Performance of Marine Vehicles at Sea

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Lecture No. # 22

Regular Sea Waves - II

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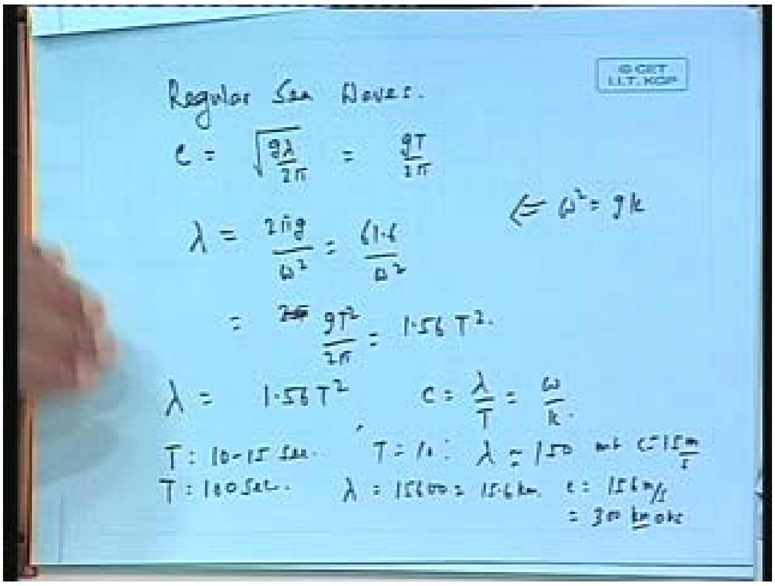

We will continue on our discussion on regular sea waves. See, this ended up at this relation g lambda by 2 pi, that is to prove that or we can also say it is g T by 2 pi. To say that longer waves travel faster and also waves with higher period travel faster. In fact, we can relate lambda and T also from dispersion, it turns out that if you go by lambda and T relation, see from omega square equal to g k, from this working back we can find that this is equal to 2 pi by g into omega square is 61.6 by omega square or this can be also 2 pi sorry, it is g T square by 2 pi equal to approximately 1.56 T square. So that means, we can say like this lambda becomes 1.56 T square. C equal to lambda by T equal to omega by K etcetera. Why I am writing this is that, let us try to find out some relation between speed, length etcetera.

Now, typical ocean waves, what we call everyday waves. A T typically may vary between say 10 to 15 second. In fact, one can say, lower may be say 8 to 15 second. So now, let us take 10 second to be a typical example. This is what is called in our terminology as every day wave. Some people call an half sheer structure, that is most frequently occurring kind of waves. Now, 10 second will be see T equal to 10 will give you lambda approximately equal to 150 meter and C will become equal to 15 meter per second, which is about 30 knots, quite high.

Now, this is actually everyday wave. Now, let me take for an example a very long wave and find the length. So, let us say that t equal to 100 second. In a very long wave, 100 second wave. Now, if t is 100 second, you find what would be lambda? Lamba is going to be 1.56 into 100 square, that is going to be 15600 or approximately 15.6 kilometer long, isn't it? That is what it is. That can be a typical length of a wave, what is called a tsunami. When a volcanic eruption when it goes far, what is a speed of that? It is going to be 15600 divided by 100, it is going to be 156 meter per second, which is above 300 knots.

So, you can imagine how fast it becomes actually like a almost like a sound wave travelling, like a plane going, 300 knots means about 500 meter kilometer per hour. This is why, when there is a very long wave, which is what happen, if there is a volcanic eruption at some place, it can actually cross the Pacific Ocean much faster because it is going at such a tremendous high speed, very long waves go.

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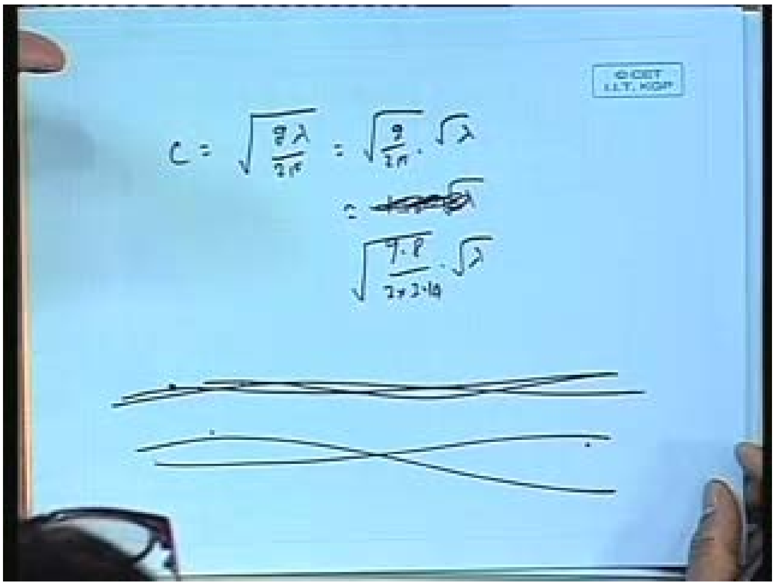

Because we do not see that. What you will see that, there is a very long wave something like that. Today it is like that, next second it will become like that, next second it will become like that, like this distance to here to here will travel so fast that you would not know, is it?

But when we are seeing as a tiny bit, we are seeing the flat surface, it will appear to be to us as a flat surface, but actually that surface just goes up and down. The entire thing is like in a tilting like that, but this tilt has tilted like that in 100, but this to this is actually few 100 kilometer. So, it has travel so much faster. It is just to give you an idea about the... These waves you cannot find out because these waves are like tide for example, it is also wave of 12 hour period, 6 hour period is a slight depression and elevation. But with a very large scale can also be called wave. If we when we see on a shape you will not feel it because the locally that vision is blurred because it is with respect to a ship it will be calm water.

But when it comes to coast, then the coastal water get flatted. This is why sea people are not bothered, ship people with this long waves because if there is a very long wave. Suppose, the wave is very long what would happen? The ship is simply going to ride it, you will not feel it they are sitting here. As far as your concern you are a shape position next to the sea, but I mentioned this only to tell you about the wave speed. But again as far as ship is concerned and offshore structure is concerned, such long wave lengths do not have much effect, they have effect on the coast, but not much effect on us. Let us now look at things like velocity etcetera. So, we start with this again phi wave ,which was given as… Now, you see the interesting part that is to be observed here.

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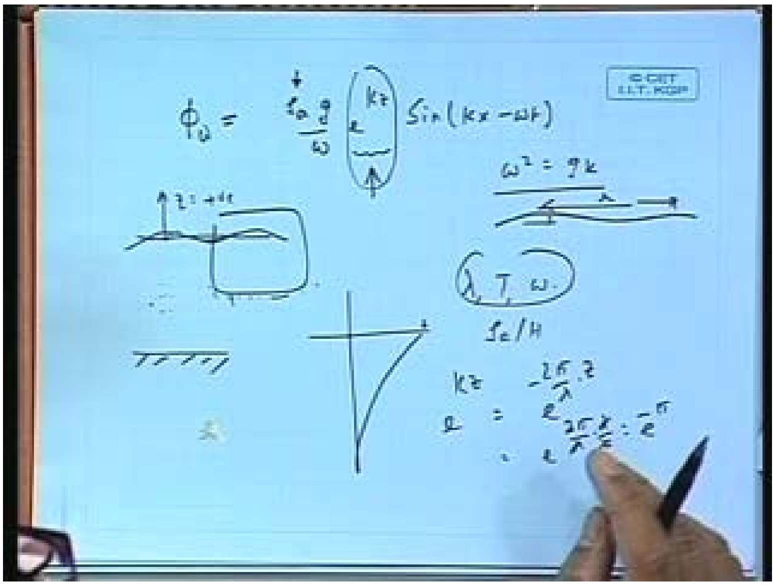

First of all you will see that this is proportional to xi a, that means all parameters will be found out in this theory, depends on the wave height. If wave height is 1 or wave amplitude is 1 meter, whatever happens, if it is 2 meter, everything will become twice, 3 meter everything become three times.

So, it is what is called linearly proportional to amplitude. And another thing that is important is that, see this relation dispersion omega square equal to g k, this tells me the wavelength. What is wavelength? It is a parameter of the wave is the x axis along the x axis, this part is wavelength, it relates wavelength this period and frequency, but no relation of that with respect to height.

So, height appears or amplitude appears always as an independent parameter. So, you can always have a 10 second wave which would be always 150 meters long, but that wave can have height of 1 meter or 2 meter or 3 meter or 4 meter, height has no relation with length. It is only length T omega is connected, but xi a or h, these are the height parameters are independent to each other. This is a another point that we must realize and we will find out later on that wave can be added. If we have to 1 meter wave, it become 2 meter wave.

So, this relation tells me that, you see xi a was say 1 meter it will be like this, that is one thing. This tells me it is linearly proportional to wave amplitude. Second thing is this part, what is Z? Now, Z here is actually is a mean surface is positive upward. So, actually there is no water here. So, see this is my sea bottom. Let us say this is my sea surface, waves are here. But you see wave does not mean that the particle is only moving here, there the water is also moving here, throughout water is moving.

When we say free surface elevation, I am only looking at this height, but this height is not the only thing, when wave moves entire particles are moving, they are having some kind of motion. Obviously, if you take a ship here, there is some particle movement that is why there is a pressure coming and that is what you want to find out. Now, it turns out that this relation tells me every parameter has a decay with respect to k Z.

Now, if you plot this k Z, it will look like in a height going like this. This I will come at more length, but what it means is that supposing I try to find out some parameter phi here, if the value say 1 unit at Z equal to 0, and when you give Z equal to minus 5, this become e to the power of minus k Z, much lower. In fact, if you keep going to down, actually if you take Z to be equal to, let us take this vector, k is two pi by lambda and Z is some Z. Supposing Z becomes the depth, you are looking at a point about half the wavelength deep, that means various half lambda, lambda by 2. So, what happen in that case? This becomes E power of 2 pi by lambda into lambda by 2, it is e to the power pi minus pi, there is a minus Z is minus.

Now, you can calculate yourself, e to the power minus pi it will become equal to 0.004. In other words, when you go down about half the water length, suppose this is this thing, you go down, everything comes down at some point here you will not even feel there is a wave there because everything decays exponentially. This is exactly why if there is a storm up, you are diving down, you would not feel there is a storm up. And second thing is that everything happens at this area most because largest velocities, everything is concentrate near the surface, unfortunately ships lie at the surface.

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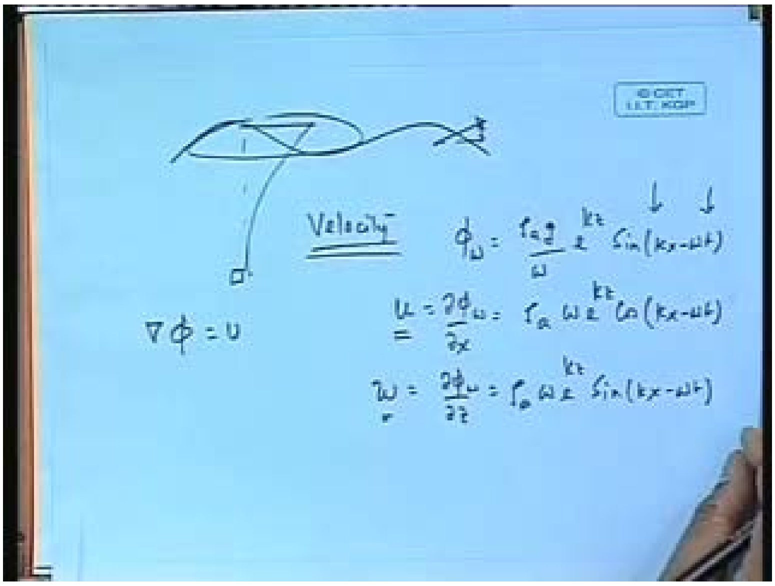

If it is a submarine all you have to do is to just dive down. There is the big storm, there it go on surface, simply dive down go little deep nothing will happened, you wont even feel water is moving. This is an important characteristic, one way to see.

Now, let us look at from this relation various things like velocity and all that. Let us see velocities. Now, phi w is given by xi a g by omega I, do we have to keep on writing this all the time? Now, velocity is U velocity, U velocity is velocity in x direction will be given by d phi by dx, by definition. Because as I mentioned phi is a function whose gradient gives you velocity.

If you take a phi function, the value of phi and if you differentiate that with respect to x. Now, differentiation is obviously, mathematical concept, that means if took a phi here, phi here then take this phi minus that phi by this distance, that is how the phi changes, the rate of change of phi with respect to x, that would be velocity. A rate of change of phi with respect to y, that is going to be velocity in the y direction etcetera.

So, by definition it is like this. Now, if you carry it out you obviously, you will get that this d by d x is there. So, this becomes K, you can work it out, probably I will just write this on, this will becomes xi a into omega into e to the power of k Z into cos. It will become like that, actually k will come here, it will become something like that.

Similarly, w which is d phi by dZ, this will turn out to be equal to xi a omega… Now, see these two are my u velocity and v velocity, the velocity of the particle. Again if you see that velocity of the particle is also sinusoidal. In fact, everything in a sine wave would be sinusoidal, because they are this function is varying with respect to x and with respect to t like a sine curve. Now, if you differentiate that, whether with respect to x or with respect to t it will also be a sine curve. Now, it turns out velocity is differentiation of with respect to this space x or Z and pressure will be differentiation with respect to t. So, there will be all sine curve. So, it will look always like a sine curve. Only thing is that we must find out how it looks like by plotting. Now, that I will do next page.

Supposing the particle at this point has a velocity this thing. So, this u means this much of velocity, v means this much of velocity. So now, we will show you actually this diagram is how the velocity look like.

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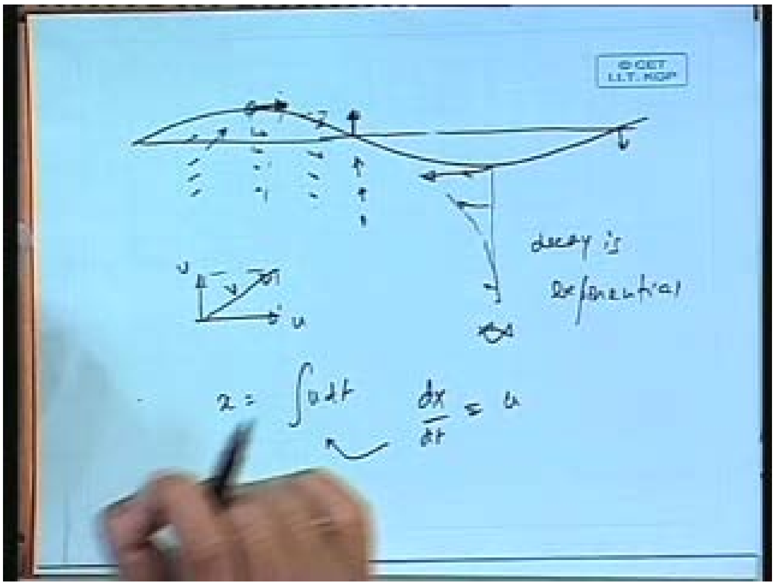

Now, you see the if you plot that, it will look like that. If you take a graph here, the velocity here will look like this, thispoint the particle you will see as only a velocity here. Let us take a graph like that. At this point the particle, we have only velocity here. One can actually show that this part will have a velocity somewhere here, one can actually make a velocity plot here. So, this velocities go down, these vectors will go down slowly these vectors will actually come down slowly. Velocity will come down very much and the particle will move like that here, at this point the particle movement will have both down and up. So, it will be something like that, it will have some velocity or some velocity here etcetera.

So, you see the velocity particle in here, the particle will actually move, in here it is this way and in here it is going to be this way particle. So, particle if you look down u and v and try to plot the u and. See, if you plot here u and here v so obviously, the actual velocity is this vector, what I plotted is this vector v, this vector. On the crest you see you will find out from this other relation, when x equal to 0 t equal to 0, then this value is equal to plus 1.

So, u equal to this term, but what is w? w equal to 0. So, when u is maximum, w is 0. Conversely, when w is maximum, u is 0. Exactly what it is. This is where particle is having only u, no w. This is where it has only w, no u. In between it will have all the velocity. So, in other points it will always be in some angle. This is not a very illustrative diagram because velocity is difficult to plot. If you plot that you will find that the velocity will look something like that and here it will look something like that.

You will find that velocity is coming down. So, whatever the velocity is here, see for example, here this if I draw, this it will come down like a exponential curve at some point it will practically have no velocity. So, a fish here we will not feel any velocity because as you go down the decay is exponential.

The thumb rule is that if you take water as more than half the wavelength, you can assume the water is calm, no disturbance. Because the disturbance factor comes down to less than something like 0.4 percent or. So, it can be taken to be as a steady water wave. So, this is my what we call a particle velocity. Now, you see if you know particle velocity, you can also find the trajectory because trajectory the x distance is nothing, but particle velocity into integration. After all dx by dt gives you u. Therefore, you can get to this. So, there is a way to find out. If a particle started from here, then which direction it moves? It is called trajectory. That is more illustrative because you want to know how the particle are moving.

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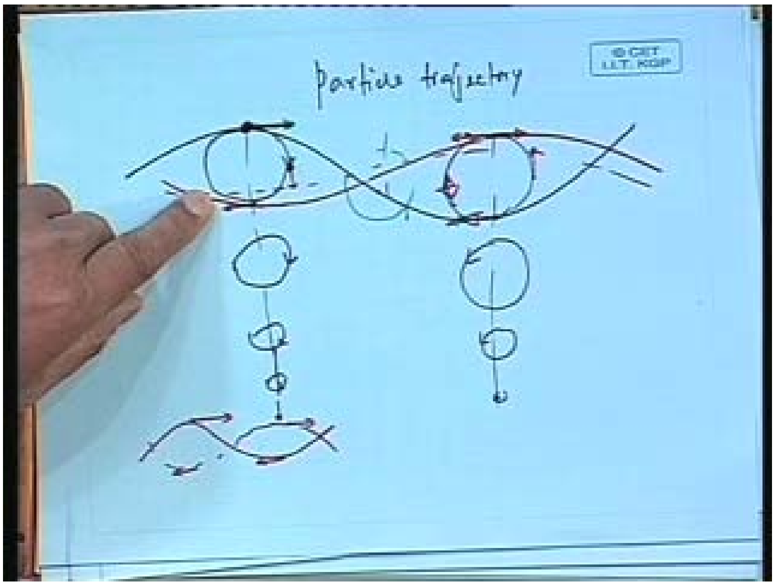

Now, that is very interesting diagram. This diagram I should draw at length. See this now we will draw a with diagram particle trajectory. Here what happened? You will find that particle trajectory are particles are moving always like a circle. See, particle trajectory very interesting, if you take a point here, this point you will find out actually moves like this. In fact, when it has come here, suppose when the particle has come here, see what would happened. This is very interesting diagram in many ways we should look at that.

See, at this instant this particle is here, so it is moving only this direction. What we saw at last time, see when the particle is at this point at this crest it is moving this direction, just what we showed in the last slide, it has no w velocity. And at this point it is moving in this direction, it was no y velocity and at this point actually it is moving in only up down direction. In fact, if you take here a circle here, it will be this difficult to say, but it will move down direction. Anyhow, I will forget this right now. Now, what happened? Little later this particles come here. See, after sometime this particle has come here, this particle has now here in a red line. And this particle as come to, this is moving in this direction, this particle have come here this line.

So, at that time if you plot, you will find out that the wave actually has become another face. So, if you draw a point by point, you will find that, you see when it becomes like this full face, that that time when the particle has come here, see full round here, then the wave crest is here, this particle by that time as actually come here, it is moving like this

No I am sorry. This also moves like this. Now, other zone this is moving like that, this is moving like this. See here interesting part is that. At this point this was moving this direction, this is moving this direction. Then after half a period, this particle has come right circle at this bottom point. And at that time this particle has come up to this bottom this point. So, at that time the surface looks like this.

So, you see that when the surface is like this, the particles is moving like this. When the surface has come to this, the particle has moved down like this, this particle is moved like this. So, it is actually moving like a circle. So, the point is moving like a circle. So, each point is actually going in circles. So, it is not going up and down, it is not going back and forth, it is going in a circle. This is why you can say that surface waves are neither transverse nor longitudinal.

See, if you look at this acoustic waves, we call it longitudinal, because it goes in this direction. Normally, we call electromagnetic waves are transverse, but surface waves, the particles move both in longitudinal direction as well as in transverse direction. So, they in fact are both longitudinal and transverse. They are not either longitudinal or transverse. There is a very famous book by Richard Feynman called lecture notes and physics, he is a novel laureate. In this book there is a chapter on waves. So, there it says that most people while talking waves give example of water wave as example because you through a stone and you find the wave. So, you say look this is the wave.

But he says that that is the worst example because this waves are the most complex in a sense because they are neither transverse nor longitudinal. The particle move like this in circles. And if you take this equation you see that there are very nicely traced out. This is actually my this deep water wave.

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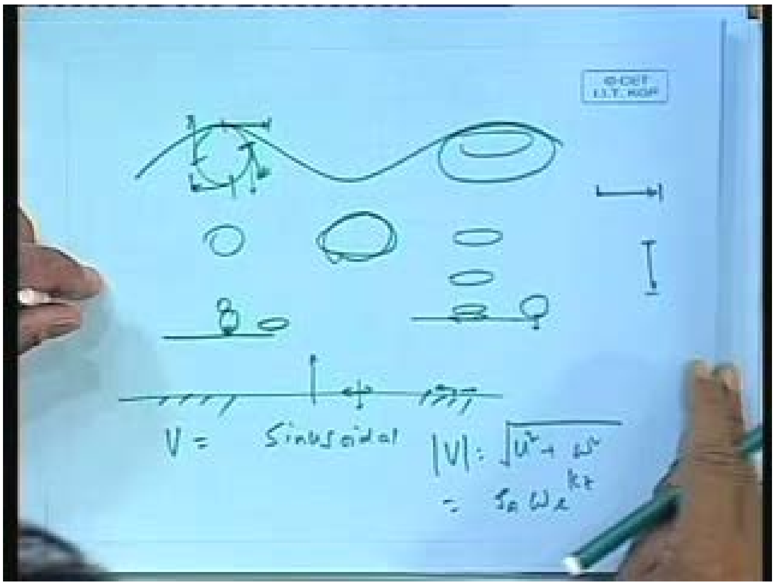

But this is another interesting curve wave I must show. Suppose, there is a bottom here. So, there is waters are moving like this. Now, supposing you are pushing this out, you are trying to gradually want to push it up, make it a shallow water. Let us say, see the one we have shown in circle is in deep water, but let us think how the shallow water will look like simply in a descriptive sense. So, I have a bottom. Let me say that I have bring a bottom and then I begin to slowly raise it, raise the bottom, reduce the height. What will happen? This at some point this will come here. Now, this cannot go like this, it will get pushed. So, this particular circle will now get pushed and get flattened. So, what happens? This particle now begin to look like this.

So, what happened? As you trying to push up, at this point see water cannot have a circle because if circle means it has to penetrate. So, it can only go only back and forth. See, on the surface water can have at based go back and forth because it cannot have any velocity this side. Why? Because if it has any velocity this side that implied that it has penetrated the surface, but that cannot happen. If there is a surface here, a water here can only go this way.

So, if I brought it up the water which has going like a circle, now I brought this, I have kind of squeezed it, I pushed it, then this will become ellipse. So, this is what happens typically in a shallow water case. Shallow water case, this particle path become to look like an ellipse. Not circle because you are, it is very simple you think there is a circular thing and you just pushed it. So, if you push that this way obviously, it will get compel and the circle will become ellipse. This is what happens in a nice way.

Now, this is about the velocity. And you see the velocity becomes important therefore, we find out therefore, velocity is sinusoidal. Everything is sinusoidal actually. And interestingly when u is maximum, v is 0 and v is maximum, u is 0 etcetera. I mean you like the two components do not have maximum at the same time. And if you take the amplitude as u square plus omega square, you will find that this has some constant number. Actually this constant is equivalent to, this will turn out to be xi an omega something like that.

So, these are all well known fact if you take, one can say that the absolute amplitude of the velocity, total velocity regard to the direction. Some point it is having only u and not omega w, sometime only w no u, other times both little bit. But the square root of them if you add them up because one is sine, one is cos, sin square theta plus cos square theta square root is equal to 1. Therefore, it becomes a constant. The velocity becomes always a constant, that means when it is moving her,e wherever it is the particle is that the net velocity with this arm, this arm this magnitude is same.

Now, whatever wave it is, the length of the vector is same. Just that it direction changes. At this point it is only directed this side and at this point only directed down side, but the length of the vector is constant meaning that the velocity magnitude is constant. Only thing is that its components are different. See, if you take a circle vector here and at this point the vector here obviously, this is vector this is same, but this has only u and no w and this is only w and no u, like that.

Why we are interested in velocity? Remember that velocity will give rise to acceleration, acceleration will give rise to force. See, if after all, if I am going to like the water is giving a push and I had to find out how much it is pushing, that is connected to how much it is accelerating in some sense, because mass into acceleration gives you some kind of a force. This is why I need to also know velocity.

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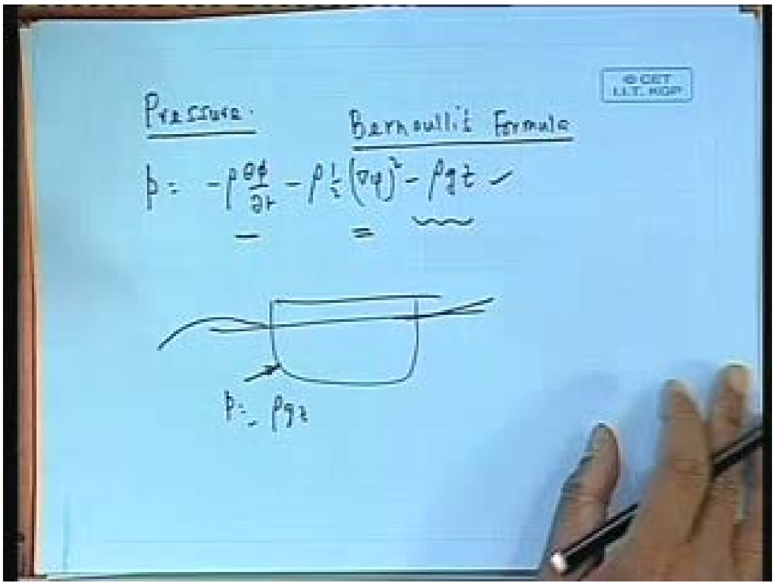

Now, let us look at pressure that is a much more relevant thing that pressure under a wave waters regular wave. Now, pressure is given by the formula of minus row into D phi by T, actually it is minus row half del phi, I am just writing a formula. See, this is a formula I am writing, we would not go through this much detail, but this is what Bernoulli has said many years back. A famous fluid dynamic is Bernoulli called Bernoulli's theorem, you would have heard this in some context.

The pressure is given by like this, of which this part is called hydro static pressure because this is the pressure that I talked in our earlier class, P equal to rho g Z and integration of that gives me everything. See, now earlier what happened? I have this body here, I want to find out what is my pressure, this is my water surface, I said that this pressure is given by rho g into z. If you recall that is the pressure, not a pressure, pressure is also the force is like pressure acts on the normal direction of this magnitude.

Unfortunately, here I have got waves now. So, I have got this pressure of course, always there, but now additional pressure is there. Why? Because the water is not static, water is moving. Now, there are Bernoulli's equation tells me two parts, one is this part, one is this part. Now, this part is called the linear dynamic pressure because this is the highest the main pressure, this is a pressure coming because of square of the velocity, because this part is velocity.

We will neglect that for time being because this turns out to be much smaller compared to this part. So, pressure can be written as minus row d phi by dt, that means it is a time derivative of phi. See, everything comes again to phi. So, look at this the phi formula and then pressure we will write. See, now again we have to look at this phi formula and then we will go to the pressure.

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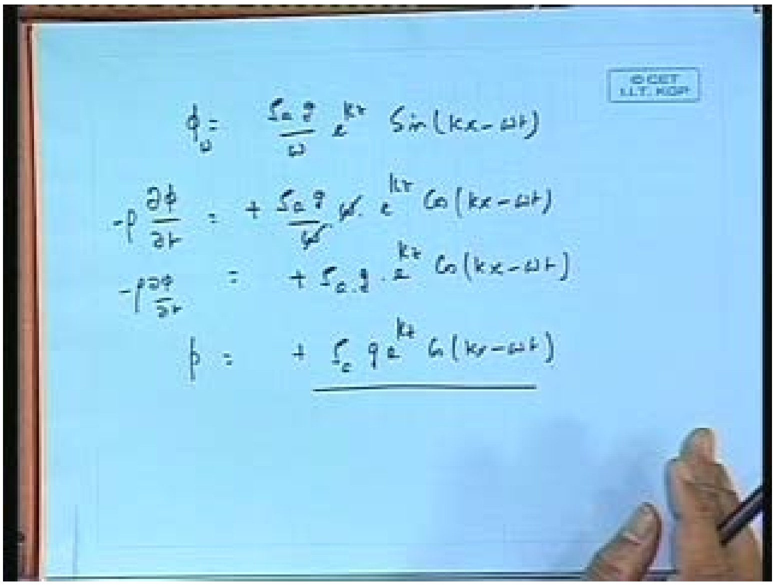

Phi omega was given as xi a g by omega sine. Now, d phi by d t. Basically, one row you can multiply, afterwards there is a minus row of that is how much? Let me take this. Dy by dt will become. See, here sine minus is there, it will become cos, sin become cos, but this minus omega will come. So, it will become minus xi a g omega into omega e k z into cos k x minus omega t, I think that is I am right on that. This will become, this minus rho d phi by dt it will jump. Sine becomes this thing, minus omega comes here, minus this thing comes here. So, this omega gets out. So, this will become minus xi a g, that is minus rho phi by this thing. I think let me see this. Now, phi is this thing, there is one more minus sign is there. So, this become plus.

See, minus rho d phi by d T become there is a minus coming from this and another minus, it becomes plus. So, this is the pressure. So, I end up getting this part pressure as plus psi a g, this is my linear dynamic pressure, that is this part that d phi by dt part of course, we have also this rho g z part. Now, this is again you can see this sinusoidal, it is a cos curve. Again it varies a time and like in a sin form, but there is a very like important concept associated with pressure, this I will like to talk to you little bit now.

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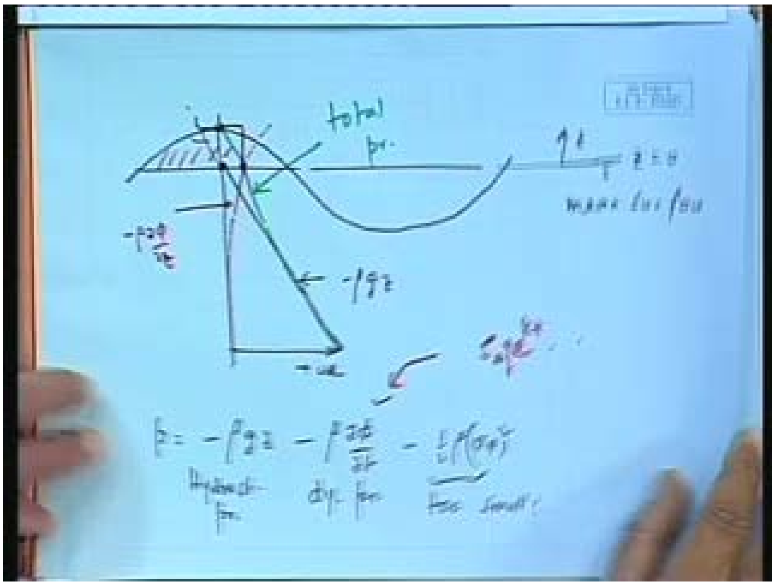

See let me take a very high wave like that. Now, what happened? Remember that pressure is given by minus rho g z minus rho d phi by dt. Now, rho d phi by dt it is and minus half rho del phi square, this part I am neglecting, too small. This part is hydrostatic pressure and this is dynamic pressure. You can call this to be wave pressure. Now, let me see how it varies. Now, what is a Z? Z is actually positive upwards, this is Z equal to 0, this mean surface, this is your so called water line.

Now, let me tell you this is a very important concept, rho g z is a pressure, that means supposing this is my Z equal to 0. So, here this value is 0. See, at this point my this value is 0, I am looking at this pressure. Let me see how this pressure looks. See, this part pressure how will it look like with respect to depth. Obviously, it will go down like this, something like that because it is 0 here and it will go down like that. See, as Z is negative, this is positive, these are all positive values, this is positive pressure.

See, suppose you take Z is minus phi meter obviously, it will be minus rho into g into minus phi, phi rho g pressure. As Z equal to 0, it is here. What about up? Now, here comes a contradiction. Now, supposing I want to take in this region the pressure, hydrostatic pressure will look like that. What is this meaning? It will tell me, supposing I take at the high, say it is wave height is 2 meter, it will tell me that the hydrostatic pressure is negative, that does not make any sense right? See, it will show me, if I use this formula, it will show me hydrostatic pressure is Z is positive value. So, minus rho g into plus 1 equal to minus some kilometer per second pressure. Now, what is the meaning of positive negative pressure?

Negative pressure does not make any sense, but let us look at this curve. What this curve looks like is that it will have exactly this much of value here and this will come down like this. I should draw it in a different, this thing, it will come down like this, this pressure. This red curve will be… and this black curve is going to be… Now, the interesting point is that which is what lot of people do not realize is that what happens in this region, this pressure is assumed as if in our theory as constant. The dynamic pressure in this region is assumed to be constant. Why? Why constant? Because if you look at the formula for dynamic pressure, I will write down this formula, this formula is written as xi a e to the power of k Z into whatever, it is having a e power of k Z.

Now, if I allowed it to go like that, it would have increased. But theory says that it cannot increase. So, what is assumed in the theory is that, as if this pressure remains constant in this region. Now, if you take now the total pressure to be this and that, what happens? This is exactly same as this therefore, the net pressure becomes 0 here. And here it becomes like this. And eventually it becomes like, this green line becomes my total pressure.

And if you look at the total pressure there is no contradiction. The total pressure is 0 here, which is how it should be. Total pressure here is equal to rho g into this height and it goes like that. Why I say that is because sometime we will be using this formula and then you forget this formula and only use this formula under wave.

Under a wave using hydrostatic pressure is meaningless within the crest, because if you took within this region, see if you took within this crest region, hydrostatic pressure alone then you will always end up getting a negative hydrostatic pressure which is meaningless. Why this contradiction comes? Because water in the first waves cannot stay like that unless there was a dynamic pressure associated. And in this the dynamic pressure that actually keeps the water in balance. It is this hydrostatic pressure becomes negative, is upset by the positive dynamic pressure, just took together makes it 0 here and going up down.

So, if I have a boat here for example, if I had a boat here and I want to find out the hydrostatic pressure, draft of the boat in a wave crest and you simply used the hydrostatic pressure there, you will get a wrong answer because you will have a negative hydrostatic pressure, the boat is going to actually fly or sink. It will not make any sense. I am saying this because this question is very critical in the beginners, there was a contradiction or not a contradiction, a confusion that arises because the way the axis is taken.

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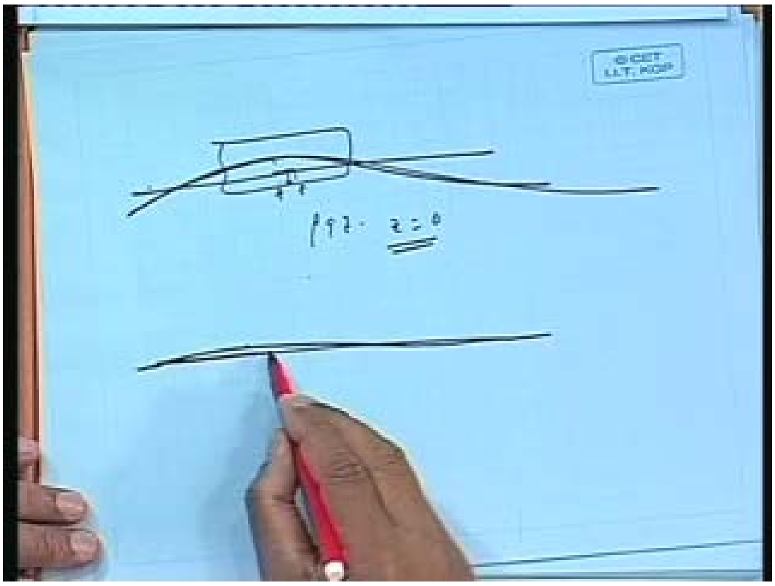

You see again I will elaborate this little bit. Suppose, there is a long wave here, there is ship going, boat here, now what happens? Typically you want to find out the pressure, a hydrostatic pressure by using this height. You say it is that draft is so and so. So, pressure is so and so, but remember that mean service is like this.

So, if you use some formula rho g Z, you will measure Z from here. And if you measure Z from here, you will end up getting a pressure based on this height, which will be smaller. And if you use that you will not be able to balance the ship. So, in this case if you have to use rho g Z pressure and measure Z from Z equal to 0 line, if you want to measure Z from Z equal to 0 line, you have to necessarily add the dynamic pressure. If you did not add dynamic pressure, you are doing completely wrong.

So, whenever we are looking at a wave, whether it is small or weak wave, you either have to take the total pressure. Or if you do not want to take dynamic pressure, assume that water is not moving and assume that this is my free water line, calm line and from there you measure.

Yes it will come the correct value. This arises because sometime the ship might have a very long wave and you may take like the heights from there, I mean, mean surface in a calculation. See, you do not normally take the height from the free surface because you typically this is here, the water is like that and you are measuring height from this point, not from this point because its axis is there. But if you are measure from this point for finding pressure, see if you measure from this mean line, that is the red line, then you have to take the dynamic pressure along with. That is a very important point. Yeah of course, it will vary, obviously it will vary. I just shown the crest because it is most. But at every point it is exactly of that magnitude, where the dynamic pressure and the static pressure when added together. Yeah positive pressure and on the surface makes it exactly 0.

Balancing because it has to be 0. In fact, that is a condition based on which this is found out, that it should be 0 on the surface. Obviously, on the surface it should be 0, basically atmospheric pressure which is taken as a determined line. So, this is an important point, we have to always sort of remember.

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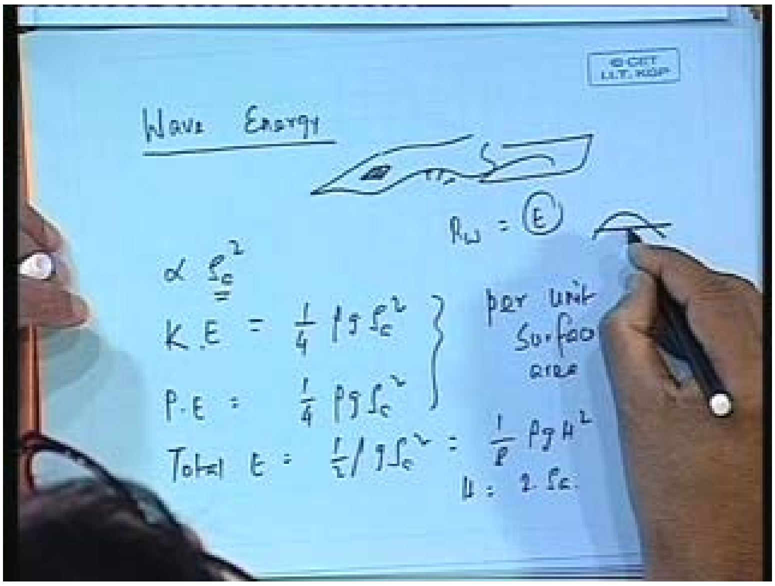

Now, I will talk quickly about an important concept called wave energy. And again energy becomes important because you will find out that ultimately when a ship is trying to move like a wave resistance you have seen, you see when the ship, I do not professor misread would have done it, makes waves. Where does it come from? It is not made by the ship. No though, but how? Why do you say it is a loss? That is the physics behind it. Why do you say that there is a loss of energy? Let the wave be made. The question is that why is that you need to have some force required to make those waves, that is the question. The reason is that, when the ship is moving you see it creates wave, now there are no waves here, wave means something is moving, something moving means there is an energy because without energy you cannot make it move, half mass v square. So, somebody has applied the energy. Who has applied? Only thing around is the ship. So, ship has supplied. How did you supply? Energy is work done, force into distance. So, there is a force that has been exerted to create that energy which is making the waves therefore, you need that extra thrust. This is why therefore, a good ship is the one that makes less waves not more waves. Yes if you can make a waveless hull, lot of researchers talk about it, then you are actually the best never I can take. You should not make waves too much disturbance. Anyhow so, the energy comes in.

There what happens? Why I say this? The wave resistance can be related to the wave energy. You can relate that to wave energy, if you could find out that what is the wave energy, energy of the waves, you can actually figure out what is the wave resistance part of it because they are intimately connected. Since the amount of energy that is gone in, that the amount of work that is done, goes into and gets spent as the energy making the waves. So, now, if you could figure out the energy of the waves, then you could actually find out what is my wave track. In fact, this is a classical way of wave analysis in finding out the resistance. So, I need to know wave energy and sound wave.

Now, what is wave energy? It turns out it is proportional to amplitude square. And actually it without this thing I will say that kinetic energy is given by 1 by 4 rho g xi a square and potential energy is given by 1 by 4 rho g into xi a square, they are same. So, the total energy, you can also write this 1 by 8 rho g into height square because height is 2 of xi a. See now, what I am saying here is that, this is interesting, that a wave would have a kinetic energy as well as potential energy, this is of course, per unit surface area.

If you take an open ocean and if you took one area, one square meter of area on the surface and average it out over a period because it is continuously changing. If you average it out about one period, then what you get is, the result you get is that it says the kinetic energy is 1 by 4 rho g xi a square, potential energy is 1 by 4 rho g xi a square.

Now, you may ask why potential energy. The reason is very simple, water is from here has been raised up. So, if you have raise it up from a determined line there; obviously, this is the potential energy. Why kinetic energy? Because waters are moving, there is a motion involved. So, both energies are involved. So, this is called wave energy formula.

That means, this is interesting, because if there is a ship moving, you see when the ship moves it makes wave. Now, very interestingly, let us say it made 10 kind of waves, a ship is moving it made 10 kind of waves. This part I should very interesting tell you. There is a ship moving and it make number of waves. Now, you do not know what length of wave it makes. So, let us say it makes 10 kinds of waves.

Now, the question is that, you are standing here, now you are looking at this point, now you are moving at a speed of v shape. Now, suppose it made 10 waves, these 10 waves would have 10 difference speeds because I mentioned that before. So, the one that is not moving at your speed would have very soon you would not see it, you have gone here and the wave is much behind or you have over taken you. So, the only wave that stays with you is the same wave which has a c equal to v.

So, you know that when the ship moves it only makes one particular length of waves, because all other waves would have dispersed way, you would not get to see them. This wave is so steady that you think that it is stuck to you. You know if you take a picture here today and tomorrow after breakfast and in the next day, it is still the crest is still at this location. So, you think it is a stuck picture now. So, the length is known. Somehow if you could figure out the height, then I could figure out the energy. And then if I figure out the energy, I can figure out the wave track.

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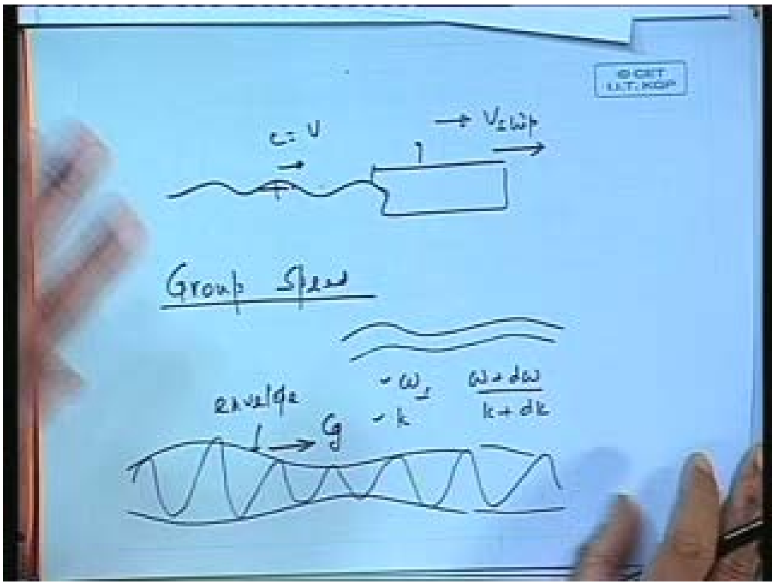

So, there is a relation between the two, very interesting relation. Now, that is another thing I have to tell you, that it is called, this is a last thing I will tell today is a group speed. This is a very interesting and difficult concept, you see if you have got two waves, a very closed by length. It turns out that if you plot them, if you take two waves, one is at frequency omega, one is this omega plus d omega so called. So, one is of length k, another is of length k plus d k, d k is a small number. See, there is a pair omega k, there is another wave slightly different, then what happened? If you plot that the wave form, it will turn out that this wave form will look like that having a beating pattern. It looks like these two, it is what is called in a sound beating pattern. If you make two noise which is slightly different, it gets modulate, I mean goes up down up down at a very low frequency. Same thing here happens. And this is called an envelope. And the speed at which the envelope moves is called group speed c g. So, if you take two waves of omega and omega plus d omega, length of k and k plus d omega, it turns out that the two waves sum together shows a modulation as if the full thing is moving up and down, but not like a single sine wave.

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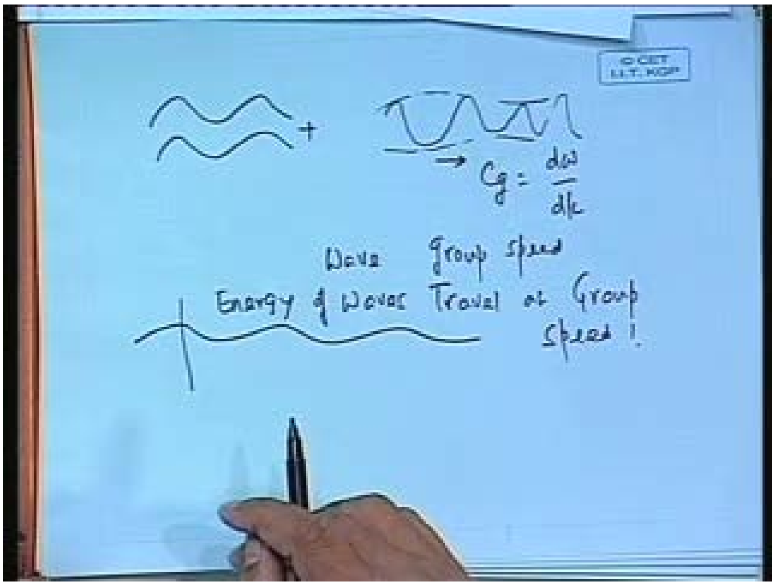

See again I will tell you. One is like a sine wave, another is a sine wave separately, but this together when you add them they look something like that. And this envelope moved at a speed and this envelopes root is called the c g, which is given by d omega by d k and this is called group speed, wave group speed.

Now, you may ask me that, what is the significance of group speed? What is the great thing about it? The great think about it is the biggest important part of group speed is their wave energies. Now, you take a particular wave here surface. You see the energy of the wave travel at group speed. This is the most important part. Now, you may say what is meant by energy of the wave travelling at group speed. It is a very typical sort of concept, but I will tell you one thing very interesting.

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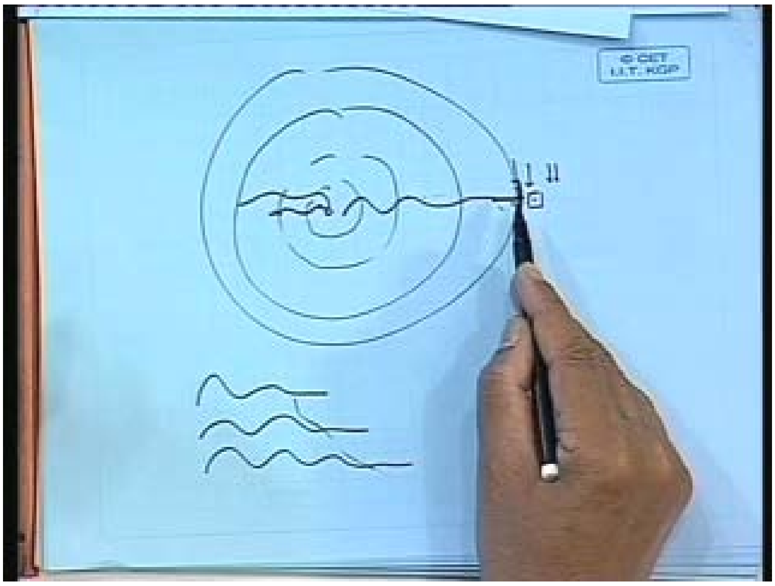

First of all c g is turned out to be half of c, group speed is half the phase speed. Now, energy travel at group speed, these two I am saying. Think of this part that you have a point where you have thrown a stone. So, it makes annular waves. Now, if you look carefully next time you should see that, you see the front of the wave it will look like that, then next second it will look like that, I mean if the front will look then next second it will look like that, then next second it will look like that. There is a front moving and there is a crest moving. You will find that the front moves at a speed much lower than the crest moves. Actually if you keep your eye on one of the crest you will soon find it has come to the edge and suddenly disappeared. So, the edge of the wave always moves at speed which is lower than the phase with the up and down speed In other words, you keep your eye on this crest, you keep your eye on that, you see next second it moves and suddenly you see that it has disappeared

Why it is so? Now, where is the energy? Now, you think of this front. This front has just the front of that, water here by definition is calm, it is just the clam water surface, just here it is the having the wave. Therefore, this particular point has no energy, this point has wave energy. Therefore, the rate at which it is moving is the rate at energy is propagating. Exactly, because if it was not so, see just this line, this point has no energy, this point has energy. So, next second this point has no energy, this point of obviously, this edge by which it is moving forward is the speed by which energy is travelling.

And this is an important part because it turns out that this speed is not same as phase speed. Another interesting example I will tell you. Before that let me tell you one thing, that the fact that the waves actually, when you throw a stone, the waves actually amplitude decays as you keep going further is also explained by the fact that the wave energy is equal to amplitude square. See, energy is proportional to square of that.

Now, what happen? Energy should be constant. Now, if you look at this circle the energy is over this area, if you look at this circle energy is over this much area. Both xi square into the perimeter should be constant. Therefore, as the area it grows bigger, the amplitude becomes smaller. This is why when you throw a stone the wave actually decays. It goes smaller and smaller as we go further. About the wave energy, the last thing I will tell you that is very important for wave energy is that, you think of this case there is a wave maker in our tank, there is a point here that moves up and down, makes wave. So, you tuned it today.

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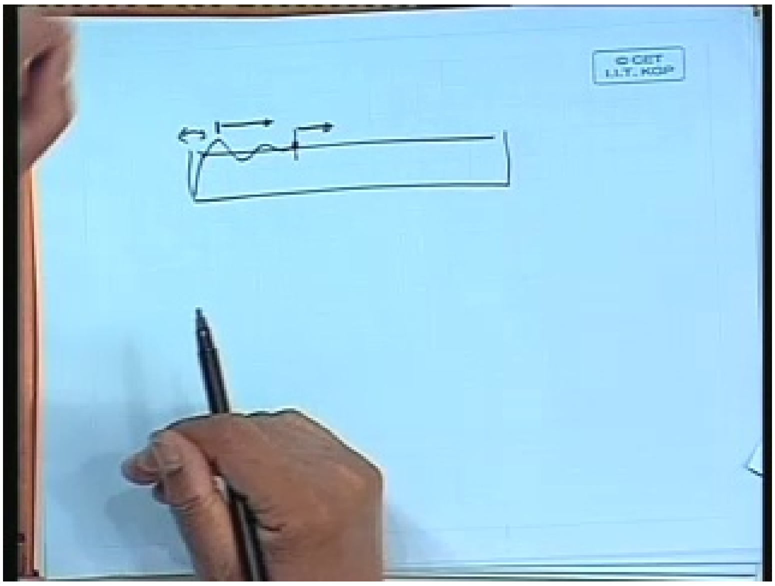

So, what happens? This wave would start like that. Now, I tell that any one of you that you walk along with the edge and another person along with this, you will very soon see that you are walking at a speed much lower than he. He is walking twice as fast. But as soon as he comes to you, collides, you disappear you become one. This is a very interesting phenomena that you will never realize unless you next time carefully observe a stone throwing on the edge. You see the edge moves much that circle is growing much slower rate than those waves are moving, but they come to the edge and disappear. It is a fascinating phenomena if you ever observe next time, you will see.

And why it becomes important for our ship case? That is because we will find out later on that the ship created wave speed and the wave speed have lot of relation. If so called the two speeds, energy of travelling of the ship wave and the waves are same, then the energy gets trapped and it is exactly same as what happens in a sonic boom.

Just like if the ship is, vehicle is moving at a solar speed, when the sound it creates wants to go at the same speed as the plane. So, the energy gets trapped. Similar kind of phenomena occur. This is why group speed is important as a concept. Anyhow, so I will today stop about linear wave part, if there is little more then next class we can pick up.

Preview of Next Lecture

Lecture No. # 23

Irregular Sea Waves - I

See, today's lecture is going to cover what we have called irregular sea waves. Some people call this actually also random. Now, see yesterday we talked about regular sea waves, nice sine curves coming.

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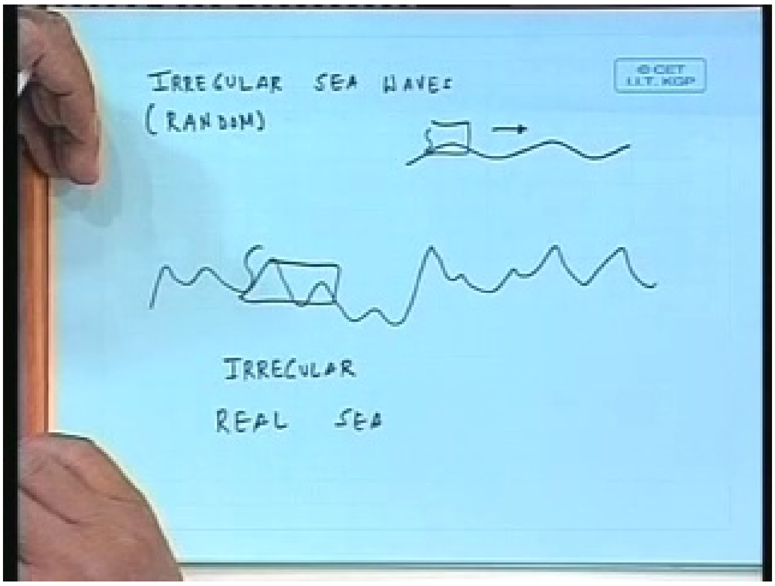

But as you all know, if you really go to open ocean you will never find a wave which is very nice and sinusoidal, you will find wave which will probably look something like something like that. If you try to monitor it at some location, it will look like that or if you try to take a snapshot also, it will look like that. It will not look like a nice and sine curve. In other words, what will you find is that there is a lot of irregularity, it does not at all look like a sine curve or one can say it is random. In fact, one can show that if you keep taking for a long time, it seems to be a never repeating kind of a signal as far as type of signal is concerned, it will never be exactly same.

This is what we called obviously, irregular waves. The word irregular is more used by or coined by ship people. People never like it as a background, but eventually when this offshore structures are coming up more of civil engineering people where concerned, they are started using the word random waves which is both are same thing. So, this what we will use it interchangeably, irregular means random wave.

Why we need to study that? Because ultimately our aim is to find out when I have going to put my ship in a sea, in a real sea, how the ship behaves. Now, a real sea is not composed up idealized single sine waves, it look something like that. So, there is no point from, I mean my ultimate aim would be not to figure out only how wave ship behaves in a regular waves, but how it is behaving in this wave. So, I need a description of this wave, that is a very critical thing because I must describe what is my environment in which I am going to put. Just like a yesterday we talked about one single wave which is environment, but the real environment is irregular waves. So, we are going to talk about this.

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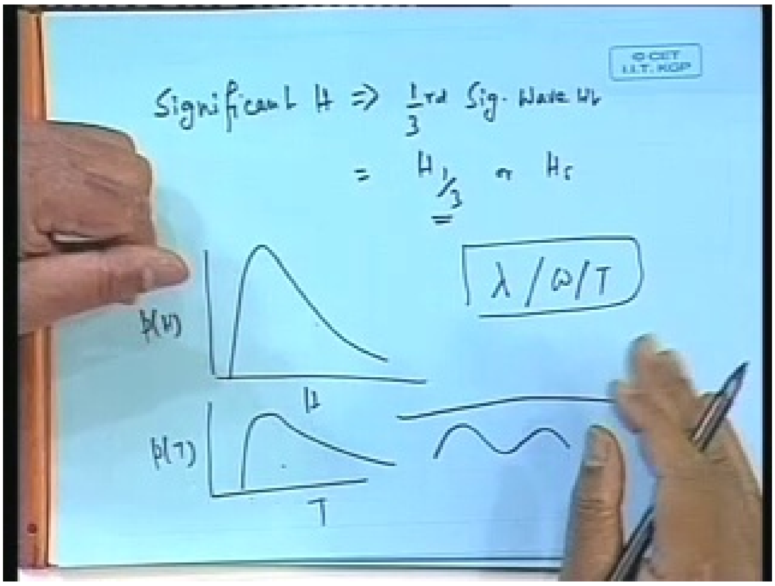

Now, let us see how we can go about it. Let me give one example. That we have found out from a wave this histogram of this thing, wave height occurring. But you see, we talked yesterday, if you look at the regular wave, you will find out that waves are not only decided by height, but also by length or frequency or period. These three are interrelated to each other.

Now, this distribution is not going to tell me this information. See, I can have another one, for say period versus p t. But I can have two different sort of a histogram. But this does not tell us the relation between the h and the t. You see, this is independent. Here I am telling how many times this height is occur, here I am telling how many times this length is occurred, but I have not anywhere telling what is that time a certain height of certain length is occurred.

Because you see after all occurrence of say 10 meter wave a 1000 meter long, 10 meter height 1000 meter long wave is very small slope is much less severe than occurrence of 10 meter wave of 100 meter long waver. Because see one would be a occurring like this wave, another is occurring like this wave.

See, if I just take height as a parameter, I do not know what the length is associated, then I do not really get a feel of the ocean. Because I can have, say 10 meter height, I can say the 10 meter of at many times fine, but the wavelength is a 1 kilometer. So, it looks still like that. No histogram cannot give that other side. It can either give only height, it can give only period, but the relation with the two is not there.

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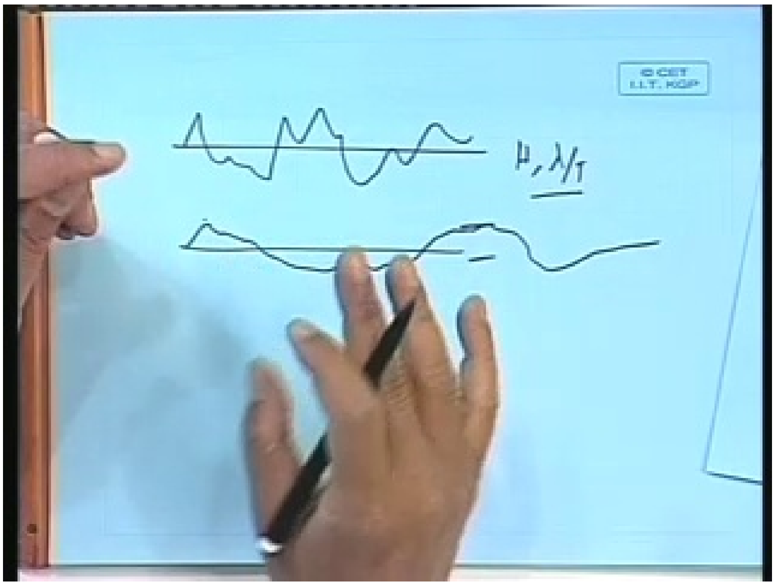

Supposing I give an example here, supposing you take the height. So, there is a height like that. Another signal with the same height, but spread. Both of them the heights are some. So, you get a same edge average. But obviously, they are not similar waves. Now, if you take a length now, you get the here longer length, but you do not know what is associated height. So, this h and lambda over t are not coupled to histogram and you do not get a feel of that. And so, we have to have another means of actually representing ocean waves, where information on both height and length are there together. This is a necessity now because otherwise you do not understand.

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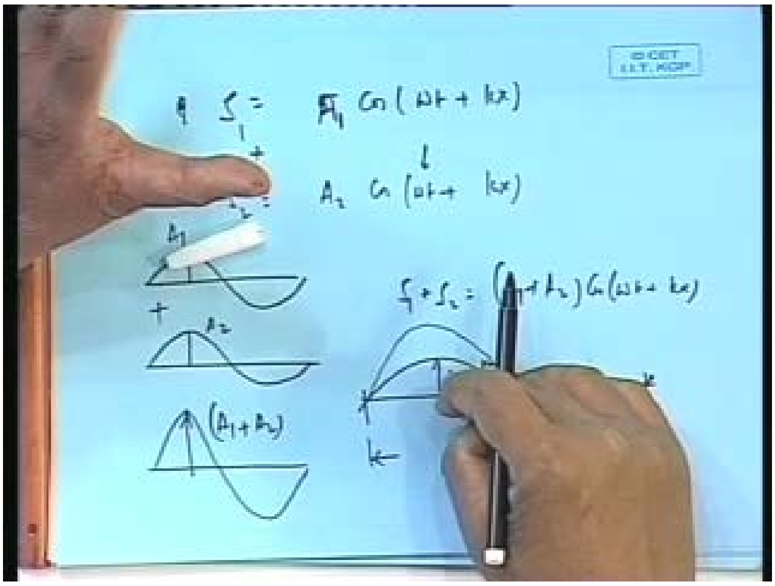

Exactly same wavelength wave, 1 meter wave, another 1 meter wave, you can add them up and say it is 2 meter wave. Now, conversely supposing I find out that there was a 10 meter long wave, let us say I have a 10 meter long wave there, 10 meter high wave. I can think this 10 meter high wave is nothing, but 10 number of 1 meter high waves added together. This is what is called breaking it up in pieces. This is only possible for linear waves. Now, you see what happens, this length is same. Now, I have another 10. So, becomes 20, then it has changed the stiffness. So, you see they although the wave was stiff, very stiff, I can think that the stiffness has come because I have added n number of linear waves.

See, supposing let me give an example. This is 100 meter. Now, in 100 meter a height of 10 meter is called quite steam, because by that time we will play. Now, suppose I have 10 meter high wave 100 meter long wave, I can say that look it is nothing, but sum of 10 number of 1 meter high waves.

Now, a 1 meter high wave is obviously, small amplitude, it follows linear theory. So now, I have adding one linear wave, small amplitude wave plus another one plus another one plus another one. The resultant wave I have got is actually not so small amplitude.

So, in conversely in this when I break it down, it just turns out that even though some time the waves are of very high stiffness, the principle of linearly superposition or breaking them to linear waves also works somewhat well.

So, it kind of it is not too bad. You just cannot get this kind of wave all right, but you can get high wave of this type by breaking it down. So, anyhow. So, this is the principle of linear superposition. I will close this lecture now and the next one we will talk about more interesting part of how we can combine and how we actually represent the wave, irregular wave. Thank you.