# Seakeeping and Manoeuvring <br> Prof. Dr. Debabrata Sen <br> Department of Ocean Engineering and Naval Architecture <br> Indian Institute of Technology, Kharagpur 

Lecture No. \# 18

## Motions in Short-Crested Sea

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See in the last lecture, we basically described a short crested or we can say same as where or one can say directional wave spectrum is all the same term. Obviously, we are only defining that, because our whole idea is to study how the ship would respond in that the response in that, now obviously today we have to talk of response.


Before I go back to this I wanted to show you little bit of this diagram see which would make it little more clear to you about this way the directional spectrum is plotted. See here, I have this side the omega or $f$ this is my minus 90 to plus 90 frequency the directional spread. So, this graph if you are if I look at this graph. This is my f theta absolute this is my f theta and of course, this side is my S omega you can see this bell shape is so, evident.

And if I take excuse me volume under that actually, it will become still area unit wise because remember this side has no unit this degree this is my top is still going to be meter square second because you know meter square second into here one by second into no dimension it is a meter square the same thing. So, although we call volume remember one of the integration is over d theta which is dimension less this side has no dimension remember this side this side is having a dimension of radiant per second. So, this side dimension is meter square second. This how just to give you a feel about that, but about the 3 D spectrum because difficult to plot it.
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But I will tell you that this is another diagram which actually is the plot for a 3 D spectrum as measured from some data that is for a typical sea. So far a particular sea for which the wave slop information or the directional spread was known you know somebody has collected the actual data looks like that, why I am showing that because you will again understand here see when you projected this side if you are going to project this side what you find here is suppose to be this $S$ omega at theta equal to 0 .

Because you know if you project just like that if I were to project this side view then what I will get is S omega in 2 D and this side if I project I should get a spread you see here in this particular case there is a peak here of course, this particular sea data appears to have 2 peaks. This might have been for a typical you know data that obtain to a typical sea which is typically not so, just this is an example, but what you find out is that there is a concentration. it is still the entire thing is still like a like a bell curve accepting that in this particular data it happen to have a 2 peaks, but normally you would have one peak type. so, you know this is how it lo s like .
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So, now we will get back to our motions in directional sea. So, of course, what we found out is very interestingly is that S 3 D not omega this omega sorry. Let me again write sorry. I am writing omega as this thing mu or theta whatever you call it. Let me call it now wave because we are going to have also ship angle coming. So, this we knew that. Now, we have this ship coming.
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So, let us look at some kind of diagram here to know this angle business before, let us put some inertial reference. So, I have now here the ship going say and my dominant
wave direction. Let us say this is my dominant wave direction. So, which side the waves are waves are essentially, the entire waves according to my spread is lies between this sectors that is coming from here up to here with a dominant wave this side. So, what is happening this ship is going to therefore, feel waves from this angle wave coming here to wave are coming here.

Now, this is my of course, dominant wave let me call this dominant wave direction to be some with number star. And in general wave direction any general wave direction is going to be this mu w. So, my wave heading of the ship is this angle or you know this is the angle which that will be between the wave direction and the ship direction. See this was a dominant wave direction I said fine. And any wave direction if you take any wave direction this is going to be this and of course, the heading angle is nothing but what is the heading angle that the difference between wind direction wave direction or the ship direction.

So, wave heading actually the ships heading mu which is what the heading angle we call. Basically, is nothing but mu w minus mu $S$ that is the wave heading angle isn’t it now of course, if I talk about the dominant direction wave heading angle it is going to be this. now, what is happening see here excuse me this is my wave heading angle, but dominant direction of weave heading angle is going to be if, I take the dominant direction is going to be this one that is this angle.

So, my wave heading is going to be from here to minus 90 degree to here to plus 90 degree this is see this plus minus 90 degree is going to give me my total heading with respect to this or I if I want to do other way round if I call this heading without that again if I call it say I call it mu star. But if I call it mu again mu w minus mu S this is my heading angle. so, I am suppose to find out response of the ship for heading angle ranging between mu w plus 90 degree minus mu S to mu w minus 90 degree minus mu S.

Because mu w will vary from this line to this line see obviously, what would happen I will take a mu w well mu w star what I mean this basically varies between mu w star plus minus pi by 2 minus mu S. this is the variation, that I am going to have that is very obvious you know that whichever way you are looking at. Now, the calculation is very simple you see the calculation is something like this.


For response R A O same formula will apply my 2 D response R A O what has happened if I call this response R A O. I had initially this is this was equal to well R A O square this is my formula earlier now, what is happening I want to have this thing 3 D this is 2 D it is going to be having omega e mu this is going to be R A O which is going to be a function of omega e into the heading into $S$ square of that, this here this mu is going to be mu w. really the process is very simple what we are trying to do now is that you if you will see what we are going to be doing is simply repeating the calculation.

See what is happening is that now I have got again looking at this diagram. if I bring that for a given theta this my this thing. Now you consider this first case I will go back to this well the case number one. so, I have got ship going this side waves range between this is my dominant wave direction again and wave range between this direction to this direction what I will do first I will take one direction some direction. Here so, I know its heading angle this angle. So, I will take some value of mu. this is my mu w some value how much I will take obviously, I will take mu w star say minus pi by 2 that is this one.

Then I will take mu w star minus 60 degree like that I will take so, I know this therefore, I know mean mu heading mu heading is of course, mu w minus mu ship this I know in fact I can make it because it does not matter waves which side it comes so, I will may be not keep it anyhow so, I will know this heading angle that is this heading angle plus in fact I can put up this also this is nothing but mu w minus mu ship.

So, for that heading angle I will find the R A O square it find out my S 2 D omega into f mu multiply that then I will get spectrum for that part. So, like that if I do for all then I will get the entire spectrum for all part. So, this is exactly what is being done in the procedure so, it is absolutely pure algebra there is just nothing to it I give an example then we will be able to understand that. So, in other words what would happen is that we will be just doing that. So, now this response is S xi we call know actually we can call this let us call let us call.
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So, we end up getting the response let me write response is $\mathrm{S} R$ rather than S xi because that is better. So, S R see here this mu we should write remember the heading comes inside this what is this omega e omega E is actually function of omega v and mu . So, this mu is actually as a functional dependence I should write w because the wave heading angle because basically see this is nothing but mu wave minus mu ship. Mu ship is fixed remember mu ship is fixed going in some direction. with respect to x y coordinate mu ship is fixed it is mu w that is basically varying.

So, I can take this in terms of like this and this is what we find out is that this is S 3 D well we can say S 3 D. you know R A O this is what we have found out. So, this is this is what we have done before. and of course, if I want to find out the area under that if I integrate that this integration is going to be 2 times because of this you know d omega d i end up getting what is called area in other words volume rather the my point of saying is
that you end up getting the volume under the response spectrum that is volume under say I am going to get a similar response spectrum. each one I multiply with that I will show that then I will get the volume.

And my response amplitude one third etcetera all of them will again become equal to say 2 m 0 etcetera. so, the procedure becomes just the same as 2 D it is just that you are going to be repeating it, so it will go like something like that you will start here. We will take mu w equal to some value say in this example,
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Let me take this example, it turns out to be say this value mu w star is let us say 60 degree then this first value is going to be minus 30 degree.

So, I am going to have mu w is coming from minus 30 degree to 60 plus 90150 degree so, I am going to take minus 30,0 like that I will do all that .so, here what I will do here say first one I will take mu as minus 30 degree then what I will get mu ship of course, I know in this case mu ship is let us say plus 45 degree say this is actually say 45 degree.


So, I have plus 45 . So, be there 45 degree then of course, my mu heading will become minus 30 minus 45 whatever that is seventy basically minus 75 degree. which is taken as 75 degree because remember for a ship for a particular ship whether, the waves come from this side or this side plus theta or minus theta they are same because you are calculating R A O for one half because of symmetry so, this is like same.

So, I will have RAO for that now I will calculate this. So, I will have for this my so, I have this. then I have this is 2 D then I sorry this I can multiply this into directly, I can do here this f mu see here, what I do remember for 75 degree I know the R A O for seventy five degree what I will do I will find out the typical that component how because I know here what is my f mu this I will find out that is 2 by pi cos square theta remember theta is 30 degree here no sorry not 3090 degree here sorry because it is measured from it is measured from this the spread theta is actually this measured from the main line.

So, I know this spreading function which is going to be 0 here. So, I know this I know this I will get this one. So, the point is therefore, I am just repeating this calculation. That is all I am just repeating that that means my table one.


I am going to have say first one first table I will I will be taking well mu S is always fixed. Mu S is say again I will want to do this. Now I will take here mu w equal to minus 30 degree f mu w I will know what is the value this value I would know it therefore, for this particular one I calculate my so, called S response this I will find out this I find out same way the way we are doing previously, you know like we have got omega a here then you have got S 2 D omega e here then of course, S 3 D omega e mu here because this is nothing but multiplying this with f mu this value see this value is multiply this into this one straight forward. then I have got R A O here for this then I have got here S R this is going to look like some small thing like that this is this applies for mu w equal to minus 30 degree repeat that for 0 degree.

So, like that you will end up getting here earlier, what we did I did not have this as a variable remember so, what I had only in fact, what I had this, but I always had this same as 45 degree because I have always assumed not 45 degree sorry I always have this mu w as mu w star that is I always had the waves.
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Which is all the waves are coming only in this direction in earlier case 2 D case I all the waves coming in this direction. So, I did the calculation only for a one particular mid of this star all that we are doing here is that we are repeating the calculation for assuming wave to be come from all direction and for each direction I have got a 2 D spectrum, but the 2 D spectrum is a 3 D multiply by that function for example, in this case for the waves coming from this direction my 2 D spectrum is actually 0 , because fmu is 0 . may be I should illustrate it even better because I think there is some little confusion still, I will may be I will elaborate slightly even better.
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The full state for a particular case of let me draw some diagram for a case just one particular case it will be easier for us. Now I will draw different colour here this is dominant wave direction. Let me call. this is the waves are going to be of course, within this waves are going to once again, I write anyway I am going to take a particular just a particular wave. So, this direction see this is I call this alpha. see what I am just see I want to go through this slowly once again then because we are going to show only one case and then you can just repeat it.

Once again ship is here in a absolute frame of reference. you can if you want you can make this as x direction, but normally, we do not do that the reason is because when we want to what would happen in nature in a particular place this direction would be fixed. For example, you would know from spectrum that there is a dominant wave in north east direction say between Calcutta to Andaman. So, this direction is fixed.

So, the ship might be heading north ship might be heading east the ship direction is arbitrary depending on which side did you ferry while going from here to Andaman it might become heading south west whereas, south east whereas, coming back it is to Calcutta for example, it will become north west. So, obviously there is a direction change in therefore, this direction should be kept variable in my calculation process that is why you are keeping this way with respect to absolute you can consider X and Y to be to north to you know west no to east say you can say north this east etcetera you know like if you want this north this is east.

Now we are speaking for a particular case. When we I know that there is a mu star here waves are coming here my ship heading is this thing. question is find out the response now, I know from the spreading that this wave is spread from this angle to this angle that means from mu w star plus minus 90 degree from here minus 90 degree this angle, plus 90 degree this angle. Now I am now going to calculate do the calculation. I took one particular I have to repeat the calculation for all wave. So, let me say I am doing the calculation for this particular direction of wave that means I have a table here first table.

In fact I will start from here, but there may be number of tables say table. So, I am calculating the spectrum for this then for this for this for this for this for this etcetera. So, this is my n-eth wave component. Let us say, direction so, for my n-eth wave let say I have a for particular one my mu wave is given by well mu wave is given some mu wave
given let say whatever. So, this is to be mu w star in other words I am doing this for let say alpha equal to repeat this minus 90 minus say 60 going to plus 90 that is what we are going to do.

So, I choose one particular alpha between minus 90 to plus 90 , so I get mu w I get mu w now remember here my spreading function is function of which angle, it is function of angle alpha. So, my spreading function a is function of angle alpha. So, what I have got now here is the case that a table, where I have got mu w star this is constant mu $S$ this is constant, and I am doing table number one number say not one say some X. So far that I have alpha equal to minus say 60 degree which of course, refer to mu w equal to some value. So, I am doing this for this alpha equal to minus 60 degree remember I have to do from minus 90 degree to plus 90 so, now I have got this, so I know now what is my f alpha.
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So, here I am having this now my alpha which is my wave spreading or whatever you can call mu that is the basically this is the way well actually, I call that mu in my earlier cases. in fact, we call this to be we call this alpha as mu in other cases this is my wave heading once again you see what is happening my wave is here my ship is here. No again I sorry no alpha is alpha only.

So, basically mu is going to be essentially this angle. That is here to here that is mu x minus mu w . this is going to be, so my wave heading is mu S minus mu w . so, that to this
is the problem my this is muS mu S minus mu w, but what is mu w. mu w in my case was mu w star minus alpha. So, this is basically, mu S minus mu w star plus alpha. so, this is the function of alpha.

So, for a given alpha so, I therefore, I have got alpha equal to something. So, I have got mu equal to something. So, I my table now here alpha equal to something. So, I have got mu equal to something. This is my angle for the spreading function because; spreading function is connected to angle measured form wave to this. That is alpha response is connected to angle of that wave from the ship that is this angle or if I were to put this way this is my yeah, this mu S mu S is on which the response depend alpha is on which the spreading depends alpha and mu is obviously connected to mu w star. so, all these are kind of connected to each other.

So, therefore, what we going to do is to repeat that. So, I have got this a particular angle alpha minus 90 or minus 60 I said therefore, I find out mu S so, I have got alpha. So, from alpha I get I will find out the mu that is heading. So, spreading is $f$ alpha R A O is R A O. omega e into mu and mu and alpha are related because. The what I we find out is this thing basically mu is what mu equal to mu S minus mu w star plus alpha this is fixed. So, this is fixed so, this is fixed for a given case. So, I have mu and alpha fixed.
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Now comes a very simple thing. So, for this particular case I once again, I if I have I have mu star given I have mu S given table one. I take alpha equal to minus in 60 degree
thus mu equal to some number I will get. Fixed now for this particular mu I have now the calculation. I have S 2 D remember this omega e also has to be done remember this important thing remember that this transformation has to be done.

This 2 D to encounter spectrum and here comes the another interesting problem because what would happen for a particular mu see I have no choice see when I take minus 60 degree I have this mu there. So, this might become a following wave it may be a following wave. So, when you do a 3 D spectrum there is a chance that you end up getting a following wave. Even, if the dominant direction is not following wave, because of this spread 90 degree plus minus spread. Some component might become following waves. We I show that to you later on in a minute.

So, here there is a chance of the remember that this transformation this omega e. what is omega omega e equal to omega 1 minus omega $v$ by g cos mu or cos mu is be being the that wave heading angle. But this heading angle mu I have we have seen is mu S minus mu w plus alpha. so, I had I do not have control remember this transformation is not constant for all so, I will be choosing one particular alpha for that I get what is my wave heading angle what is my spreading function.

Therefore, I have this transformation this transformation may not be necessarily, this transformation may turn out to be as equivalent for following waves because it may turn out that this particular alpha as you chose this value has turn out to be equal to more than between 0 to 90 degree that is possible. Because after all whatever heading you will understand the heading will also be over all spread over an radius of pi. This mu range of mu will also be pi. because you after all you are going from one side to other side.

So, there is a chance of its becoming some component following waves. So, you have this from there next I multiply this with f in this case f alphat that spreading function. So, I end up getting this which is S 2 D into f alpha then I have R A O then multiply that to I i get S 3 D response for that particular alpha. So, this I will repeat it and when I repeat it I end up getting ah the graph. So, it will look something like this therefore, for example, we did that I just show it to you it may look something like just show a particular this it may look ultimately, something like this. this I do not know how I can put the picture here.


The RAO may look ultimately you may end up getting a look like this. The spectrum may look response spectrum; very difficult to plot it I mean what I mean is that what is happening is that this is my response. you know like actually it is not that way I will put it this way this is my R A O let us say, what we are putting R A O see the R A O you will be putting the R A O for over the entire you know like spread of a particular see this is my pure R A O because I would have calculate R A O see I will need now this tells me. I would basically need to calculate my response amplitude operator from 0 to 9180 degree however, not all waves must be existing.

Typically this 3 D spectrum therefore, requires me first to precalculate R A O for 0 to 180 degree. Why 0 to 180 degree because symmetry tells me that 0 to minus 180 degree is same. So, I will have the entire 0 to 180 degree saying that, for any angle mu heading angle I know the R A O. So, I must calculate my mu the R A O for all angles. so, this is what typical R A O given.

Now this picture again sees if you see. so, I have my R A O the that means whichever direction the my wave comes I know my response comes at second step I chose spectrum number one with a dominant direction here then I know this spectrum have waves coming from this angle to this angle. So, I will find out R A O for this angle when I chase then I repeat that calculation. So, what would happen is that ultimately, I will end up getting a series of graph for different angle.


So, I will end up getting basically S R the response for different angle alpha you can say here, we call alpha we can call anything this alpha being the angle of the wave with respect to, the dominant angle that is this one that is this one this with respect to the wave the alpha.

This alpha would be ranging from minus 90 to plus 90 and I would have been doing this for say S R omega e for say minus 90 then minus 60 minus 300 like that. For each one therefore, I will then I will get a spread I can always plot this graph the more important thing is that then I will find the areas. So, what I will do I need to find out this volume under S R omega e alpha d omega e d alpha obviously, this is very simple.

So, I will essentially do this each one then it is like your displacement calculation know we find out for this section the areas and then add the sections, so get the volume so, you first find out this for each one.
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So, for each one this picture, I wanted to show this picture this is it comes out in a short form it will come out like that. for a particular angle I will end up getting this, this I let me say that this is my essentially, this wave this is for alpha equal to minus 90 degree then I have well of course, here will be.

Like that you repeat see what I have done here, if you look at that you are getting for a given minus alpha. First of all, remember I wrote it in one step, but there are more than one step involve. First of all you have to fund out well 3 steps involve. S 2 D omega. Then see for getting this one. first I have got S 2 D omega that is lo that like that then I have got S 2 D omega e. then I multiply that with $f$ omega or $f$ alpha to get this one so, this itself has got 3 steps. So, well may be I will just show this once again.


So, what is happening here to get this I have got S 2 D against omega from there I end up getting, now actually this is not a graph here then let say then, see this f alpha is a number know see for example, cos square 60 is how much cos 60 is root 3 by 2 . So, 3 by 4 give cos 60 is 1 by 2 . So, 1 by 4 then. So, whatever 2 pi. So, you know this what my point of view was what I am saying is that see this particular one is therefore, this one this is a graph is the one that you are plotting here this is the graph you are plotting here.

Remember, getting this graph involves your alpha. Alpha is involved in this transformation is this transformation this to this. what are angles it involve it involves not only alpha also ship heading because connected to ship heading this omega e comes out omega e is equal to you know like basically, this is a function of mu and mu is a function of mu S mu you know like w star and alpha. So, all are involved you just have to keep your mind straight that is all.

All I am trying to say is that the procedure therefore, is absolutely algebraic there is just noting to it only what we are doing is we are repeating the graph going back to this very first diagram that all that we are doing is now is very straight forward.
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I would be knowing for each spectrum for each angle this one is for example, here this is for 30 degree this is for 30 degree of angle alpha like that.

Now I would find out I will choose this one so, I know f theta but then I will find out this causes what heading angle with respect to ship that means these waves are coming at certain angle to the ship say 45 degree then I will choose R A O for that particular 45 degree use omega e for that particular waves then I will find out the spectrum for that then I will get response for this like that I will go in other words
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What is therefore, happening my ship is travelling here one set of waves are coming this way. so, I will find out what is the response.

Second set of waves are coming this way I will find out that third set of waves are coming this way. So, like that, I will vary this from here, this is a dominant direction somewhere here from there to there. So, I will choose this I will choose this I will choose this I will choose this each one, I will choose wave coming that is all there is nothing to it and therefore, now the question is that when I do that you see that I cannot avoid some time following waves very simple reason I have the ship going this side.
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Now let us consider this case in this particular case. So, if I had dominant wave is head wave dominant wave is head wave then my waves will be coming this direction this direction this direction through this circle. So, here what is happening, I have actually all the angle heading angle between minus pi by t 2 well actually, if I come this side well the angle is 180 minus 90 to 180 plus 90 . So, it is all head sea condition, but suppose, any other angle see this is this is a point any other angle supposing I had the wave coming dominantly this direction what is happen therefore, my waves are spread over this angle this. So, when I consider this angle see this particular angle my ship is here my wave is coming this say this particular one. so, my angle is less than 90 degree.

In fact you can therefore, what I am saying what my point of saying is that when I take 3 D spectrum. then necessarily if my dominant direction is not 180 degree because after all
if dominant direction is 180 degree in other words if you look at this mu is equal to mu ship minus mu w star plus minus pi by 2 this is by range. So, if this is this one is pi then $i$ end up getting pi plus minus pi by 2 is how much pi plus this head sea.

But if this is anything beyond pi anything else than pi I always will have an angle which is having a value between 0 minus pi by t 2 plus pi by 2 which means which will have a following wave condition. For example, once again if for example, dominant weave is beam direction.
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So, this is another way of looking at see my dominant waves were coming this side this dominant wave. so, what will happen there will be some wave this side some wave this side.

You see here I am getting a completely a following wave one component. The dominant wave is this side I will get wave from here up to here. So, this is the reason why although I said when I was talking about 2 D spectrum that look the 3 D spectrum transformation that is omega 2 this is following wave and in a head wave sorry head wave following wave it can become very funny.

I was saying we do not pay much attention to that, but you know if we have to do in automatic program. You are writing a code or something you are doing a calculation for 3 D spectrum then what would happen you would normally feed the dominant angle as
input data. you would feed the ship angle as input data or you may also call some time you may if you want you may feed that dominant angle with respect to ship as input data 45 degree.

Immediately, you find out I have some waves which is actually 45 minus 90 to 40 that is 45 degree itself that is a bow stern quartering wave up to the bow quartering wave. So, I cannot avoid doing following a wave what should say a spectrum for heading angle which correspond to following wave, so the shape as I said yesterday one of the components.
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Therefore, in case 1 so, I have component one when alpha if once again if I take minus 90 degree. I end up getting a spectrum shape which look like that, but when I take alpha equal to plus 90 degree. Then, I end up getting a shape which is nice this is my S R

Then what is happening remember I have to find the area under this and also area under this because I need to need the volume. So, I may end up getting problem here because I have some singular values coming and that is exactly, I was trying to tell that we could use for such cases integration over omega domain you that is much better to take integration over omega domain for finding area.

So, there is no avoidance of actually knowing how to handle the spectrum in following wave condition. If you are going to determine response in 3 D waves. So, this is one
point that I wanted to make the other point is that which I said yesterday that you see look at this volume part once again this volume part now response spectrum is going to look something similar to that because you are multiplying responses.
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Now what is happening if I were to take only 2 D spectrum. So, my 2 D spectrum would have looked a graph which will which would be this one only, but higher remember this is not you can push down. so, now suppose my response the I am doing it for different mu S. so, I had a particular mu S my response is higher on this side after all I am doing for see I am doing the responses for mu S equal to 0 degree ten degree twenty degree etcetera. in one particular case what would happen the mu $S$ will be such that my response R A O was very high after all response R A O also keep changing with respect to angle.

The simplest example is beam waves roll you would expect that if obviously, the ship is like that and waves are coming this side you expect the largest roll. So, roll you expect high if heading is pi by 2 . now what is happening let us take roll as an example, if I were to do a 2 D spectrum calculation I am presuming all the waves coming from this side and I have a very large roll so, I end up getting an estimate of roll say spectrum whatever roll one third or whatever some value say 15 degree.

If on the other hand I were to take this is a dominant wave direction and wave coming also from here and here remember see remember in this case I will have wave coming
this side, but also me wave coming this side some wave coming this side now these 2 waves do not give roll at all. Because they are coming exactly symmetric direction. So, therefore, what happen the one that is giving roll high is less number in number in the 3 D wave.

So, what would happen if I get the volume I will end up getting the roll which would be probably less than that 12 or whatever may be around 8 second 8 degree. so, what happen if I were to use a 2 D spectrum my prediction is that there is a case or direction of waves in which my roll maximum is become twelve degree one third roll maximum. If, I take on the other hand 3 D spectrum. I find out that the maximum roll considering all angles is nine degree or whatever. Therefore, 2 D spectrum normally ends up giving me higher estimate.

You may say why should I go for higher? That is the different question, but you know that normally that is what is obvious. This is one of the reason that I mention yesterday that 2 D spectrums are used, but having said some people want to do 3 D spectrum. now we learn today and that 3 D spectrum calculation as far as calculation is concerned is nothing but sum of 2 D somebody told me long back you know that this I want to I will end this thing by that.
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Say if, supposing a function there say let me call it y is a function of x you need only one graph if it is function of x 1 and x 2 you need series of graph, if it is x $1 \times 22$ and x 3
what you need is same thing repeated. So, this is what is happen my 2 D case it was function of omega mu v, but $v$ is constant. So, what happen for these 2 I need one graph but now what happen I have to repeat that for different angle also, that is it?

So, my 2 D spectrum is page one and 3 D spectrum is going to be number of pages. So, simple algebra nothing to it. So, with that I am going to end this 3 D spectrum you know motion in 3 D spectrum.

