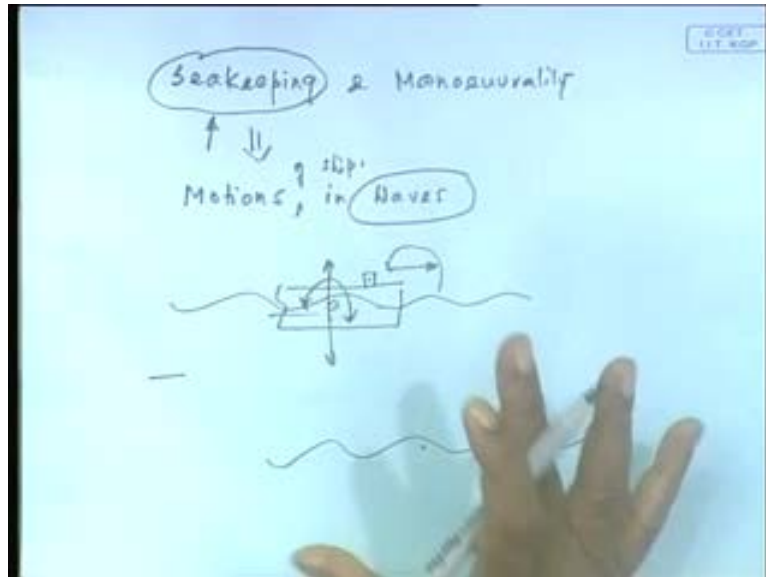


**Seakeeping and Manoeuvring**  
**Prof. Dr. Debabrata Sen**  
**Department of Ocean Engineering and Naval Architecture**  
**Indian Institute of Technology, Kharagpur**

**Module No. # 01**  
**Lecture No. # 01**  
**Regular Water Waves – I**

The **this** course, the name of the course as you have seen is, I just write it once again.

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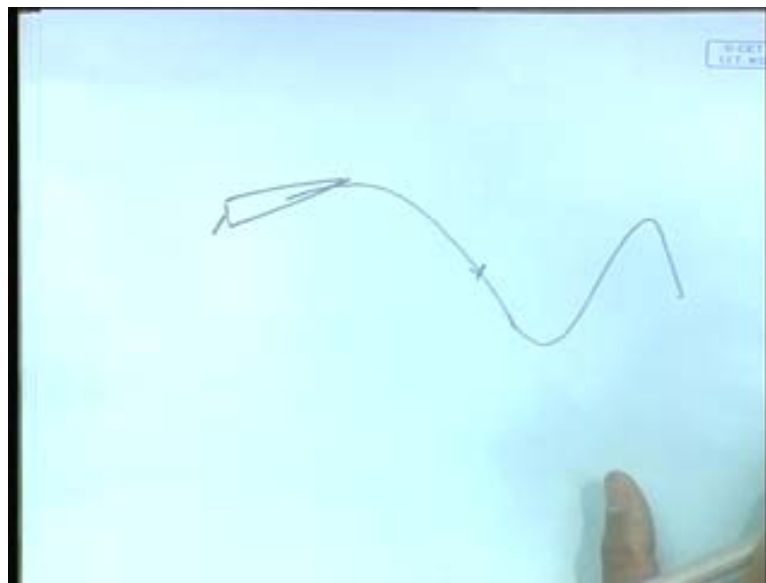
(No audio from 00:27 to 00:37) Actually it is in two modules, **I can we can** we can say that, it is in two large modules one is Seakeeping, which essentially deals with how a ship behaves in waves or rather one can say motions in waves.

Now, by that what I mean is supposing you consider an ocean surface which will have existing waves and you take a ship which tries to move, it undergoes all kinds of motions up, down etcetera. Now, not only the motions as a consequence of the motions there are many effects such as, the bow of ship might come out of water and bang, water may go on top there can be large acceleration in one part and here can be an equipment which undergoes a large pressure large force, because of that; even human beings are subjected

to acceleration and therefore, they tend to fall sick and have motion sickness; all these aspects, which are a consequence of the fact, that the ship is moving in waves and therefore, undergoing motions is the part of seakeeping.

So, the first module of the course is basically this part which we will see later on is connected to essentially the ship motions in vertical plane, that is this way this way and about the other axis, x axis this way.

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The other part of the course, the second module which would be approximately half the course, manoeuvring is connected to or related to the behavior of the ship, in the horizontal plane, supposing this stage you consider it to be the ocean surface.

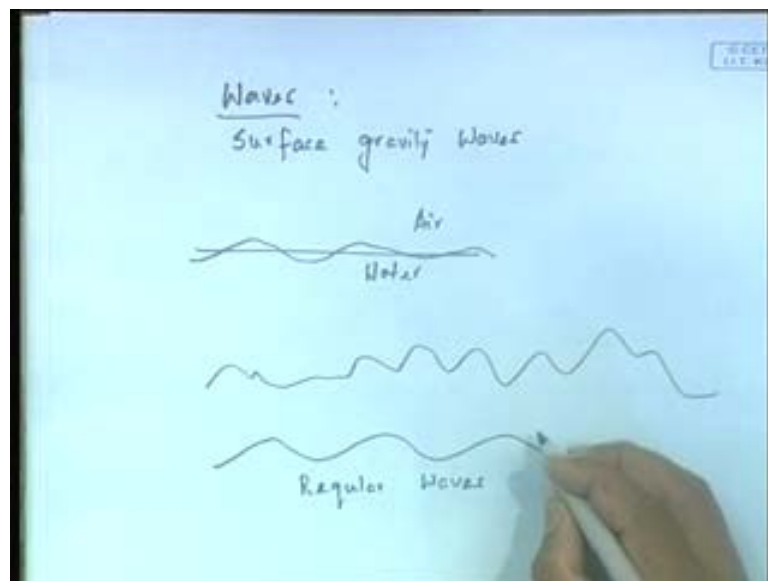
Now, it wants to turn, make a turn or it want to take a trajectory which is zigzag, how will it do that how would it manoeuvre by which instrument or equipment part of the hull, it will under **you know** it will induce this kind of motions, these entire subject is manoeuvring, which is what we will do in the second half of the course.

Now, what you will get out of the course is, after the first half for example, what the course will lead to you is an ability to assess the, so called sea keeping qualities, it is what sea keeping if I again bring the first one back the word has come from breaking it as how the ship keeps a sea.

Supposing a ship is excessively undergoing roll or undergoing pitch, people would not want to be on the boat; supposing you have an air craft area and **you know** it could not operate or air craft could not land, if the deck was undergoing excessive motion. Therefore one would have to have some kind of a limitation this assessment is the ship good can it withstand. So, much of sea severity of sea, does the role exceeds certain amount of degrees, etcetera all these ability you will be able to do at the end of the course or at least have an idea how to go about.

So, this is the general instruction, now having said that, since this course you will see here it is motions in waves, motion of what of ships I can add here, but where in waves, so therefore, at the very beginning we require to understand how do you define a wave. Waves becomes **the** you might say my input, my I have a surface where I got waves, I must first understand this waves which is my environment in which I put my ship. So, therefore, the rest of the today's class and tomorrow's class, we will spent trying to recapitulate and summarize, how do we characterize, what is known as regular waves.

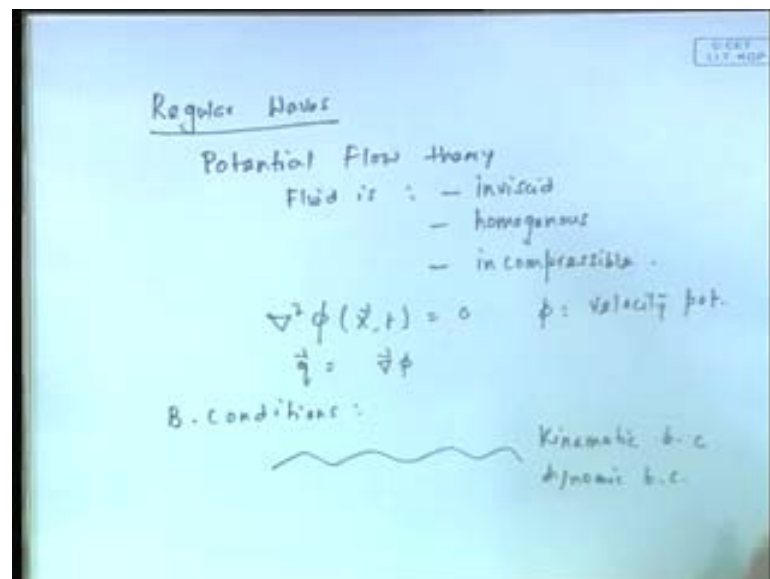
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So, I will now go to this fact of waves, strictly speaking one should write this with a additional **you know** like qualification, surface gravity waves, now by definition if I have a free surface; free surface meaning an interface between water and air and if I cause a disturbance on the surface, any kind of disturbance under action of gravity, the surface undergoes an undulation and this is known as waves this is the definition of waves.

Obviously now, there are many kinds of waves for example, if you went to an open ocean you may purely find a wave surface looking like that, but to before we get there the first part of the study is if I consider a single periodic oscillation, which gives rise to a wave of this nature, that you might have seen many times depicted many books, many **many** places this is what is called as regular waves. Now, this regular wave has already been studied before this course as a pre requisite in other courses like main in hydrodynamics, but I need to still recapitulate certain parts.

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Now, regular waves we know theoretically that, regular waves are studied based on potential theory, which means that the fluid is assumed in viscous and the flow is assumed irrotational free surfaces is also assumed homogeneous and incompressible that means (No audio from 07:41 to 08:04), actually this gives rise to this classical governing equation called Laplace equation as we know, we end up getting, where phi is known as the velocity potential, such that velocity vector q is actually grad phi.

This is a very basic definition of irrotational motion, fluid flowing irrotational motion now, for surface gravity waves if you want to study turns out, you have to have certain boundary condition imposed on the fluid.

So, there are, so called boundary conditions there are two boundary conditions that become **(C)** one is what we called kinematic boundary condition and one what is called dynamic boundary condition.

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$$\phi = \frac{g \cdot A}{\omega} \frac{\cosh k(z+h)}{\cosh kh} \sin(kx - \omega t)$$
$$k = \frac{2\pi}{\lambda}$$
$$\omega = \frac{2\pi}{T}$$

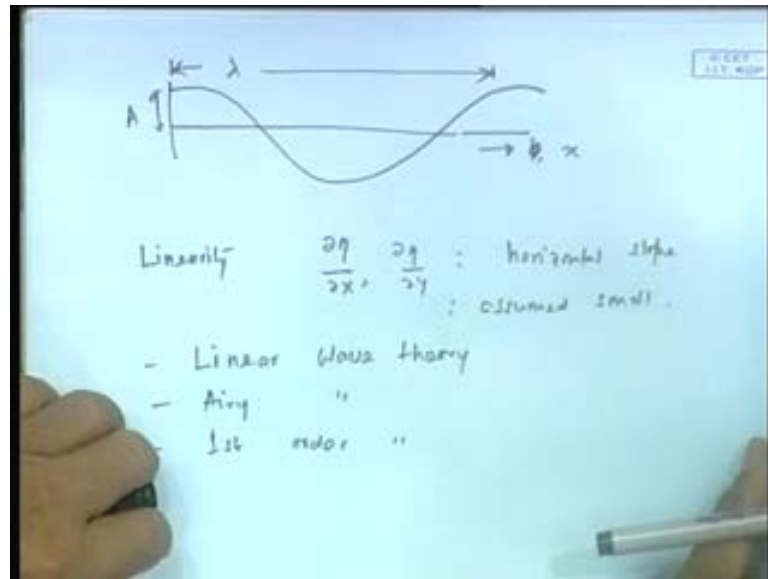
A = Wave-amplitude

$$\eta = A \cos(kx - \omega t)$$
$$\downarrow = -\frac{1}{g} \frac{\partial \phi}{\partial t} \Big|_{z=0}$$

Now, having gone through all this it turns out which is what I will simply recapitulate, that the surface gravity waves potential turns out to be equal to without prove we are saying, because this is supposed to be in a pre requisite to a course and you would have known that it turns out for a linear surface gravity wave the phi turns out to be equal to  $g$  by  $\omega k$ . Where well this is a kind of solution that comes where let me specify this parts,  $h$  is water depth in this formula,  $z$  is positive upwards, origin is at the mean free surface that is  $z$  equal to  $0$  would imply the mean free surface,  $a$  is the amplitude of the wave  $\cos h$  **cos h**  $k h$ .

So, in this case  $k$  is known as the wave number given by  $2 \pi$  by  $\lambda$ ,  $\omega$  is known as the wave period  $2 \pi$  by  $t$ , wave here is actually,  $A$  having an amplitude and the wave profile  $\eta$  is given by  $A \cos k x$  minus  $\omega t$ , actually  $\eta$  is obtained from the fact that is minus  $1$  by  $g d \phi$  by  $d t$  at  $z$  equal to  $0$  you can confirm that this if I do this operation on this it becomes like this.

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So, here the wave profile therefore, would look like a cross curve, if were to call this where it can be actually  $t$ , it can be also  $x$ , it can be either time axis which means you are looking at one point or it can be at a given time  $x$  axis, avail depending on that if I were to call this  $t$ , then the **the** time taken for it evolve is period  $t$  rather than I want to call this  $x$  here, then this becomes my wave length  $\lambda$  this is my  $A$  etcetera, etcetera.

What it means therefore, you see is that according to the theory **according to the theory** in which we have already made an assumption of smallness of the amplitude with respect to length, which means in evolving this particular getting back to that, in evolving this theory we had made an assumption of linearity. The word linearity implies here physically, that we have said that its slope that is  $d\eta/dx$  or  $d\eta/dy$ , that is it horizontal slope is assumed to be small this was an assumption inherent based on which the formula, that the profile is sinusoidal has emerged.

So, therefore, what we can say is that the sign profile is a theoretical solution of a regular wave, based on the affect that the slope has been assumed to be small, this is what is called as the number of **number of** terms it is we can call it linear wave theory, you can call it airy wave theory, you can call it first order wave theory or whatever.

So, all these are basically synonymous all these means, that you made an approximation that the slope is small and if that is the case then you end of getting a profile given by  $\sin$  curve and what is most interesting to see here is that, you will find that the  $\phi$

expression back here is sinusoidal with respect to special axis x, that is with respect to the special coordinate and with respect to also time coordinate t.

So, therefore, if I were to take a gradient of y with respect to either time or with space it would also be sinusoidal and it turns out all quantities of physical interest are actually gradient of phi.

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$$\vec{v} = \text{grad } \phi$$

$$u = \frac{\partial \phi}{\partial x}$$

$$w = \frac{\partial \phi}{\partial z}$$

$$\dot{u} = \frac{\partial}{\partial t} \frac{\partial \phi}{\partial x}, \quad \dot{w} = \frac{\partial}{\partial t} \frac{\partial \phi}{\partial z}$$

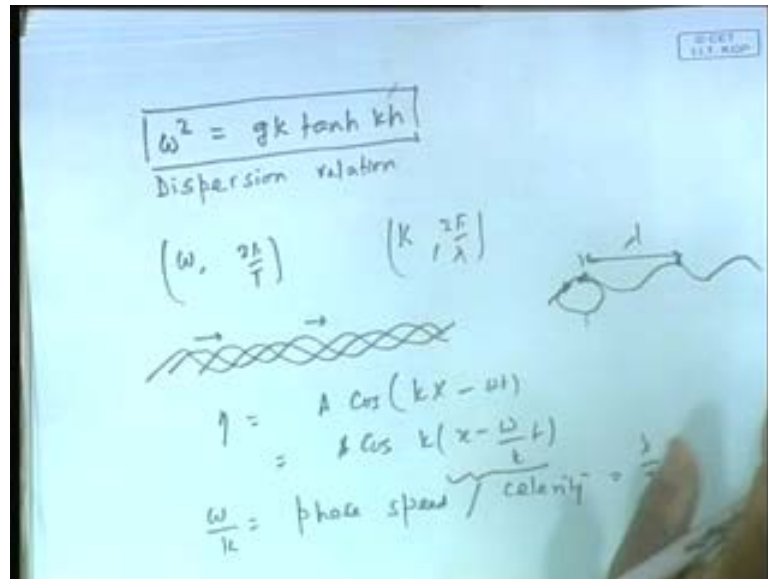
$$p = - \underbrace{\rho g z}_{\text{Hydrostatic pressure}} - \underbrace{\rho \frac{\partial \phi}{\partial t}}_{\text{Linear dynamic pressure}}$$

For example, if I want to find out velocity say v of the practical it is given by grad phi; that means, u is given by d phi by d x, w vertical it is given by d phi by d z. If you want to do u dot that is acceleration that is d by d t of d phi by d x, if I want to do w dot horizontal acceleration it is d by d t of d phi by d x other important thing is pressure if I want to find pressure; the pressure is given by these expression in here, this is my hydrostatic pressure and this is what is called a linear dynamic pressure.

Remember linear dynamic pressure the word linear because there was another term, which was second order which we neglected according to our linear wave theory, now you find out that, this linear dynamic pressure is gradient of phi in time.

So, if you were to do this differentiation of my phi curve obviously, all them is going to be sinusoidal in some form or other, so therefore, a turns out that in **wave** linear wave phenomena, be it velocity, be it acceleration, be it pressure, everything has a periodicity of frequency omega and time period t as **you know** omega is 2 pi by t etcetera.

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So, having said that this is only the physical characteristic, but one most important thing of the linear water wave, which we have not said is the fact that, it follows a relation called dispersion relation given by this expression, this is called dispersion relation.

Now, this is a very, very important relation as far as water waves are concerned why, so you will find out that, here my left hand side is omega, now omega which is nothing, but to pi by t, so omega T the time period is related on this side to K. What is K? K is nothing but 2 pi by lambda that means, essentially if for a given water depth h which means that if for a given water depth, I have a wave length lambda it is necessarily of certain variate T I cannot have for example, in a given water depth a wave period of 10 second and 20 second of same length.

Now, this you can contrast for example, with electromagnetic radiation, where you can have different wave length, different frequency, but there is a speed is constant, we will find out here that, now what is my speed of a wave see, when I have this wave here next instant it goes like that, next instant it goes like that, the form of the wave is traveling. In fact, I can show that if I were to write this eta to be A cos k x minus omega t this can be written as A cos k x minus omega by k t and you will see that if you have to travel at the speed omega by k, with respect to this wave to you it will look stationery, which essentially mean the form is traveling with the speed omega by k.



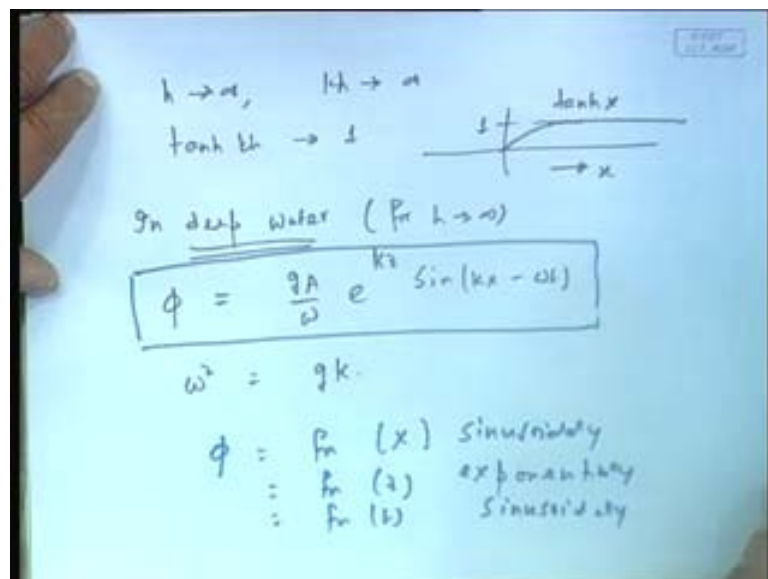
So,  $\omega/k$  becomes my phase speed or some people call it celerity and it is interesting, because  $\omega/k$  is nothing, but  $\lambda/T$  if you see because  $\omega = 2\pi/t$   $k = 2\pi/\lambda$  and obviously, it makes sense, because what is happening is that, this is  $\lambda$  and the time for it to the form to actually go down one cycle and come up which would imply as if this has gone here is  $T$ .

So, therefore, you would think this form has travelled from here to here in time  $T$ , so this is what is phase speed, remember it is the form that is traveling not the particle itself, we will see later on that the particle here, they move in a circular fashion we will come to that in a minute.

Now, having said that this  $\omega^2$  etcetera, now I will actually come to one simplification, what is happening here you see that in this expression this is all for water depth  $h$ , now in if for example,  $h$  was very deep in other words  $kh$  is **you know** like  $2\pi$  by  $\lambda$  into  $h$  is very high,  $\tanh kh$  will tend to actually 1 like this relation this  $gk$ , this  $\tanh kh$  this will tend to actually 1 and even this relation will all simplify.

In such cases what happens my relation this relation becomes  $\omega$  is equal to  $gk$  and in fact, this relations, in fact or become exponential  $kz$   $e^{\text{power of } kz}$  minus  $kz$ .

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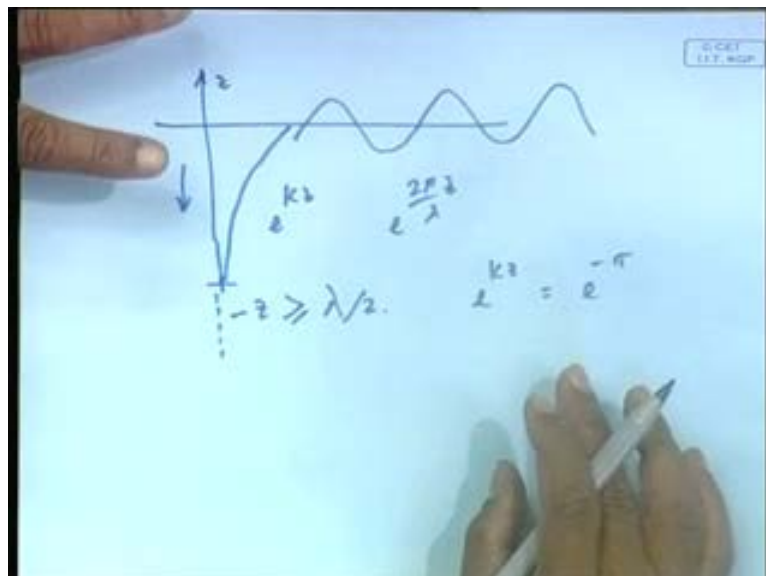
So, in deep water when I assume  $h$  tends to infinity then  $kh$ ; obviously, tends to also infinity because **you know**  $kh$  is  $2\pi$  by  $\lambda$  into  $h$ .

Then  $\tanh kh$  tends to 1, because  $\tanh kh$  actually if you **if you** write  $x$  here  $\tanh h$  graph looks something like that equals to 1 that is  $\tanh x$  and there are similar relation for  $\sinh$  and  $\cosh x$  for large values of  $x$ .

So, if I put those thing it turns out in deep water well, the word deep water implies for  $h$  tending to infinity for now I am saying this, now we will now find a practical name of what is  $h$  in a minute it turns out  $\phi$  reduces to  $g A$  by  $\omega e$  power of well actually this will become depending on which side the  $z$  is the  $z$  is plus opposite. So, therefore, it is plus  $kz$  and of course,  $\eta$  remains same and my dispersion becomes  $\omega^2$  equal to  $gk$ .

Now, this is another thing what I will like to tell you here, let us look at this **this** relation you will find here two things, one is that  $\phi$  is a function of  $x$  sinusoidal is a function of  $z$  exponentially and function of  $t$  sinusoidal. So, therefore, what is happening when I differentiate this number 1 is that, it always remains sinusoidal, because when I differentiate it supposing I differentiate it with respect to  $x$  it is sinusoidal, if I differentiate with respect to  $z$  also it is sinusoidal, because this term get differentiation if I do it against  $t$  also sinusoidal.

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But other thing is that its  $z$  dependence is exponential; that means, that if I were to see a property of a water wave along  $z$ , every property is reducing sinusoidal and every property is harmonic in the horizontal plane, whether it is with respect to space or with

respect to time. This is very important because, we will find out afterwards then when I put a ship here; obviously, it is it would also respond to this harmonic excitation **you know** something is being pushed every 10 second, so it will also have that every 10 second oscillation.

So, this is extremely important from water wave that things come down exponentially, how does it come down, with  $e$  power of  $kz$ , what is  $kz$ ,  $e$  power of  $2\pi$  by  $\lambda$  into  $z$ . Now it turns out that normally if  $\lambda$  or rather I will tell you if  $z$  is more than equal to  $\lambda$  by 2 then  $e$  power of  $kz$  that is becomes  $e$  power of minus well, minus  $z$  I will say because  $z$  is plus opposite minus  $\pi$ , this value itself is something like 0.04. And normally just like, when we say **you know** the  $\sin \theta$  is  $\theta$ , if  $\theta$  is less than 4 degree similarly we will say that normally, when  $e$  power of the exponential part has become more than  $e$  power of minus  $\pi$  it is almost 0.

So, the rule of thumb is that, **that** is how this word came that is water depth is more than half the wave length, then anything that is happening below that is, so small almost 0 then you can ignore it. So, we therefore, we say that if water depth was more than half the wave length, we consider the water depth to be deep water or it is a deep water case.

On the other hand if the water wave is less of course, that would not hold, in fact, we call that too intermediate water, now why I am saying is because in our course of seakeeping normally, we always deal with deep water cases why because a typical wavelength would be about 100 meter, 200 meter, 300 meter, may be 400 meter, but ocean depth were ships operate runs in 1000 of meter.

So, mostly the operation of ship would be in deep water, whatever the water is therefore, rest of the course when we talk we will only consider deep water cases, because ship operating deep water, when we are looking at ship, you can always refer to the exact equation with  $h$ , it is an unnecessary complication; there is no point of doing it, because when you actually put numbers it will turn out that  $e$  power of  $kh$  **you know**  $\tan kh$   $kh$  will become almost 1, 0.99999 something.

So, having said that this is my property of water wave, now I will try to tell you about the practical **you know** velocity etcetera **etcetera** well before that let us now spend little more time on dispersion relation still.

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$$\omega^2 = gk$$

$$c = \frac{\omega}{k} = \frac{g}{\omega} = \frac{gT}{2\pi}$$

$$= \frac{g}{\sqrt{gk}} = \sqrt{\frac{g}{k}} = \sqrt{\frac{2g}{\lambda}}$$

$$\left. \begin{array}{l} c \propto \sqrt{\lambda} \\ c \propto \frac{1}{\omega} \\ c \propto T \end{array} \right\}$$

$$\begin{array}{cccc} \omega/T/\lambda/c & & & \\ | & | & | & | \\ 1 & 1 & 1 & 1 \end{array}$$

So, I have this omega square equal to g k, now speed c what is c, c is omega by k, so that is g by omega, now g by omega is what g by, now you see it is g by omega means speed depends on frequency inversely, so if I want to write omega T it will become 2 pi by T.

So, therefore, speed depends on period, but if I want to write in terms of the length g by c omega, I can represent as root over of g k, because omega is root over of g k, so this will turn out to be root over of g by k it is root over of g by lambda by 2 pi, so what do I find c  $\propto \sqrt{\lambda}$  with root lambda, c  $\propto \frac{1}{\omega}$  with 1 by omega, c  $\propto T$  with T.

So, this tells us that celerity of water wave or phase beat depends on length and of course, length and frequency are connected therefore, it depends on length or period or frequency, which means in deep water if I were to tell omega or t or lambda or c all of them, in deep water and also in shallow water for a given h would immediately relate to each other, there can be only one value for this and this and this (Refer slide time: 27:46).

So, you cannot have for example, two different wave lengths travelling at two different speed, this is where I was earlier mentioning water waves differ from electromagnetic radiation, because in electromagnetic radiation or in sound **sound** waves for example, sea is constant for example, in sound waves if sea was not constant you probably would not hear me, because the ear's frequencies would have reached your ears differently.

If the electromagnetic radiation was not of same speed, you would not see the picture white, because all colors would be different, but in water waves it is, so **so** this is why water waves are called dispersive, because if 10 water waves start of 10 different length at 1 point, then eventually the longer one would have travelled this much shorter one, longer one would have travelled further.

So, **you know** it would have separated out this is what we called dispersion relation very, very important relation and what happen, now coming back to this **this** part (Refer slide time: 28:53), this pi we are writing this  $c = g \lambda$  by omega, some people will write in terms of some other parameter, because I can now write omega in terms of t in terms of lambda in terms of k.

So, there this part can be rewritten depending on what parameter you want to use as independent parameter it can be something like  $c = \sqrt{g \lambda}$  for example, I can write  $c = g \lambda$  by omega, as  $c = g \lambda$  by well not in this case, in the deep water case  $c = \sqrt{g \lambda}$  and I can relate that root over of  $g \lambda = 2 \pi \lambda / T$  etcetera. So, this is a question of only convenience of how we proceed, it really depends on the way we want to look at it.

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The image shows a handwritten derivation on a light blue background. At the top, the wave period  $T$  is written. Below it, the wavelength  $\lambda$  is expressed as a function of  $T$  using the dispersion relation for deep water waves:  $\lambda = \frac{2\pi g}{\omega^2} = \frac{2\pi \cdot g \cdot T^2}{(2\pi)^2} = \frac{g T^2}{2\pi}$ . A note indicates that  $g \approx 1.56 T^2$  in SI units. Below the equation, three examples are listed, grouped by a large right-facing curly bracket:  $T: 8 \text{ s} \rightarrow \lambda \approx 100 \text{ m}$ ,  $T: 10 \text{ s} \rightarrow \lambda \approx 150 \text{ m}$ , and  $T: 12 \text{ s} \rightarrow \lambda \approx 200 \text{ m}$ .

Now let us, look at some of these numbers therefore, **you know** like relation of this number, now typically let us say, let us take a wave period of  $T$  before **before** that let me also see this other way round, now lambda it turns out therefore, equal to  $2 \pi \sqrt{g \lambda}$  by

omega square, this is from the dispersion relation omega square is equal to g k straight forward.

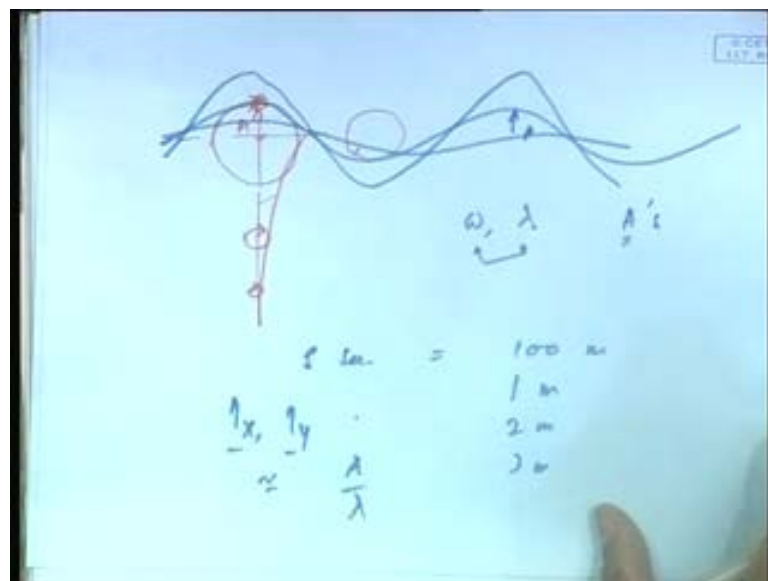
So, this we are trying to relate, now a relationship this turns out to be if you are to put a SI unit that lambda in meter g is you know like meter per second square and period T in terms of second, actually this I can write this way g by 2 pi square by T square, so this turns out to be g T square by 2 pi.

So, this in the this thing becomes almost 1.56 T square in SI unit, so that tells me that if T was 8 second, lambda would have been 8 square into 1.5 around 100 meter approximately if T was to be 10 second, lambda would be around 150 meter, say T was about 12 second, lambda would be approximately 144 into 1 and half about 200 as, so let say and it is all approximate number.

So, this gives me a feel about the relation between these number and this let us look at the speed value, say for this what is the phase speed very simple 100 by 8 because lambda by T, it would have been something like 12 meter per second here, you will see 200 by 12 it would be approximately 16 meter per second.

So, you see speed is increasing this is of course, a reason why a long wave like tsunami you have travel speed, so high phase speed of travel, so high you know it can be in in many kilometers per second per hour.

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So, this is exactly the **you know** like part connected to dispersion relation, now I will just want to also tell about the particle path, that it takes it turns out if we were to do a study in deep water this particles actually travel in a circular way, this is my circular way. And obviously, since I mention earlier that everything diminishes with this thing, in here the particle will move this way in here the particle will move this way etcetera.

So, the particle motion is basically circular path that means, if I were to put a particle here, it actually stays in same place going like that, it does not move forward according to linear theory, so there is no net mass moving forward as per linear theory it is only the form that is moving forward.

So, the particle are just moving like this, you can also get a feel about the particle velocities for the simple reason that see, this is  $A$  this circle distance is  $2\pi A$  and a particle is traveling is  $2\pi A$  distance on average in period  $t$ , so therefore, **you know**  $2\pi A$  by  $t$  would be a kind of **you know** like speed.

Now, one thing in dispersion relation that is very important is that, we come back to dispersion relation is because see, this vertical axis which is my  $A$  this has no connection with the dispersion relation I could have another wave this much or another wave very small all of them will have same frequency  $\omega$  say length is same.

So, this two are connected, but for the same  $\omega$   $\lambda$  combination I cannot different  $A$ 's, so in other words the dispersion relation is connecting my quantities in the horizontal axis, time axis and  $x$  axis, but nothing in the vertical axis.

In other words if I have an 8 second wave, it is 100 meter what is the height, well 100 meter wave can have a height of 1 meter, can have 2 meter, can have 3 meter this is not specified why it is so, linear theory because remember that I have made an assumption that well  $\eta_x \eta_y$  is small.

What is this approximately, they are equal to  $A$  by  $\lambda$  in **in** that order, so what is happening  $A$  by  $\lambda$  has been taken as small, but no specifically of  $A$  what is small one can be small 0.02 is also small therefore, if I take 0.01 for example, my  $A$  would have been certain number, 0.02 would be twice the number. So,  $A$  cannot be specified, so the theory applies for small  $a$  as long as  $a$  by  $\lambda$  is small, but not for in specifically of  $a$ , this is very important to understand.

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$$p = -\rho g z - \rho \frac{d\phi}{dt}$$

$$\phi = \frac{gA}{\omega} e^{kz} \sin(kx - \omega t)$$

$$-\rho \frac{d\phi}{dt} = -\rho \frac{gA}{\omega} e^{kz} \cos(kx - \omega t)$$

$$= -2\rho g A e^{kz} \cos(kx - \omega t)$$

$$\eta = A \cos(kx - \omega t)$$

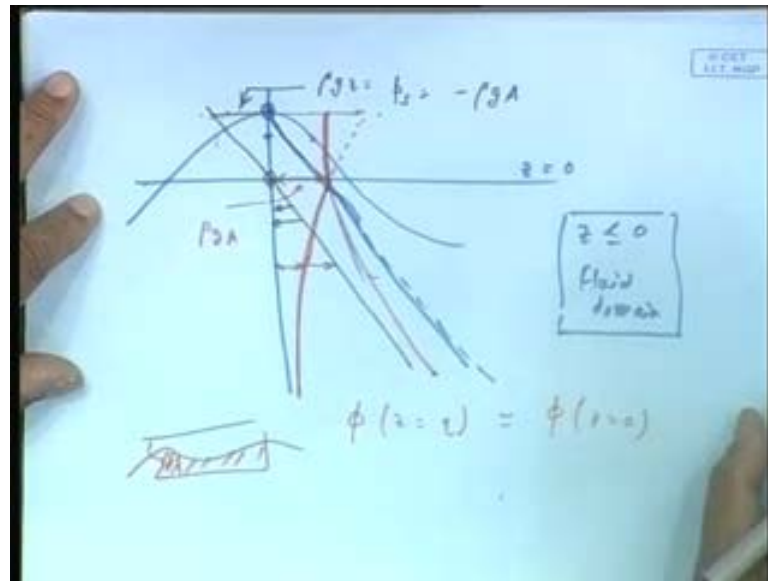
So, now the next point very most important point is actually to talk about is pressure, the other part of water wave would be pressure  $p$ , now what did we see before,  $t$  is minus  $\rho g z$  minus  $\rho \frac{d\phi}{dt}$ . Now, let me write the  $\phi$  expression again, because this is important for us to realize this it was  $\frac{gA}{\omega} e^{kz} \sin(kx - \omega t)$  now much is  $\rho \frac{d\phi}{dt}$ .

Let us work it out see, minus  $\rho \frac{d\phi}{dt}$  this linear dynamic pressure, if I do minus here, so this 1 minus let me just write it down, this becomes minus  $\omega$  comes in  $\frac{gA}{\omega}$  by  $\omega$ , that is right, so that means, it is minus  $gA e^{kz} \cos(kx - \omega t)$  etcetera.

Now, what is happening obviously, you can understand this that again, now if I see this  $\eta$  it is something like  $A \cos(kx - \omega t)$ , so naturally there is a highest dynamic pressure, either positive or negative occurs under crest of trough for this, but remember in all this formula my  $z$  is 0 here.



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Now, let us look at the two part of the pressure by a plot, the hydrostatic pressure and the linear dynamic pressure, now let us look under trough, so I have got here say now, under this see remember this  $z=0$ , how **how** does the hydrostatic pressure vary well it is minus  $\rho g z$   $z$  is negative down.

So, therefore, this becomes positive here this way, giving a pressure we can say is positive pressure; obviously, pressure can be positive, but the interesting point is that what is happening here, well you end up getting a negative pressure that does not make any sense, but any how let us put this way you end up getting here,  $\rho g z$  that is static pressure to be a minus  $\rho g$  into  $A$  at this point.

Now, what is my dynamic pressure, you will find out that if I were to work, this is also well see here, actually there is  $\rho$  here, that I this  $\rho$  has to be there, this  $\rho$ , this  $\rho$  here, so this will be also  $\rho g A$  because  $z$  is  $0$ .

So, if you work it out here it will turn out to be like this exponentially down and this merge is  $\rho g A$ , everything is fine remember that we have defined in our theory and that is the most important point when you are doing this application, my  $z$  is  $0$  at this level this is my  $z$  equal to  $0$  level and my water is only below  $z$  equal to  $0$ , my water is assumed in theory only  $z$  equal to less than  $0$ , this is my fluid domain.

However please understand this that I have now a crest here means, this part is water, but my theory I always have only in a valid for  $z$  equal to less than 0, what happen here I know, I want to know physically what is the pressure of this point for example, how do I find out.

Let us say, this is 5 meter I want I want to find out what is my pressure 1 meter below the crest, now if I were to apply only  $\rho g z$  by this formula what do I get a completely absurd result of a negative pressure.

(( ))

No, that is not, but it is that is true, but then this wave is of course, existing here see the wave is existing because the particles are moving liquid cannot sustain sheer, question that is ask if perfectly correct, so why that there was no motion there see obviously, if you look at the wave it is sustaining this ship it is of course, sustaining the ship, if it did not sustain the ship, I would not have a wave, why it is sustaining the ship, because the particles are moving.

So, there are dynamic involve and this is a question you cannot presume this mass of water to be hydrostatic and if I take an hydrostatic pressure, I end up getting minus  $\rho g z$ , that is that absurd result come because, hydrostatically it is it is absurd to consider that water mass can stay like that, it can only stay along with the hydrodynamics.

So, now if I were to add the dynamic pressure **there** there is also a problem supposing I add dynamic pressures, so I apply this formula, so what would happen it goes like that, since  $z$  e power of  $k z$   $z$  equal to plus 1 whatever, now if you look at this point what will happen this; obviously, is more than  $\rho g A$ , now this is  $\rho g A$ , so this will end up getting some pressure that is not correct, because my pressure of this point is suppose to be 0, where is anomaly?

The anomaly is because that in developing linear water wave theory, what we have said is that this height with respect to the length is very small a consequence of for that was that, every quantity that is on the surface is assumed to be same as it is on the  $z$  equal to 0 line; that means, if I were to take a point here and if I were to take  $\phi$  at  $z$  equal to  $\eta$  this is taken same as  $\phi$   $z$  equal to 0.

Now; obviously, you look at a pressure, what is pressure here,  $d\phi$  by  $dT$  at  $z$  equal to  $\eta$ , but that I have said is same as  $d\phi$  by  $dT$  at  $z$  equal to 0; that means, I have said the pressure in this region dynamic, all the dynamic quantities are same as which means I made a presumption, that this actually pressure is like this, this pressure is constant in this region.

And if I were to do this then, there is no problem because now if I add this line, with this line what do I get I get a pressure here, this way because you see here up to this much if I add this black line with this line, I get the pressure A and here I get the pressure like this this is how much this is nothing but this plus this, that is this much plus this much, gives you this much (Refer slide time: 42:41).

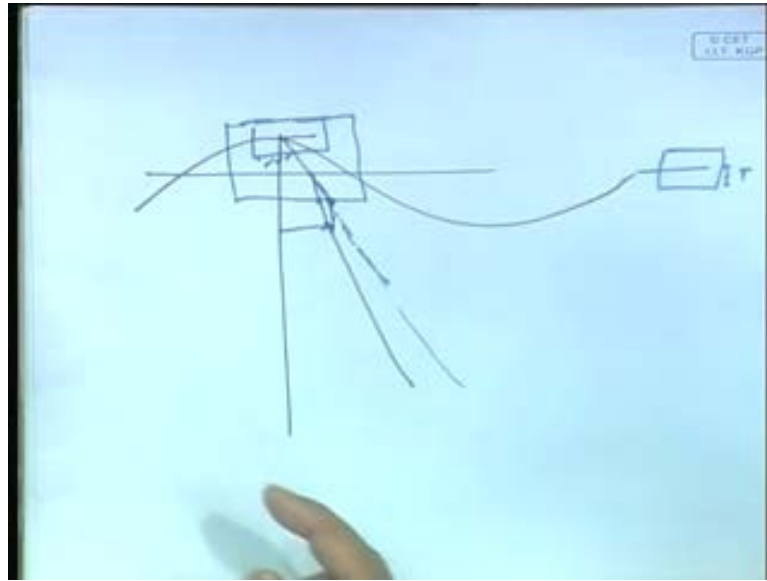
So, this is exactly how the pressure varies under linear water wave that this is extremely important for us to understand, because you see you must have a physical pressure here and the pressure dynamic pressure is having like this, once again I repeat this whereas, hydrostatic pressure is having like that, now if I were to evaluate a pressure in this region or anywhere under wave you cannot use hydrostatic pressure by itself, because if you did that you end up getting a wrong result, but if you were to find pressure, you have to use hydrostatic and hydrodynamic pressure simultaneously.

Supposing I want to use hydrostatic pressure as you have done sometime in ship strength, in ship strength what we do if you recall you simply take a ship, you have this profile and you find this buoyancy curve based on buoyancy that means, you are using only hydrostatic pressure.

What here you can see is that, supposing I use this my datum line this was my  $z$  equal to 0, what would my hydrostatic pressure it would have been like that, well this is closer to see now, if I were to see this color, this line is one and reality is this and this, so at least this is not that bad in a sense that you will not go completely off, your going to make this much of under over prediction fine, but you do not go completely off.

So, if you use only hydrostatic pressure, so you know that if you have to use only hydrostatic pressure, then you must use you cannot use  $\rho g z$  with  $z$  equal to 0 was the mean line, then you must use a instantaneous water surface.

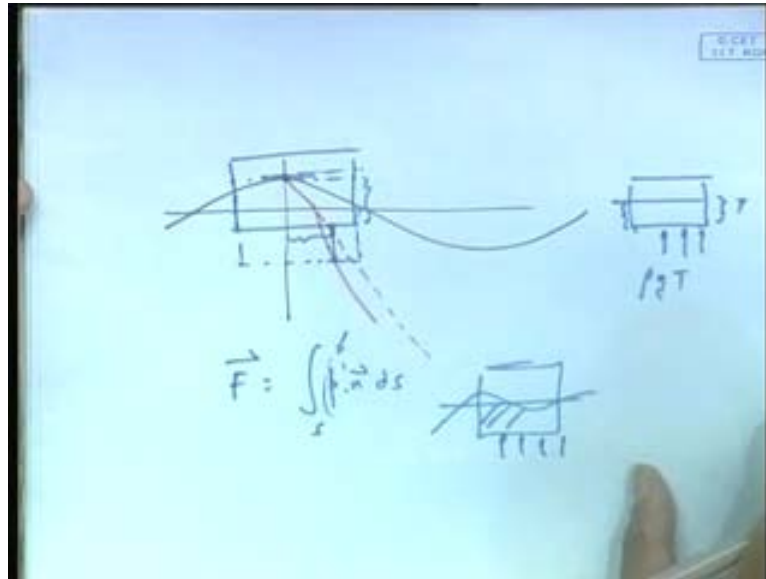
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I gave an example of this with respect to a both now, in a minute see **we**, so we end up finding this, that if I was to have a this thing, the pressure depth is basically coming like this **this** my pressure, actual pressure and if I were to ignore the dynamics and I use this as  $z = 0$  when my pressure would have been this, now you think of this a boat what happens. Suppose there is a boat here, now what happens, but let us consider in a calm water first the boat, it has a trough of  $t$ , which means that below  $T$  I have this pressure  $\rho g \rho g T$  is supporting it, now what is happening here you see or rather I should make it bigger see trough like this much.

Now, thing is that in this case what happens tell me, if the trough go down or go up under crest, what would be the **the you know** the situation remember that you see that the trough would have been same if the pressure was this much, this much here, but actually my pressure is this much. So, what would happen the body will actually sink down in crest, because I must have the pressure equal to this much to support that, I will explain this **this** picture again once more.

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Now, perhaps I will take a better diagram once again, because this diagram, I will take this case here, see the pressure actual pressure on the wave under a crest looks like this **this** is my straight line and actual pressure looks like that, that my actual pressure. Now I have a boat which has a trough of some value, what does it mean; it means that in this here I had a hydrostatic pressure of  $\rho g T$  which supported my mass.

So, I must go down that much where my pressure is  $\rho g T$ , it is a flat bottom plane because; obviously, I must have as much trough were here, because only this phase is contributing to my buoyancy in this case.

Now, here what has happened I have this body, now you see here, in this case my trough here is this much, but my pressure this is my  $T$  and this is my  $\rho g T$  this much, so if the pressure was this much my trough will be in  $T$ , but actually my pressure is only the small line.

So, my  $\rho g T$  pressure would come when this bottom surface is here, which means the body must actually come down, because the pressure below the crest have reduced rather than increased, so therefore, the ship must come down further in order to support itself and the opposite would actually happen in trough.

So, therefore, what would happen of course, this actually happen small **you know** numbers are not, so large, but there is a tendency for the trough to come down and go up

more, because of the hydrodynamic contribution dynamic pressure contribution, so the dynamic pressure has therefore, an extremely important role to play.

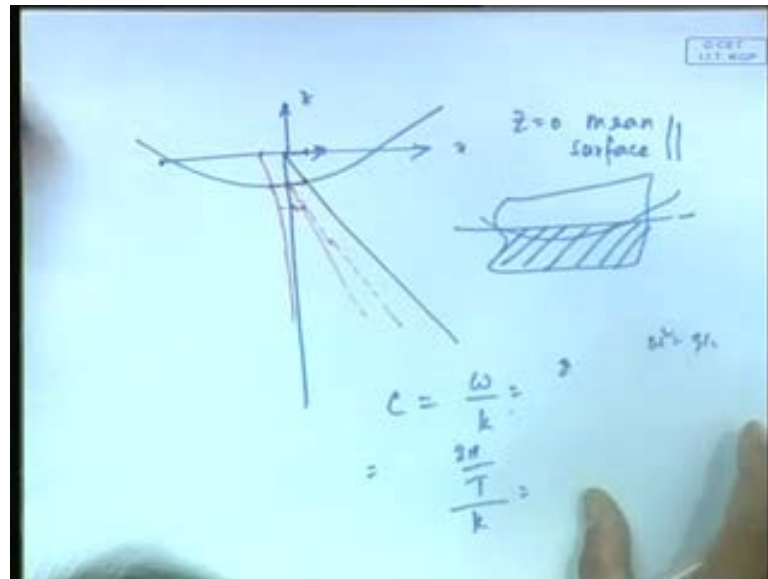
Now we are interested in dynamic pressure, so much more, now let us look at this in the same case also hydrostatic balance case, ultimately I want to find force for any dynamic system I would like to find out the force. The net force acting on the body, because the module seakeeping is because to study, what the waves do to the structure, what does it do.

Well in a hydrostatic case **you know** if there is a body there it gives you hydrostatic pressure, that gives you buoyancy and that makes gives you a float, everything is based on that, now in a dynamic cases same thing, I actually have to find out what is my pressure well, what is the force and force is nothing but integrational pressure over the surface.

So, I need this pressure very well and therefore, I need to find the pressure very well of course, the fact that, if there is a body there the wave itself would change the different issue altogether will come to that afterwards, but the fact that even if I did not assume that, we still have to figure out the pressure under wave. And therefore, I need to have a very good understanding about pressure and if I do not take the  $z$  etcetera properly I end up getting pressure wrong, so I were to only use hydrostatic pressure I will repeat again, you must use  $z = 0$  to be the variable axis from measured from the instantaneous water line.

If I was using a theoretically consistent value of  $z = 0$  to be the mean water line, then I have to use hydrostatic and hydrodynamic pressure together not in isolation, because in isolation any of them cannot exist, this is one of the most important part that **you know** we **we** end up finding here.

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Now, let us see now, the same similar thing will actually happen in a **in a** trough case I were to look at the trough case **you know** like, let me see this trough case here **here** this pressure is opposite see this was like that coming whereas, other pressure well this a part **a part** here it is this side. So, we have to actually now add this two, so therefore, the situation become different and there what would happen is that, see here again if I were to see remember there is no water here this part.

let me just see this, see here, essentially what would happen therefore, this **this this** and this will make it zero here, so I will end up getting if I were to have this **this** part, but with this respect I have this added. So, I end up getting this minus this because, this on the other side will end up getting some value like that.

So, what I mean is that again the similar kind of thing will occur **you know** like in trough like in crest, the only problem that happens here is that, because there is no water here you do not have to really consider, because this region there is no water.

Now, I tell you why these things are important, because we will find out that in many case of floating body structure ultimately, you will have to find the pressure over the mean water surface even though the water is like that, theory will tell us that we have to determine on the mean water surface. And that is why it is always good to know the pressure and use it consistent theory of  $z$  equal to 0 on the mean free surface, if  $z$  equal to 0 on the mean free surface, then you will have no problem.

So, for our study purpose, we would always take  $z$  this mean this is  $x$  and this is my  $z$  equal to 0, that is  $z$  equal to will be always my mean surface, so essentially today I will, in fact, close on this pressure part and we will next lecture we will have one more lecture on linear water wave theory, we will discuss about the other aspects of energy, energy flux group speed, etcetera and perhaps we will also work out some simple elementary calculations.

**You know** who these numbers are used to determine, you can actually see from here itself as an example, this speed as I said  $c$  is equal to  $\omega$  by  $k$  and the wave, we have done is  $\omega$  by 2 in terms of **in terms of you know** like let me shift time it was something like  $2\pi$  by  $t$  and  $\omega$   $g$ ,  $\omega$  square is  $gk$ ; so well **well** I was trying to write it in terms of time let me just figure it out.

**(( ))**

No, no that is

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Handwritten notes on a whiteboard:

$$c = \frac{\omega}{k} \approx c \propto T$$

$$\omega^2 = gk$$

$T$ : 8 sec.  
 $\lambda$ : 100 m  
 $c$ : 12.5 m/s

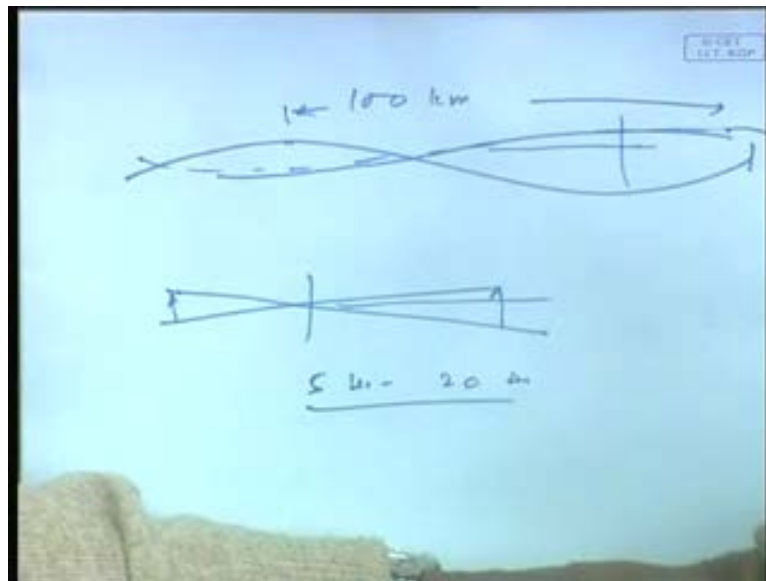
$T$ :  $30 \times 60$  sec  
 $\lambda$ :  $\dots$

See,  $c$  is  $\omega$  by  $k$   $\omega$  square equal to  $gk$ , in any case you can relate that  $c$  to be in proportional to something like  $T$  that we have done at some point of time. So, what I am saying is that now, if see it turns out that if  $T$  was 8 second my  $\lambda$  was 100 meter and  $c$  was about 12.5 meter per second.



But you will see that, if my T is equal to say a tsunami wave 20 minutes or 30 minutes say 30 into 60 second, you work out lambda you will find out, it will turn out to be order of several 1000 kilometer and you will find out c will turn out to be, in order of something like 100 kilometer per hour, 500 kilometer per hour, this is exactly why the speed is high. But people do not realize and you must realize and I will tell you, I will end this term with this, that how do you explain physically, so high c, what is the meaning of this, so high c what you like **you know** physically see.

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The reason is very simple, if the wave is very long like that, let us say this is actually **you know** like say 100 kilometer, what happen after 20 second is water has come down here and this has moved up here.

So, water till move out from here to here in 20, the form appeared to or moved up from here to here is 20 minutes, so after 20 minutes this place would appear to be having trough water. So, this it is like an oscillation, so this oscillation was like this at one time next instant it become like that, so you see around this water appear to have come move to this place and; obviously, if there was a coastline here you suddenly find out 20 minutes as water moved up and rushed it.

So, therefore, the concept is on the form only which can travel very fast like a long line you just shift it this way, so this much distance is moving in that time, this is how you explain never the particle moving, particle cannot move 100 kilometer per hour.

Anyhow, so this is about the speed we in our course we will find out that our range of wave speed will lie between something like 5 second to may be 20 second.

We are not interested in tsunami, we are interested in capillary wave, none of them have any effect on the ship, we will close today's talk of case and begin tomorrow on the other aspect of water wave theory, thank you.