

Seakeeping and Manoeuvring
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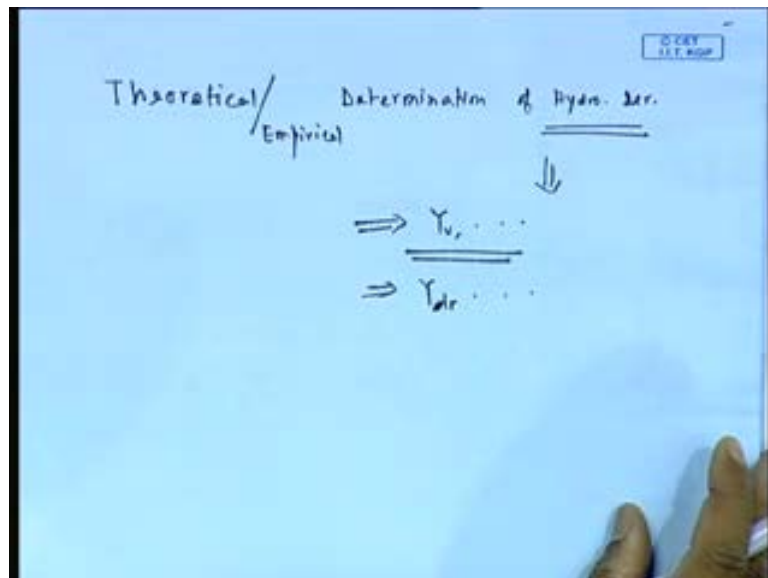
Module No. # 01

Lecture No. # 39

Theoretical Determination of Hydrodynamic Derivatives – I

Today is the last but one lecture (No audio from 00:23 to 00:38).

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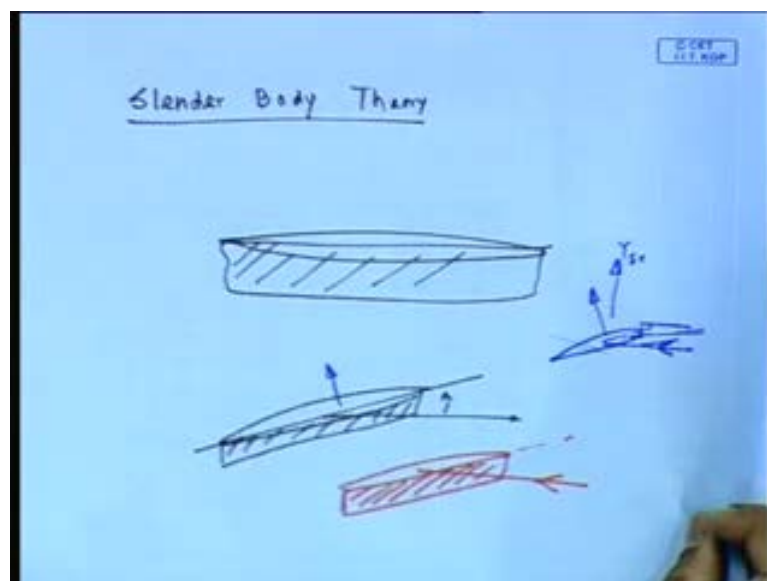
Actually I keep this gap here to say, it is theoretically empirical etcetera, what I meant is this see, now we understand that when you want to study maneuvering characteristics. I require to characterizing the hull by means of quantities called hydrodynamic derivatives. So, these are the quantities, basically they are quantification of the characteristic of the hull, rudder characteristic also is there which of course, is given by rudder derivatives **right** this is may be Y_v etcetera **etcetera**, and rudder derivatives would be etcetera **etcetera**, **can the** I can call it hydro derivatives, I can call it control derivatives.

Now, control derivatives we have also talks slightly Y d r **sorry**, how we could estimate them using CDCL etcetera, **you know** very simple form we have shown, because essentially it is a force on angle **angle** of attack. We discuss about this based on experiment **right**, how we can determine that and in fact, that is still the way of doing, we will see at the end of the class, if you want to do reliably.

But, **you know** when you are doing a design; say today you are doing a design. Now, for design you want to have an estimate for the derivatives, because that will tell you how the ship is sluggish or bad or good etcetera **etcetera**; you want to know essentially its control characteristic, because we have seen that if a ship is highly maneuverable, then it tends to be little unstable. And is very stable means it does not want to turn, very sluggish like large tankers are designer sluggish, because you are only going trade, but whereas, a coastal vessel, coastguard vessel or a naval vessel, you want to have a very **(O) you know**; if you want to chase a for example, for **you know** security purposes you have to be very agile, so you need to have an estimate.

Now, I **I** want to talk about that, because this subject is not very straight forward, you will find out there was a wide variability that may actually come in, **just one second I also close this**. Now, let us look at this see, **one of the best**, one of the method which is used to find out this theoretically, we will first talk theoretically, is what is called slender body theory?

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We will of course, not go through this theory, but I will want to just tell you, how this theory is developed. In fact, what happen that, see if you look at this hull, well you can say slender body theory and **you know** like strip theory together. Look at this hull, look like that just look at the center plane, the center plane is this one, now it is say from the top side I say it is yawing the ship is going like that in this direction.

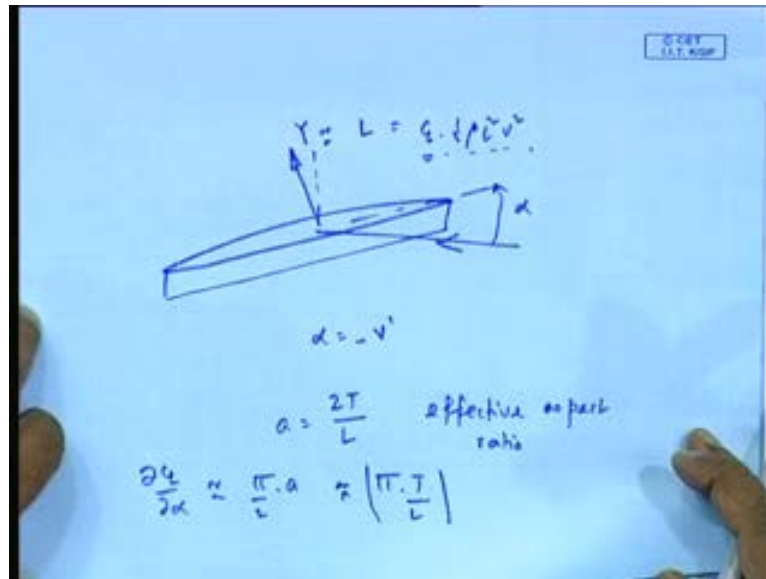
Now, what happen this is like a section a foil section, if not foil a particular section going with an angle of attack, remember it is something like well the draw, may be drawing is not clear, but if you look at this side see ship is going to turn like that, now I have this is an isometric view, cross section and the flow comes like that, ship is like this.

So, **you know** what happen essentially, because of this angle of attack, this is this angle of attack which is caused here some kind of a force, just like what we have done for rudder, not clear. See, in a rudder plate what happen, rudder plate was like that, flow came this way and I get a force this side and of course, this force would be viewed in terms of normal to the ship, and we call this to be Y_r , $Y_{\Delta r}$ **right this was** this was the main ship.

Essentially understand the physics, the force coming in the Y direction here was, because of lift and drag, because it is a foil section similarly, when a ship hull is moving with an angle of attack, the ship hull if you look at the center plane can be viewed to be a very long aerofoil type or lifting device. Because, it is going in a **you know** this that even a flat plate goes of an angle of attack, the it produces a lift may be very low lift, **it is not an efficient lifting device**, it is not an efficient lifting device. Because that is why you have aerofoil section; but, if you look at this older boats, country boats you will find they simply use one flat plate, just make an flat plate angle.

So, a flat plate when the flow comes to it at an angle of attack also produces lift, now the ship here the theory is developed based on the fact or you can assume that the ship center plane **you know** l by t , if you look l is this side t is draft, as if it is an l by t flat plate. **You know** this is l by t , you can say this is l , this is t and flow comes this way, so what happen if the flow comes this way force, so you see this can be exploit, this is what is done exploit.

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I will just, if I draw **draw** it again here, the ship is now like that flow comes this way with an angle say alpha, what is alpha **alpha** actually you will find out that is v dash, because by definition minus v dash whatever, so here I have this force coming this side, actually this align here, if I call this force coming this side. And I can call this force Y approximate same as essentially lift, because what happen this angle is small. If I basically it will be $L \cos \alpha$ etcetera, but if you assume this angle to be small for small value good approximation **right**.

See, I can say Y equal to L , where L equal to actually C_l into whatever **you know** half **you know** $\rho l^2 v^2$ etcetera **etcetera**, so from there I can easily find Y and **you know** like Y for alpha **alpha** is v dash. So, I can find out dY by $d\alpha$, same way we have what we have done, so this procedure of assume and no, **that** that is one thing.

Second thing is that, see **there are** there are theoretical estimates, the theoretical estimates that is the says that actually I can use this, if again if I draw this way, the aspect ratio, the effective aspect ratio for this particular situation has become what, effective aspect is basically given by 2 by L , that is what we call effective aspect ratio.

$2T$ is taken, because you have to take a double body on both sides, because it is in a free surface and that is why it becomes $2T$, that is the by definition and what **what** why I am saying is, because there are lot of **you know** like theory that tell us that basically this lift

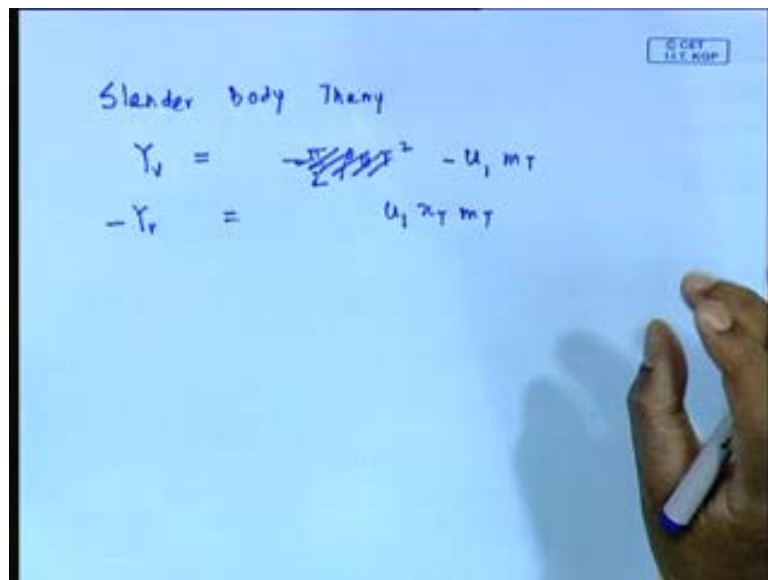
for a **low aspect ratio**, low aspect ratio lifting device $d c L$ by $d \alpha$ can be approximately by, what is called as π into a .

This is one approximation that in fact we showed that in one of the classes regarding various formulas for rudder, but this normally valid, is valid for low aspect ratio you can say in fact, this theory all came more from aerodynamics. When there was this thin wing theory etcetera, this is like a thin wing, if you assume the this to be a wing, thickness is very small compare to length, the breath is very small it is like a thin wing of a loop, but the difference being is a low aspect ratio.

So, now, so this becomes something like π into ρ **no no**, π by 2 a by this thing it becomes πT by L of this order, so **you know** what happen **you can** you can have one kind of a like formula based on this, but I will tell you that similar idea is used and the slender body theory is developed.

And the result of a slender body theory turns out to be, I want to write down to you this one, so we will come back to this part they all connected slender body theory, then there is also added mass theory, then there are also **you know** ultimately they are to be modified by empirical coefficients etcetera **etcetera**.

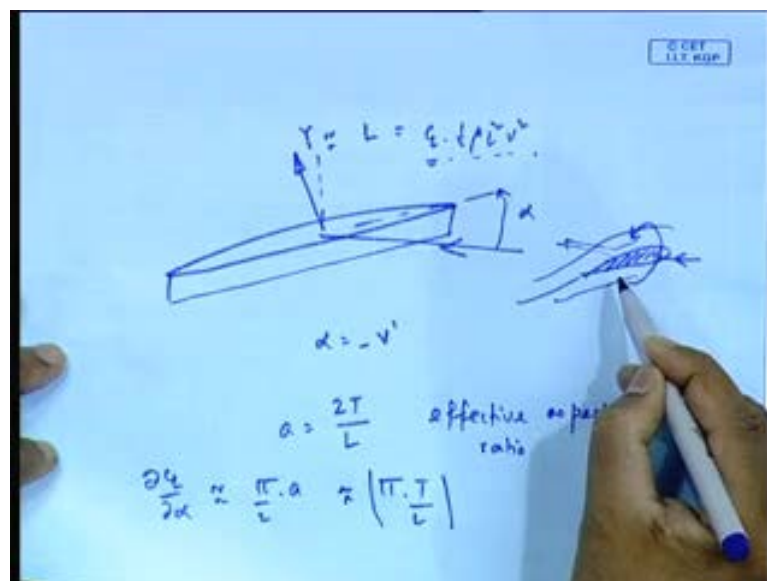
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But, the slender body theory result purely theoretically based on thin wing theory turns out to be this way, and I will show you that this two actually becomes this I mean this

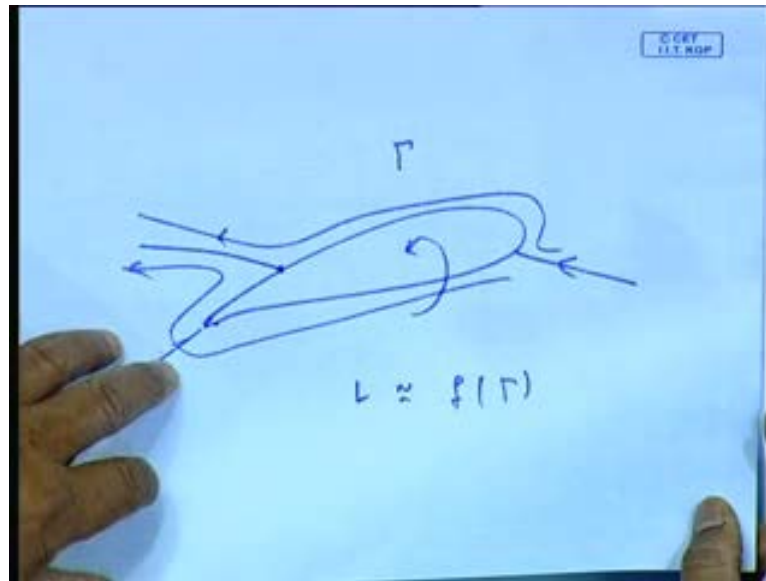
difficult to say, where slender body theory, where thin wing theory used both are related. This **this** results in this way, γ is comes out to be these all dimensional value $\rho u T^2$ no I am **sorry sorry** no **no** making a mistake, **the** this will come, it comes to be minus u I will tell you what is $m T$, γ I will put this here, turns out to be $u \times T$ this is actually, not exactly this the what I am saying will be connected to this after, but slender body theory is more fundamental. It actually presumes the body to be a lifting surface finds out the kind of lifted case generated, because of the **you know** I will just give another example **you know** this lifting theory.

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The flow comes this side there has to be a circulation, so that flow is coming out from this side this is the fundamental requirement for a lifting surface theory, **do you** did I make it clear. See, if there is a aerofoil, if flow comes this side, if you presume there is no circulation present, this is the fundamental of any lifting surface basic hydrodynamics, then the stream line would have gone out like that, flow would have come and gone out like that or **or** let me draw this part I will come back to this, because **you know** this is all.

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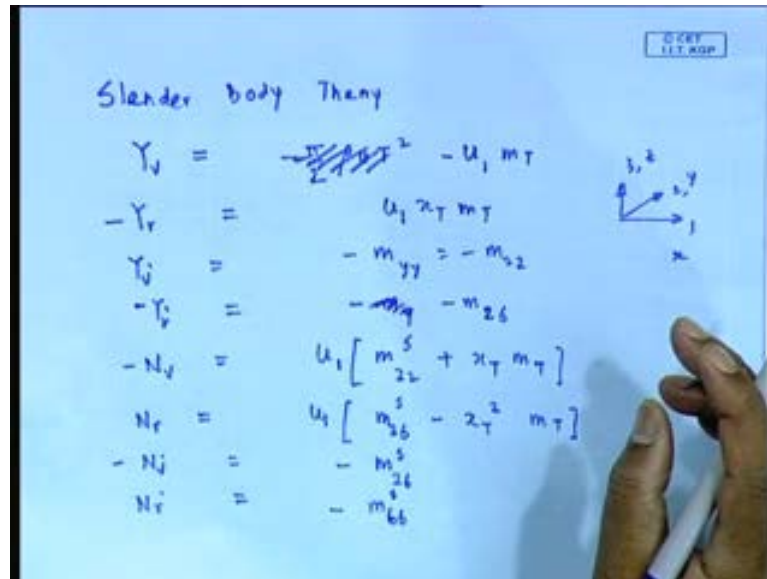


See, if I had a aerofoil, if I have a flow coming like that, if I presumed that there is no circulation, then what happen it turns out that my flow pattern become like that, but that is not possible this has to be pushed down here, **you know** that it has to be like tat here. So, for that you need to put a circulation, so push it down here, so there has to be a circulation tau and **you know** that lift can be made in terms of tau, function of tau you can, I can call, this is the fundamental theory of how lift is produced **right**.

Now, same thing when I apply for a hull, for this kind of hull then, but assumed the lift that this is a slender body which produces lift then, the result I get is what I am writing, we are not talking the basic hydro dynamics, but the theory of what we are writing here slender body is based on that fundamental principal. That the ship section, this section produce as lift, **it is this** it is a low lift generating device at an angle of attack, so there is a circulation and because of the circulation that can be connected to certain quantities, which results final to this what I am writing.

And we will see that, this will become same as eventually what I wrote about T by L aspect ratio part similar to that, so according to this it is like that, let me complete writing it here.

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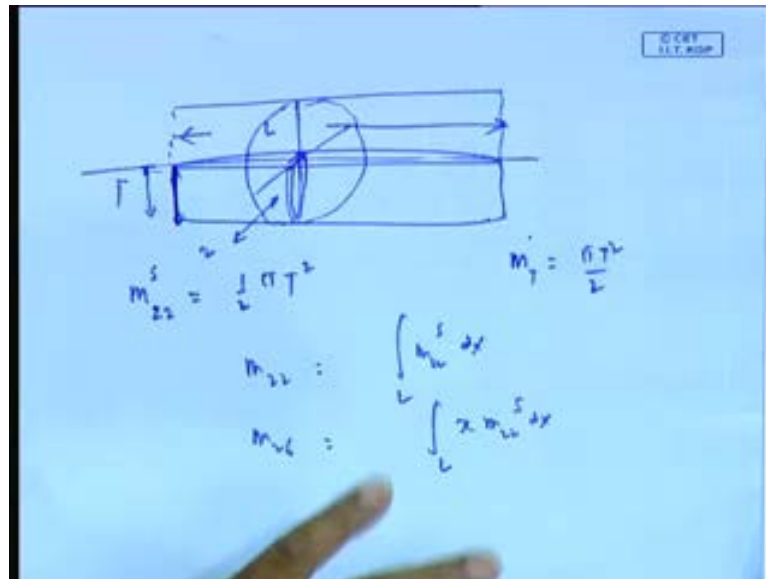


Then Y_v dot actually added masses are of course, are much easier, it is given by minus m_{22} means, if I call say let me write down this direction 1 here, 2 here, 3 here, 1 is direction x, this is direction Y, this is direction z, generally we are actually having 1 and 2 direction, rotation is about z.

So, you can say m_{yy} that is added mass in I will come back to that, then we have got minus Y_r dot this turn out to be, **I we are** I am just writing it from here as minus m_{yy} this is $2y$, we can see it is difficult to say, I will write it as m_{26} is a couple added mass 26 , let me call it as minus m_{22} only or then we have got minus N_v , let me write it down then I will tell you this $u_1 m_{22}$ a stern plane it is called (No audio from 15:12 to 16:07).

See here, what here actually this **this** suffix x_T basically implies, the stern section this last section, this stern section, **this this section** this section is called this distance is x_T and m_T are essentially the 2 d added mass of that section (Refer Slide Time: 16:20). This m_T is the two-dimensional added mass of the stern section, I mean basically never mind the origin of this equation, but what I wanted to tell you is that this origin of this equation, basically from theory it comes out to be like that, now we I just show you one or two example of that, then you will see, how we can go from there.

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Now, you see here take a ship, let us take a ship, we will assume it to be like that, it has got draft of say this is **you know** like I mean the weighted hull L is length, now **what is** what happened here is that see, added mass sectional added mass, this direction that is, the added mass in Y direction, it is going to be given by what, it will be given by as I told you from steep theory, in estimate of that will be circle. But, remember there is a free surface here, so I have to actually make a double body hull make a circle for this full one and take half of it, this will make it m , if I take sectional this is sectional, m sectional like m_{22} is given by π this is T remember **right**, t square half of that **right**.

m_{22} because, I am calling this, see **see** m_{22} is because, this direction is Y (\circ), this is Y direction, I am talking of sectional added mass in sway direction, with that is what the formulas are actually all these things here, this m_{22} comes m_{26} come etcetera, say m_{26} is going to be simply, the sectional added. See, the **the** point is like that, what is m_{22} , m_{22} is going to be the added mass, sectional added mass integrated, see m_{22} is integration of $m_{22}^s dx$ over L similarly, m_{26} is going to be integration of $x m_{22}^s dx$ over L from the center plane like that.

So, the point is that, this is given in terms, this have fundamental theorem given in terms of your sectional added masses and stern quantity, this T implies stern quantity, now you see here m_{22} is added mass of the stern section, and **if you are** if I take the stern section to be of thickness T , then I have got this added mass of this much, this $\frac{1}{2} \pi T$

square, that means $m T^2$ here, is also πT^2 by 2, that is basically it is a sway added mass. So, what I mean is that, if I now actually if we put it here, for example here this one, but will we find Y_v you will find minus u 1 into half $\rho \pi T^2$ for example, you will find Y_v as that, so let me take this and take one by one.

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Handwritten equations on a blue background:

$$\begin{aligned}
 Y_v &= -\frac{\pi}{2} \rho T^2 U \\
 -Y_r &= u_1 \cdot -\frac{1}{2} \frac{\pi}{2} \rho T^2 U = -\frac{\pi}{4} \rho U T^2 L \\
 Y_i &= -\frac{\pi}{2} \rho T^2 L \\
 -Y_r &= 0 \\
 N_v &= \frac{\pi}{4} \rho U T^2 L \\
 N_r &= -\frac{\pi}{2} \rho U T^2 L^2 \\
 N_i &= 0 \\
 N_r &= -\frac{\pi}{24} \rho T^2 L^2
 \end{aligned}$$

I just take one by one this Y_v , you will find Y_v as minus π by 2 ρT^2 into U , let us test the other one, let us take Y_r minus Y_r let me put it here, because that is what I have minus Y_r you are going to get how much, see here this is interesting. What is x_T , x_T is minus l by 2 remember, because x_T is the origin of that location **right**, x_T is this, but remember this is my positive x , so x_T is actually minus l by 2, so if I put that here it will turn out to be here, this one u 1 minus L by 2 into π by 2 $\rho T^2 U$, this will turn out to be equal to **minus π by 4 $\rho U T^2 L$** , minus π by 4 $\rho U T^2 L$.

So, similarly, if I do Y_v etcetera, I mean I will just, if you just put it back see m_{22} , what is m_{22} from strip theory, remember here what we are doing is an interesting thing, we are using a slender body theory for m_T , m_T you are not given slender body theory, m_T is a two-dimensional section, but m_{22} is sway added mass, what is sway added mass, is going to be the added, sectional added mass into length. That is strip theory approximation that means, I am using for m_{22} a strip theory approximation, because **I am** if see or rather in another piece of paper this one, m_{22} if I this is by strip theory.

So, according to the strip theory here, what is my \dot{Y} , see \dot{Y} may be you should go to tell me, m_{22} is π by $2\rho T^2$ **right**, that is sectional added mass multiplied by length, so therefore, you end up getting here simply minus π by $2\rho T^2$ into L . This much remember, this **this** part is sectional added mass into L gives you this, so similarly, if you went through that, **you know** the results **II** you will end up getting is now say minus \dot{Y} , this will become 0 **you know**, because \dot{Y} , because this has got here m_{26} .

m_{26} is as I told you this sectional added mass \times into m_{22} since, it is both side similar, if you carry it out it will be 0, see this 1 if you carry it out, actually you are to carry out minus 1 by 2 to plus L by 2 this is become 0, m_{26} is the **yes** added mass, coupled added mass of sway yaw, means if we see theoretically if I give a **a** unite displacement in direction 2, what is the moment coming in direction 6, this is what we have discussed **in the** in the previous semester **on** on sea keeping know. See, m_{26} , m_{62} or m_{xy} are what is called m_{ij} essentially means connected to force in direction j , because of motion in direction i , this called cross coupled added mass they are normally small.

So, **you know** then I will just write this full part out then comes N_v , minus N_v this will turn out to be π by 4 in fact, this is very important $\rho U T^2 L$, you see here, now here N_v , see m_{22} , actually m_{22} is basically for the ship, that is what we are **you know** this **this** m_{22} is anyhow, if you carry it out you will end up finding out, that you end up getting that result, basically if you carry it out, because $m_{T \times T}$ is just, now what we have done, that is this **this** term, that is ρ by minus π by 4 $\rho U T^2 L$.

So, if you carry it out you end up getting this **this** result, this **this** part then N_r (No audio from 24:47 to 25:00) is you will end up getting this (No audio from 25:03 to 25:13), this not N_r , this is (No audio from 25:15 to 25:27) something like that you are going to get.

See, now this is one set, what we say basically these results we have obtained essentially from assuming the ship to be slender and therefore, slender body theory is applied and then apply strip theory for the added mass parts estimation that means it is a combination of slender body theory, strip theory. Actually this one is what I showed here, is a fundamental theory, but **you know** that same result, same result is obtained actually and that is the result, that is given in almost all text books, **which** which will look like this,

which I will try to this thing, **the** that means combination of slender body theory and strip theory results to me like this.

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The image shows a list of handwritten equations on a blue background, enclosed in a hand-drawn box. The equations are:

$$\begin{aligned}
 Y'_d &= -\pi \left(\frac{T}{L}\right)^2 (1) \\
 Y'_r &= -\pi \left(\frac{T}{L}\right)^2 (1) \\
 N'_d &= -\pi \left(\frac{T}{L}\right)^2 (0) \\
 N'_v &= -\pi \left(\frac{T}{L}\right)^2 \left(\frac{1}{12}\right) \\
 Y'_v &= -\pi \left(\frac{T}{L}\right)^2 (1) \\
 Y'_r &= -\pi \left(\frac{T}{L}\right)^2 (1) \\
 N'_d &= -\pi \left(\frac{T}{L}\right)^2 \left(\frac{1}{2}\right) \\
 N'_r &= -\pi \left(\frac{T}{L}\right)^2 \left(\frac{1}{6}\right)
 \end{aligned}$$

Y well here, I am just writing in a non-dimensional form I will tell you, this is what you end up getting in any text book, this is 0 this all non-dimensional values, this is what you will find PNS book (No audio from 26:44 to 27:12). See, I have a reason for writing this I tell you, then you will understand why a T by L is the most important term rather than Y r is this N dash v this is also very important, and N dash r I hope the space there minus pi.

You compare this, this is the result that we get again and in fact, you can get this I can tell you same result, if you use also the slender body formula for lift that Y v that I discussed and use the aspect ratio T by L. See, what **what** you are finding interestingly is that see here, if you look at that you find d c by d alpha is function of T by L, that is what you are finding **right** d c L **L** by d alpha is function of T by L that is draft by length ratio as per the first approximation based on low wing aspect ratio lift thin.

Now, you see you will find out that, if I were to proceed with that I end up getting at least for Y v Y this **this** coefficient, this damping coefficient same result at this and number one, if you actually proceed this way also, so what it means theoretically then I can estimate that all of them comes to the same thing. Slender body theory, strip theory or low wing theory, after all slender body theory is nothing but, a low wing theory of

aspect ratio essentially **you know** theoretically it is the same thing, we are what we are saying is that, if I have a thin body small aspect ratio, what is the lift produce and that lift is basically estimated from slender body theory, assuming body is slender.

So, what has happened, now you end up getting this, now **if you** if you compare you will find out **you know** that all are same, I just take an example see Y_v , now actually this is $Y_{dash v}$ is Y_v , see $Y_{dash v}$ is Y_v by $\rho L^2 U^2$ with a half, that is the formula. Now, you see here take, this divide by if you do this half of $\rho L^2 T^2$ you will see U will go off and T by L^2 will come, and this half will go off, so you end up getting actually $\frac{1}{2} \pi T$ by L^2 into 1.

Now, you see here another thing interesting is that all of this has T^2 , see this **this** is a T^2 , this is $T^2 L$, because this is Y and r , N and v will be $T^2 L$ and N rare will be $T^2 L^2$, but if you divide them accordingly, because here I have to divide by half ρL^4 here I have to divide by half ρL^3 etcetera, you will find out all the terms will be in proportion to T by L^2 .

Actually you **you** should see this term by term, see N_v , so if I non-dimensional **(O)** it N_v dot here, it is going to be see $N_{dash v}$ is N_v by half $\rho L^3 U$, so **if I if I do that** if I do that L by L^3 gives me L^2 below, this half will give me, this half term that is why this term become half π into half, because here it is $1/4$ and U divide by half, so becomes half you can check that **you know**.

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Handwritten mathematical derivations on a blue background, showing force coefficients for a slender body. The equations are:

$$\begin{aligned}
 Y_v &= -\frac{\pi}{2} \rho T^2 U \\
 -Y_r &= \frac{1}{4} \cdot \frac{\pi}{2} \rho T^2 U = -\frac{\pi}{8} \rho U T^2 L \\
 Y_i &= -\frac{\pi}{2} \rho T^2 L \\
 -Y_r &= 0 \\
 -N_v &= \frac{\pi}{4} \rho U T^2 L \quad \left(N_v = \frac{N_v}{\frac{1}{2} \rho L^3 U} = -\frac{\pi}{2} \left(\frac{T}{L} \right) \cdot \frac{1}{2} \right) \\
 N_r &= -\frac{\pi}{2} \rho U T^2 L^2 \\
 -N_i &= 0 \\
 N_i &= -\frac{\pi}{24} \rho T^2 L^2
 \end{aligned}$$

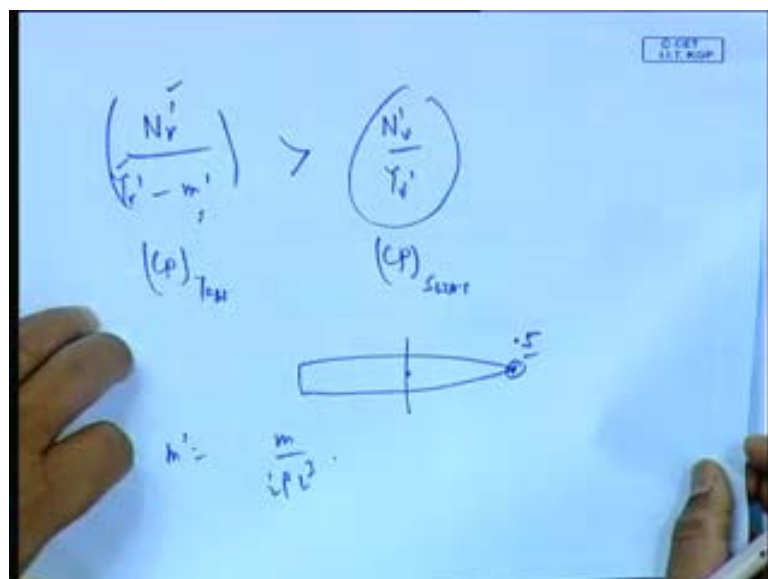
If you take this $N \dashv v$ this you will that is $N \dashv v$ you will end up getting exactly same value as this minus πT by L square into half same thing, so you see why I am saying that, may be this is a good idea, what I showed here this one or this one is more formal theories, slender body theory, apply strip theory approximation for sectional added masses.

You end up getting a set of approximation, presuming of course, here another presumption is there, the stern section is having same draft T , the stern section makes the very **very** important contribution, it is the stern aft section where the flow has to be separated out for this **(O)** condition the stern added mass becomes the most important quantity.

So, if I assume the ship is of uniform draft which is a very reasonable as approximation, if I assume the stern is pointed that is, what actually is one of the important consequences of this, for on which this is derived. Then I end up finding this relation which converges to this relation, which is same as the relation that we find out in a any other text book that even suggested by many authors.

If **if** you look at this, this is interesting because, what happens you will find out that all quantities are based on T by L , now is it stable is it unstable, how good it is, how bad it is we will just talk in a minute, but this is a important point for you to realize, tell me if I the question is asked, how do **you know** the ship is stable or unstable.

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Well actually I have the criteria remember, the criteria of course is something like that, see that N_r by Y_r dash minus m dash must be greater than N_v dash by Y_v dash, this many times we say that, because this is actually condition of C_p in sway, this is you can say C_p in yaw I can call it that way and some authors call it.

If you look at that here, N_v dash by Y_v dash no **no** not **not** this one, N_v dash is this $1 N_v$ dash by Y_v dash, how much did you get you will see it is a 0.5, so this point I know is 0.5, this neutral point. Because, here this thing regarding this of course, I know this and this, but it depends on the mass, so depending on the mass, because mass has to be m dash **you know**, m dash is suppose to be m by half ρL^3 , so you have to put that to realize what is the condition of stability that is exactly how you can do this here.

Now, let me just give an example, how good are these estimate **are they very** are they quite good, how much we differ if you want to compare that with measured values just to give an idea, rough idea **you know** this values basically there is a hull call mariner hull.

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	Slender Body Theory	Error %
Y_v	-12.3	± 3.6
$-Y_r$	-2.62	± 0.72
Y_j	-6.57	± 0.93
$-Y_r$	0.25	± 0.01
$-N_v$	3.69	

This mariner hull extensive, **you know** like measurement have been made for the hydrodynamic coefficients, and then example of this is that based on this I both are same as I said that slender body theory or this theory is same.

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	$\times 10^2$ Slender body	Exp
Y_v	-7.1	-13.3 ± 2.6
$-Y_r$	-3.6	-2.62 ± 0.72
$Y_{\dot{v}}$	-2.1	-6.55 ± 0.93
$-Y_{\dot{r}}$	0	0.25 ± 0.07
$-N_v$	3.6	3.69 ± 0.78
N_r	-1.8	-2.40 ± 0.50
$-N_{\dot{v}}$	0	0.22 ± 0.08
$N_{\dot{r}}$	-0.6	-0.36 ± 0.12

The difference that came from mariner hull is that **you know** this is slender body just to give an idea which is as I repeatedly telling, the slender body theory that I mentioned here is slender body theory modified assuming that, the body has a uniform draft and sectional added masses can be determined using strip theory of half rho pi T square, this what we are talking here, which is which resulted into this.

So, here the values came something like that I will tell you why I am telling you Y_v there is a the error **sorry**, let me not write the error from experiment this turns out to be approximately minus 13.6 in percentage plus minus 3.6 minus Y_r turns out to be, I am just giving this some kind of result 0.72, not too bad, Y_v dot turns out to be minus Y_r dot turns out to be minus N_v turns out to be **no no actually no no sorry**, I am making a mistake of this is not the actual the **the no the not not** just **just** let me take it off.

Because, this is not the actual value of the error no, the what is this is slender body and this is experiment, not **not** really this thing all multiplied by 10 to the minus 3 I am making a mistake here, Y_v slender body theory gives you minus 7.1, this is actually actual numerical value **sorry sorry**, actual numerical value I should not have.

Because now, these things are much different, that is why I was trying to tell should not have said there **there** is a point of **you know** like saying this because, as a class you need to understand, kind of number you get in hydrodynamic coefficient, **you know** you have dealt with many subjects where you might get results in 5 percent, 10 percent here you

might find result sometime 200 percent off and yet you get somewhat good result, that is what I wanted to bring it to you and I will show you some other results of that type.

There is always a point Y_v , Y_r , $Y_v \dot{}$ we have done $Y_r \dot{}$ this is 0, this is plus minus this is 0.25 plus minus 0.08, then minus N_v this is from a this is 3.6, N_r slender body give you minus 1.8 minus 2.40 plus minus 0 this plus minus is the experimental inaccuracy **you know**, you can see that **you know** when you measure also there are, so much of inaccuracy may come (No audio from 38:23 to 38:43).

See here, why I **I** mention to you **you know** that have an, let us have an appreciation first of all these numbers are into 10 to the power minus 3, that means this will be 0.0071, this really 0.0013, 0.0133 plus minus whatever, look at this Y_v almost twice **twice** low **right**. Y_r well 3 and 36, 26 may be 10, 30 percent, this is not too bad $Y_v \dot{}$, actually $Y_v \dot{}$, $Y_r \dot{}$, $N_v \dot{}$, $N_r \dot{}$ you will find out that the N_r , the added masses 0.25 and where is this other one $N_v \dot{}$.

Normally added masses are not too bad normally but, damping is the one that is much more difficult to estimate, so here you find here something like twice as much then where is the bigger difference then, let us see Y_r the last 1.6, 0.36 you can say almost half, in fact Y_v and N_r are connected to each other **you know** you can find Y_v into sometime as N_r , so these are the kind of numbers you get.

So, therefore, do not expect from using such theories and in fact, I will continue little more on other theories, do not expect to get results that are very close by 5 percent **you know** this gives an understanding of the appreciation of the kind of numbers. So, this is based on one obviously lot of difference comes in, so now what is happened people have tried whole lot to evolve formulas for this coefficients, for your design purpose, so there are number of available.

What you do, you will collect, and you go to your company they **they** would make a proprietary, they would have collected 100 of data made a recreation fit and then try to tell you a formula, like that there are some formulas which I will tell, but before that there are even simpler formulas. Let me just tell you or rather let me tell you this only, what happen is that we look at this formula again, the first thing would be that see this was based on this **this** formulas, you would expect number one, number one that you must realize.

All the formulas that would have evolved, you would expect them to be having a dependence on T by L, because T by L dependence is fundamental comes from basic theory, remember empirical formulas are never out of your head, normally you choose a form, based on certain fundamental principal. Then you try to tune the formula to find out what is the coefficient that come, what is the correction that comes, I find this is not, so I would multiply with the factor that is the way you will do it normally.

So, all formulas you will end up finding the T by L square will come obviously, there will be formulas evolving where geometric parameter of the hull, block coefficient etcetera **etcetera**, would come in, **you know** because this is insensitive where is the location of CG suppose it is forward etcetera; so you see the first set of modification that is suggested is multiply this with some kind of a coefficients.

(Refer slide Time: 42:19)

$$Y'_j = -\pi \left(\frac{T}{L}\right)^2 \int_{Bow}^{stern} C_H dx'$$

$$N'_v = -\pi \left(\frac{T}{L}\right)^2 \left[\underbrace{(C_H x')}_{stern} + \underbrace{(C_H x')}_b \right]$$

So for example, you will have something like this, I **I** think I will look at this, see something like this added masses I just give you minus pi T by L square, now here I have some kind of a coefficient from bow to stern, some kind of sectional coefficient. Because, here there I assume you say, remember there I assume my sectional added mass is constant T here, what I will do I will introduce the term of course, I had to determine that means I had to find the sectional added mass of actual sections.

Like that, if you carry on for example, **you know I can** I can show you the more complicated form will come something like it will be minus T by L square, it will turn

out to be C H that this factor x dash stern bow to stern in fact, this part **you know** is nothing but, actually going back to the original form of this (Refer Slide Time: 43:25), you look at this here, the look at the form of this minus pi T by L square C H x dash. C H basically of stern that is, this is connected to the added mass at stern and total added mass you will see the formula, this C H is connected to the added mass part you will find **no no sorry** this part is connected to this part, this **this** x T m T this part see here.

Minus pi T by L square C H x dash plus this **this**, C H is the sectional added mass coefficient, so this part essentially is this part, total added mass of ship, C H x dash, x dash is location of the stern this part. So, **you know** basically what happen the first set was presuming that the draft is constant etcetera **etcetera**, now you are modifying it by taking actual sectional added masses, you have to find it out by some means, if you have it, but that nobody is going to do that.

So, there have been several I will mention this, several formulas that were evolved there is no **no** unique one, the several formulas that have evolved eventually, where you can use a prediction, one of the simpler. Before, I go to this **this this** coefficient, let me let me let me tell one of the formula, because we will probably have to go overshoot the next hour to talk about the other formulas.

(Refer slide Time: 45:11)

The image shows four handwritten equations on a blue background, likely from a presentation slide. The equations are:

$$\frac{-Y_v}{\pi (T/L)^2} = 1 + 0.16 C_B \frac{B}{T} - 5.1 \left(\frac{D}{L}\right)^2$$

$$\frac{-Y_v'}{\pi (T/L)^2} = 0.67 \frac{D}{L} - 0.0033 \left(\frac{D}{T}\right)^2$$

$$\frac{-N_v''}{\pi (T/L)^2} = 1.1 \frac{B}{L} - 0.041 \frac{B}{T}$$

$$\frac{-N_v'''}{\pi (T/L)^2} = \frac{1}{12} + 0.017 C_B \frac{B}{T} - 1.33 \frac{D}{L}$$

See, one formula suggested widely uses like this, this is important minus Y dash v by pi this is written always same all are non-dimensional with this 1 plus 0.16 C B B by T

minus 5 this is one, this given by one set up, look at this I will write it down the rest part, but I will tell you why I am wanting it to look at this \dot{Y} is $Y \dot{v}$. Look at this here $Y \dot{v}$ by the simple formula is $-\pi T$ by L^2 into 1, that is this term only what people have done using empirical regression analysis, there have been one suggestion which is popularly used for usual commercial vessels; used an additional modification of this type, if you work it out you will find out they are not actually too much away from one but, some part will come.

So, like that there are some formulas there which may be useful for example, we have $-\dot{Y} r$ divided by π , see the reason of familiarizing you with this is that tomorrow when you are doing a ship design, in your design project for example, what happens \dot{Y} you will need to ensure that it meets with the regulation. Let us say for turning circle there is a regulation that, the turning radius should be less than 5 times length r by L or d by L or r by L I do not recall and advances so and so.

Let us talk of turning radius only what you can do remember r by L also can be expressed in terms of $\dot{Y} v$, $N v$, $\dot{Y} \delta$, $N \delta$ etcetera, just expression at least for your preliminary estimation, I was telling him before it is not just the rudder coefficient for finding r by L \dot{Y} you have that steady turning radius per unit length or divide by ship length is a function of what $\dot{Y} v$, $N v$, $\dot{Y} r$, $N r$ as well as Y rudder and N rudder.

So, you remember out of the 6, 4 are hull derivatives, 2 are rudder derivatives, so only rudder derivatives does not tell you supposing you have a ship, where same rudder Y , if your rudder is same my Y rudder, N rudder, remain same, but those things can be different, so you have done a design and your r by L turns out to be 7 obviously it is not acceptable.

Now, my point is that, these formulas can be useful for you to estimate those Y values which will go and from there you can find out r by L they may not be correct but, at least give you some idea, so that is way the use of this comes in your design. And remember, when you are doing a design it is not only ruder, it is the hull that plays a very important role of same rudder behind ship number one own make it turn ship number two of same length may make it turn anyway, so that is why this may be useful to you.

So, let me write out this other formula also then $-\dot{N} v$ this is not dot \dot{Y} \dot{Y} this is all no this dash here, only by πT by L^2 , there is another reason for

writing that also see here, you look **look** at the, what are the parameters other parameters coming in, other geometric parameters coming in (No audio from 49:13 to 49:34). Let me just complete that, because again say look at this **you know**, this is are added masses **right**, what **what** are the parameters came in C B or I will **I will** get back to this after I finish writing then we will go back to this, minus Y dash v by pi T by L square 1 plus 0.4 minus Y x dash (No audio from 50:07 to 50:36) and minus (No audio from 50:38 to 50:53) 5 6, anyhow you can look at that.

(Refer slide Time: 49:53)

$$\begin{aligned}
 \frac{-\dot{Y}}{\pi (T/L)^2} &= 1 + 0.16 \left(\frac{B}{T}\right) - 5.1 \left(\frac{B}{L}\right)^2 \\
 \frac{-\dot{Y}'}{\pi (T/L)^2} &= +6.7 \left(\frac{B}{T}\right) - 0.0033 \left(\frac{B}{T}\right)^2 \\
 \frac{-\dot{N}}{\pi (T/L)^2} &= 1.1 \left(\frac{B}{L}\right) - 0.06 \left(\frac{B}{T}\right) \\
 \frac{-\dot{N}'}{\pi (T/L)^2} &= \frac{1}{12} + 0.017 \left(\frac{B}{T}\right) - 0.33 \left(\frac{B}{L}\right)
 \end{aligned}$$

See, I have two questions to you, number one is that suppose somebody ask you a question which is the most significant parameter of hull that decides the hull derivatives, what is answer, which is the obviously T by L, so you see T by L has the most important influence on the maneuvering characteristic or hull derivatives, that also make sense, if it is very deep **you know** then you have more forces coming to turn etcetera **etcetera right**, very thin like a **you know** brought ship different kind of thing.

So, what is a general conclusion for example, if I have a very large and thin ship deep draft is it more maneuverable or it is a flat type of thing that is a barge type of thing it is more maneuverable, anyhow I leave that to you for you to find out. But, there is other thing is that, **in this formula** in this formula whoever has devices formula it is of course, if also **you know** like recreational, what are the parameters used.

Remember, primary parameters uses B by T coming here, the other one they have used is B by L, T by L of course, is already there and also C B, if you look at that here B by T, B by T here, B by T here, B by T here and of course, B by L here, B by L here, B by L here (Refer Slide Time: 52:25). So, what and of course, C B is there to an extent, so what I mean therefore, if I were to ask you off hand you see what are the like people think obviously, **you know** when you start making an empirical formula you have to find out what geometric parameters are important.

So, obviously it turns out that draft by length is of course, most important parameter, if even if have others ignored that cannot be ignored, but then this formula I will tell about second and third set of formula we will continue this discussion little more time next class, other **other** parameters are B by T and L by B basically, the principal particular ratios, but they are not that strong, if you look at that B by T seems to be stronger in terms of some of these parameters.

But, whatever and of course, to an extent C B also comes in, so when you have a ship you can do that but, I **I** will tell you in the next class, that you have to take all these with the pinch of salt, because then we will talk about **another formula** another formula. And if you use these formulas for a given ship you may end up getting the values of coefficient widely differing, then you do not know what to do **right**.

So, the full purpose of my telling you is this, will be this is that rudder derivatives you will rather be able to find out more precisely, because it is an aerofoil section CDCL for that are more widely known; it is the hull derivatives where the uncertainty is much more usually however it so happens that the trajectory that you predict, have less uncertainty than the uncertainty in the coefficient itself. So, anyhow with that I am closing this hour, and then we will continue that in the next hour.