

Performance of Marine Vehicles at Sea

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Lecture No. # 28

Ship Motion in Irregular Waves – I

Somehow, see now, a natural sequel to motion in regular wave would be motion in irregular wave.

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Ship Motion in Irregular Waves

Regular Wave

Natural Period : $h_{nat} = \sqrt{\frac{\rho g A_{wp}}{(m+a)}}$

$= \sqrt{\frac{\rho g \cdot V \cdot B \cdot C_{wp}}{(1+C_a) \rho \cdot C_a \cdot B \cdot T}} \approx \sqrt{\frac{g \cdot C_{wp}}{(1+C_a) \cdot T \cdot C_a}}$

So, although I am going to call this ship motion in irregular waves, I will still pickup little bit on the regular waves part and continue. See, again motion in irregular wave; obviously, should follow the study of motion in regular waves, so in fact we have to continue little bit on motion in regular waves, because some of these parts are very crucial in order to study irregular wave part.

So, I will still continue some small discussion on regular wave motion in regular wave, what I will do? See, let us talk about some simple thing of natural period, why it is

important? Now, natural period is turns out to be equal to for in heave equal to $\rho g A w p$ by mass plus added mass, this is the formula.

Now, if you take a typical ship, in fact we can work it out various waves, but if you take a typical ship value, let me work it out here $\rho g, A w p$ is actually L into B into water plane air coefficient, mass plus added mass can be written as 1 plus C_a into mass, mass is ρ into C_B into $L B$ into T , root over of that. See, I am just breaking it down just for benefit C_B into length, breadth, trough into ρ gives you mass into 1 plus added mass coefficient give you this, this is ρ into $g, A w p$ is $L B C w p$. So, if you do that, cut this, if you cut that, then ρ gets half, then it becomes proportional to g by 1 plus C_a , g by 1 plus C_a $C w p$ by $c B$.

I think now there is T there, $g C w p, T$ will be there. Now, if you work it out for a typical ship you know, you will find that this is almost like a 1 , see this will be almost like a 1 now T , let us say C_a is actually also 1 , so it is 2 . Let us say it is you know 1 , so it 2 , so $9, 10$ by that is 10 by you know it is 10 . So, you can say if g , if you take g to be $10, 9.8$ you know approximate $10, 1$ plus C_a to be 2 , this it is becomes 5 .

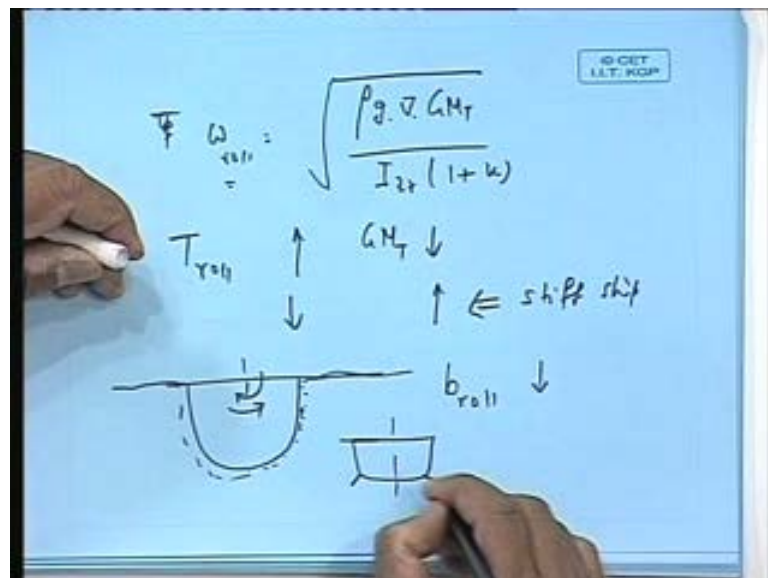
So, it is 5 by T , root over of 5 by T , you take any T value, say T of 8 meter or something. So, root over of 5 by 8 , if you do that, that frequency will come out of 0.8 or so, 2π by 0.8 if you do, you know period, see this will turn out to be say 0.81 like that may be this also, may be point I will put it at 0.6 to say 1 , that is remember frequency 2 by ωe , what is t ? 2π by $\omega e, 2 \pi$ by 0.6 , how much, it is $10, 12$. So, T turns out therefore, natural period of heave in an order of 8 to 12 seconds something like that, I mean it is not really a wave from the typical wave frequencies, every day the wave you find is is of that order, so you see it becomes a real problem.

In other words, what I am saying is that, supposing it happened that your forcing frequency waves, where often period which is very far from that typical natural period, you would not bothered, it will never (0) , so you why care, but it happens it is not so. Another thing, you will find that here it is inversely proportional to T ; you know opposite of T , you can see from here. So, if the ship is shallow as would happened to a barge, you would have here a high frequency low period you know, 10 second, 6 second, 8 second like that, which happens to a small boats normally.

On the other hand, if you have a deeper ship, it has a low period. So, if area of water plane is very large, this is high, then you have an obviously high frequency, low period. Now, that would happened to this, the (O) is that you know, that goes on Calcutta then Howrah to you know, that much this large water plane area small tape.

You can never; you should never take them to see is going to go up and down like a car very bad. So, this is one one point to that the kind of range because later on, we will see that in a spectrum, we will find out in a irregular sea which one is occurring most and you must try to have ship design if the the natural period is not close to that.

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I want to just briefly talk about roll here, roll natural period t not t omega roll it becomes equal to rho g into volume into G M T, you know actually just like there it was rho g into A w p into Z, this becomes actually this way and here it becomes your moment of inertia I z z into 1 plus that factor, you know that, say that moment of inertia something like that.

Never mind this writing, what I want to tell you is that, this depends inversely, I mean this thing with G M T you see, if G M T is increased, roll period is increased, roll frequency is increased, period is decreased. So, you see period T roll increases as G M T decreases and it decreases as this increases; that means, if you have a smaller G M T that met a centric height, you have a larger roll period higher, lower frequency larger period.

You will find out, again if you do that typically this number will come to be on 8, 10 seconds or something like that. If you make G M T very high, you will see it will be come to 6 second, if you make a G M T smaller, it will say it may be half a meter, it may come out to be the 15 second, this is where the contradiction comes in a vessel which is having G M T high this is called a stiff ship very stable, but ship stiff.

So, stable it makes sense you see, you are let us say the ship has tilted by 10 degree, its G M is so high that the restoring force is very fast, quickly bring it back, where the restoring forces are small means it **will** may be brought back slowly. So, you see safety requires that you must have high G M T, but high G M T makes it more uncomfortable. So, you have to have a compromise in a **in a** vessel, now why roll, it is more important.

Now, you take an example, suppose I take a semicircle, this is my last thing on this regular wave, now I rotate it. What will happen? See, this water particle they will simply it will be like a bottle at the vessel simple will go up and down, what kind of wave you think it will make? Practically no wave, because it is you see it is a circle, so it is like a group is there.

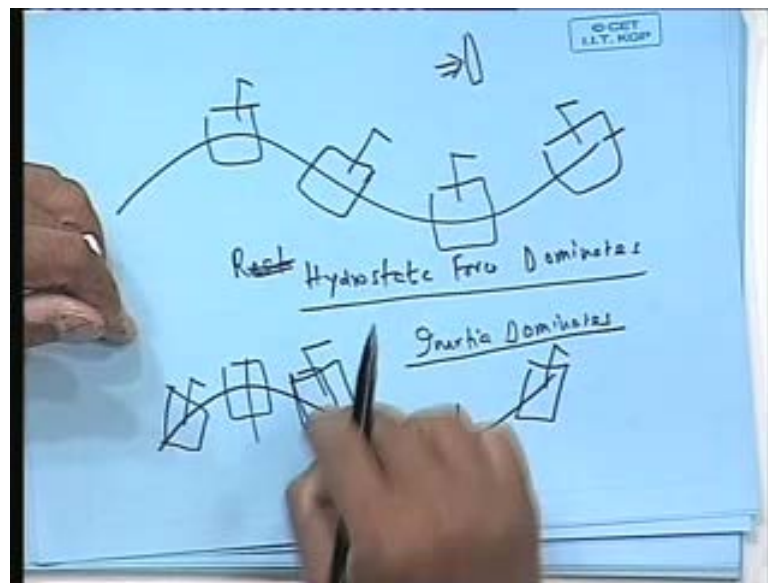
So, you **you** know supposing I pushed it down, then the particle curve push down, but here I am rotating it remember like a cylinder. So, I am rotating it there may be some surface stresses they are very small which we are neglecting, but the net pushing of water is just not there, it is now you know you can say that I **made a** made a cutting in the water and the thing is going, that is why the roll damping is very low, there is no roll damping. Therefore, if roll damping that is b roll is very low, you do not expect any I mean like this **this** sort of, in a resonance a lower motion it is very high.

So, that is why you have got the, if you do not have anything you have of course, normal ship will have some damping because it is something like that, but I am saying still it is much lower when you rotate that the damping, the wave mechanism is much lower, this is why when you take a model and if you just tilt it for a long time it will actually oscillate, you know you can take yourself, you take a bucket and anything that float and you tilt it, it will for a long time go, but you push it down, it will go to and stop.

You know, you take in next time a tank if you go, you just take a body and push it down, it will go just twice and stop, you roll it, it will go for a long time, why because there is

nothing stopping it, no damping. So, what you do, will you put you put all kind of stuff you know, anti rolling device this is why you put that because rolling first of all it is you know dangerous, large rolling means it can be dangerous and on the other hand, you do not have been natural damping mechanism. So, you have to induce mechanism by putting (O), so that you do not get, see you may not change the frequency, remember you cannot change that natural period, you say 12 second, you cannot change it, but you can at 12 second, there is no damping it would have gone to 30 degrees, if you put a damping it will go to 10 degree.

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So, you have to put this damping, this is the mechanism, second another thing that is most interesting thing I want to share you about this thing is this thing, if if supposing restoring force is very important then the ship will always be actually following the, you know like like this, I was telling you another two extreme, when the restoring force dominating and when the inertia force dominating.

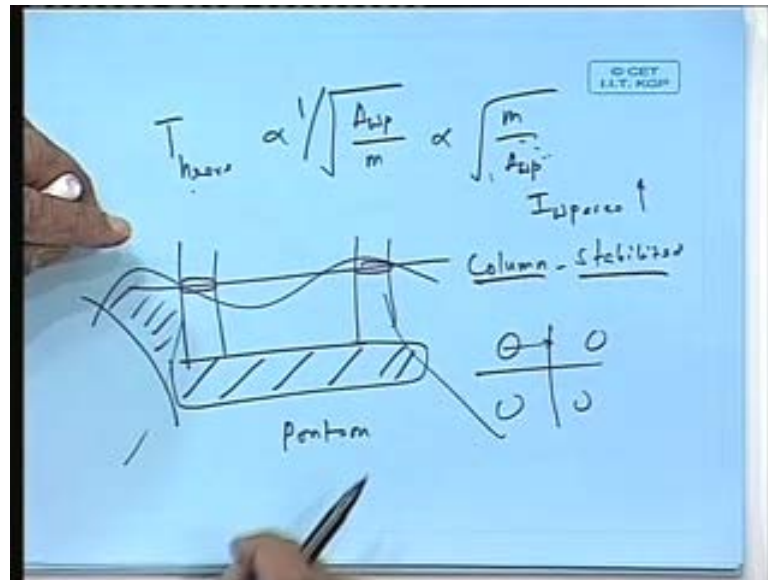
When restoring force dominate that is restoring force, that is hydrostatic we can say hydrostatic force dominating, then typically what happens? It actually follows, you know I am showing a beam wave, there is a there there is a ship there, waves are coming from the beam side, you know there is a ship here, waves are coming from this side.

So, it will go like that more or less. So, I told you in hydrostatic case, you can afford to make mistake in added mass damping exciting force, you can afford to make mistake, but then the point is that, even if you made a mistake situation is not dangerous. Unfortunately what happened? When you do this, other extreme end the ship behavior is actually opposite phase that is this goes like this, this goes like this, this goes like this, (Refer Slide Time: 11:00) I mean what I am trying to say that it goes opposite phase.

So, what happen, this is when the inertia dominates, the the interesting part here, interesting or unfortunate part is that where my situation is more dangerous which obviously, in this case because you know you can see that if the if if the you know, vessel is bent like this you know, more dangerous because the (()) is closer to the water. Then if it was falling, see in this, if you are standing here is of course, if you make as long you will not feel you are moving, because the water is earth same depth below you, but here you will see that you know water is coming up you are going closing down, you you know water is coming up when the wave is rising, you are going down here.

So, it is dangerous, unfortunately this happens when inertia dominance and and that is the case when you need to find out the hydrodynamic more carefully. So, you know in roll case, it turns out that you cannot afford hydrodynamics, because the more dangerous situation arises where hydrodynamic forces are more important. So, whether we like or not mathematics, we have to have some estimate of hydrodynamic forces that is what I want to tell here.

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Then I want to tell you one another interesting thing about the semisubmersible. See, natural period in heave say heave is proportional to area of water plane and mass something like that. Now, you see semisubmersible, **you cannot get away from** you cannot get away from a particular sight, unlike ship because it is anchored you know, it is anchored at one place, you are **you know** getting oil from there, unlike ship a bad weather comes, you cannot run away from there. So, good or bad it must be surviving and if it heaves very much, you cannot dig oil, it must be small heaving, what you want to do, you want to reduce heave.

So, what you want to do? You want to make sure T_{heave} is far away from that typical wave period, it is about 25 second, 25 second is equal to, 20 second is equal to a wave of something like kilometer long, that does not occurred, you know mostly the waves that, what is used in offshore, as every day wave is about 100 to 300 meter long, 8 to 12 second or 14 second period, 20 second waves or 25 second wave do not occur very much. So, your idea is here that you design a structure so that its natural period is far away from the usually occurring waves, so it is like you make it 25 seconds.

How do you do that? You can of course, make to another side that is not possible. So, you know I made a mistake, this is proportional to $1/\omega$ because ω is proportional to this T is proportional to actually mass by A_{wp} .

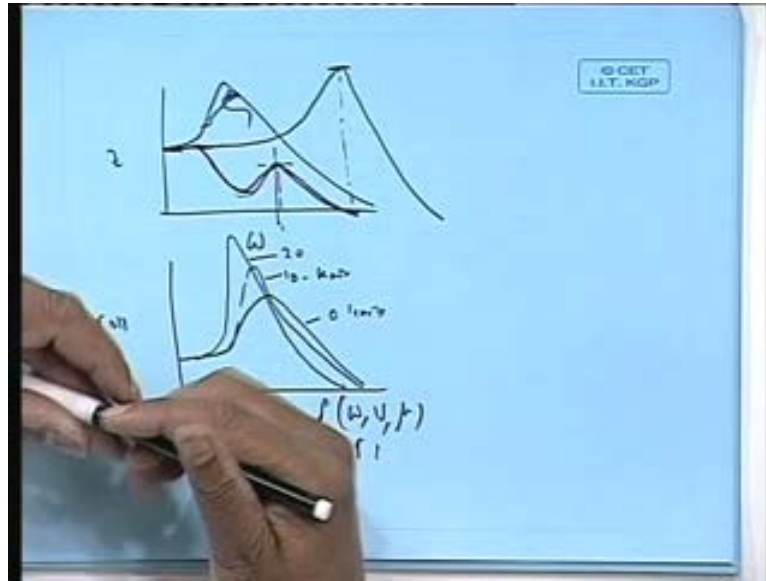
See ω is proportional to $A w$ by m , this is $[FL]$ opposite. So, if you want this high you want to make it low, this is exactly what you do, you make it low by only two circles or four circles, column that is why they call column stabilize. Now, what happened if you have this thing going out is small, **you** but unit mass, large mass. How do you get the large mass? You put the large mass underwater pontoon, so you put the large mass under water, below water. So, you have got your mass high, but you got $A w p$ small. So, you got $A w p$ small by just putting the one that is penetrating small.

But there is a safety issue, you want $G M T$ be more or less good, for $G M$ unit high of the water plane area high, how do you get that? All you do is that, you simply make this away because I is proportional to area into distance square.

So, you would have small water plane area, but like a catamaran hull, you just space it out, this is exactly how this design evolved. The entire idea of semisubmersible is that **(O)** will come from and there is a submerge part where you know it does not contribute to water plane area, water plane area is by the smaller part columns and the columns are far of apart to stabilize the body to get high **high**, that is why you call columns stabilized.

One more thing is there that, if you see the wave part, the pressure is always exponential. So, if you have a small part penetrating out, the net force pushing out is much less, exiting forces also reduced, because you know the pressure that comes here from wave because it is much below, it is much lower.

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So, you you So, this is the entire concept of semisubmersible, you know that what you are doing and you know, typically it will look very interesting that in a typical ship, a typical ship ω versus a heave it will look like it goes high and then resonance goes like that, it goes more than 1, a semisubmersible will go like that its resonance period is going to be oh sorry I think this will be high period this will go like that.

See a ship would go having a resonance period high, but it goes up, but here what happened? It actually resonance is somewhere here, but at the resonance the excitation is small. So, although it is resonating the motion is smaller than even 1. So, you see, it will resonate, but because exciting force is small everything is small it resonance, but heave is very small, this is this is an idea of you know, this is semisubmersible type and this is a typical ship ok. Now, this is where I will go to enter this regular wave part, but I the reason this I say that after all irregular regular is integrated connected to we will find out, we will see that how a vessel behaves in irregular waves is completely decided by how it behaves in regular waves. Therefore, it is very more important actually to solve this part, then to solve the irregular wave part.

So, we will come to that in a minute, may be before that I want to say another interesting part that typical are you say, roll versus say frequency, you may now because it depends on encounter frequency, you can actually this is a function of ω , v and μ . It turns out that by changing speed and changing μ , specially changing speed, you can actually

change this supposing at v , at a some v it is looks like that, if you went to another v , it may look like that it will another v it may look like that, because what happened, changing v will change ω will be making it closer to resonance.

So, if there is a relation between ω v and this roll, you see this is an example of a container ship in waves. Now, there is an exciting force coming now, this particular wave condition, it turns out that at some frequency give force that some force, it is frequency dependent. Now, obviously, if you change the speed, the characteristic changes and this is another thing that is important, because this is a typical phenomenon that one use, in this example it was 20 knots, this 10 knots, this is actually 0 knot.

You know, all of you probably experience more than I would, that if you slow down, normally it has a beneficial effect on motions, high speeds happen larger motions because you know frequency is you know, going somewhere etcetera **etcetera** and you know forces are faster etcetera. So, this also shows that the moving ship with depends on frequency, but frequency depends on heading and speed and therefore, changing heading and speed, you can change that the tuning the resonance can be called that as if the ship has got tuned with the wave, you know that can be change and you know, like that you can reduce the motions.

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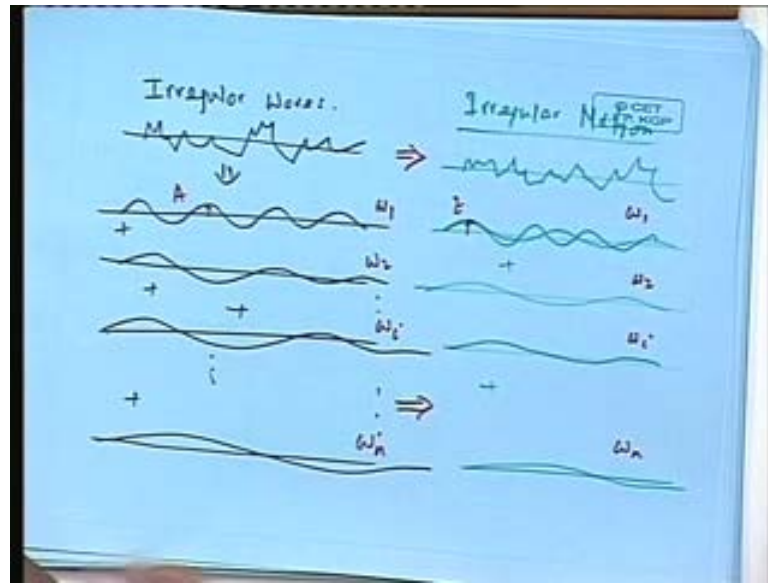
So, let me now **now now** forget this, but now let me go to this irregular waves, this is very interesting and **and** I will request you to just get this once in your mind, and if you get that you will find out that this concept is so **so** interesting that it applies finding out not only motion, everything that you want in a regular waves, you want to find out pressure, you want to find out bending moment, you want to find out anything that you want any response in irregular waves, the procedure will remaining exactly same.

So, that is why we need to sort of pay little attention. Now, see here we have got, what do have if you take a ship in a irregular wave, the irregular wave η or T looks like very irregular. Now, you have put your ship there and you want to measure your let us say, heave or some response. Let us say heave, we have measure Z T , Z T also look something, like irregular isn't it? See, if you have an irregular wave, then let us say you went down to irregular wave and put as β to measure how much it is heave, so it will look irregular record.

What is our requirement is that, given this find this; we will like to have to predict this given this, why given this, because obviously, we have talked earlier how wave sea spectrum is defined. So, I have a sea spectrum from there, I can find out an irregular or in other words, let us say, I have this. So, I measure my input, after all this is my, you can call, this is my input and this is my output.

Why it is input, because this is the existing wave environment, you can say this is the environmental disturbance that is existing; there this is going to cause a disturbance that causes my ship to move up and down. So, I have to predict from here to here, given this find this, this is my objective and that is what is meant by how I find out motion at irregular waves. So, this is a very interesting part and I will tell you how we go about it.

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See here, now first of all look at this part, this irregular wave, what we had, we had an irregular wave I have to draw it slowly, then this what we did, we say that it can be broken down to a number of sine waves this plus, this plus. This is what we have said earlier, isn't it? what we have said earlier if you recall correctly is that, if there is an irregular record we say it that, if you do a flourier analysis or something if you analyze that, you can assume that this is composed of number of sine curves of different frequency, if you add them up you will get this or if you have this, you can break it down to this pieces, this is what we said.

In that we said this looks random, but actually there is nothing random about it, it is random of course, but nothing complicated or confusing about it, because you can on the assumption that they can be broken to many sine curves, the point remains how to break it down and there are now absolutely standard mathematics, Fourier analysis etcetera to do that, this is irregular wave input.

Now, I have to have an output, this is my waves, this is my irregular motion say motion this also now this also looks something like that. So, this can also be broken down like this, this can also be broken down like this. Now, you see **oh sorry** I step broken it down in the same frequency. Now, let us say that this frequency, we have broken it down to a frequency ω_1 , this ω_2 like that, this is ω_n , say I like that, this is ω_n , n number of frequencies. Here also I can break it down to the same 1 ω , n ω .

Now, what happens, this gives me this, this input gives me this output. Now, I say that this input is nothing but (Refer Slide Time:23:44) this plus this plus this plus this, which I have done by spectrum.

Now, what I do is that, I find out that if this was my regular wave, what will be my output, that is if this was my see my, you know A, what is my z here; that means, this by this for that frequency because that is what I have done. See to get this from this was what I so far talked, that is how it behaves in a regular wave; that means, supposing this is my regular wave amplitude A 5 meter, what would be my heave. So, going from here to here this way is finding out response in regular waves.

So, what I did, I break it down to this sine components, for each one I find out if this was a wave then what would my regular, the if this is regular wave what is my regular heave motion. So, I break it down to this then add them up to go there. So, you see the algebra, this is the picture that you must I mean, keep keep in your mind this is broken to this piece. So, this way the next step I go from here to here, then next I go from there to upwards.

So, you see I have this, break it down to these pieces. What is this breaking down? This is basically this breaking down is algebra pure, you know what is this going from here to here? This is actually response in regular waves. A regular wave given what is the output, what is this to this? Algebra, just adding them up. So, you see getting here to here is not so tough, it is actually in your mind, very simple because if I can break it down to this pieces which I have done in a spectrum analysis, you know spectrum, we say that actually it is nothing, but this the broken pieces, then I can actually go from here to there.

This part here to there is what we have done so far, that means, for each frequency I must find out what is my frequency response, actually why the trough that RAO graph, then add them up, this become so very simple, this is actually you know, this this is a point that has to be clear in your mind, because this is not only heave, it this thing will appear for any response.

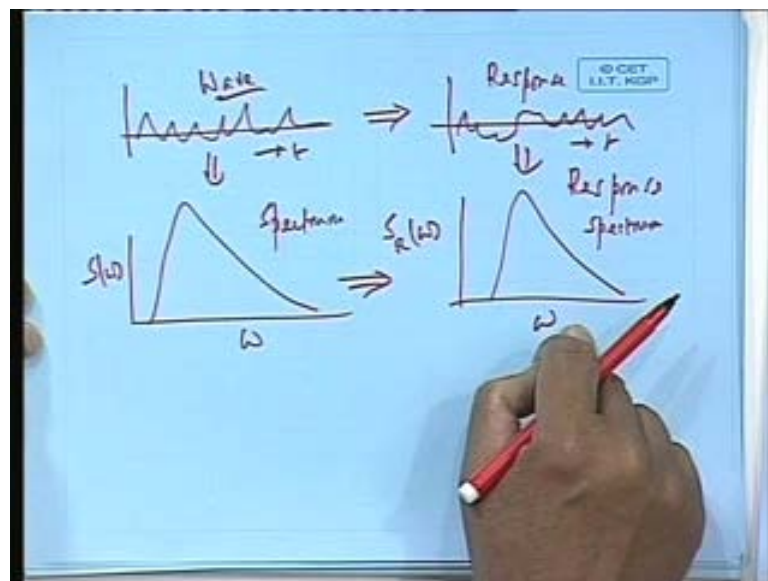
Given a wave, find out the response; may be heave, may be roll, may be pressure at a point, may be bending moment at a point, what we are doing given a wave, what is the it

is given a wave what is the response? Regular wave, I am trying to find out. Irregular wave is nothing, but sum of regular waves ok.

Each regular wave induces me a response, regular response, add them up. So, you break it down, go add them up, this is a picture that must to clear in your mind. Because, it is getting here to here appears very confusing, not correlated as if you know, there it is very confusing, but it is actually that, this is the most simplest thing, this part and this part is pure nothing, but algebra, just you cannot make mistake unless in a in a typing mistake type, there is no hydrodynamic mistake in there, hydrodynamic comes finding out in a single wave what is the response, which is what we talked so far.

So, you see the complication for us 95 percent is here to here, breaking on this, people call this and this as post processing, not even analysis in in to this algebra, you see. So, this is a picture we must keep in mind, now I will I will elaborate that more getting from here to is RAO that we will we will we will elaborate more.

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Now, let us see the same picture how I can do in a spectrum form. So, I have this, this thing you see, I said you know this this diagram, see this thing I said can be written down like a spectrum s omega omega.

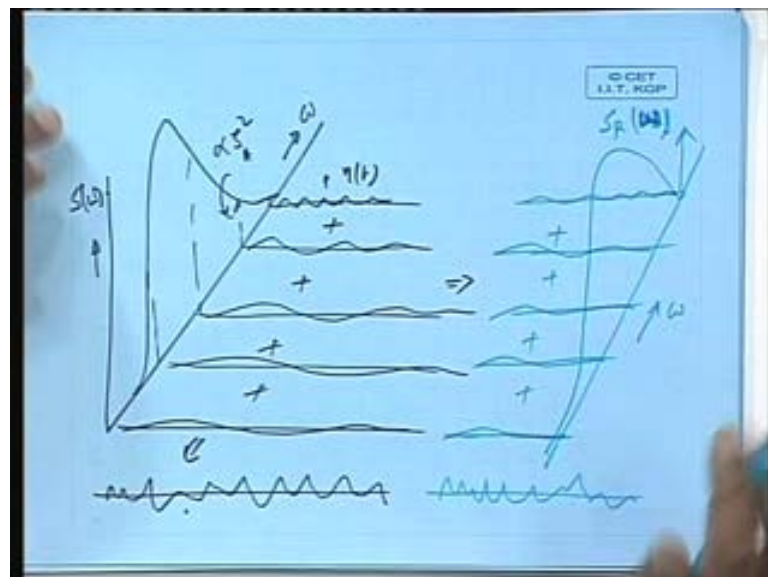
After all the spectrum is nothing, but the representation of that, I will draw another picture of that. Now, here I have got basically this is my wave, this is my response, this

also looks like that, this also must be can be written down like a spectrum, because any irregular waves response can be broken down to spectrum, this is a responsive spectrum I can call, this is omega and this may be response, I can call it R omega something like that.

So, what happened, going from here to here is actually going from here to there. In other words, I have this, I have to find out this, but normally I do not represent my waves by this way, I represent by a spectrum, you know standard ITTC spectrum. So, similarly I do not represent by this way, I will represent by this fashion. So, basically what would happened, I would be I would be given this, given this I must find out this, which is same as given this, I must find out this.

Remember one thing, this is a time, this is time. So, what we are calling given a time domain input of wave, find out time domain output of response same as given a time frequency domain input of waves, find out frequency domain input of spectrum. This is the kind of picture that we are looking at, I will there was a very nice picture in the the reference that I will give you later on, but I will try to draw it here and see whether it can be sort of combine, this full thing can be combined in one picture now ok.

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This is the picture that we must remember; this may not come in one graph, so I try to see. So, here let me see and try to draw this in a in a very complex fashion, this is my

spectrum here of wave and each of them represents a wave, this is my omega, this is my s omega, these are all my eta's.

And when I add them all up, (Refer Slide Time:29:56) this plus this plus this plus this, this way I have got. See, please understand this, what we have done, this is how I have done, see there is a irregular wave, this one is obtained by adding this one or you can reverse it, there is a this is why I discuss before in irregular wave many times, there is an irregular record can be broken down to regular record square of the amplitude, you put them here, this is proportional to amplitude square, say amplitude square ok.

You you have made them proportional, I mean if you plot them here, a term proportional to amplitude square by delta omega, you have got this; this is what I discuss always. That means, I have got this is still the wave part, I have got this part or to started I have got this. This is broken down to this frequency response which is squared and I got this plot. Now, here you see, how this relation will now come to this, will I have to draw here another one same.

Now, see this one now gives rise to a particular response, this one gives rise to a particular response, this gives rise to particular response, this gives rise to a particular this like exactly now I can take same thing square of this amplitude and plot them here and then I get here the so called, the response spectrum. So, this has become now my so called omega, whichever we like to look that and if you then add them all up, you have got this this thing and this is my s R, let me call omega, actually I can call it omega, let me call it omega right now.

I will call this omega e part later on, which is another complication that will come. So, this is a very very interesting and very very important way of looking at that. See, how this diagram is more clear you will see, what we had done is that a record is nothing, but sum of these waves which is represented like that. So, you can see either way, this is time domain this is time here, this time domain is broken to this pieces all all all these are times, but it can plot them like a frequency domain here, each of them gives rise to a time domain response, some you will get a time domain response here.

But if you plot the square of the amplitude you will get this, what is this amplitude here? This is another thing that is important, what is the amplitude here; let us say I am taking

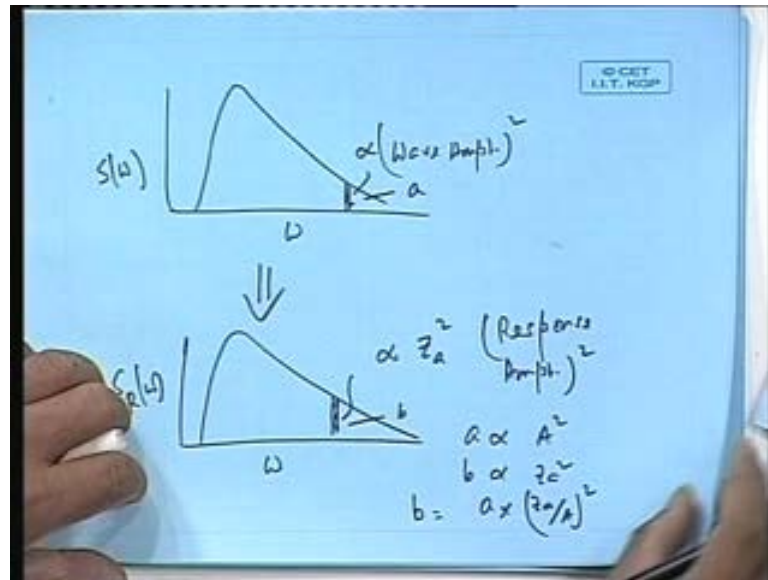
heave. So, it is z a is the response amplitude **ok**, what is **the this is a** this is wave amplitude, instead of a , I mean I can call this to be say, wave amplitude. See, here I have got square of wave amplitude and here I must have square of response amplitude, here I have got a square, if I call a as wave amplitude, here I must have that; **the** I will come to the relation.

See, what we have to do, give this I must find out this, because to in when you do an analysis you are not going to go down and measure, this would be when you measured it in actual case, but in our case what happens, see you have a ship there. Now, you have earlier I talked you have a spectrum, ITTC spectrum; that means, you are somebody has given you this, **you** they said that your significant height is equal to 5 meter, sea state is 4, therefore you went to the formula you have got this.

Now, you have to find out what is my significant heave motion; that means, I must find out from this, this curve and then again I will show you from there how I can get everything else. So, I will be given this, I must find out this, but this is the physics behind it, the **the** algebra behind it as I again see that this to this, breaking, getting sum to this, or here to here is algebra, how because, this is a formula nothing, but a formula.

Similarly, you will see that the sum to this is also algebra, because all we are doing is that for each frequency find the amplitude square and plot, you know square of the amplitude over that period, you just plot them, and only this part is important **ok**. This picture should be very clear in the mind, this is the **the** most fundamental picture. So, this part to this part, that is ratio of this is to this, give me amplitude square **sorry** response amplitude operator that is the response amplitude per unit wave amplitude, if this picture is clear now, **now** I will show you the way we go.

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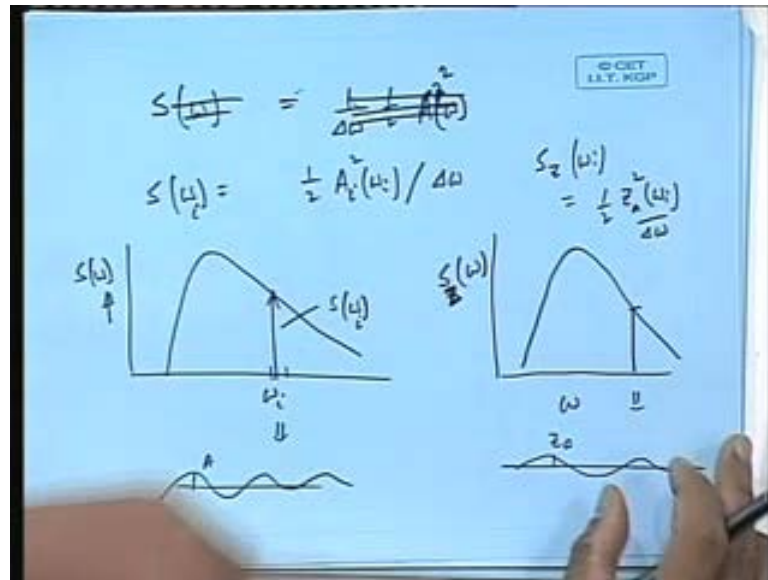


Then now, starting with that I have got this S omega wave, I have to get from there I have to get from there response spectrum, I must give from there, now what is this proportional to, this part is proportional to response say **say** response amplitude square, square of response because this one is proportional to wave amplitude square **square** this proportional to that, isn't it? We have say that this part is nothing, but half ξa square divided by delta omega.

So, obviously same notation, the spectrum is energy, see it is any spectrum is nothing, but square of the amplitude of the signal. So, here the signal amplitude as a , the wave amplitude here the signal amplitude plus the response amplitude square. So, this must be ξa square, how do I supposing I call this to be a and this particular term to be b , how do I get a from b ? See, a is proportional to a square amplitude, squared b is proportional to $z a$ square.

So, I can say therefore, b we can obtained by a multiplied by $z a$ by a square, you see the I mean now this is a important part, but see a this is proportional to amplitude square wave amplitude square, this must be proportional to response amplitude square. Therefore, I can say b is equal to a which is equal to proportional to $z a$ square, because see this, if you write **write** that this a square into $z a$ square by a square that becomes $a z$ proportional to $z a$ square, see a is nothing, but a square. So, it becomes proportional to that.

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So, you see this **this** theory if you see again, then I will tell you that S wave is nothing, but equal to or you know, that the original of that is equal to 1 by delta omega, half of A square, if you A omega square.

oh Let me write it again, S omega equal to this, let us say, S omega at any some **some** point S omega i at omega i, let us say at some omega i, what is the value I am trying to find out, S omega i is given by **by** our formula, half of A i omega i square divided by delta omega, this is what I have done before, if you recall that this part is nothing, but square of the amplitude of that incident wave square divided by 1, that delta omega that the delta omega has to be taken because area becomes constant.

This is what we have done, see this ordinate, now again this we have to understand, how did we get the ordinate we say the ordinates are proportional to wave amplitude square. Of course, it is proportional because if you just took wave amplitude square, you could not get. So, you have to divide by some kind of bandwidth delta omega **ok**. Now, I have to have this one, this will be my S say response, let me call it z response, I mean I am talking of one particular response is z omega. So, this must be this is S z omega i, this must be equal to half of z s omega i square divided by delta omega, because z a omega is the amplitude of the response of that frequency.

See what **what** it means is that, this particular wave is a wave of this frequency with **with** a certain height of A, what is this? This is equal to same frequency with a height of z a, so obviously, what you are plotting here is z a square, what you are plotting is a square.

Question is how do I get this from this, now you see, here if you look at this, now I will have to go to the next, but please remember this formula this one and this one, you will see that now I have to get this from this, see I have to get this from this, how do I do that is the question we are we are talking.

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The image shows a handwritten derivation on a blue board. The equations are as follows:

$$S(\omega_i) = \frac{1}{\Delta\omega} A_i^2(\omega_i)$$

$$S_z(\omega_i) = \frac{1}{\Delta\omega} Z_a^2(\omega_i)$$

$$= \frac{1}{\Delta\omega} \frac{Z_a^2(\omega_i)}{A_i^2(\omega_i)} \times A_i^2(\omega_i)$$

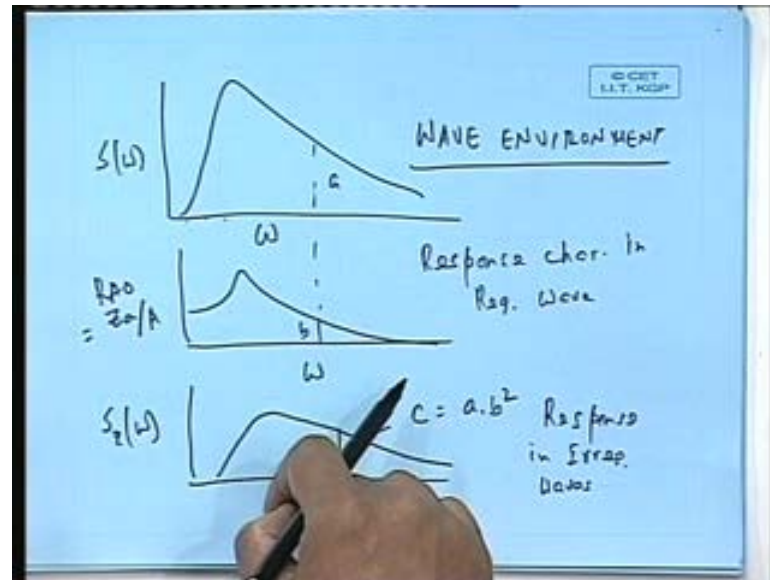
$$= \frac{Z_a^2(\omega_i)}{A_i^2(\omega_i)} \times \frac{1}{\Delta\omega} A_i^2(\omega_i)$$

Below these equations, there are annotations: a bracket groups $\frac{Z_a^2(\omega_i)}{A_i^2(\omega_i)}$ and labels it as $RAO^2(\omega_i)$. A horizontal line is drawn under $\frac{1}{\Delta\omega} A_i^2(\omega_i)$, which is labeled as $S(\omega_i)$.

So, **we** I have to go to the next slide and I will again show you that S omega i, therefore is we will write again this way, amplitude square omega i and S z omega i equal to half delta omega Z a square omega i, this is obviously, equal to 1 by delta omega A i square omega i by z **sorry sorry** z i square omega I by a i square omega i into A i square omega i, this is equal to z i square omega i by A i square omega i into 1 by delta omega A i square omega i.

What is this? This is equal to S omega i and what is this? This is equal to RAO omega i square because it is Z by A square of that you know, you can call that if you want z i by A i omega i **omega i** full thing square, same thing this **this** part. This part is nothing, but this. So, you see what we are doing is that, we can get the response amplitude **sorry** the response spectrum at a given point simply by multiplying the RAO square with this.

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Now, I will show you this in a very most schematic wave very nicely that then you will understand this even better. So, I have got here $\omega S \omega$, again I have got here it looks like that and I have got here ω into RAO what which is Z a by a which will look something like (Refer Slide Time:41:26).

So, you take any point, here a any point here b you will get this as simple as that you see the algebraic the algebra is. So, simple that it is embarrassingly I mean like embarrassingly simply I may say. So, what is this **this** is my wave environment because this is what as represented by irregular wave, how irregular wave looks like, what is this? This is my response in characteristic in regular wave response characteristic in regular wave.

So, you get this and this added up to combine what you get response in irregular wave. So, you can see that that what I showed earlier that breaking down is a physics, but mathematically it is so simple, it is all you do is that, you take the spectrum, take this regular wave response square that as part, I mean if you do in a table form, just take in a number of them here you know 1, 2, 3, 4, 20, twenty multiplication, that is all you know, it is like Simpson's rule.

So, you can see that given a spectrum, given the response in regular waves to find out irregular motion spectrum becomes equivalent to absolutely it is a real algebra, you

cannot make a mistake you know to c equal to a b square, you cannot any nobody should make a mistake, it is simple too, isn't it? I mean, this is as simple as that. So, why I say this repeatedly is that, **that the** it always appears very confusing that how will I get irregular motion from you know, irregular waves because both looks very complicated and I showed this **this** earlier, may be I will bring.

I mean, when you see this sort of thing here that irregular wave to regular wave, this you know here to here, this looks random this looks random, how do I get from this to this, but this I have already found out given significant height I have a spectrum and now if I have a spectrum, this to this transformation is that RAO. So, given RAO, given spectrum the other part is just one line multiplication. So, you see this is, this part this is environment ocean of graph as give me this **this** part.

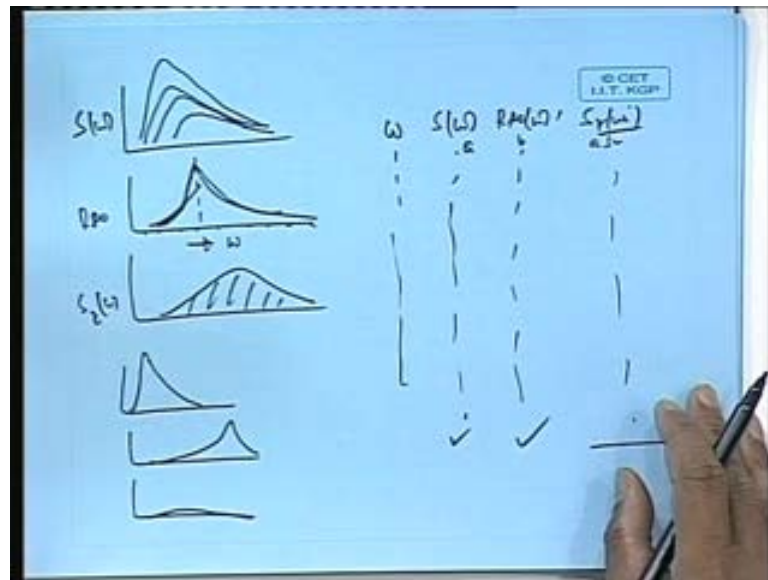
Ocean of graph as give me this, this is what this gentle men is talking, we need the environment this is what we do how a ship behaves in one single regular wave and most complex part comes here, because that is where you have to have added mass damping exciting force.

If you have this, if you have that getting this is piece of cake, so called you know, you just square it, there is nothing doing and you have to understand this, this particular method is what we call spectral analysis and spectral analysis is nothing, but algebra. An interesting part of that just like in our subject, that the important results are connected to how this graphs looks like etcetera **etcetera** that I will discuss eventually, the interesting practically important results come out from here.

But to get to this curve is the very trivial job and the **the** in between path of getting this is the most difficult part, this is given, this is what you have to find from some source if you have that getting adding this half is no problem at all **ok**. This is something that we must basically sort of realize that here to here this, this part is absolutely straight forward part, I will talk little more, but the idea that has come here is that, this is a random signal, energy of that should be also proportional to its amplitude square. So, I must write something is amplitude square of the response, this is the amplitude square of the input, but then response by input, I already found out from here. So, I can get this directly from here and here **ok**.

And this is you know, now if I want to do a table, I mean I will spend time only on this little bit, see I want to do a table. So, what happened, I have got again, I draw a small curve this and this is response looks something like that and this you know this is a, b, c. So, this is S_ω , this is say RAO, this is actually you know S_z .

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You make a table omega values **ok**, S_ω values, number them, say start from 1, 2, 3, like that, then RAO values, this is given, this is given from regular wave, this is given from environment, S_z omega i, just square of this into this, if you call it b, c, a this a b square. So, what time it takes you, you know, you can feel it and then you plot that against this if you want. So, you see the **the the** process is very simple absolutely simple, of course, we will pick up on that next class more, I mean I will not do it today.

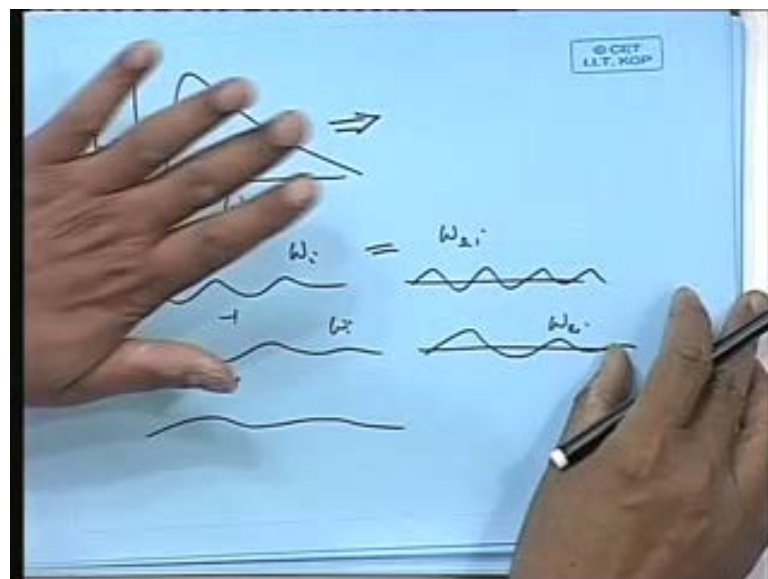
Then, what how do you find out the things that you want to know, that is you would know you do not want spectrum only, you want to find out finally, what is my average heave or averages? What is my significant heave? Is it good or bad? Suppose, again this very bad how can I tune it, how can I make you know like, now what happened? Typically, what we will do? This remains constant for a hull, does not change because given hull at what given shape at what frequency, how much it moves is constant. But this depends on weather type today's weather is like that, tomorrow's like that, day after tomorrow is like that, it is sea state dependent.

So, this part, the most difficult part you do once for all frequency freeze it, but then this procedure, you can repeat for given environment, you will find out for example, that this is my peak, it has lot to do with whether this peak match with this peak, because obviously, you can make out if this is high this is also high, you are squaring this into this, it will become very high.

But supposing it turns out this peak is somewhere here, this peak is you know this wave has a wave peak is here, my response peak is here, you square this with this, you get almost this zero here, this is what we will discuss that is the most important part. This is this will explain to you why natural period is important, why a tanker behaves badly in typhoon, but does not bothered small weather, you know like 10 knots, 20 knots when we just feel it, if you are in a 20000 what 200000 ton tanker.

But if you on a 2000 ton barge, you will feel that 20 knot wind equally badly and may be then **then then** the other one that is why small boat behave bad, because most of the time occurring waves are actually may be 3 meter, 4 meter, 2 meter like that, you know sea state three type even there it behaves bad, but a tanker you do not feel it, you know it does not get, but then on the other hand, if it hits a large typhoon type thing, it will behave badly, it is all connected to this which I will discuss at length, that is where the practical you know implication comes in very **very** much, that is extremely important of the practical part.

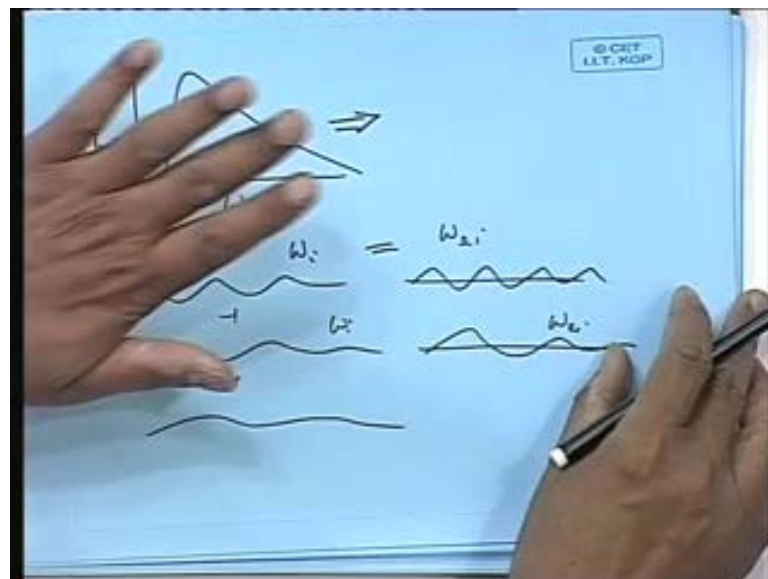
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Now, there is one thing that is very important, that also I will talk is that although I say this procedure is simple, there is one pitch here, this is ω this is $S \omega$. However, you see this is what is ω , this is as you observe the waves from a fixed reference point because you have taken the record at a fixed point, but ship is moving.

So, the all those irregular wave what the ship sees, each one see, this one is broken down to number of waves, but the ship that the same wave would feel to the ship to be something else, because this ω will feels to be this ω_i will feel at $\omega_e i$ **this ω I will be feel as $\omega_e i$** . Because, supposing this is composed of let me give an example, a 10 second wave plus a 12 second wave plus a 14 second wave. For the 10 second wave, when the ship is moving at 20 knots, we will feel like an 8 second wave the 12 second wave if we feel like something else. So, first of all although I said that this has to be transformed initially to how the spectrum would be if you are on the ship or as the ship would feel it **ok**.

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I will actually go to that next class, but I will just tell you why it is important. See, again there is a ship here, it is stationary here, there is an observation point here, the ship is stationary and if you observe the wave looks like that **ok**, but if you now move, then same observation point is changing. So, if you are moving now, then this same thing will gets squeezed. So, this is in kind of ω domain this is an ω_e domain.

So, first of all, **if** this looks like something like this, this should look little more spread, because it will be stretched omega, this each omega is suppose to changed omega e i **i**; that means, I have to first change the wave measured that is spectrum given, which is for a stationary point to an observer sitting on the ship. So, there is a transformation there, before I can start this procedure that I mentioned last you know, before see I mentioned this to this I have one step, I have to first change this to omega e, first I have to change that to omega e.

So, first from this to omega e that I will discuss next class, but that is also algebra, but you have to know that, you know because after all depending on how you move, this itself changes and when this changes this combination will call the change. So, this also as an effect on how you actually get tune or you know like just last thing is that supposing a ship sea state 4, there is a peak wave coming at 10 second. Now, if you are going at a different speed that 10 will appear 8 second, and suppose your resonance is 8 seconds, and then you are going to be bad, badly hit, change the speed you make it 9 second, it will not be so bad. So, we will discuss that in next class the **the** response part at more length. So, today we will end it here, thank you.