Performance of Marine Vehicles at Sea

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Lecture No. # 24

Irregular Sea Waves - II

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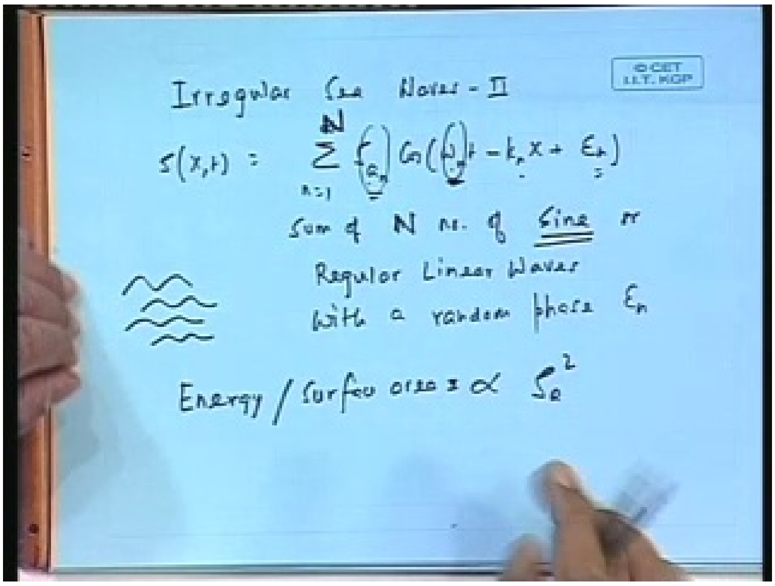

See, we are going to continue our talk on irregular sea waves. So, what we found out in the last lecture is that, you can have by Fourier analysis a random wave signal represented as sum of n number of sinusoidal waves. See, this represents this side, sum of n; basically, here I wrote infinity, but we can make it n; n number of sin or regular linear waves. See, in other words, the random signal that we had, could be thought or could be broken down into n number of sin curves, each of them representing a small amplitude regular wave with a random phase; this is my random phase… with a; this is my random phase.

In other words, phase we have no information on. You are adding n number of waves; one is sin omega 1t, sin omega, 2t sin omega 3t, but the phase, meaning the starting point is different. You know one is starting from here; another may be starting from here;

another may be starting from here etcetera, etcetera; that is what decides the phase. So, we can get that, we can get a random signal by synthesizing that into this.

Now, if you look at that, you will see here that this has got the amplitude part; this has also got the frequency part. See, K is after all connected to the omega, because K… omega square is g K. Therefore, this and this are connected. So, here, you have got this. So, I have got omega; I have got this; both information are embedded. Supposing, therefore, if I took a random signal, and if I could find out this and this together, then I will know for which omega what was my corresponding psi a.

Earlier, I got what is my height or amplitude, just number of amplitude, that is all. Again, I had separately found out what are the number of omegas or you can say period or length, but I did not know which wave at how many omegas. So, but in this one, if I do, did that, you see I will find the… say look, I have got in wave number one height so and so, omega 1, so and so; wave number two height so and so, omega two so and so. So, I have that information of both. This is number one. This Fourier analysis, therefore tells me that the component waves, when I break it down, each has a corresponding length or period and a height; only the phase I do not know; that is one point.

Second point is we should try to think in terms of now physics, energy. See, now, what happened, earlier I mentioned a wave energy per unit of surface area is proportional to xi a square, amplitude square. In fact, yesterday, we have had the formula half rho g xi a square is to be the, your energy. Now look at this part of how an ocean wave gets generated.

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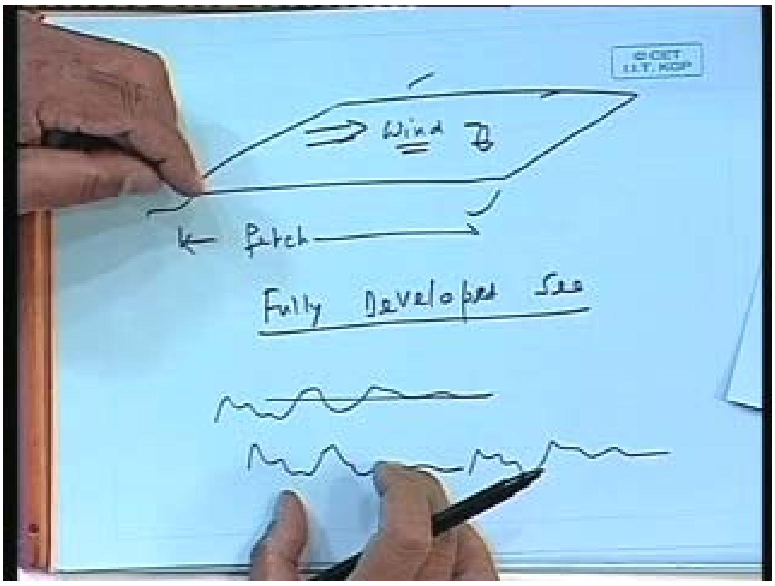

Now, you see there, there is an ocean. I just draw an ocean surface. There is a typical ocean surface. Some part, you know, large ocean surface may be there. Now, wind blows; as wind blows, from where wind blows, that we do not know or some depression somewhere etcetera; this is all connected to the solar energy somewhere. But as the wind blows, wind imparts some energy to the water surface. Certain part of energy gets absorbed by the water surface, and therefore, the water begins to move up and down, and begin to form waves. You will find out that if you take a bucket of water, and if you did, you know, it ripples. If you give a larger one, you have larger height. So, more energy you give, more energy gets transferred.

Now, in this kind of sea description, what we are assuming is that, we assume this an over a distance - is called fetch - over long, some 100 miles or so distance. A wind has been blowing continuously for several hours; this is what we are, we are describing here what is known as a fully developed sea. Why I say fully developed sea? It means that wind has been blowing for quite some times there. So, therefore, you know whatever energy the wind has to impart, you know, whatever energy from the wind, the water could absorb, they have already absorbed; it has a as a saturation level. So, it has now created certain waves.

So, you can assume conversely that the energy of the waves remains constant for a given wind speed or a given wave state, because energy is a conservative principle. Now, see initially it starts, wind starts, blowing and certain energy gets absorbed, certain waves form. Now, more, sometime later, more energy gets absorbed, but beyond a certain point of time energy cannot keep on getting absorbed; it has got saturated. So, and that time I call that sea to be fully developed. Statistically, now heights are not continuously increasing, it is now random, irregular, but having a statistical average. This is what is called a fully developed sea, and we are describing here why I come to this point? The main point is here that, I want to describe where if I took this random signal and break it down in components, per unit area energy of that surface remains constant, because energy is conservative principle.

If wind was blowing at say 20 knots for long time, then the wave that has got created would be of such in nature where the total area over a certain, you know, certain surface area, unit surface area, remains constant. It may look different; it may have a different kind of distribution; it may you now like the it, it today it may look something like that; then it may look random.

But sir according to the wind, the energy fully what is there, it is not spent.

Fully.

What energy it is observing and taking in…

No, no whatever now, that is okay. It is not the energy of the wind, that is fine; that is not what we are saying; see nobody knows the mechanism, but what it says is that, this is called wind generated waves. Let us say that 20 percent only gets absorbed. So, it is going to be that 20 percent only; that 20 percent does not change. So, the amount of, see if the, if there is a wind of 50 knots, let us say a certain x units of energy can get transferred to the water, maximum. So, it has got transferred. Now wind keeps on blowing and it is only x, whatever that random wave you find, will, should contain, should represent an energy content of x units per square this thing.

So, although the nature looks random, statistically they should be such that, you know, that their total energy is something like x units; this is the point. So, the wave looks different today, tomorrow, day after tomorrow, or today, and next hour, and next-to-next hour, but they are all more or less having x units of energy. So, in other words, the one that I broke down to various components, it should be such that, normally that the area under that remains constant, sorry, the energy of that remains constant, the energy of that. So, this is the principle; therefore, it would be much better to plot the energy distribution. Energy, see I have broken it down to various component wave.

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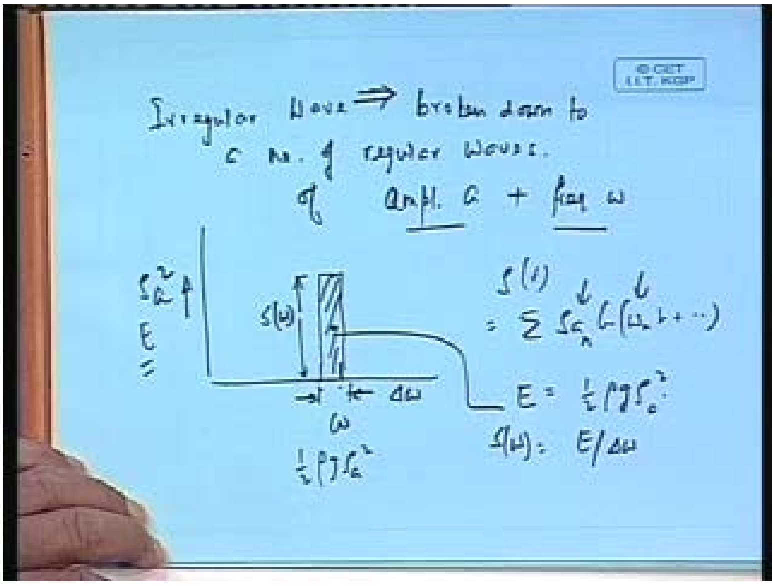

Again, I will say that, I have an irregular signal, I broke it down to a number of certain this thing, certain amplitude a plus frequency omega. Now, if I can plot instead of that here, see instead of plotting here amplitude and omega, if I plot here something proportional to amplitude square or proportional to energy, then it becomes much better from a physical point of view.

See I have got this random signal, right? I have got the random signal, where I have got this to be sigma of, you know, again xi a amplitude into cos omega n T plus etcetera. So, I have got this; I have got this. So, I could have actually plotted omega n versus a to find out, you know, in a random signal, which omega and which a is existing, I can have this plot. Instead of that plotting, the logic says that, that plot is less meaningful, than to plot on this side something which is proportional to amplitude square, because that would be proportional to energy.

Half square.

Yes.

So, instead of plotting amplitude against frequency, that is what is called amplitude distribution against frequency, if you plot square of amplitude distribution against frequency, you are actually plotting energy distribution against frequency, energy of component waves. So, what I mean, suppose there is a wave here of this frequency, now it would not be of one frequency. So, all the waves between this to this frequency have certain energy.

Now, if I plot this, this area, so that this area represents energy of all the waves. In other words, this area would represent half rho g into xi a square of all waves between this to this frequency band. That would imply, that this is my energy. Such a plot, if you make, that is, on the vertical side, if you take amplitude square divided by, see I will call this to be delta omega. So, if this shaded area, if this has to be energy equal to half rho g xi a square, then this height – s - it is called s omega, s omega then therefore becomes e divided by delta omega; because, I have taken delta omega to be this distance. So, if I plot that, that means, if I plot that….

Now I will come to the next slide; getting little confusing here; instead of plotting, see that what are the options, the option was that I broke it down to number of regular waves. So, I know what my omega is and what is the corresponding amplitude; I know that. So, I could have plotted omega against amplitude, I could have plotted omega against amplitude, I will again come to that.

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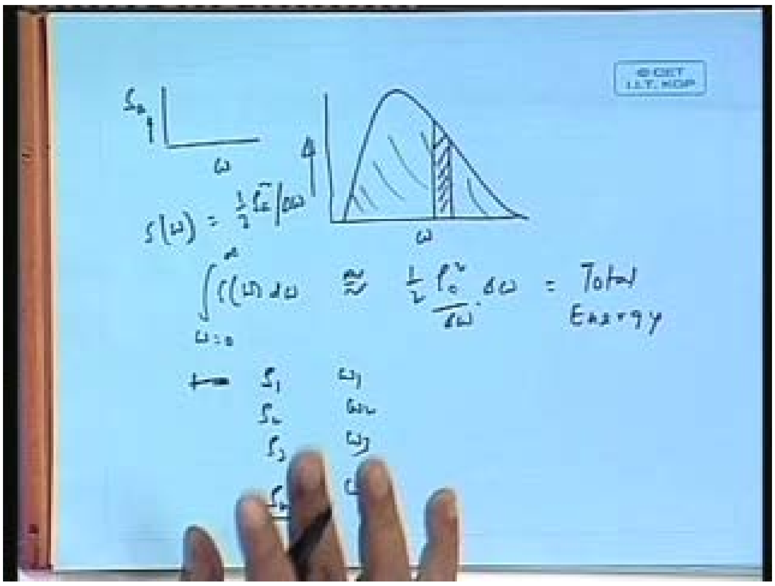

See I have got broken down. So, I could have plotted number one omega, what is this? xi n; xi n; I could have plotted that, but that, that would be called frequency distributions of amplitudes, but people are not plotting that. What I could have plotted square of that. Instead of that what I am plotting here, this side is the term proportional to half of xi a square divided by delta omega when it is omega, because why I say is that, this plot, this plot will represent such that it will basically represent the area under the curve between certain frequency band to be the energy of all the waves of that frequency band. So that if you take the entire area, the entire area would represent the total energy of the waves.

Supposing I call this to be… we call it s omega. Now, if you take, see s omega d omega over the entire range omega equal to 0 to infinity; what is this? This is going to be half xi a square by delta omega into delta omega proportional to; this is going to be that total energy. I think it will require little more discussion. See, once more, I will come to that. What we are doing, see, I want to plot here something proportional to energy, what, means what? I must plot here something proportional to xi a square instead of xi a what is omega I should put something xi a square versus omega, but if do xi a square by itself what happened? This side can get spread; you see what happened. Let us say, I have done the graph for 10 omegas; I broke it down to 10 components; you broke it into 100 components; somebody broke it down to 1000 components.

If you just plot here, xi a square, you see, let us say, what…. I will give an example. Suppose, I broke it down to 10 components. So, I have got see one meter sorry xi 1 for omega 1, xi 2 for omega 2, xi 3 for omega 3 etcetera, xi n for omega n. Now, if I just square them up and plot them this side, you see, then if you add them all up together, there will be xi 1 square plus xi 2 square plus xi 3 square etcetera, etcetera, and they will not be same, because tomorrow I can have instead of n 100, I can have 1000. So, what the convention is therefore, is that to plot here something proportional to xi square all right, but xi square divided by delta omega.

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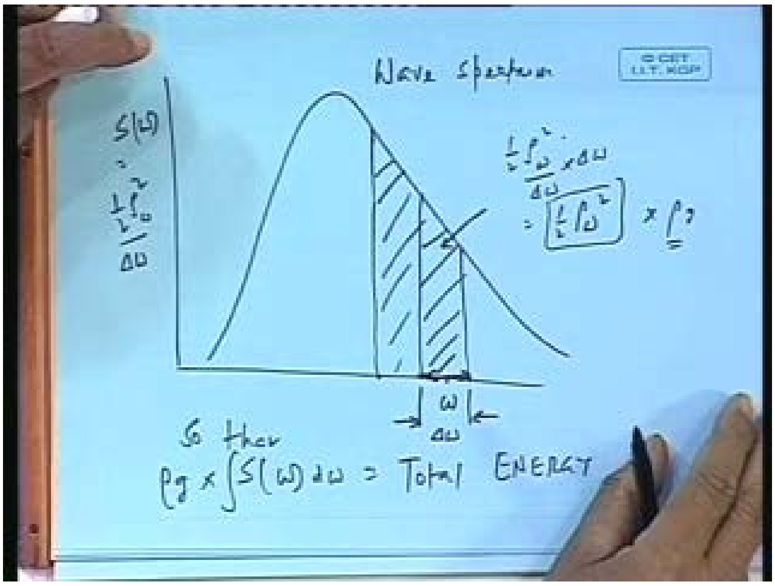

Why the logic is? Again next one, in a big one, we will, we will have to go through that. See, I am plotting here. Now, this is here, what I am plotting is half xi square divided by delta omega. What is delta omega? Delta omega is the interval. So, you see, that is my omega. So, this distance is my…. Now, if I plot that, if I plot this, what happened to this area? What does this area imply? This area, this shaded area, would be half xi. Actually, we can call it to be xi omega here, omega square divided delta omega into delta omega that is going to be half of xi omega square.

This area is going to be half of xi omega square, which is equal to area, energy, because energy actually is multiplied by row g will give you the energy. So, what I am saying is that if you plot this, then this would be, if you have plotted this quantity, if you plot this quantity, that would represent what is the energy, that would represent a quantity proportional to energy of all the waves of this frequency band, instead of just amplitude square.

Now, now, supposing I take this frequency band here, then what is this area? This area is energy of all the waves between this frequency band. Like that if you keep on adding this entire area under the graph will therefore represent the total energy of the surface - water surface. This is why you have this coming here, that is or I can call this to be s omega equal to… so that s omega d omega integration of that into rho g gives you total energy.

The purpose of drawing this curve is that nothing but an energy distribution alright, but such that the area under the curve represents the energy proportional to. See always rho into g is a constant. You do not want to deal with that rho g, because it is not a nice number. So, you take it out as a constant. So, you see this is very important, very important concept that you actually are plotting this side; essentially the square of the amplitude divided by bandwidth, because you know, you cannot get waves exactly at a particular frequency.

So, again if you look back at that, what we will be doing, that we have a spectrum, you do Fourier analysis. So, you say that wave between omega 1 to omega 2 so many numbers of heights so much; omega 2 to omega 3 height is so much; omega 3 to omega 4 height is so much; you are averaging it all out. So, now, I know that my waves come in between this to this; all the waves between this frequency bands. Just like when I said there all the waves coming between this height bands - 0 to 1 meter - so many times occurring. Here what we are telling is all the waves between frequency omega 1 to omega 2 or omega plus minus a bandwidth within that, within frequency band, the average, average amplitude is an; all the waves of this frequency band, average amplitude is so and so; all the waves of this frequency band, average amplitude is so and so.

And what I am plotting here is that square of the amplitude divided by the frequency band. Why? Divided by frequency band, because by dividing by the frequency band, it makes sure that the area under this becomes the energy of those waves. If I did not divide by the frequency band, then the area would not have become so, because it would have gone higher. I want to make sure that the area under this represents the energy of all the waves. So, this is a, is a very standard representation; it is called energy density spectrum. This is also there in electrical and all other subjects you see, because it tells me, that this spectrum is such a curve, which tells me the ordinate will give me a measure of the energy of the component wave.

But area under the curve gives me the total energy, and obviously, the area remains constant. Why this area is important? Because, now you take this for example, the wind is blowing for long term, say one full day. Now, you took a record for one hour today evening five to six “o” clock, you got a record. So you broke it down to various components and you plotted this graph. He did that after evening, nine to ten “o” clock.

Now, what he gets and what you get are totally different set of supposed to be amplitude and frequency, but the area under that should be same, because the total energy is constant. That is the basic philosophy, that although the signal looks different, signal looks widely different, the signal still represents the same energy and you will find out this is a - we will come to that later on - that the frequency distribution of energy also are actually same. In other words, this graph will remain almost same whether it is today or whether it is tomorrow in the same place; that I will come to, that why it is same, later on.

But let us first understand this distribution. So, the wave spectrum, this is called a wave spectrum or some people call wave energy spectrum, you know, either you call wave spectrum or wave energy spectrum; some people call energy density spectrum; all various terms, I will next slide write.

But this essentially represents a distribution of the energy against frequency of the component waves. So, again if I have to repeat, I have an irregular signal, I broke it down to number of regular waves. Now I have my next job is - how I can represent that in a very convenient graph? I could have actually plotted frequency versus amplitude, but people think it is much more illustrative or instructive to plot energy against frequency.

But energy by itself cannot be plotted, because energy depends on how you broke it down. So, you find out such a way that the ordinate represents a quantity energy by a frequency bandwidth, sSo that the area under the graph represents energy; this is the main purpose. So, this thing we will… now the typicality of this graph. See now if the graph will look something like that.

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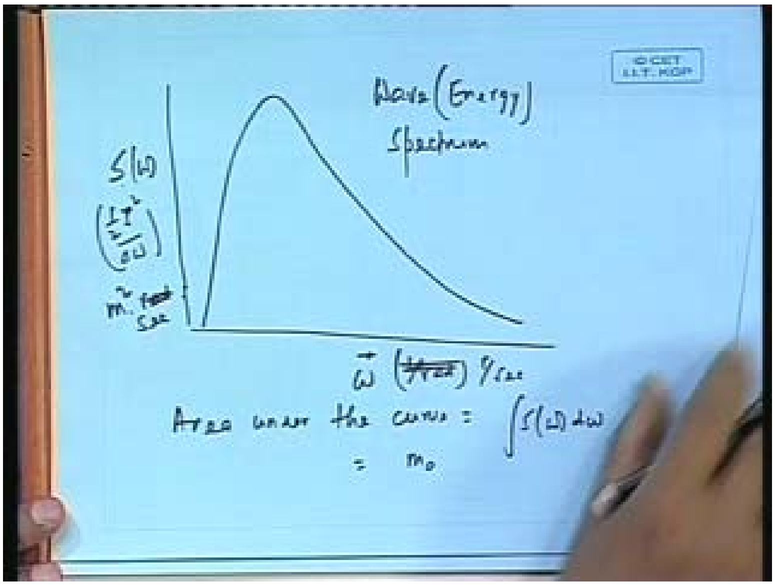

So, this is now s omega; this is we can call this wave. Some, as I say, some people call energy spectrum. So, it is called omega is called s omega. This is actually proportional to half xi amplitude square y delta omega. This is how this side is, as I said. So this multiply by rho into g, will give you energy per unit surface area, again energy per unit; it will have a unit of energy per meter square; we will, we will come to that later. What is a unit of that? Obviously, it is unit of meter square divided by unit of radian omega; that is this is meter square, no, by 1 by radian, that is meter square into radian, and this is, this is 1 by radian; no, sorry, sorry I am sorry

Meter square second

Meter square second

This is 1 by second or radian by second whatever; you can call R by second meter square radian by second by radian; radian has no unit, so it is meter square second and 1 by second. If you multiply, take the area under that, what is the unit? It is meter square. If you, if you multiply this, it will turn out to be the area is, the unit of that is meter square. So area under this curve, I can call, some people call this to be as m 0; you know, by definition you can call this m 0, you see; you call the area under the curve to be m 0; that is… that I will come to the little later on about about this m 0 and m 1 part.

Now, the question is that this is the most fundamental part is to therefore realize that I have taken the component waves, I broke it down, and now I will draw a nice diagram, in a… try to draw, where this shows the energy of the component waves.

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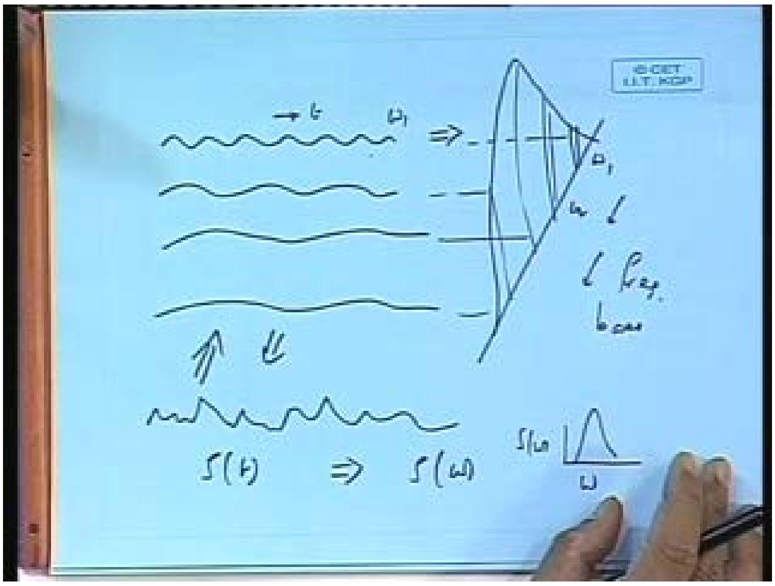

This diagram is given… later on, may be the book will be better. See here, let me try to draw. There is a wave here. I am drawing the component wave; this is one component wave; next one is… next one is… next one is…; you have broken it on this component wave; this you are representing in this graph. This, suppose, there is a there is a spectrum graph is there; this you are representing here; this omega, this omega 1, this you are representing, I mean when I write that, basically the area under that, this is here.

See basically here, I have got this omega signals against time, this is time and this is omega. What we have done is that, there is a signal which was against time xi against t, random signal I. This, when you sum them up, you have got this. So, this signal was broken down to these components. See this signal was broken down to its components, frequency omega 1 with xi 1, omega 2 with xi 2, omega 3 with xi 3 etcetera.

Now, for each omega, the xi square divided by delta omega is what you have plotted. See, this you have broken it down to various component frequencies omega 1 and corresponding amplitude 1, omega 2 corresponding amplitude 2 etcetera. So, this can be thought as if it is composed of this, this frequency with the corresponding amplitude etcetera. Now you are plotting this frequency as the base; this is my frequency base; the amplitude square, a quantity proportional to amplitude square.

In other words, what you have done is that whatever you have got xi against t, you have transferred that somewhere into xi against omega. In fact, what you have done is that, a signal you had in time domain, against time, you have actually represent them in frequency , against frequency, because this you are plotting against omega versus something. So, this is a typically a... one can also say, that energy spectrum representation where a real time, you know, real signal in that, in real time is represented in frequency domain, in frequency, against frequency, this is what we have done. This one has been done against frequency; this is the transformation that you have actually, we have been doing it.

So, as long as I can break it down to component waves, I can actually go back to the component and just plot the square of this amplitude here. So, if by doing that I get this graph, this is what we call energy wave spectrum. And area under the… what is the area under the graph? It will turn out to be proportional to half in… see it is proportional to half into xi square; the area under the graph become proportional to half into the xi n square; I will come to that later on.

In other words, area under the graph becomes in proportional to the statistical average wave height square. There is a… you know, it is connected to the square of the height. Now we will come to this, the relations, little later on, in a minute, but we should understand this part first.

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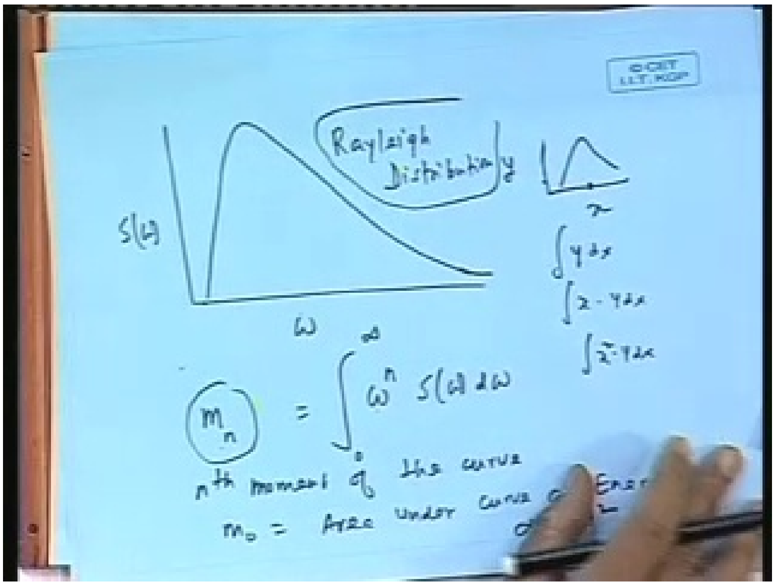

Having understood that spectrum, now what we have, we have done is that we have got this spectrum, s omega again omega. Now, area under that curve. How do I represent area? See there are number of ways of representing area. Let me take this quantity define moment. What is m n? Now, you see, tell me, what is m n? m n represents the n th moment of this area, because see, this see, if there is a graph here, this is x here, this is y here, integration of y dx is the area under the graph; integration of x into y dx will be moment of the graph about this line; integration of x square into y dx would be second amount of area about the graph.

Therefore, integration of this ordinate power into y dx, that is why this ordinate power y to the power m into this dx this d omega represents n th moment, n th moment of the curve. Why I say this n th moment of the curve, because we will find out that many of the statistical parameters of this graph are related to that n th moment.

Now, you see when you take m n to be 0; that means, if you take m 0, what is m 0? It is nothing but the area under the graph; m 0 is nothing but the area under the graph, because m 0 is s omega d omega. So, m 0 which is proportional to energy, which is proportional to height square, so the m 0 becomes proportional to height square. Now, we will come to that this in a minute; this is very, very important. See this now we must realize that, obviously, this is true, because what happens now? I took a spectrum, I took a… I synthesized that, and I found out this graph, and I know now that the area under this graph is supposed to be proportional to that total energy of that random wave; that means, if I took an 1 square meter area and I kept on measuring the energy of the content of that at the, I mean that surface, the energy would be equal to m 0. That area under the graph. And that energy is obviously proportional to the average height square or some statistical height square, because more energy means there will be higher waves, less energy means lower waves; they may not be one or two, but there will be random waves. So, average wave height or significant wave height would increase if there is more energy. So, we will find out later on that this m 0 is directly connected to various kinds of wave heights.

Let me just concentrate right now, concentrate right now, on this part. Now it turns out people, you know what people have done? People have actually been collecting data; you know, this data - this oceanographic data - has been there for ages. People have been for many, many years hundreds of years collecting data, and they have been actually synthesizing, and they have been plotting this graph.

So, suppose it turns out that you plot this graph say Bay of Bengal you are taking. So, 1950s, you took 60, you took 70, you took 80, you took everywhere you take, it turns out that this graph follows very closely a Rayleigh distribution; that again makes sense; means, it is like that not that… means there are… this is actually, I mean, the graph will look something like this - less number of high waves, this is actually more number of this, omega high means small waves; there are more number of small waves there, goes like this. This is what has been found out.

Now, if you can actually have a Rayleigh distribution graph, it turns out that all the statistical formula connected to the average wave height - significant wave height - all possible properties becomes connected to this (Refer Slide Time: 31:01 min); that means, if you know the spectrum, and its area and the moments under the graph, specially area; in fact, all heights becomes proportional to the area. So, if you… first of all, I have done a spectrum, then I find that it follows Rayleigh distribution like this, then I can say that I am going to assume that it follows a Rayleigh distribution, because Rayleigh distribution is a statistical property some kind of graph. Then I can say, if that is so, then the area under that graph m 0 can be connected to all statistical property.

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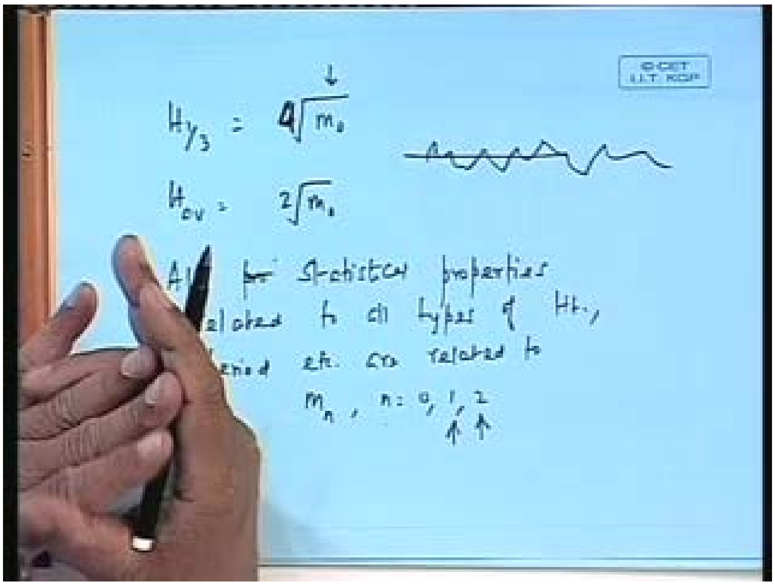

For example, now we will come to this example. H one-third will turn out to become, I will just tell that from here… I do not want to wait now… h one-third will become 2 root, no 4 root m 0; significant wave height; h average becomes 2 root m 0; like that, you know, I just, I just give you the an example, that it turns out that if the spectrum was following a Rayleigh distribution, and if the area under the spectrum is m 0, then the h one-third - the significant wave height - would be 4 times root over m 0, wave height, amplitude would be 2 m 0; height would be 4 m 0. What happens, therefore, if I know m 0, then I can find put h one-third.

In fact, we can find out all statistical property with respect to all kind of period; all properties, statistical property, all types of height, period, etcetera, are related to m n, n equal to 0, 1, 2 typically; that means, basically, they are related to the area under the graph, first moment under the graph and second moment under the graph. Basically, you know, this is first moment. See n equal to 1 represents the moment of the graph and this represents the moment of an (( )) the graph of the curve. Any how, I will not write the various formulas here, but it turns out that, that this is how it is. So, therefore, life begins to get sort of simple.

So, first of all I have a random signal, then I find out that it is nothing but sum of regular waves. Then if I can find it out, I can find out what omega and what a are associated. Then, I plot that graph; it turns out that the graph represents, , you know, Rayleigh distribution. So, I make an assumption, look I will consider this graph to be Rayleigh distribution. Now, I have to find out what is my average wave height or significant wave height. All I will do, take the area under the graph, use this formula. People do not believe it. You take this signal that from where you started in the first place, take all the heights there, find out those like what we did before, see the average, you will see that they will match, if the signal was Rayleigh distribution.

So, basically, this is how now it is allowing it to become simple, because any time that you try to take a signal, a record, at a location it turns out, obviously today, and tomorrow, and day after tomorrow, they will look different, but you expect a statistical property is same.

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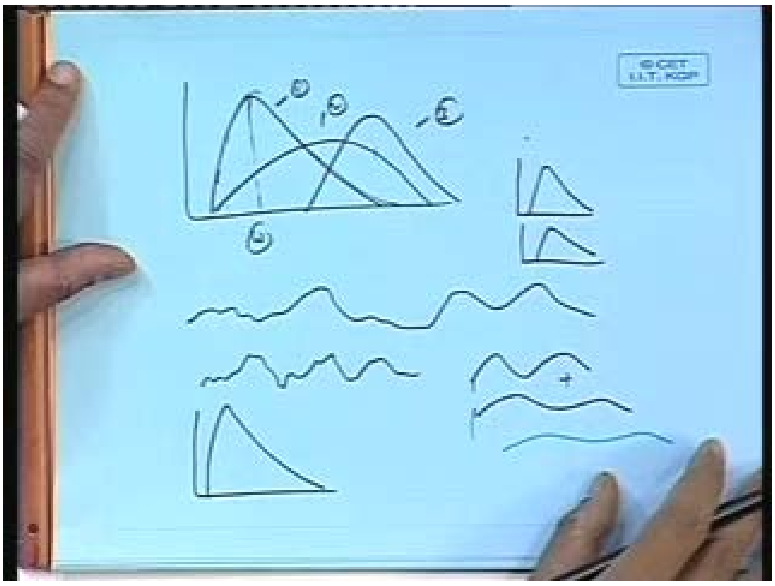

Here comes the question that I earlier said - that see, suppose this is a 1 having area as same; there is another one is look like that, area is same; another one looking something like this; all of the this… this is one, this is two, this is three - all of them the area is same; that means, all of them represent same significant wave height, but they are different, because this one is having more number of, you know, more number of low frequency wave; that means, it will look more low frequency longer wave; this will look more number of shorter wave; this one wave, three, will look like that etcetera. So, they are not the same. So, only energy or only height, do not tell us, as I said earlier, about the content. You need the frequency distribution.

So, in this graph, therefore you know both of them. Now, the question is that - if Bay of Bengal in month of July, if I today measure and I got a spectrum like that, should tomorrow also become like that - is the question. The answer is that, just think what, what all parameters might cause the waves to be to excited? Number one - wind speed, that is for sure; number two is obviously the local bathymetry. You see if you take a particular location, and wind is coming from particular direction, at particular speed, waves will be random, but you would expect still the statistical quantity of waves to be almost same, because supposing it is creating more number of long waves, you would expect tomorrow also more number of long waves, because they are because of the bottom profile or some kind of bathymetry.

So, you see the statistical distribution, for the same wind speed. you would expect same? Just that they look different nature, you would still expect 10 number of 100 meter wave and 20 number of 1000 meter wave. Only thing is that the, the signal looks different simply because, they do not, you know… if you just take sin waves are at the phase differently, you will see that they look different.

See if you take a sin curve, another sin curve, add them together, you will get one picture. Now you simply shift this sin curve slightly; you shift this sin curve, instead of starting from here, you start from here, and add this and this, you will get a different signal. So, although you get different signal, they may still represent same energy because this plus this, and this plus this, both will have same frequency, same energy; both of them will have same spectrum, but they look different. Why they look different? Only because their phase is different.

So, now, conversely in a same location, for the same wind speed, you know, same time of the year, let us say, for you know, all the other condition being same, you would expect that all though the signals are different, if you have synthesized them, and if you plotted this graph, it will be same frequency distribution; that is very logical to assume; and this is exactly why the spectrum concept is even more appealing, because it tells me the frequency distribution of the heights or amplitudes, and that distribution, is more or less constant for a location and for a given wind speed for a various excitation. After all, if you take a … it is a very simple example, if you took a bucket of water - shallow bucket - and give a certain, you know, like wind, it gives certain kind of wind, you take a dipper bucket it may be something else, but if you take shallow water today, and tomorrow, and day after tomorrow, it will you will expect same kind of waves.

So, this is exactly why, this in a given site, even people have taken for a long time, you find out that the signals are same, but the spectrum that you find can be actually similar. And then, you find that, number one - you find the spectrum is similar; number two - you find that it follows Rayleigh distribution.

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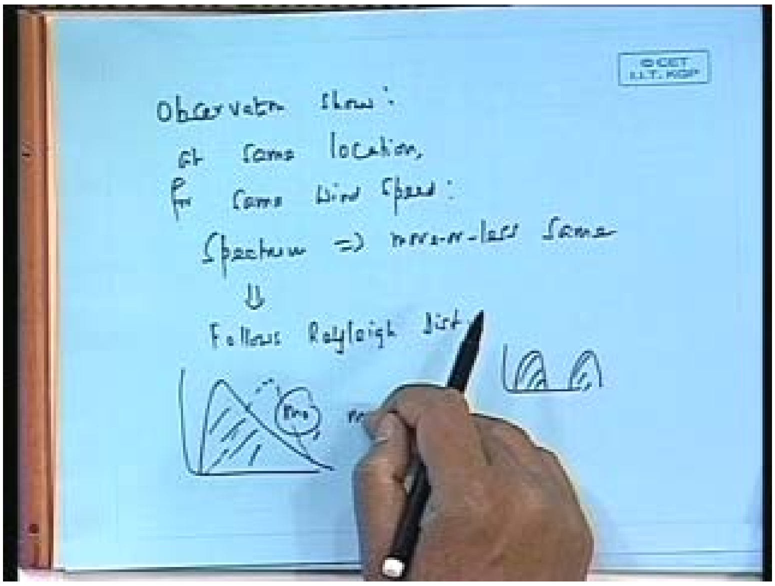

Immediately, let me write it down here, observation show, otherwise we are probably… more or less same, I am writing. So, the spectrum is same. Now you find also the spectrum follows Rayleigh distribution. Therefore, you can say that all I have to know is that whether today, or tomorrow, or day after tomorrow, I end up getting this graph only, all the time, because you know the spectrum turns out to be same.

And now I take this area m 0, and you know other moments, and I find out that, you see, that the moment comes… here comes the question of moments. See, suppose the same area, see if you take now this graph, you will find that m 0 is same for both cases, but m 1 is not same. So, that is why m 1 and m 2 comes in picture, because you know, if you take one graph – this, another graph - say this, both of them might have same area, but moment will be different; this is exactly why m 1 and m 2 again fixes up its location actually. So, if you have m 0, m 1, m 2 same, so this is the same graph, you will end up finding that, you know, the as if the irregular sea is same.

In other words, a random signal, all though looks very random, can be statistically same as far as this is concerned, and this is where it, I mean, we are now, that we find out that whenever you are synthesizing them, most of the places it is similar.

Now, comes the second question, you cannot keep as… see we are engineers, we cannot keep going and measuring wave heights at every point; what we need is somebody to tell me - look I have a ship going from here to Bay of Bengal, what is my weather at Bay of Bengal in a spectrum?

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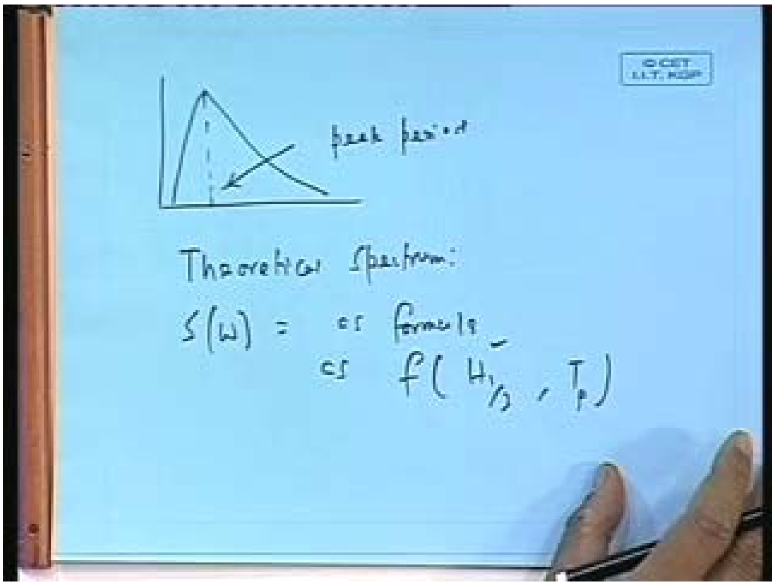

So, people what they have done is that, they have actually come up, no they have actually fitted this. Oceanographers have been doing that wave modeling and all, they came up with certain what is called theoretical spectrum.

In other words, they found out that, this spectrum can be fit to a formula where unknown may be the significant wave height and so…. In other words, let me say, if this can be written as a formula, as function of either one-third or T some kind of T, may be T peak period or something like that, some peak.

See there have been number of formula, therefore suggested by people, that has come up, where you can say, you can determine s omega as a formula, where the unknown are h one-third - sometime only h one-third - earlier days it was wind speed, but now-a-days more of h one-third, and or sometime this plus one some kind of a period, T p meaning… see this is my the peak period. Some formulas are there with only this; some are with both; both are more common, basically; this is what is called as theoretical spectrum.

Why this h-one third therefore, come in? Therefore, what happened? See, that supposing there is a wind blowing, strong one, then obviously, you would expect it will be rougher weather; rougher weather means, it would have a higher h 1, significant wave height. So, therefore, conversely if you say that if my, I have a certain place significant wave height of 5 meter, then I can go back to the spectrum and I can put to this formula; a standard formula.

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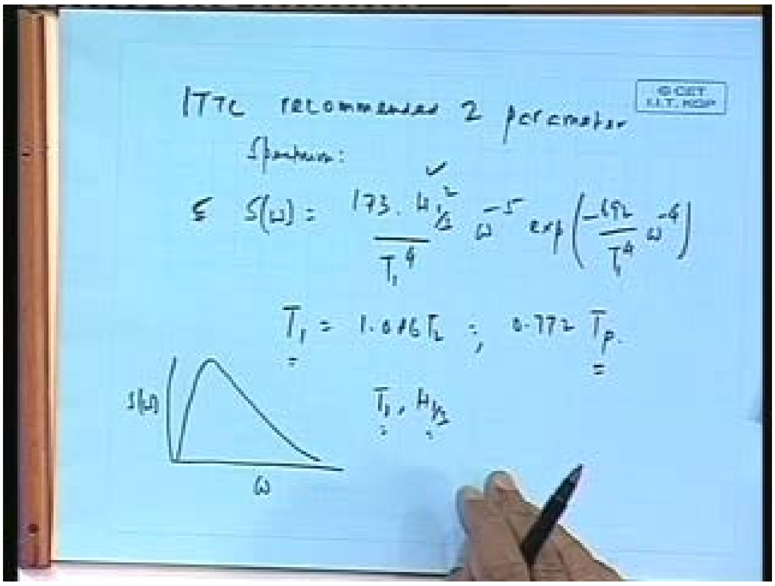

For example, I will just write down ITTC formula. ITTC recommended two parameters. It will look like s omega is 173 h one-third square divided by T 1 4; T 1 is some period; omega minus 5 exponential minus 692 by T 1 4 omega equal to minus 4 etcetera. When T 1 equal to 1.0862 equal to 0.772 T p. So, you see, if you know peak period, you can find out this T 1; T 1 is actually connected to what is called an average period, one of the period.

So, what happen if you know h one-third and if you know T p, then you can get T 1. Then you can find out for every omega, what is s; any omega what is s. Therefore, you can actually for, for a given say T 1 and given h one-third, you can find out this graph, omega versus s omega. See, in other words, there is a formula given for s omega as a function of h one-third and T 1 in this particular formula. So, therefore, if you know h one-third and T 1, they are again connected to each other, then you can find s omega.

Now, it turns out that oceanographers have also related use the what, what is called sea state, you know. It is something like, if there was more wind, long back it started, if there was a stronger wind for aircraft people, Beauforte - one British scientist or engineer - used a scale called Beaufort scale; he says that if wind is so and so… I will I will call it Beaufort scale so and so.

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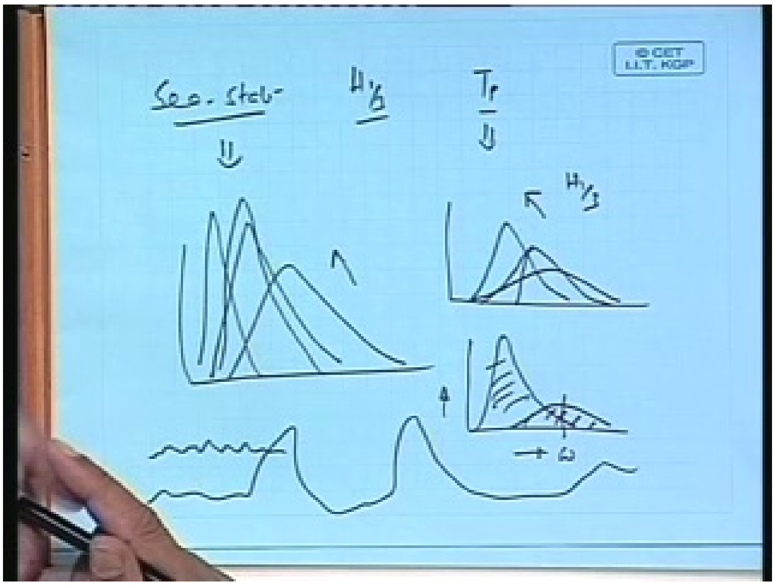

So, from that, later on, I mean, we people like ship people, you know, what ocean side people started calling instead of Beaufort scale a sea state. So, if we say that if h one-third is 5 to 6 meter, might be it is 5. So there is a scale between the what sea state and h one-third. So, now we have got a table, sea state versus h one-third versus t. So, what happened? I want to find out - what is my wave spectrum in sea state 5? I will go back to the table and find out what is my h one-third T p; then I go to the formula and I draw this graph; this become my spectrum for sea state 3. See what happened? I go to this picture, I find out for sea state 1, my h-one third and T p is so and so. I go and use that formula.

It turns out that actually what happened, you know, the spectrum, as the wind blows… this the diagram is not good, I should write just last time and then quit. This, as the wind starts growing up, the spectrum actually begins to shift on this side. In other words, what happens, as h one-third goes up, as wind starts blowing more, your significant wave height will be more; you will end up seeing a spectrum which will having more number of long waves; this… I will pick up on that little bit on the next class.

Energy content will be come more. So area. So, you take two things, small height here and very high wave here. So, the area under that is much smaller, because obviously, area under that is proportional to significant wave height, which is say 2 meter, but there will be more. See when there is a 2 meter wave, it is all connected. I have got a wind blowing at 10 knots; 10 knots will give me number of small length wave, 50 -100 meter long wave with a smaller height. So, average height is less. So, the spectrum is shifted on higher frequency side, small wave length side.

As wind begins to blow more, I will have higher waves also longer waves. So, it will shift on this side and area will also become more; this will happen. This has a very significant effect on the ship motion, we will find out. See small boats are operating in very moderate weather, but they are operating this small sea state. What happens when the typhoon starts to blow, you will have more number of 200-meter long wave or 300-meter long wave, very long wave. In one case, you will have more number of small waves; another case you will find much more number of longer waves, you see. So these are the two extremes and we have to… typically that happens… we will discuss more of that on next class.

So, we will, we will stop today here. We will, we will do a little bit of this elementary spectrum part also in the next class, thank you.