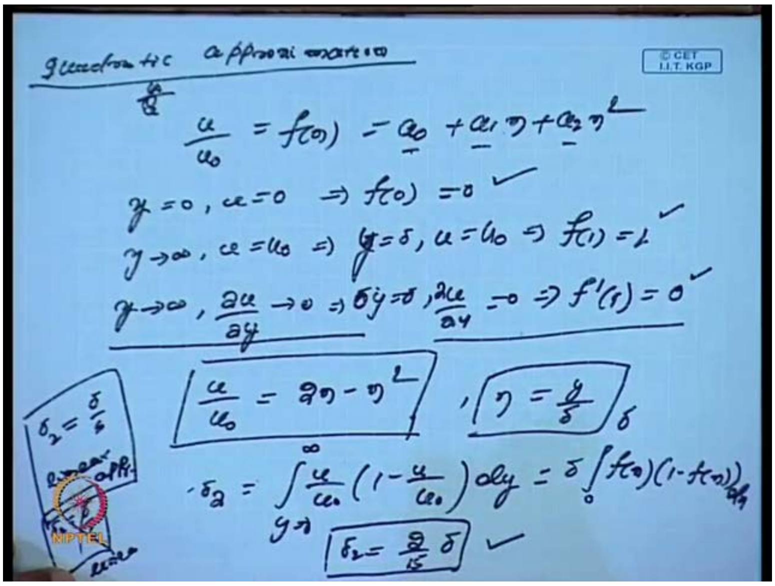

So, as I say that I will take u by u naught for a quadratic approximation just I will stay the intimated steps, how I go for this particularly, how I am adjusting the boundary condition. So, I have been given u by u naught is equal to f of eta is a 0 plus a 1 eta plus a 2 eta square. So, in this case what I do, I have the boundary condition because I have same they have the same semi infinity plate. So, y is 0 and u is equal to 0 and that gives me f of 0 is 0 and y tends to infinity u is equal to u naught which implies y is equal to delta y is equal to delta u is equal to u naught because beyond delta y is equal to it is same as above we write infinity.

So, that gives me f of 1 is equal to 1 then I have another one as y tends to infinity del u by del y also will tend to 0, it can be because as y tends infinity u becomes u naught so I will and it is a function of x. So, I can also say that y tends to infinity by del y tends to 0 and that play same as y, y is equal to delta u del u by del y 0 and this gives me f dash 0 f dash 1 equal to 0. So, earlier I had when I am going for a I was going for a linear approximation I used to take two terms and these were my boundary condition.

Now, when we are going for a quadratic approximation I am adding this condition, so that is there is no harm in adding this condition. In the process I am getting a better approximation if I going that and I will find out because I have no stream condition one two and three conditions on eta and this is my f of eta. So, I can easily find a 0, a 1 and a 2, so what I will if I take substitute for this conditions in this explanation for the polynomial f eta then I will get my u by u naught is equal to 2 eta minus eta square. This will be the approximation I will get this is my quadratic and in this case eta is equal to y by delta.

So, then again now I substitute for this in the momentum equation if I substitute thisin the momentum equation I will get when del 2 is equal to y is equal to 0 to infinity u by u naught 1 minus u by u naught d y. That will give me on delta times 0 to delta f eta 1 minus f of eta d eta and this gives me nothing but 2 by 15 delta. So, delta 2 becomes 2 by 15 delta you can just see that in the earlier case in case of k 1, I have I had a relation then I had seen that my delta 2 was 1 by 6 delta by 6.

There in a case when we have a linear approximation I mean delta 2 as delta by 6 in case of linear approximation, sorry delta by 6 linear approximations. On the other hand, we have delta 2 is a 2 by 15 into delta in this case of quadratic approximation, in a similar manner I can find out what exactly is happening the momentum I have.

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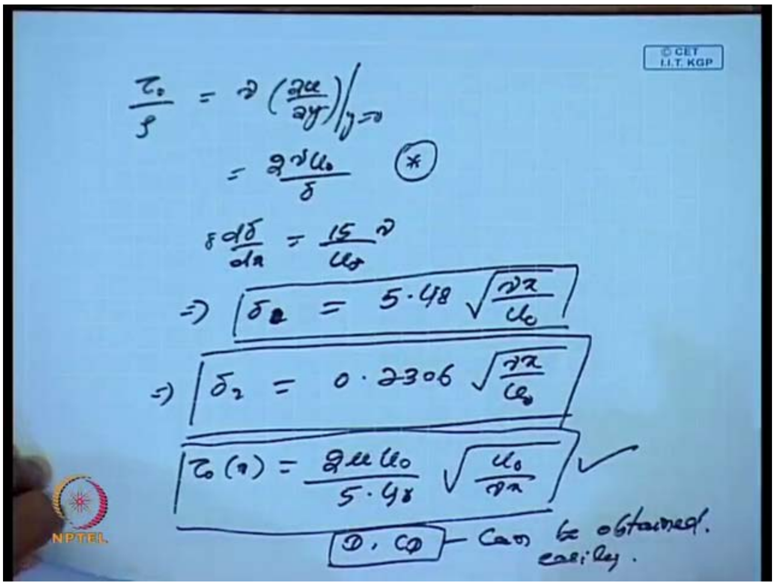

What will happen to my tau naught by rho, that will be a nu del u by del y at y is equal to 0 and this will give me if you calculate it that will give me 2 nu u naught by delta. And then again, if I substitute this delta 2 and tau naught by rho in the momentum equation then I will get delta d delta by d x is 15 by u infinity into nu and which gives me delta 2 is been delta as 5.48 to mu x by u naught and which in place delta 2 is 0.7306 into nu x by u naught. This is a method and this is by delta boundary layer thickness further, I will have tau naught x you can always find 2 mu.

So, tau naught x it will give me, this will be 2 mu u naught by 5.48 into u naught by mu x in case of a quadratic approximation. This is by shear stress and thus the other quantity like the drag coefficient C D D and C D can be obtained from this because we know tau naught x these are the can be easily obtained. So, these are the drag coefficient and then these are drag for square that is acting on the upper side of the plate that will be obtained easily. So, this is a way we can apply the momentum integral equation as I have mentioned that this is momentum integral method is suitable not only for boundary layer problems, but also for turbulent flow problems and in the process this becomes like a on the other hand.

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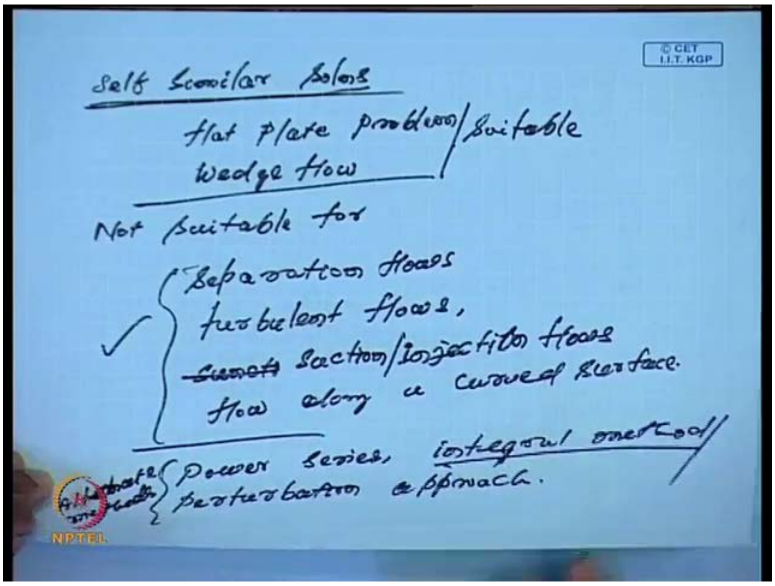

We look into self-similar solution, the self-similar solutions have only restricted is only restricted to a simple class of problems. It is not suitable for all kinds of problems like self-similarity this is only when we have flat plate problems or two flows this is more suitable self-similar solution is suitable. On the other hand, this kind of self this kind of self-similar solutions is not suitable for separation flow to analyze like separation flow, turbulent flows.

So, flow near suction or injection flow, sorry suction or injection flows are flow along a curved surface for these self-similar solutions. I am not saying, it rather the most appropriate are in this case a power series solution method integral method or perturbation theory or perturbational approach. So, self-similar solution only suitable for these, but not suitable for this class of solution and in this case we always go for the power series method alternate methods.

So, these are the methods we apply for this class of problem which is not possible for which at the self-similar solution approach will not be applicable and here the integral method, the process the integral method become it becomes one of the robust method to deal with a marginal class of rho. So, this understanding of the methods on this solution to deal with boundary layer theory will a not go further will not go into the details or further flow problems associated with a boundary layer. But we have just as I say that we will briefly discuss about what is boundary layer theory and the two dimensional governing equation and then two approaches we have talked about.

One is the self-similar solution up to and then the momentum integral up to equation method that is based on the residue method with a residue method and two examples we have worked out. Particularly in both the cases we have taken the same example to show the solution by the solution of a boundary layer flow equation by two methods. We have seen how the solution is easily obtainable without most of a difficulty, but there are last classes of problem for which solution can be easily obtained.

There is a large a series of lectures by under this NPTEL program by faculty of civil and mechanical engineering departments of I I T's. So, those lectures can be looked into for further detail and various types of rho boundary layer approach here is now I will just briefly mention because today we are almost completing our series of lectures. So, I will have few these things to mention that what about flow separation.

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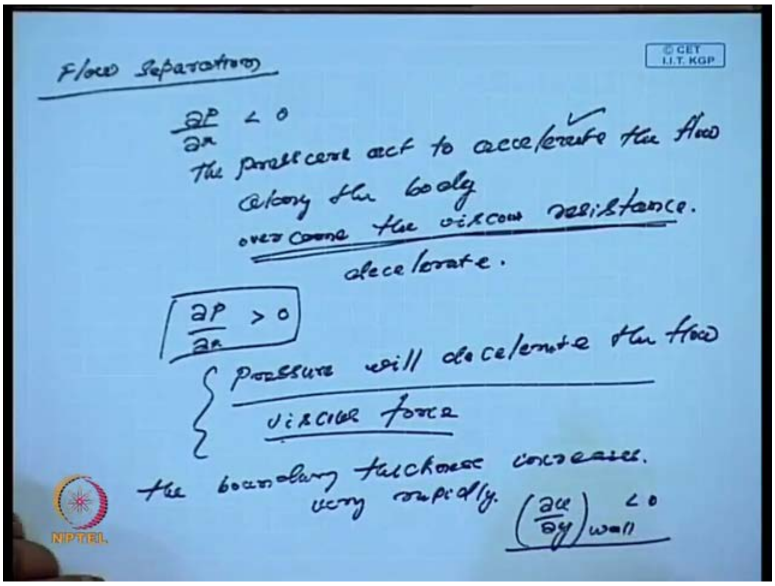

Already few points to mention before I conclude this flow separation, I have already told when the pressure gradient along a body surface becomes negative when del p by del x is equal to less than 0 the pressure act to accelerate the flow. Then the pressure act to accelerate the flow along the body and what it does it acts to work on the, in the process it overcome the viscous resistance because the viscosity will provide as a resistance of in the pressure act to accelerate the flow. So, it overcomes this viscous resistance is viscous percipate and in the process while this is viscous resistance what it does.

So, in the process the flow decelerate, on the other hand on the other hand what happens when the pressure becomes when del p by del x is greater than 0 when del p by del x is greater than 0. So, pressure gradients become positive once the pressure gradient is positive. So, here the pressure was acting to accelerate the flow, so and thus when the pressure gradient will be positive in this process the pressure will act to decelerate the flow and we will decelerate the flow.

One should decelerate the flow what will happen because viscosity is again is a viscous force, which is always acting on the body and that is decelerating flow and again this is also pressure reserves acting on it. This will also decelerate the flow and as a result what will happen the viscous portion the boundary layer thickness, so the pressure. Since, there are two negative two processes which are decelerating the flow, so in the process the boundary layer thickness increases very rapidly and the flow. So, reverse why in the process what are when it the, and in the process the flow del u by del y near the wall becomes negative.

That means the flow decelerate and moves in the speed the flow decelerate and in the process what happened that is a point at which the deduction of the flow in the flow will come to a stagnation point that means there will not be any flow on flow separation will occur.

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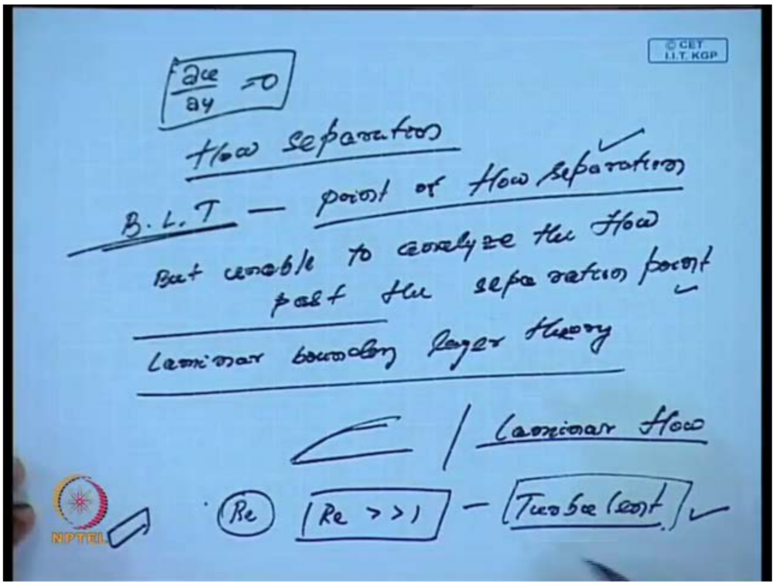

So, that will be a point when del u by del y becomes 0 and in the process, there will be the flow separation will takes place and up to the flow separation what happen after this if further it decelerates the flow fluid will flow in the opposite direction. In the process and there the pressure gradients again because we will change the direction from positive to negative and in this here what happen the boundary layer theory, this will able to determine the point of flow separation.

On the other hand, it will not able to tell the flow pattern a flow characteristics it talks about the point of flow separation it will credit the point of flow separation, but unable to analyze flow past. This point analyze the flow last this point past the separation point this is one of the very major important point of the boundary layer theory the boundary layer theory only talks about the point of flow separation. It can identify, but it will not able to talk about the flow pattern at the separation point there were several there is another thing.

So, already we have talked not about the laminar boundary, boundary layer theory and theory again this laminar boundary layer theory holds when just the fluid becomes viscous just adjacent a wall boundary and beyond the wall boundary in the provision we said. On the other hand, we have also talked about laminar flow for directional flows intermediate from intermediate Reynolds number, but in case of a Reynolds number laminar of boundary layer theory holds only for a class of problem. And again beyond when r e the Reynolds number r e extremely large becomes large much larger and then boundary layer theory results a boundary layer theory will not hold good then the flow become turbulent.

In this region of once the motion is turbulent it is very difficult to predict the pattern on the flow and the flow becomes random and it becomes very regular and the methods which we were using to analyze. It will becomes very difficult to analyze the flow the classical methods in many situation fails. However some of the methods like momentum integral equation methods can be applied, but the theory that is developed to deal with boundary layer equation no more hold.

It has to go for a the theory has to be again looked into the flow pattern and other things there are various types of equations which will be again derived from the navier stokes equation. So, in that situation we are not going to those details of the theory of the turbulent flows, but we just stop here about this laminar flows boundary layer theory. Turbulent flow theory will not talk about in this series of lectures in marine hydrodynamics itself will lead to a different type of analysis and we are not going into detail.

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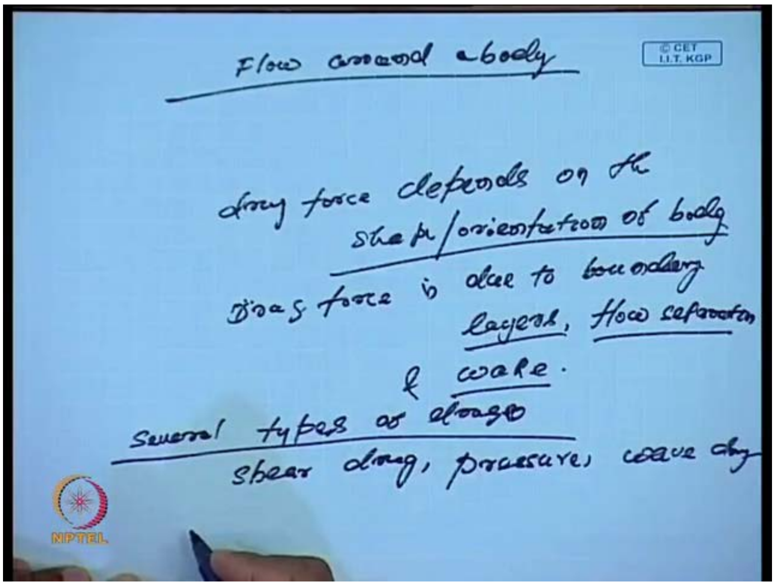

However, before concluding I will just talk flow around a body with this I will conclude my lecture, flow around a body what happens when a body moves through a fluid or when a body fluid flows past a body either a body is moving or the fluid is moving in a body fluid is moving body is fixed.

So, there are two types of resistance that appears one of the resistance is encounter the body encounters one types of resistance the other kind of type of resistance is the fluid encounters resistance that is a body encounter encounter resistance. The other is a fluid encounter resistance some kinds of resistance for example, when a aero plane flying in the sky particularly in the atmosphere or a submarines that moves a through water. The propulsion system has to exert a force that it should be sufficient enough to balance the forces like the drag forces that is called the one of the force that is called the drag force.

On the other hand, for a body in most in a fluid stream that drag force depends again depends on the shape of the body and orientation of the body shape and orientation of the body. The drag force is due to and this drag force is due to again due to the boundary layer drag force is due to the boundary layer boundary layer, due to flow separation and wake then what is wake.

So, there are several type when this is the drag force is due to the boundary layer flow separation and wake then another thing is here there are several type types of drag types of drag force one. Finally, experience by the body some of them some of them are shear drag one is a pressure drag and one is called wave drag. So, there are three types of drag basically face drag by a body when it is a must in the water or in any fluid and when the flow takes place with the separation.

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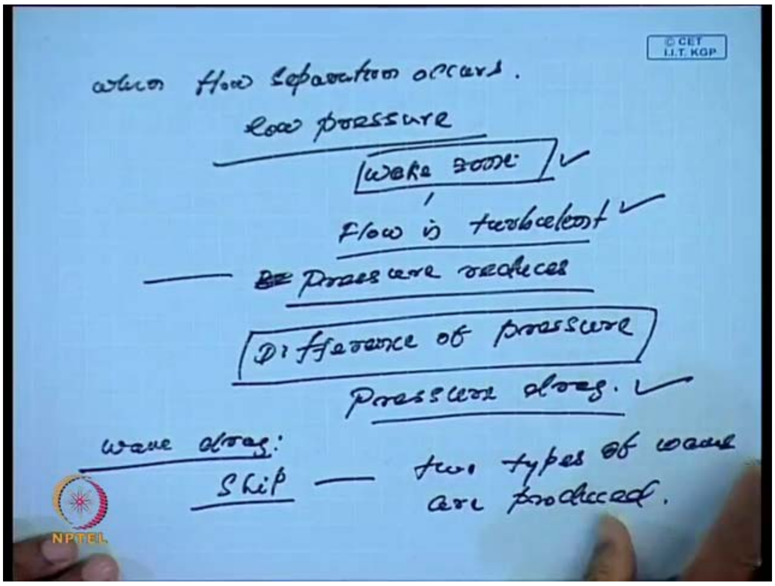

When there is a flow separation what happens, it gives rise when flow separation occurs when flow separation occurs it gives rise to a zone of negative low pressure it gives rise to a region of zone of low pressure. And in the low pressure this region this zone of low pressure is called the wake zone wake zone just after separation flow occurs this wake zone is developed.

Then the flow in the wake zone is highly turbulent, in this zone the flow is turbulent, it is very difficult the laminar cloth area the viscous flow is turbulent. It is very difficult to analyze by using the classical boundary layer theory here and again what happen here as a result because there is a flow becomes turbulent. So, in the process energy distribution takes place at a higher rate with a reduction in pressure.

So, the pressure reduces up to this, so as I said there is a low pressure is an un pressure reduces, but what happen in the process in the initially the body there is a different separation is developed around the body because initially the pressure was high. Once one side of the body on the front side and on the back side different separation is developed and this difference of pressure leads to the pressure drag. So, the front side of the pressure becomes higher than the back side of the pressure and that is a result.

So, this force are arising this to arising due to this pressure difference is called the basically the pressure drag then and again it is this pressure drag again depends on the nature of the structure or shape of the structure. Basically, it depends at the form of the body then on another aspect of this what I say that wave drag what again happen in this case. If you look into particularly this happens in case of a ship when body like a ship which moves partially in the water and its open partially is immersion in the water. So, there are two types of waves, two types of motion produced two types of a waves that is one wave is produced as a front.

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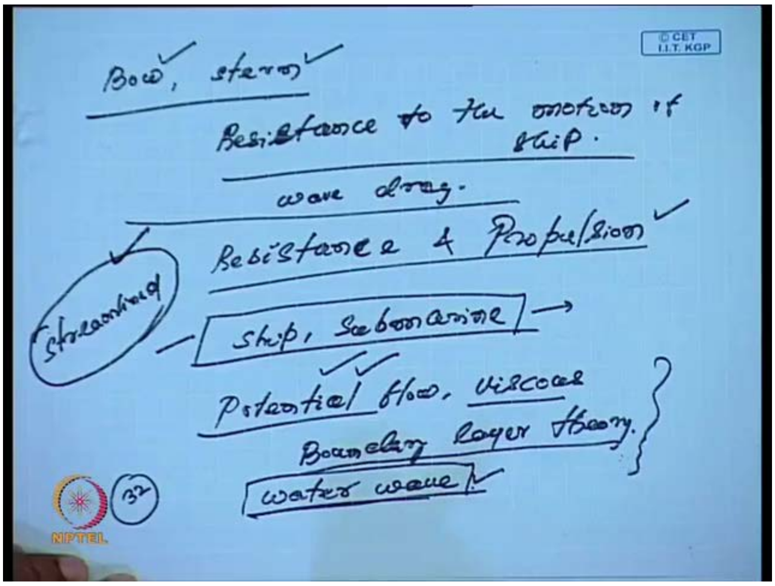

So, one is at the both the ship and yeah at the both there is a type of wave, that is produced and another wave which produced the stern one of the ship this type two types of wave. These waves, one wave produce the bow the other wave produce at the stern these wave provides a resistance they provides resistance to the motion of the ship motion of the ship and these kind of forces this forces are called wave drag.

So, the details about the behavior kind of forces acting on a ship particularly it drag forces and resistance forces will not go to the details just I have given you a brief introduction how these things happens. The details will be cover in the course called resistance and propulsions is a normally taught in the basically these kind of courses are taught to students in sub hydrodynamics ocean engineering or in the aerodynamics students, those who in the department of various spacing engineering hence the understanding, so what we understood here.

So, with the point of a talking about this flow are only body is that that there are so many kinds of we have two took into wave forces. We have to look into the pressure forces that is acting various types of resistance and again the boundary layer the flow separation a boundary layer the wave formation and again the viscosity characteristics as well as their rotational motion characteristics of the fluid.

So, this example itself gives us a background that the importance of this course of marine hydrodynamics because in this course we are able to we will able to deal with problems of last class. Problems associated whether it is a motion of a ship or a submarine ship or a submarine or any other person related vehicles or even if a structures you can always know what are the forces that is acting on this. Again, we have talked about wave forces in detail and wave characteristics we have understood various wave characteristics.

So, all these will give us a good understanding basic understanding about the various aspect of marine hydrodynamic various aspect of the marine fluid in marine environment. Basically, the subject what we call them marine hydrodynamics, so here we have problem one potential flow we have also introduce to the potential fluid for other potential flow then we talked about viscous flow viscous flow then we have introduce to boundary layer theory.

The important part, all this search in a taught in a very scattered manner, but finally the part which is more important to particularly to students of ocean engineering naval architecture is the wave part, water wave part. In this way this potential flow though there is communality for a class of students of aerospace and engineering. But major part of this I did not spend most time on the viscous flow as well as on the boundary layer theory because this courses that in detail various flow problems, which will be which are being taught to students of various engineering departments particularly in physical sciences.

So, the theory can be borrowed from their when it requires, but I have spent more time the potential flow theory as well as on waves. In fact we have taken almost thirty two lectures on this potential flow and the wave problems where as we have just introduced in eight lectures around seven or eight lectures about viscous flow boundary layer theory.

Another important point here I would like to highlight is that this drag reduction particularly when it comes to design upper ships or some marines or any ocean related to vehicles. We unless consider the body as a streamlined body and the idea is that because there are so many drag force act on the body one of the main objective of this particularly for the designer is to reduce the drag on the structure.

So, the body when the design when they look in to the design of the structure the streamlined body one of the basic objective of the design is to ensure that we are minimizing the drag that is on the body. So, that will have less the, you will have less resistance is on the less drag force that will be acting on the structure. And in the process the ocean going vehicle, so will be moves without full consumption and other design related life and life of the vehicle are even if and rather robustness of this truth will be there.

It will be one of the best the better design and better design application the most of these structures which are ocean or even if in now a structures as are urban vehicles they are one of the main thing is that we are loosely look into streamline body. These streamline body are such that that will help us in introducing the drag with this understanding and these much details, I will stop here this course. This is last class and if there will be a need I will again take few classes, but we will for time being we will stop here.

Thank you.