Seakeeping and Manoeuvering

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Lecture No. # 14

Ship Motion in Irregular Waves – I

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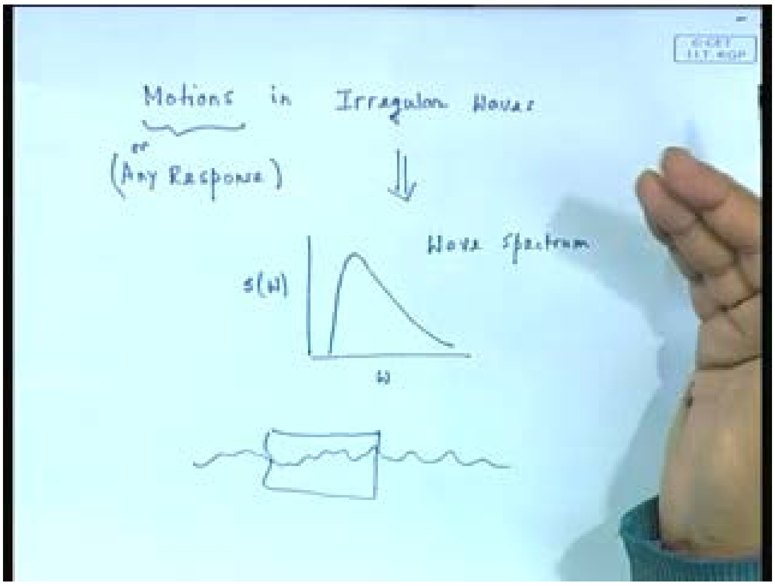

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See today, we are going to talk about motions or ship motion. Let me just just write motions actually. Although, we are calling motions, it could be any response. What I mean is this. See, we have seen in the past lectures that irregular waves can be finally represented by means of a wave spectrum and this is nothing, but a sum of regular waves. So, I know my environment that, have talked in last two lectures.

So, my next question which is the pertinent question is I put my ship in that wave. How does it respond? So, here I mentioned it as ship motions, but let me understand or let me clarify that this will apply to any response. For example, it could have been pressure at some point, it could have been acceleration at some point, it could have been bending moment in the mid-ship shear force at some location etcetera. Any response whatever output that comes out, whatever you want to measure for putting the ship in a irregular waves, how do we get it? It is this is the question that we are asking. Now, obviously, this is most important because our interest is not describing irregular waves. We have described it already, so that I can find out how my ship would behave in that. That is my purpose.

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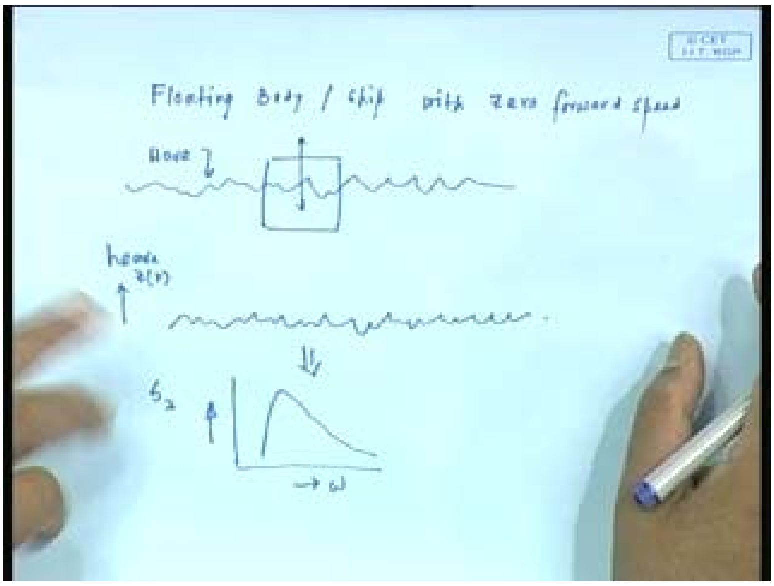

Now, let us, so we are going to talk about this procedure which is called spectral procedure. Now, let me look at it. First we are going to look at a simpler case. We will consider a floating body or a ship you can call, with zero forward speed right now just for simplicity of the expression which means that we are not considering something moving forward because that takes one more modification which we will come in a minute.

Now, let us take this example. I have this wave and I put a body. Now, I am measuring its heave motion. I have a stick and I measure heave motion. How will it look like? Well, if you take the response z, see this is wave and if I take the heave motion, let me call it heave motion or z t. It is also going to look like random naturally, because random input is going to give me random output.

So, this also I can exactly same way synthesize. I can exactly same way as what we have done in wave, say that it consist of number of sinusoidal waves and end up representing a spectrum for z against omega which will look something like that again because after all, if I look from a different point of view, this is another random signal whether you call it z t or eta t, it does not matter. I have a random signal and the random signal or irregular signal can always be processed and can always be represented another spectrum.

So, this I can get. In other words, if I have only this signal to start with, if somebody gave me this signal and I am told that you represent the signal, I can always do that. Well, of course, the fundamental question is how do I get? Given this, I get this that is the question to ask. The procedure given by irregular waves, how do I get my irregular response? This is the question that we are asking, but I wanted to tell here is that I can always get this to this. Now, see there is a very simple procedure and very interesting and nice procedure. Now, let us take one simple example. Well, we have been talking of motion.

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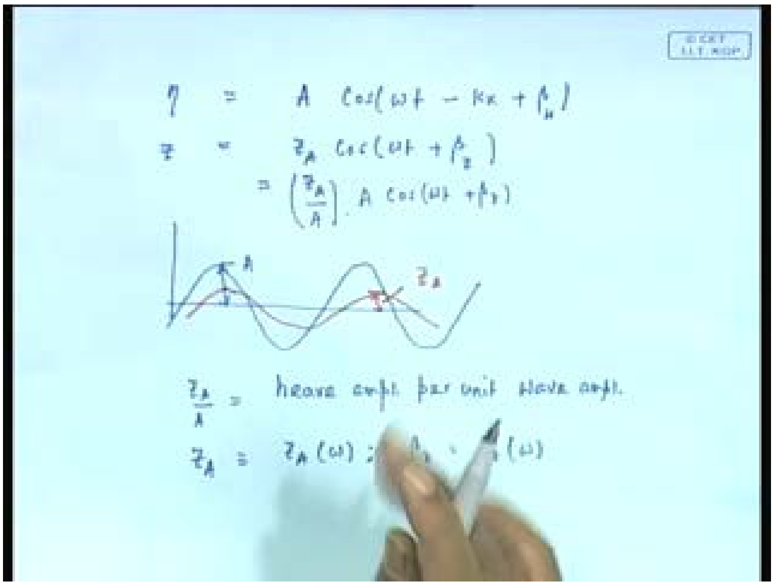

So, I will take a sample as heave motion. Now, see let say I have a wave, particular wave one frequency wave eta one or say, well I will put that one later on. So, it is given by a cos omega t minus k x. Let us put it this way or even I can put a beta if I want.

Now, this wave gives me a response. This wave is some kind of A sine wave. It gives me response Z. It is expressed as a amplitude into let us say cos. Let me call this beta wave beta Z. This we have done before. Isn't it? We have always seen that in a regular wave. Well, when we talked about single degree freedom system earlier, what we have found out in a regular wave? My response is also regular. Its sinusoidal. In other words, if eta was in this case looking something like this with an amplitude A, my heave here would have looked something like say with amplitude ZA.

We have always said that and of course, what we could represent this as going by this, I can always call this as ZA by A multiplied by A. I can call like that just in what is ZA by A amplitude of heave per unit amplitude of wave, ok. Now, obviously, for a given wave of given frequency, this is a frequency omega. What will happen is my ZA will remain constant, ZA and beta beta Z. So, that means, these things become a function of omega.

So, what is happening for a given wave, regular wave you take a wave of length 100 meter which would be having a given omega. So, what would happen for that omega? If I put my ship, it is going to have a response amplitude some value. So, obviously, ZA is a function of omega and so is beta Z explaining onwards. See what is happening. Now, let us take one particular wave of some length, say 100 meter long wave I put my ship there it response.

Say wave height is one meter for that. So, it will go up by say two meter. ZA is two meter. Now, if I have three meter. then it will be six meter because in that particular wave of given frequency, my response is linear because as I said two one meter wave is equal to one two meter wave. So, ZA by A is constant. That we know. My my point here was to say that ZA is a function of omega. Why? Because it is very common physics. If I have a 100 meter long wave, the amount it will move up and down is not going to be same. If I give a 200 meter long wave or if I give a 50 meter long wave, I give an example in a minute.

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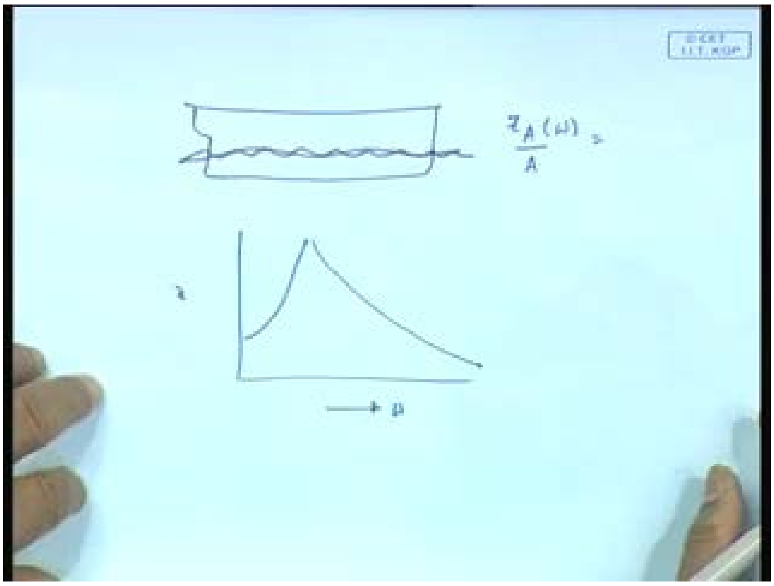

Suppose, you have this large ship. So, I have a very small wave, very small omega let say. So, what happen it has a heave for this omega. This is going to some number, but now if I put the same ship under a much longer wave of same height A, same A, do you think that this will be same? Of course, it will not be same because it depends on the wavelength. So, therefore, ZA by A obviously is a function of omega or wavelength.

In other words, what is happening is that for a given wave of frequency omega, I have an output heave of amplitude ZA and phase beta Z. Obviously, the output depends on the input and here, well number one is that output per unit of A remains constant because as far as A is concerned, see here if I have eta one A cos omega t. Eta 2 is A 2 cos omega 2 t i can add A 1 plus A 2 amplitude can be added at the same length, but length side, obviously the response will depend on omega.

So, therefore, ZA by A is a function of omega. Naturally different wavelengths would give you different response. This is exactly what we have done. In fact, to say this, there is more interesting way of putting that. If you remember our regular wave response, what did you find out? Omega versus you remember omega versus here I have got this Z and how did it look like. Obviously, the very graph showed that the output how much response depends on omega. So, there is no like question or doubt on that fact you see.

So, this ZA by A is my amplitude of response by amplitude of incident wave which you can call transfer function or I can call this to be say, some function h. Obviously, it is a function omega I can call it because what transfer function essentially implies output by input. Yes, output by per unit input, but it is a function of omega. That is what I was trying to tell here.

Now, we will come back to this. This we have discussed several times when we talked about response in regular waves. This we are talked in the very beginning you know single degree freedom equation etcetera. For example, obviously, if omega you know matched with a natural period, then you would expect ZA to be very high. If it is very far from natural period, you would expect ZA to be very small. Therefore, ZA is a function of omega. It depends on what you know wavelength is there, but the question we are talking today is of course, motion we are talking of this subject motion in irregular waves.

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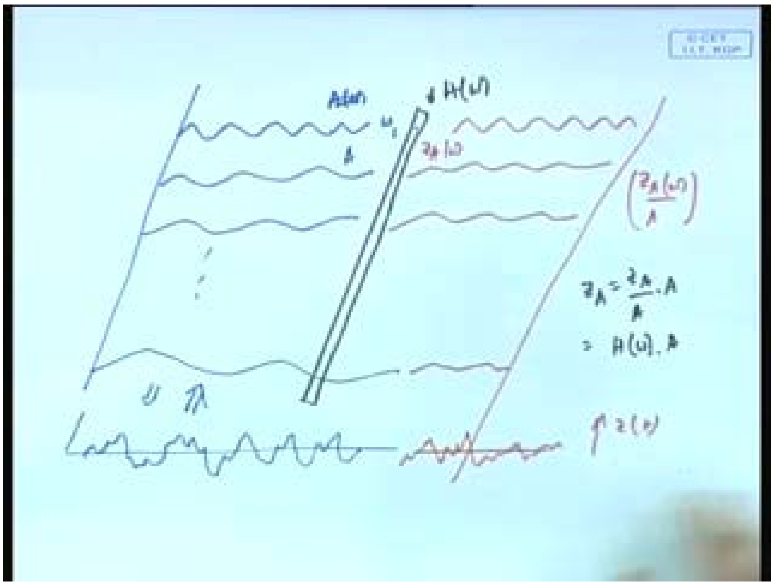

So, what is happening now is this is the most interesting part. See here I will draw a different wave. Now, I have a incident wave of this omega 1. Let say this is omega 1. It will give me a response of the same omega. So, I have eta here, I have Z here and now, I take another one. See here remember this thing. Irregular wave what we have said in an irregular wave. An irregular wave is a sum of sine curves. So, I have started with this and I say that this is nothing, but basically, either I can add this to get this, but to start with, in fact here I started with that and I said this is actually sum of this. Now, I broke it down. Now, each one of them is going to give me a response here. So, this gives me this of the same frequency. So, if I draw a line here, this is going to give me this response. So, if I add it up, I get.

So, what I am saying. Therefore, see the interesting part is very simple that this I have to predict this from this. See I have to predict this line, this red line which is my response output which is my Z and t. This is actually my Z t. That is my how which is heaving irregular wave for this eta t. This is what I have to predict. My question that I am asking you is that given this what is this now? I have seen that this is nothing, but sum of this which I have done by spectrum.

So, what I do is I break it down to this sum now. Remember the response are linear. So, this one and this one, that is if I call this to be here, you know like any response amplitude here A omega and well, let me call it only A and here I calling ZA omega. My ZA omega by A is constant for a given omega that is linear. So, what is happening therefore is that see first of all, I will tell the formal procedure in a minute, but you know you must get in your mind very strongly the concept behind it, the physics behind it because afterwards you will find out the application is pure algebra. There is nothing much to it.

Once you get the concept of how it is, the responses can be done, you will find out that the procedure is algebra. In fact, I am very fond of telling this line that the first paper which proposed the method of determining response in irregular wave, it was called something like motions in confused sea and I am fond of telling that there is absolutely nothing confusing in finding motions in a confused sea because it is a pure algebra, but you have to get the idea first. That is what I am trying to spend the time on. So, you see it is very simple. I am breaking it down to A sine waves. Each sine gives me A sine output. This output by input if I were to this diagram in come properly, if I were to draw a block here, I will do that in a next graph. You know if I were to draw a block here, it passed through this, like this to this. Let us say it is a black line. That means, this multiply by something gives you this.

See for example, ZA we have seen this ZA is ZA by A into A, that is h omega into a. So, this is my h omega. So, A into h omega gives me ZA straight forward. So, I can pass it through this to get this. So, each one gives me this output. So, I broke it down this way, got this output, sum it all up and I get this line. So, it is like I had a black box, where I had this. I shook it out all, this filtered out for each one I found out the response and put this block number to again shaken them I get this out.

So, you see the procedure, how simple pure algebra. So, getting here breaking it down here, the only hydrodynamics is in this black line. What is the black line find out? How much it respond in a regular wave? That is nothing else this part is algebra. We have seen this part is algebra. So, you see this is confused sea and this is motion in confused sea, this is is motion in regular waves. So, you see this no confused as far as confused sea is concerned. You are simply adding it. What is difficult is this part going to the black line.

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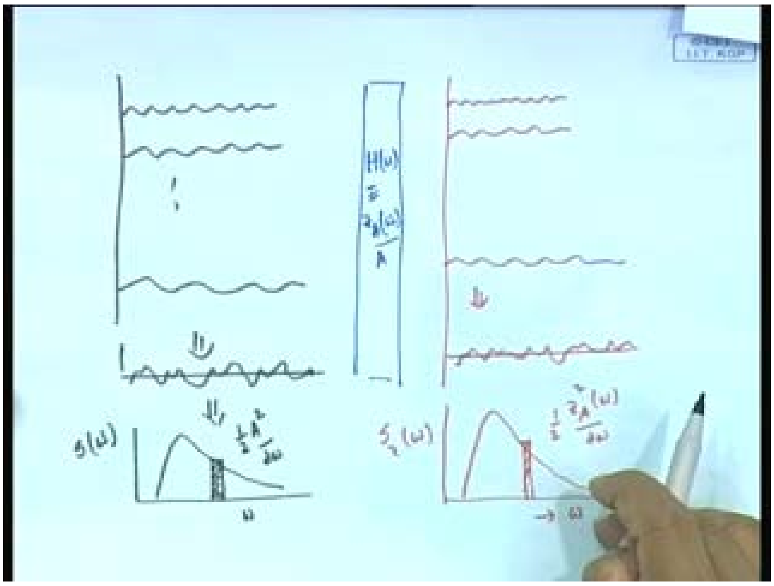

So, now having said that, we will come to another graph. Now, it is like this response. Again we have found out. Let me draw it this way. Now, this is easier to draw probably we see that these waves gives me, but what is happen that I have represented this in terms of a spectrum. Remember, this I have represented in terms of a spectrum. What did I do? I plotted here this, this and actually any point here happens to be half of a square divided by d omega. That is what we have done. Isn’t it? This is what the definition is.

In other words, what we have done is see this I represent this way, but now what has happened, I have an output here. So, each one has given me this output. So, as far as representation is concerned, this also I can plot in this way where I have this equal to half of ZA omega square by d omega, same thing. After all, ZA omega is the amplitude like A. What we do is that we put this is how we we can call it to be s z omega versus omega.

What I am trying to say, therefore, is that see I know how to get this from this. See this goes to this h omega, this black line is that filter z h omega which is equal to ZA omega by A. The amplitude of response per unit wave amplitude. This is this line. So, it goes to, I break it down this way, go through this, add it up. This part we understood. So, this part is I can call response in regular waves. This is simply algebra of breaking down irregular waves into sine components. This is simply getting the output sin components, add it all up to get this, all right.

I will tell the procedure in a minute, but once again I am emphasizing that understand this physics. Now, I know now how to get from here to there, but I obviously do not want to plot a graph like that. How did I represent this? I represent this in terms of a spectrum. I do not want to put t versus you know like eta. I put omega versus s omega which is same thing. I am just representing this in terms of a spectrum or this time signal is represented in terms of an frequency signal over an omega.

Now, exactly these are same. You do not look at that. Let us say, this black line is I just drew this black line in a red pen. So, I have this instead of eta, I have here z with an amplitude ZA. Instead of A, I have ZA. So, I have exactly the same spectrum. In other words, what does it mean? It will imply obviously half of ZA square by omega. In other words, if I add this area up, I will end up getting essentially a quantity proportional to energy of this spectrum into rho g because energy of any signal is proportional to half of amplitude square.

So, this is a question of representing it. So, our question therefore is that given this, find this. In a sense, now you see this there is such a one-one line procedure that will come out in a minute. So, what is this half ZA omega square by d omega? What I can write this? I think I will draw that in a bigger graph because this will take time. So, draw the red only. This is just bigger one to show you.

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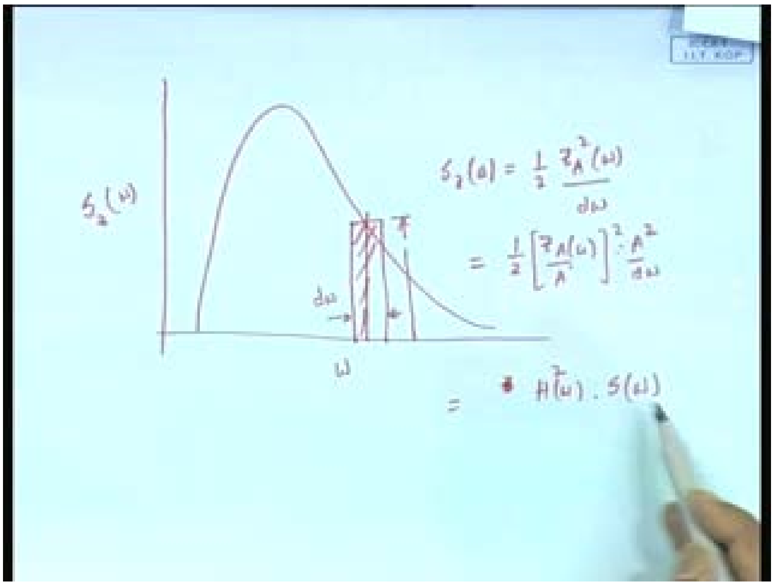

So, I have let say this is SZ omega and I have this and this part is or rather SZ omega. I can call it is half of this. We know that this is how we have done. Now, see here this is equal to half of ZA by A which is equal to right or rather we are going to write h because you see this half A by B, sorry here A squares A square, right. So, you see this is you understand that ZA by A is h omega half A square by d omega is s omega. So, what is happening is it become simply this to this look back at this. Therefore, this amplitude is this amplitude multiplied by square of this at the appropriate frequency straight forward.

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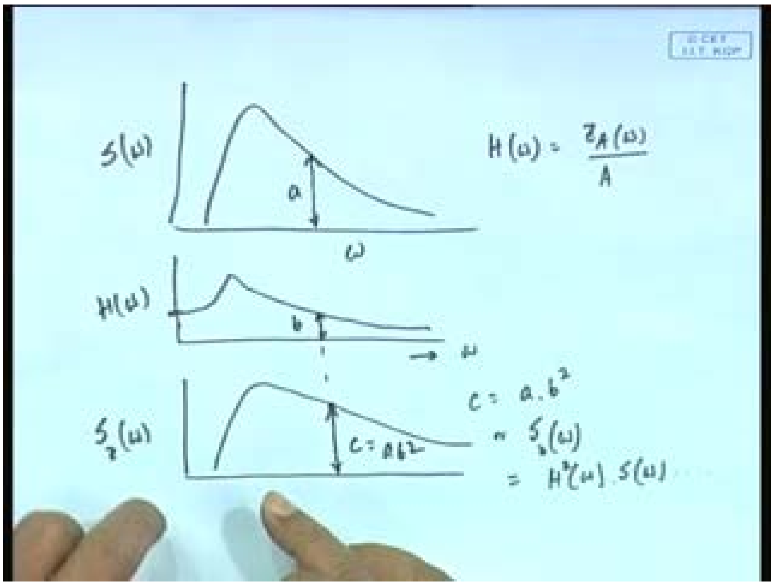

So, the procedure is as so simple that is actually embarrassing in some sense. What I mean therefore, the procedure therefore is something like that. Never mind. The time signal I have this spectrum s omega versus omega. Somebody has given me this h omegas. This is what we have done in regular wave. Remember? That looks something like that. Take an appropriate place, say this and this same at omega. Say I call this to be say A, I call this to be B. My responses become now B square A into this. Sorry, C is A into B square or SZ omega h square omega into s omega.

So, what is h omega? H omega z amplitude of response. The other frequency divided by a, that is nothing, but this because you are plotting. See remember here also we plotted. You can say it is a non-dimensional way of plotting it. If it is linear motion, basically you are plotting the output per unit input because it is a linear response function. After all, if A as i told you that for a given omega, let say given wavelength 100 meter, if my heave was two meter for one meter high wave, then it is going to be four meter for two meter high waves because a two meter high wave is two one meter high wave is together. Both will give to plus two four. So, it is as simple as that.

So, you see. Now, tell me I have a formula for this because we have discussed that last class throughout you know this formula theoretically I have got this discussed in the very first beginning lectures. How to get this the transfer function? Let us you know single degree of freedom you have a solution if you could solve it whichever way. Let say this is given. This is the in fact, we will find out afterwards that the most complex part is this. This is where the hydrodynamics lies, this is where everything lies. These are formula as of now. This is where hydrodynamics on everything lies added mass damping various methods and let say, you ended up somebody gave it to you either experimentally or determining it.

How do I get this? If I know these two, this into square of this, this one line of calculation. So, you see that is why I keep saying that getting, therefore the irregular motion this picture form. This picture is as trivial as simple as this, but because it is say, but very important. Everybody in industry, everywhere you go, people are going to ask you that tell me the response in irregular waves because actual waves are irregular. If you got actual ocean, nobody is going to find a wave like that unless you make it in a lab. So, therefore, when you go to an open ocean, any practitioner or any person will tell you, but you are taking sine, but I have never seen sine in my life when I sail for 20 years in ocean.

So, you see the question is that people will tell you that how do you get this from, but what we find out is that getting this from this process is one percent of the effort getting this. This is 99. I would say 99.5 percent of the effort getting this, to this is 0.1 percent of the effort, but that 0.1 percent if you did not complete it, it is like you did not add the final spice on your you know like food. People will not understand. You have to finally give you, give them this. Not only this filter quantity is this which I will come in a minute. So, you see people are going to ask you, but you have to tell me how does the ship respond in sea state three let say. So, what would happen? You started sea state three. So, I know h one-third. I have a formula for this. Then, I have got this from a different source. I combine, I get and then, I will say in sea state three, my heave motion responses this spectrum is this.

This is what is known as response spectrum. Now, you will now having said that we will find out that this spectrum also actually satisfies all the statistical quality, quantities same as Rayleigh distribution, narrow band rayleigh distribution. Therefore, we will be able to find out what is my average amplitude of the response, one-third amplitude of the response and all quantities that I want just like what I could do for this. See here I could find out what is the most probable wave height in thousand second. I can find out this also. What is the most probable roll? If it is roll angle in thousand cycles or second,

everything is same because this graph is assumed to be having same form of this. I will come to this in a minute, this formulas for that, but we can find out. This is what people will ask you that tell me that my roll average, roll does not make 6 1. So, you cannot tell peak roll because you know average means certainly one can be high, but that is we will come to that in a minute. There is a fundamental question. The question is coming back to this. You see here I started with saying this line body floating with 0 forward speed, this procedure, but actually ship is not zero forward speed, ship is moving forward.

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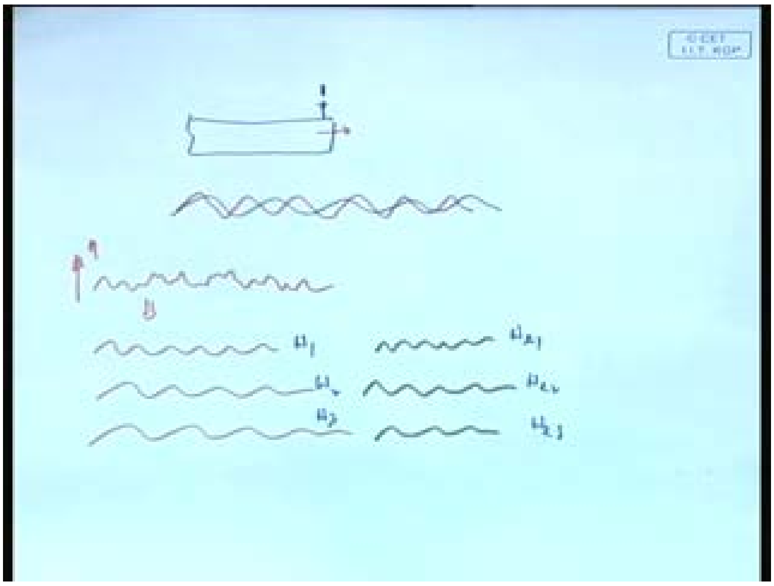

So, now, what happen, what changes happen because of that forward speed? Let us talk about that. Now, you see we have seen that there is a ship here. You are standing here. Now, you see there is a wave coming absolute like that, but if you are standing here, you are going to see this wave coming and you are going forward. You will. So, the wave will appear to be like that. Remember the frequency will change. So, what would happen is that the wave that this ship encounters are the encounter this.

Now, I have this spectrum, wave spectrum eta which I broke it down to, but the ship is not seeing this. The ship is seeing this as this as this as. In other words, if this was omega 1, ship is seeing omega e 1. If is omega 2, ship is seeing omega e 2. If is omega 3, ship is seeing omega e 3.

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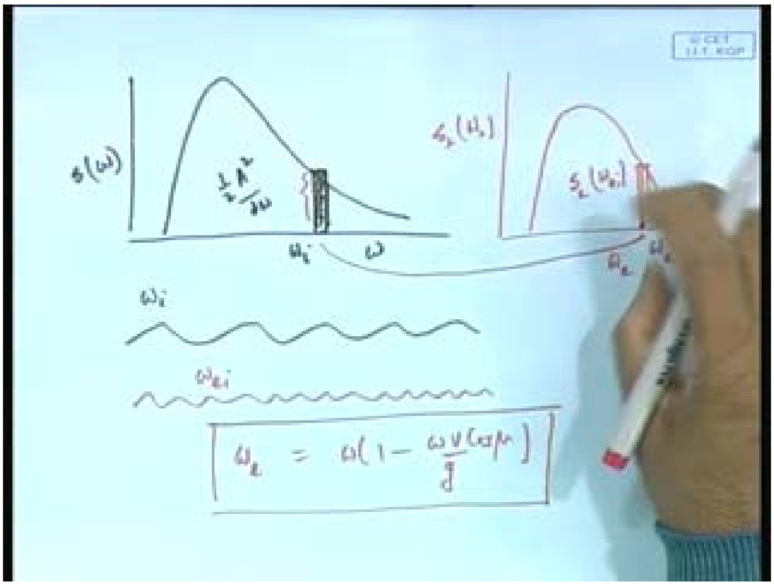

Remember the one that is feeling is not the omegas. So, what happen when I broke it down? This spectrum to this omegas, that is not what ship is responding to. So, I have to take one more step. I have to first break it down, then this to omega to omega e. Then, that I have to filter to this output. So, that is why in forward speed, I have this problem. So, that means, what I have to do is that see if I see in terms of spectrum, see I have this spectrum, wave spectrum. What does this mean? Say this is certain omega. Let me choose one omega I. What this mean? This means I have a wave which is this particular omega I.

Now, as far as the ship is concerned, the ship is going to see this here as these are all sine curves like it does not look like sine, but this is only for mu hand writing problem. So, it is going to look like omega e i. So, what would happen? You see I must transform. So, this is my what we call this is my s omega versus omega. I have to transform it to s omega e. Well, you can call it s e omega e to omega e. This should be my spectrum whichever way it is which basically is what my ship feels.

In other words, basically the waves that I find out you know you can say in this fashion. These waves that I find out are measured with respect to an absolute frame of reference. You have to transform that to a frame of reference which is moving with the ship because you are seeing the same waves as you move. So, in a sense, it is nothing, but transferring the same waves to a moving frame of reference and obviously, if you move at different speed, you encounter the same wave at different frequencies.

For example, if you went at the same speed of the wave, you would not even encounter. Nothing is going to hit you that we have seen zero encounter. So, I need to know this though. Now, tell me an interesting part. What is this mean this, this side? It is actually half of a square by d omega, but what is this area means? This area essentially means energy of all the waves of the frequency. Now, you see this frequency gets transformed. Let say omega e gets transformed to omega e I, omega e gets transformed to omega e i because I know that omega e equal to omega into 1 minus omega v by g. This way we know this know from beforehand encounter period.

This formula we have done it before in our very beginning classes. If you look back the lecture encounter frequency, frequency of encounter we have done even problems you know how and if the formula look like this, we have done this. So, I know that this omega e, this wave of frequency so and so will appear as so and so. My next question is how do I get this? What is my s e? This value, right. So, my question is how do I get this? Now, you see here this block should represent energy of all the waves of this, of all the waves which are same as this just that you are viewing it from a different reference frame.

So, energy will remain constant. Energy cannot change whether you see, whether you run and see the waves or you stand and see same wave would have same energy. So, therefore, this red block and this black block must be same. Area wise this is where gives me the interesting point.

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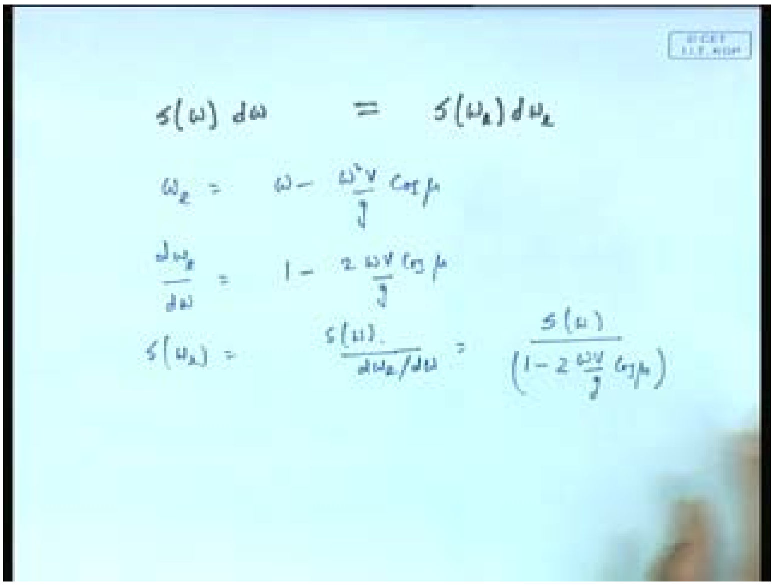

So, what is the red block? See the black block, the energy is here s omega into d omega, whereas that is s omega. Well, let me call it A with a particular I or let me put it this way.

So, this must be same for a given bandwidth. Remember I mean I just drop this because I is A. This thing, this is s omega d omega and this is s. So, I end up seeing this. So, now, it is only a mathematical manipulation we just do that. So, now, you see I have omega e equal to omega minus omega square v by g cos mu. So, d omega e. How much is d omega is coming to by d omega, let say one minus two. That is right.

What is from this formula s omega e? You agree with that straight formula this. So, this becomes so you see you get this formula here and you get this formula here. So, it is now a simple procedure to transfer the absolute wave spectrum to what is called an encounter wave spectrum. That is wave spectrum seen by the ship. How do we do that absolutely straight. Once again I do that every point.

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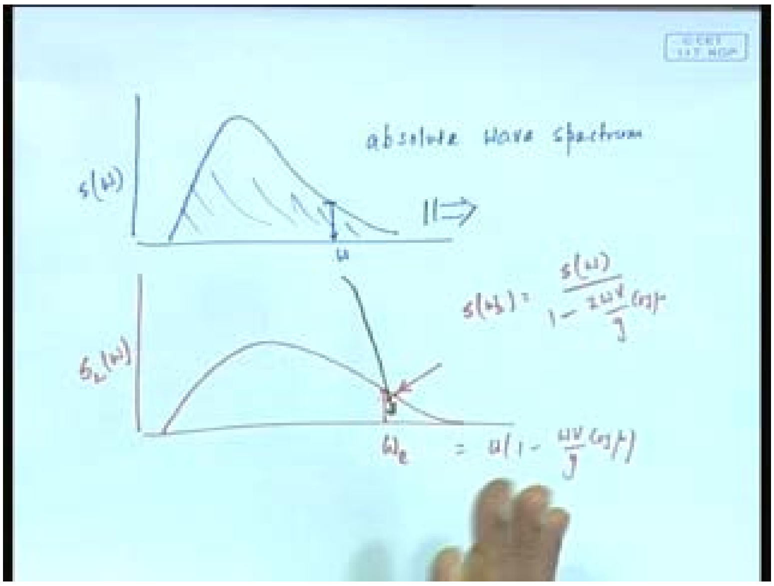

See here, this any point at omega, we will transform to put this. Why this formula is something like that and the height, this height will transform back to a height here. This height is going to be s omega e. What did we write one by, so what is happening now you see. That means, for each point, I can get this. That is very simple, right as far as calculation is concerned s omega and I am calling it s e omega.

So, you see now let me, so what we call this? tTis we can call absolute. This is the one that I got from the formula. This is the one that is been actually you know obtained at a stationary point etcetera which is what the observers gave. This is same wave, same spectrum as viewed on board the ship. Obviously, this cannot be have a formula because it depends on v, depends on mu, depends on in what angle in the wave and what speed you are moving.

See why I shown interesting point, I have to show in this transformation before I go any further. Look cos mu in a head sea, that is when the waves are coming towards the ship cos mu is within 90 to 180 degress. Let us say just simple case of head sea and following sea 180 degree, what happen to this? This value cos mu minus 1. So, this becomes larger one plus. So, s omega comes down because s omega e. See this is more than 1, right. If this is more than 1, obviously, this is going to be less than this.

So, this value will come down and omega e of course, becomes more because this is negative. That means, this stretch this side. So, it is something like remember the area under this graph net. What is the total area under this graph? The total energy of the waves that must remain same. Obviously, that is true because if you integrate you know like if you simply integrate this and this zero to infinity, total energy remain same because component energy is same.

So, there is no doubt on the area under that. You know that is the fundamental principle energy cannot change, but what happen is interesting that if I have a head sea, it is getting stretched this side and the top is coming down. So, it is something like you are standing here and kind of push pulling. It is like you are standing and pulling it. So, there is a change in head waves, but look at the following wave which is going to be very you know like complicated in some sense. I will discuss that in a minute.

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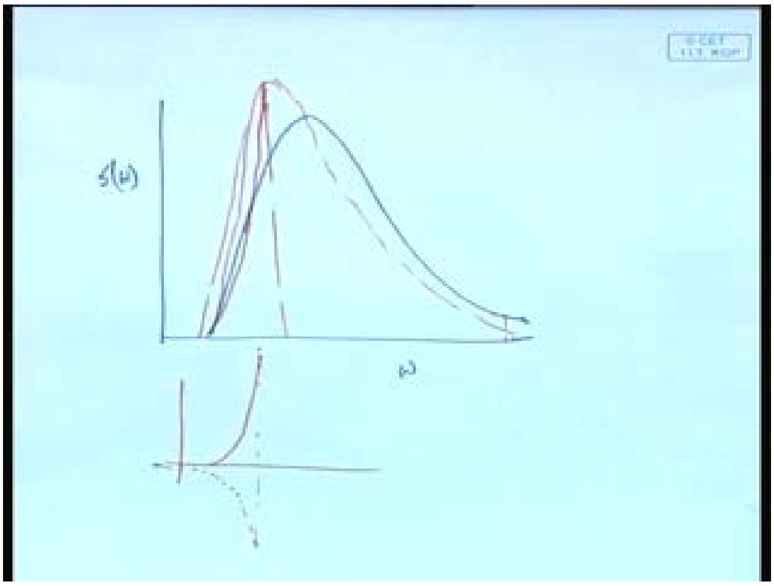

You see let us take the straight forward case of mu to be 0. Then, my cos mu is equal to 1. What is happening? Number one is that this becomes this or let we should put this another diagram because it is going to be, I will put in one only then. So, now, what is happening if I want to get omega s omega e? Now, each point is getting pushed. Remember this formula or rather shall we write this formula again here or rather we will, we can just transfer that.

See omega e is becoming less than omega because my cos mu is equal to now plus 1. So, it is getting squeezed and s omega e is going to become more. That means as if I am pushing here and these things are blowing up. So, you see what happen. It is becoming something like, well here only getting blown up. So, each point is getting pushed and getting blown up. Now, the most interesting point is that what happens? What would happen? Remember there is a case when omega is equal to 0. Well, before that let us look at this. This become 0 at some point. What happens when it become 0? It becomes infinity.

So, it becomes 0 at omega equal to g by 2 v cos mu. So, at omega equal to g by 2 v cos mu, see this. This part become 0 at omega is equal to 2 v cos mu. At that frequency, what would happen? You are going to find out that. This thing has become infinity. So, actually, thing would have gone to something like infinity. After that what happens? You will find out. I will leave it to you to work out after that. What you will find out is that see omega e. If you increase more becomes begin to actually reduce down. That means, omega e, although omega is going up, omega goes this side. So, the graph becomes very interesting. It actually if you work it out, it will become something like this. Let me draw here. This goes to infinity. Then, for the next domain, it goes to minus infinity. Then, it basically goes like that and goes like that.

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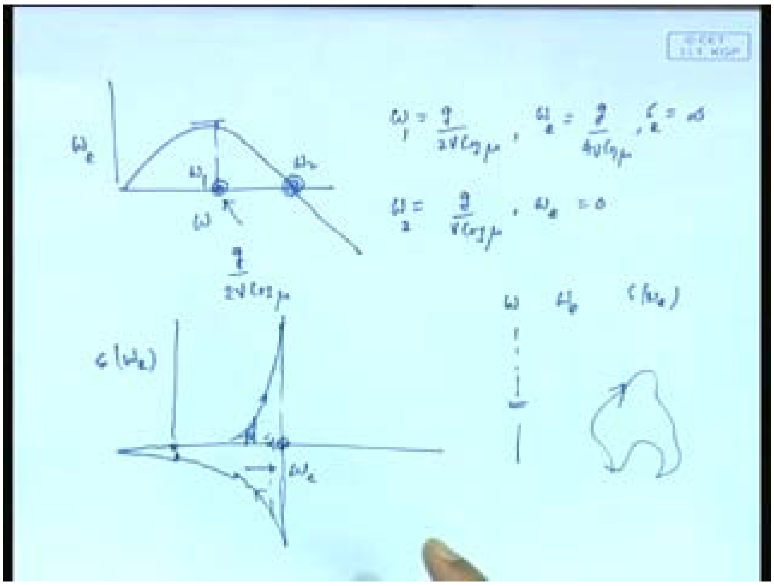

So, the shape looks very odd, very very odd. Very different because let me just put this in a slightly better perspective. Just let me put in a little better perspective because otherwise, it may become little confusing. In following sea case part, I am going to draw this with respect to this formula for omega omega e and if you recall my formula looks like that and it goes like that and this part happens to be omega one which is happen to be. This is omega one divide to be cos mu no and this part or rather let me put this way omega s e infinity.

Now, if you take omega equal to g by v cos, that is this point or rather let me call this as omega one. Call it omega 2 0. Now, the question is that when I do this transformation here, it is like that at this frequency equivalen in this goes to infinity. We can just check it yourself. After that see what is happening. My omega is increasing. See this is my omega e. Remember this is my s omega e. See up to this part, see omega is in see I have this graph omega and omega e and s omega e.

Now, omega is increasing here to here. So, omega is increasing here to here at this point of omega equal to g by 2 b cos mu. I have gone to omega e g by 4 b cos mu and s has become infinity. After that I am increasing omega. Further, what happen is omega is coming down. So, omega is coming down this side and if you see the formula for s, it will become negative. So, actually invert to that and go, it will become negative infinity and begin to come down. I will come to that. I will come to that physical is that you are actually going this way and the area becomes energy.

See if you take absolute value, you have to reverse it. Physical significance is very simple. When I integrate a graph in a cyclic wave, it is the area under the graph. That is physically important. That remains positive because d omega become negative. You see if I go I will show that to you in in a minute. So, physically there is no contradiction if you use that way, but we normally use reverse.

Now, take take this point and this point. What has happen omega is zero. So, I come to this point, but after that if I go further, I end up getting a graph like that. So, I end up getting a graph like this and what is now? Now, I will answer your question. See what has happen is I am going this graph like that, going like that, going like this. Now, if I want to what is the energy on the wave, it is this area. So, here if I take an energy. See take from this point to this point. Since, I am travelling in a negative direction, I will end up getting the same energy. See area will be positive.

See if I take a graph you know something like this graph, if I go this way, you will always find the area whether it is which whichever way you do, you end up getting the net energy under the graph. So, here when I go from here to here, see I have this energy.

Now, here when I go here to here, I have this to this, I will have positive energy because my d omega is negative and my s omega is negative. So, I get positive.

So, of course, this is very confusing. So, what we people will do is that you reflect this band. See this is also very difficult. So, what we people will do is you will reflect this that on this graph. So, you will say this is my jon one coming because of this part that is I will call this as jon one. This is jon one, then this two. Two is this part. Remember this part. So, I can do that by reflecting that area wise this way and three is this part which I can because I am taking absolute value like that. I can reflect it like that.

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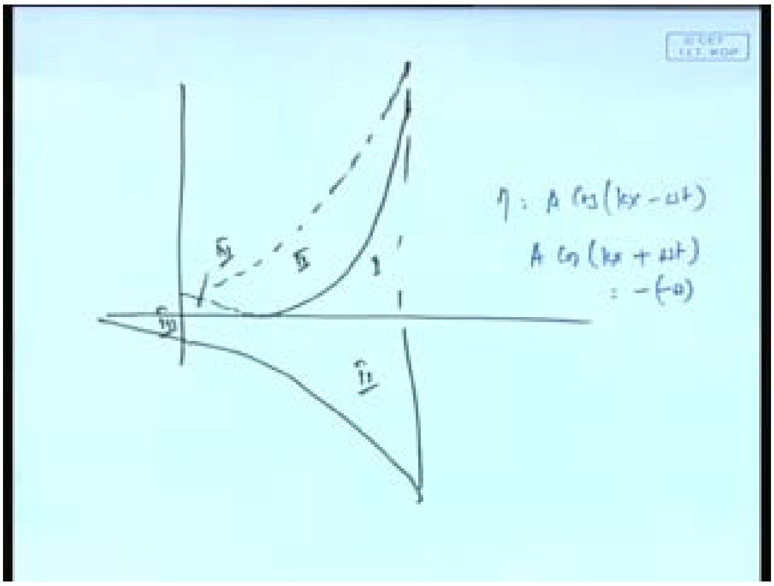

So, what would this is? This is becoming very confusing to you, but what would happen is you end up getting something like this infinity. This is infinity. So, what I do is this jon two, jon three, jon one, this two I will I do not plot it this way because people always are confused. So, what I will do is reverse it and make an absolute value just for the sake of plotting. So, when I reverse it, I end up getting here this jon two and I reverse this, I will end up getting jon three. So, this is my reverse jon two and this is my reverse jon three and I add it all up to end up getting the graph. No no no not necessarily. Not at all. Not because that depends on the response. It need not be symmetrical at all g by b cos mu is this. Yeah that is that we have that we have discussed long back in earlier. No negative frequency essentially implies that you are moving faster than the wave. Therefore, wave would appear to be as if going receding. See you are standing on a wave. See it is like you are here, wave is here. I do not know how to give this demo here.

So, it is like you are here, wave is here. Now, you go faster and wave is going like that. So, as a result to you, wave appear to be going on the other side. That is what is negative frequency. That we have discussed at length at earlier time you know. See that means, physically negative frequency. See it is like if you look at this graph also a cos k x minus omega t. It implies a wave moving in plus x direction, but if I have a cos k x plus omega t minus omega omega t, it is this is actually minus of minus omega t which means it is moving in the other direction, same thing.

Now, to you it appears to be moving in the other direction because you are going slowly. You have seen this relative speed in a train and everywhere you are passing fast, you think the other train is going backward. Same thing, exactly the same thing. Physically they both are moving forward. So, the physics, the dynamic involve is for forward, it is only that it appears of going backward. That is why we talk in terms of absolute because that is the question you ask people have a difficulty of thinking in the negative negative side.

So, here I have got negative omega e d omega e negative s omega e. So, it is better make it both positive. That is what is done. So, this is how this is the very special case of what we can call wave spectrum in you may say, in following waves which becomes more complex you know.

Now, looking back to this part, you know here if I were to look back to this. See the interesting part that I want to tell people is like you assume this way. You have a bar here head waves, pull it this side, pull it this side. So, it becomes fat following waves push. Now, you keep pushing at some point. This will shoot up to infinity and then, push further you go on the other side. So, physically you think that way you know like it is very easy. I always I am fond of telling this because you make a mistake in the shape.

So, you know like you pull that in the same graph, the area must remain the same like a wire here. It will just flatten down, push up. You have a block here. You cannot go beyond that. Then, it will go shoot up. In fact, you will find out later on that this is equal to this infinity is equal to group speed, group energy of the wave. In fact, it is very interesting. I will tell you this part. You remember that this part is resistance score. This you will be equal to you will find that the group speed of the waves which means the speed at which energy travels and you will find out actually that this is a phenomena of energy trapping.

I am surprised, you did not ask me what is the meaning of infinity s going to infinity. What is meant by infinite energy? I thought you would have asked me, but the question is that energy is not this thing. See energy is always remember this into this. So, this goes to infinity, but this goes to zero you will find out that therefore, there is no mathematical contradiction.

Otherwise, it is difficult to understand why and in a program computer program, you have to make sure you do not keep that point because your program will stop on the flow error, but there is no physical contradiction, remember. So, what you can do other techniques analytically integrate and all the stuff.

So, anyhow with that I am going to stop this lecture. The next one we will now talk about the responses including this transformation and then, we will find out that for ship we have we are so lucky that if you encounter bad waves, you can simply change direction of speed and basically become better. With that, I will end this lecture. .