Strength and Vibration of Marine Structures
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Lecture - 10
Statically Indeterminate Structure – IV

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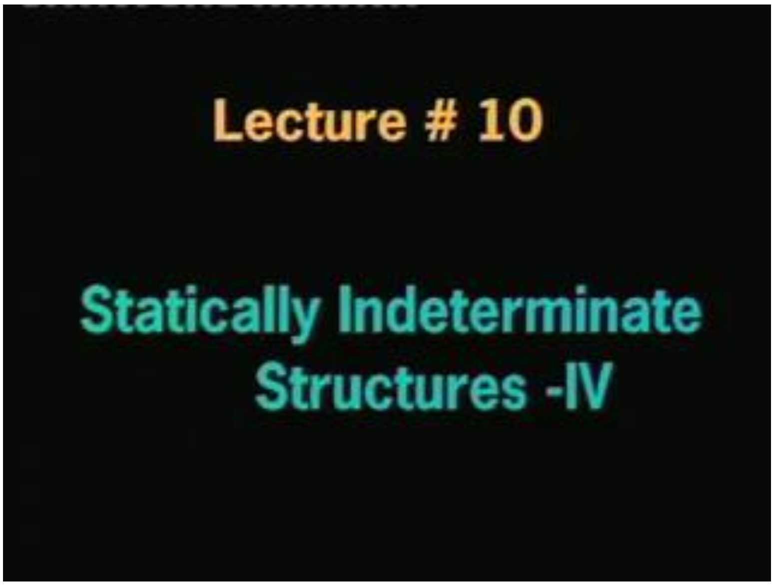

So, last class we talked about energy method and I was talking there is a method called unit load method. It is basically derivative of your unit energy method. So, energy method, if we look in a separate angle, we will get this method, which we are trying to define as unit load method.

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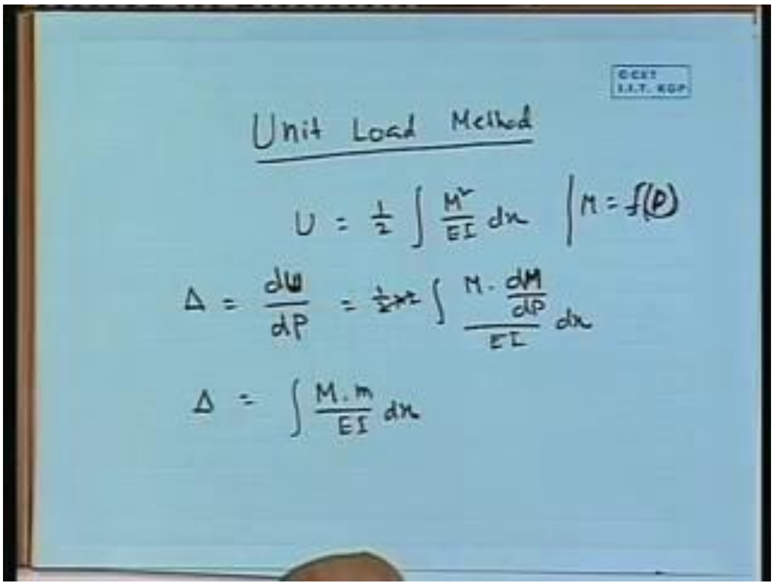

So, it is not a separate method. It is basically energy method, but looking - the whole thing is looked at in a different angle. The idea is to make the treatment much more systematic or much more mechanical. So, we need not start from the very basic thing. So, energy principle is basically its background, but in a different form we want to apply; rather I was trying to explain you here.

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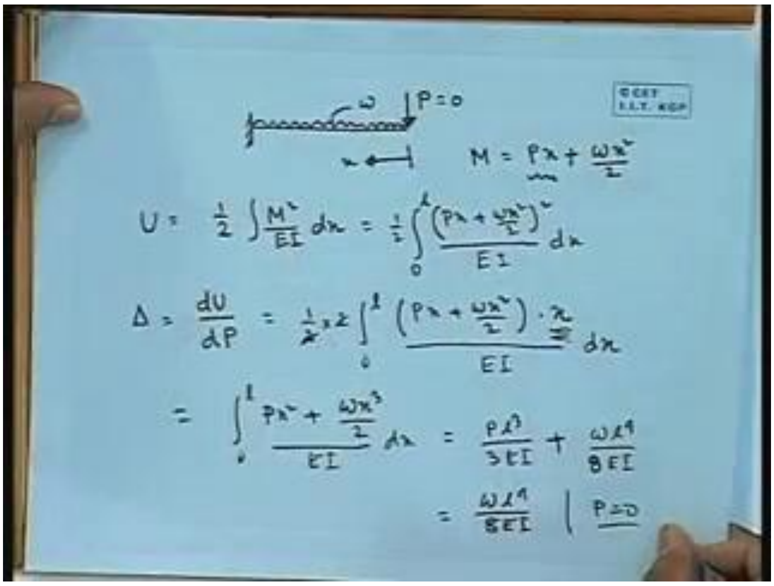

This problem we have solved. M square part we have put there, after taking derivative, we are getting this plus another component. So, this part was basically the moment and this part is the derivative of that moment about that P. Now, if we write here, in general, U equal to half integral M square EI dx. So, delta we are trying to tell, this is du, dU by dP; it will be this half 2 M dM dP divided by EI dx.

So, U equal to half M square EI dx, where this M is… M was same function of P. So, P I am writing it is not the P what we have considered earlier; P may be P, MB, or any other quantity. So, it is a variable. So, if P is some function of… if M is function of P. So, M square by EI dx, this part if we take derivative, so it will be dU by dP. So, here, this M square will be 2 M, 2 it will cancel, it will be M, and this part will be dM by dx EI into dx. Now, this part, we can write as this is capital M, and this derivative part if we write small m divided by EI into dx. You try to go through that expression. So, delta, finally, we are getting integral M m EI dx capital M small m EI dx.

Now, you can… You may have… take this problem. So, I am keeping both the sheets here, or this I got, the earlier sheet I am putting it here. So, delta. So, this is basically M and this part is small m. What is small m? Derivative of that with respect to P.

Now, for that problem, this is basically the M and this part is derivative of that with respect to P. So, P here it is MB. So, if we will generalize. So, M is what? M is the bending moment generated by the actual loading; say here, at this level, this MB we can drop. So, this is the actual bending moment expression due to the applied load, or here also, this part we can drop, because x already we have obtained. So, this is due to the actual applied load. So, this capital M is basically the bending moment due to the actual applied load. So, we need not bother about taking some artificial load and all those thing.

But what is small m? Small m is derivative of the moment expression with that artificial load and taking derivative about this artificial load. If there is no load there actually, we are talking in terms of artificial load or load something. Now, try to see the expression of that. Same way it was P and we are taking expression about that. So, for P we are getting Px and its derivative about P we are getting x x. So, what is x? Because when you are taking about P, this part will not participate; only of it is derivative about P. So, other force will not be appearing. So, it is continuation of the P and we are taking about P. So, for moment generated by P, if you take derivative about P, means it is basically if you put a unit load here. So, 1 into x will be the derivative. So, P there, some moment will P into x; if you take the derivate about P, so Px will be x. So, what is x? It is basically if you put a unit load there, whatever moment we are supposed to get, that will come.

Now from this concept, we are coming to unit load method. So, it is basically derivative of the energy principle; nothing else. The first component is N. It will be generated by actual load. And second part - small m; this is the moment expression, generated by unit load. And unit load where we will get? Where you want to get the deflection, there we have to put 1 unit load. Say, here, this cantilever problem, we are interested for finding out the deflection here; there is no load at this point. So, just capital M part omega x square by 2 due to the actual load; this is the capital M, because P is 0 here. So, it is omega x square by 2 is the bending moment expression for the actual load.

And x is what? If you put a unit load here, 1 into x that should be the small m. So, this small m is due to the unit load applied at the free end where we want to get the deflection. Or in that case, the slope calculation, if you come at that level, so this part is the bending moment where MB is equal to 0. So, this is basically the expression for capital M. Capital M is moment generated by actual load. And this is what? This is the moment, if you put a 1 unit moment. So, anywhere moment will be 1 unit. So, it will be 1. So, this is the small m; small m is the moment generated by 1 unit moment at the free end where we want to get the rotation.

So, it is just 1 unit.

One unit, because we are putting some value. If you take the derivative, so it is P into something, if you take the derivative of that, so P part will be going there, So, that will be defined in terms of 1 unit. From there, we are trying to define it is a unit load method, because it is coming just like putting a unit load there, unit load or unit moment. If you are interested in terms of slope, we have to put 1 unit moment; if you are interested for deflection, we have to put 1 unit - 1 unit of force there.

So, we came to much more systematic formula energy method. So, what we have to do? We have to take the structure; we have to add the expression for the bending moment, for the different zone due to the load that is capital M.

What is small m? Where we want to get the deflection, there we have to put 1 unit load or 1 unit moment, and find out the expression for the bending moment for the different segment. Then capital M, small m we have to multiply, divided by EI, integrate for the different zone, and go on adding, we will get the corresponding deflection of slope there.

Small m is the expression of moment generated by 1 unit force or 1 unit moment, where we want to get the deflection or slope. Say, for a structure, at any point, say point A, we want to get the deflection; so, we have to give 1 unit displacement at A. If you want to get the rotation about A, we have to give 1 unit moment at A. That is the only force on the structure; no other force; due to that will get the moment expression; that is basically small m. Because small m it is derivative of dm by dp; dm by dp m involves all the loading plus the applied load - what we are trying to say pseudo load or some artificial load something. So, if you take the derivative, all the effect of pseudo artificial load will be reflected. Now we are taking derivative about this means, that times it will come down. So, if it is P, P will create the moment P into x, if you take derivative it will be x or if it is Px square by 2 it will be… just x square by 2. So, it is just becoming, we are taking derivative about P; automatically it is becoming the effect of 1 unit load there.

So, at this level, if you want to forget what we have done and try to follow the steps, what we have to do? Take the structure, due to the actual load different segment we will find, write down the moment expression - capital M

Now, next question will be - what we want to do? We want to get the deflection at point C. So, we have to apply 1 unit load at point C; only 1 unit load; no other load; and different segment we will get the moment expression, that will be small m. And segment wise capital M small m divided by EI, we have to integrate. And all the segment if we take the contribution, we will get the deflection at C.

Now next step. We may be intersect for finding out slope at B. So, we have to take the structure, just put 1 unit moment at B. We will get another set of expression for small m due to getting slope at B. Now, the capital M part is fixed; that is the expression of moment due to actual loading. So, capital M, and the new set of m, due to 1 unit moment at B go on integrating in that manner. So, we should multiply divided by E and just integrate. So, we will get the slope at B.

Now you can say at this timing, thing is becoming much more systematic or mechanical. So, if you just follow the steps, we are in a position for finding out the deflection and slope.

Small m it will be for the different, different cases; cases means, if I want to find out deflection at two places, slope at three places, so there will be five set of small m. Then, we have to calculate separately.

Now, this problem can be… we can take one problem to explain how we can utilize that? Main part is it is dependent on the expression of m, and it may be the different from segment to segment; it may vary; in one segment there will be one expression; another segment it may be different expression. Apart from that EI is also there. If EI varies, automatically segment number will be more.

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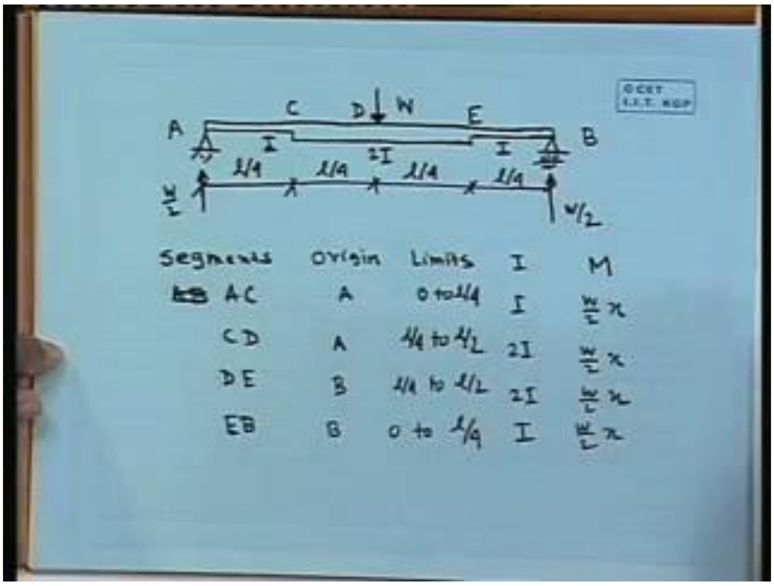

So, I am taking one problem intentionally of that type. Say, total length of the beam is l, and its central part this l by 4, l by 4, that is l by 2; it is bit heavier. We know the bending moment will be more at the mid span. Accordingly, say, the designer thought the middle part will have a heavier type of section. So, the moment of inertia there it is 2 I, and that quarter span, left and right, it it is half the value of I. So, it is simply I - second moment of area.

So, middle part is double line and at the both the end it is I and I. And the whole thing is subjected to a load W. And we are interested for finding out the slope at one of the support. So, if it is A, if it is B; say here, if I say C, if it is D, and if it is E. So, beam has a symmetrical type of load and geometry. So, load is at the centre; though the cross section is different, but it is symmetrically placed. So, slope at A and slope at B will be identical. We are interested for finding out the slope. We may be interested for slope at A or we may be interested at deflection at the middle point D.

Now, say, we are interested for finding out the slope at A. The slope is little bit different in idea, because slope means we have to put some moment and there is no load here. So, we have taken in that manner. Now, moment expression, first of all the reaction will get here, it is W by 2; here also W by 2. So, A to point B, there will be a moment expression of W by 2 into x. And here also, here to here, there will be another expression; it may be W by 2 into x minus W of x minus l by 2 something. So, it will have a distribution like this, bending moment, central beam with a central load; it will be linearly go to the peak; come down. So, we can say there are two segment A to D and D to B; but we are calculating capital M small m divided by EI.

Now, EI part is different; E is same; I is different. So, ultimately, the problem will be reduced to four segments. So, one segment, two segment; moment wise it is two segment, but that EI part is different. So, earlier, we are taking the break when the moment expression was different; here at least it is same, but EI part is different. So, here to here - one segment; here to here another segment; here to this point one segment. So, four segments are there.

Now, this problem can be solved if we write in a tabular form; it will be much more systematic. Say, we write segments. So, first is segment AB; not AB; B is at that end; it is AC; then CD; then DE; and your EB. Now, origin, limits, I, and say, M. Now, AC, CD, DE, and AB. So, there are four segments. So, we have to just put for the different segment.

Now, say, I part we can put; say AC I is I; CD I equal to 2 twice I; DE is also twice I; and ED is I. E, you can also vary. Now you could write EI. So, it will be EI 2, EI 2, EI I. Now, the moment expression, we have to write, and here, the moment expression will be in the form of some x, at this one we can write as - the first one - it will be W divided by 2 multiplied by x. Second one, we can give some expression. So, it will be W divided by 2 x. Which one? Say W by 2 is the reaction; reaction into x, reaction into x; expression of moment is W by 2 into x, which one?

So, we are writing there. Now, here we have to put the limits. Now, if we take AC, its origin is A; we are taking x from here; and limit will be 0 to l by 4. Now if we take CD, origin again it is A, because x is starting from A, but limit is your l by 4 to l by 2. Now, this limit we could change 0 to l by 4. In that case, origin should be C; in that case, moment expression we could write W by 2 into x plus l by 2. x, we could start from here. So, we could make the origin C, from C. So, if it is x. So, x plus l by 2 should be the moment. So, moment expression we could write W by 2 x plus l by 2, and this limit we could change 0 to l by 4, and the origin we could make it C. You will get the same expression.

So, which one will be convenient, that we normally follow I think. So, if the, one of the limit is 0, most of the cases that part will not contribute, I think. Now, here this expression will be more. So, somehow, we have to compromise.

Now, the much more interesting part will come, it will come to the right side. Right side if I come here or here, so expression will be this or W by 2 into that distance; this into that distance. The other alternative, we can start calculating from this side and that will be much more easier. So, here, so we will take the last span. So, last span origin will be your B; not A. And limit will be, so x we will take counting from that side. So, it will be your 0 to l by 4 and expression will be your again W by 2 into x. So, I have tried to filled up this origin limit little later, I think, because you write the expression of M, so which will be your convenient, accordingly we have to pick up the limit and origin.

Now, here, we can again follow it is B; if we follow in a similar manner. So, it will be your l by 4 to l by 2 and expression will be W by 2 into x. So, entire structure, we can start the origin from this side or that side or if we take much more complicated case, say, if we take a frame, so we can start from here; next stage we can start from there; next stage we can start from there. So, at different level we can start, and accordingly our limit will be adjusted, and all moment expression will be also little bit different depending on the origin. So, origin is very important; limit is very important; and depending on that, your moment expression will be different.

Now, once that part is there, a small m, in a similar manner we have to write. Small m here, we are interested for finding out the slope at A, means we have to take the beam, and put a moment here, and we will get the moment expression. So, there entire beam will get one moment expression, but as M has these four segments, there also M we have to just take in four segments. Now, we can put it, we can put the small m here. So, if we are interested for finding out only one deformation, we can just put the expression of small m here, but normally in a problem, it will be required to find out slope here, slope here, deflection here, deflection here, many cases.

So, there will be a separate table just for small m. So, there you need not write origin and all those, because that part are common. Only small m expressions you can put in a separate table or if the small m number is small, if you have sufficient space, you can write 1 M 1 for this, M 2 is for this like this. So, we can put MA here, small a here, small c or small d; something you can go on adding. So, it is, there is no rule here; rather we will follow certain norm, which will help us to solve the problem. So, we can make in that manner. So, small m part we can put it here, but rather draw the beam, and put the load.

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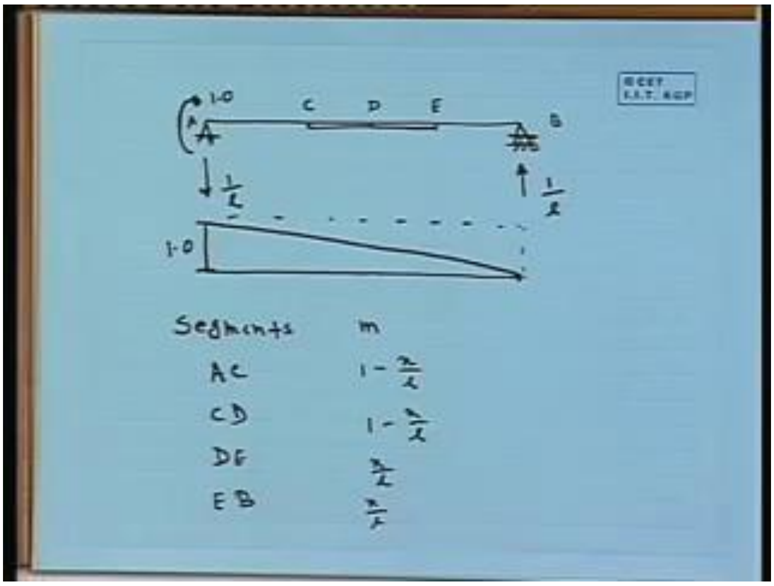

So, it will be... So, here, I am not putting all the thickness; just try to indicate this part is heavy compared to this. Now, here we have to put just 1 unit moment; that is all. So, it was A, it was B, it was C, it was D, it was E. So, you can put all this l by 4, l by 4, l by 4, l 4, EI, everything we can put; it is identical; already we have written. Now, due to the moment, we have to find out the expression of bending moment, and those are nothing but the small m. How we will get it? First, we have to find out the reactions. So, what will be the reaction here? A simply supported beam, there is a moment M, total length is l.

Say, there is a…. the beam is subjected to only one moment - unit moment. So, that unit moment will try to rotate that member; who will prevent that? The reactions. So, reactions will be… there is no other force. So, this reaction and that reaction should be equal and opposite; otherwise force equilibrium will not be balanced. So, this will be one force, just opposite force, and they will give a couple, and that value of that couple should be equal to the applied couple, applied couple, applied moment. So, one is there. So, these two forces will give in a reverse manner. So, this is clock wise; it will be anticlockwise. So, this side reaction will be this side; this will be that side; and they will produce a moment of 1. So, this force will be total length is l. So, it should be 1 by l; it should be 1 by l.

The moment part is little bit confusing; sometimes, force we can handle much more conveniently; moment sometimes creates problem, because this moment is applied here, and reaction we will get in that manner. So, there is a applied moment; if you take the free body of the beam, so it is a simple bar; there is a physically applied moment; and there might be some reaction; there might be some reaction. So, if I take moment about this, so there is a applied moment; so, it is 1; and the force into that distance, say, R into l it should be equal to 1. So, R will be 1 by l. In that way also you can know. Or physically you can think there is a moment, it should be balance by this; this moment, if I put it here, then also you will get the same reaction.

If the moment is applied in the middle, anywhere if you apply, because moment is… basically. this moment will create same reaction. So, here, here, anywhere if we put, it will be balanced by this reaction; it will generate a moment; and that moment and this moment should cancel. Only moment expression will be different; if we keep the moment here, means applied coupled here, and applied coupled here.

So, let us draw the bending moment diagram. Here, moment will be how much? 1. And here moment will be 0; it will come like this. Anywhere, if you pick up, it will be 1 minus 1 by l into x. So, moment expression will be 1 minus 1 by l into x; when x will be l, so 1 by l into l, 1 one cancel, it will be 0. And it should be 0, because it is a free end; there is no applied moment; it is also free end, but there is a applied moment.

You can just think in this form: there is a cantilever beam with a load, so there is a load. So, shear force will be that force. So, it is a free end; there is a moment. So, moment will be that applied moment. And this moment, and inside there is no load, so it will vary in a linear manner; it will come to 0. If there is a moment here, we should get in a different form. So, it should be like that. If we put a moment here, we should get up to this, then there should be a change over by 1 unit, and go like this. So, here also, if we just write segment… it is the moment 1.

Now, one option is we can put a column M, or here, say, space is not there, again, we have to write segment and M, we can put origin limit I, or already it is there, we can just follow it. So, small m… AC will be how much? So, A to C it will be 1 minus x by l. So, 1 minus this reaction 1 by l into x. So, 1 by l into x. So, here also same origin. So, 1 minus x by l, because both the segment we have taken origin here. So, this line will be valid. So, it is basically what? This is 1, minus there is a linear curve x by l. So, if x equal to l, it will cancel, it will be 0. So, if we make a box – rectangle - it is basically the diagonal, but this one EB, we have taken origin from this side. So, it will be just 1 by l into x. So, it will be x by l and it will be x by l.

Now, earlier case, if the origin was only A, we could retain the same expression, but whatever we have decided, that will more or less follow. So, for capital M some origin, some limit, we have fixed up. So, for small m also same origin, same limit will be valid. Now this capital M, say capital M, small m, the sets of expressions we obtained. This small m is due to the applied moment at A; means, we are going to calculate the slope at A. So, with these two tables - this table and this table - basically these expressions and these expressions plus this I value, with that limit, we can write our expression, what we have derived for unit load; that is M small m EI dx for different segments.

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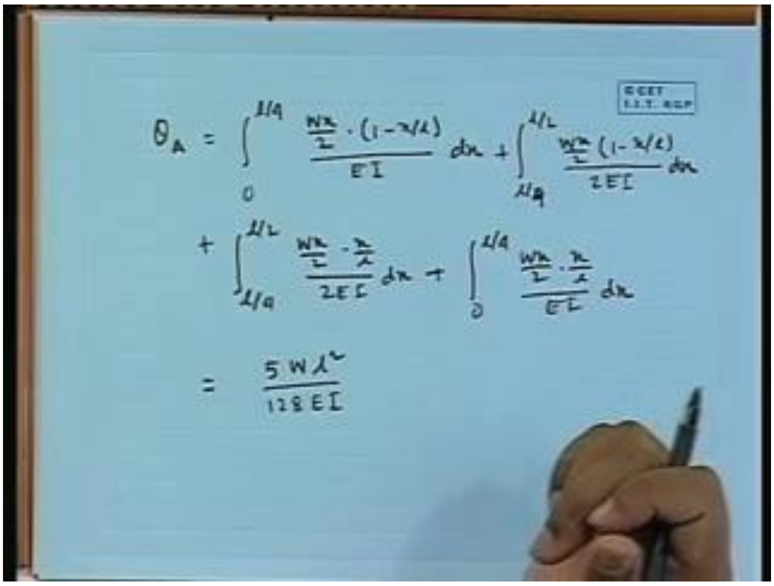

So, we can write theta A; it should be integral 0 to l by 4; capital M was Wx by 2; small m was 1 minus x by l divided by EI dx. This plus it was l by 2 l by 4 to l by 2 Wx by 2 1 minus x by l 2 EI dx, plus it is your l by 4 to l by 2 Wx by 2. And here x by l twice EI dx plus 0 l by 4.

So, there are four segments; first segment 0 to l by 4; second one is l by 4 to l by 2; third one is l by 4 to l by 2; fourth one 0 l by 4. Wx by 2, this capital M expression, is identical. For the left, small m first two span is 1 minus x by l, 1 minus x by l; right part is x by l x by l. Here it is EI 2 EI 2 EI EI. Now, this part if we just integrate and put all limit, we will get the slope at theta A. Now, if you are interested for theta B, we have to generate another set of m - small m. So, we have to put one unit of moment here, and we will get some reactive force, and find out the expression for small m. And this small m we have to put here, here, here, and here. So, if you calculate in that manner, so you will get theta B. Now, which one?

Now, this problem we can solve, and the value, whatever you will get, I can supply you what should be the answer for that. It will be 5 Wl square 128 EI. So, because everywhere EI will come out, W will come out, and here x into 1 x square by 2, it will be some l square some factor, so for those factors if you go on accumulating. after calculation it will come 5 W l square 128 EI. So, we can check this value, later on, after integration.

Now, the method I have shown with a beam, where there are different segments and the loading is acting at the middle, we are trying to get the slope. Now, you will find much more use of that method, if we handle a little difficult type of problem, we talked about frame. So, one vertical member, one horizontal member we have taken. If we take a much more generalized form of a frame, there is one example here, if we pick up.

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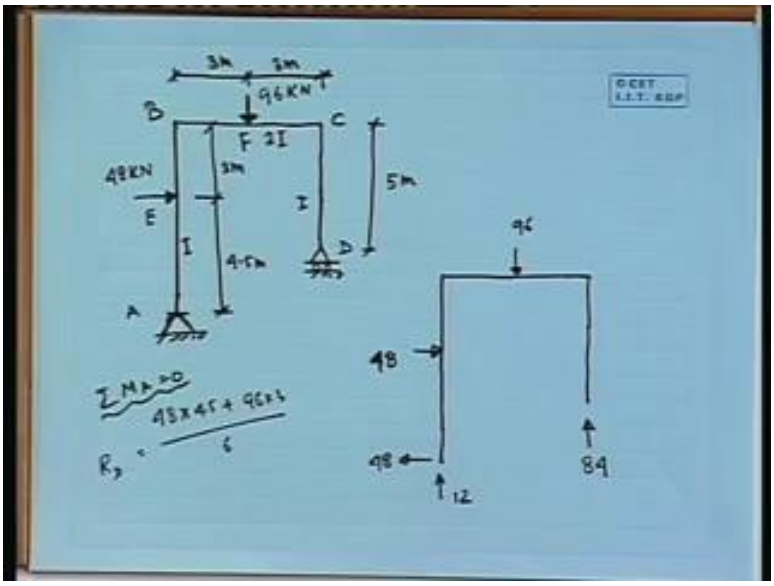

You can nicely analyze that problem with this unit load method. See there is a frame like this; there are two vertical members; not only that their lengths are not identical, but there is a horizontal member. Horizontal member is subjected to a load 96-kilo Newton. On horizontal members, there is a load 48-kilo Newton. This is a roller type of support; this is a hinge type of support. Now, the length here, it is 5 meter, and here there is 4.5 meter; that is 3 meter. So, this is 4 and half meters; this is 3 meter; this is 3 meter; this is 3 meter; this is 5 meter; these are the loads. And say, this is I, this is I, and this is 2 I.

So, horizontal member has a heavier section; I will twice I. And vertical members are both I and I, but height is different; here it is 4.535 meter, 3 meter, 3 meter, 96 kilo Newton, and 48 kilo Newton. So, this is hinge; this is roller. Now, this structure if we try to tell you, if you try to solve by moment-area theorem of differential equation technique. The problem theoretically it can be solved, but, it will very, very difficult to solve. So, many steps we have to carry out to get the solutions of that.

Now, this can be nicely handled by your unit loop method. Now, I can write the different points set: if is A, if it is B, if it is C, if it is D. So, E we have to take. F we have to take. So, this is A, this is B, this is C, this is D. So, there is a load here. So, we are taking E; there is load it is F. Now, we may be interested for finding out - what will be the horizontal displacement at point B. Due the load, whole thing may undergo some moment. So, the horizontal moment of B and C will be identical, because they are connected by horizontal member, and we are not going to take any extension of this member; only bending part we are considering. So, horizontal moment of BC will be identical.

We may be interested for finding out the horizontal displacement of D; vertically it will not come down, because extension we are not taking. So, B and C, it will have a common displacement horizontally. D can move horizontally. Now, rotation we may get here, here, here, here, under this small load, under this load, at any point we may get rotation. So, there are so many possibilities, I mean, so many displacement components we can find out from there. We can say this is a structure, you find out horizontal displacement at C, horizontal displacement at D, rotation at A, rotation at B, rotation at C rotation at D, rotation at E, rotation at F. So, there will be a long list. So, small m you will have a series, and capital M, one. So, capital M and one set of small m, you will get one displacement component. Capital M and another small m, you will get another displacement; like that, you can go on calculating.

Now, I have taken this problem, because this problem is little complex compared to the earlier problem. Unit load we have just applied for a beam problem. So, this is a frame with at least three members with some loading this side, that side. So, that structure, if you pick up, first of all we have to define the different segments, their origin, their limit, plus due to the external load we have to find out the expression of capital M. So, that part we can write.

Now, the number of segment: we can say A to E there will be a segment; E to B there will be a segment; B to C there will be a segment; F to C there will be a segment; and F to D there will be another segment. So, one, two, three, four, five segments will be there. Now we can write the segments. So, next page if we write the segments.

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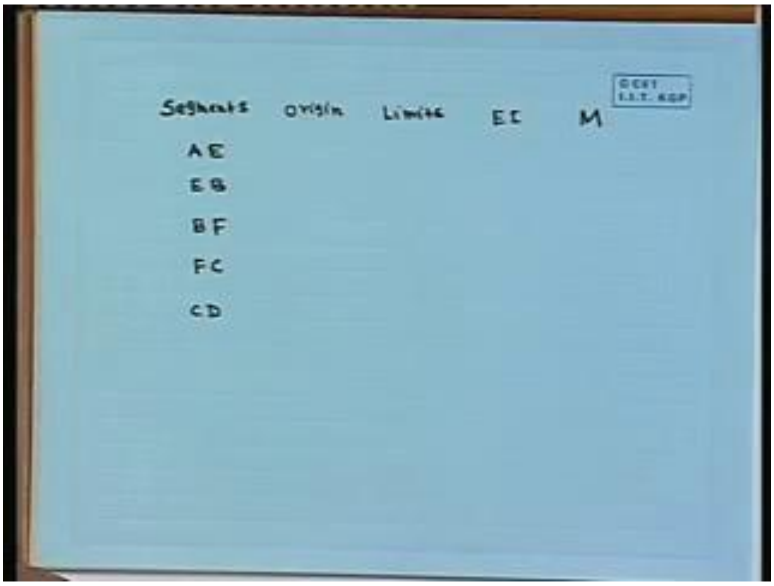

Say if it is AE, and here say EB, BF, FC, and CD. So, one will be your origin, limits, EI you can write, capital M you can write. Now, our main job is we have to find out the expression for capital M, and it should be taken in much more convenient form. So, that limit will be more or less in reasonable order, moment expression will be little bit small, and for that purpose we can change our limits. Now, for getting that moment expression, let us return back to our earlier problem. Here, our first job is to find out the reaction and that will help to find out the expression for the bending moment.

Now, here there will be two reaction components or rather I can draw a member without this detail 3 meter or 5 meter or EI. So, if I put it there, this is up to this. So, see I am putting only 96, not kilo Newton, in order to make the diagram as much clear as possible. Now, this point is hinge, so it can carry axial force, so it can carry horizontal as well as vertical; this will carry only vertical. So, all the horizontal force will be carried by this support.

Now, there will be two reaction components; vertical here; you can take moment about this or moment about that. So, if we take moment about this, it will be a 48 into 4.5 plus 96 into 3 - that will be the moment; that will be balanced by this force. So, it will be both clockwise. So, there will be a reaction. So, it will be how much? It will , say, it we write, should be 48 into 4.5 plus 96 into 3 divided by 6. If we take moment about A. So, 48 into 4.5, it will give a moment; 96 into 3 it will give a moment; that moment should be balance by your reactive force at D upward, that in to the distance 6, it will be your reaction at D. So, it is just if you take moment about A, that will give this value, and this value will be how much? This value will be, this will be 84 kilo Newton.

Now, the next option is, if it is 84, but the vertical force is 96, this is 48. So, there should be a another force here. So, it will be 12 kilo Newton; 12 plus 84 equal to 96; summation of all upward forces. Now, if you have the reaction 48 12 84, we can proceed for finding out the moment at the different segments for capital M.

So, this is the actual structure. We have removed the support and tried to find out the reactions; the objective is we have to get the expression of capital M for the different segment. So, reaction part we have determined. So, next job is for the different segment we have to find out the expression of capital M, and we may vary our original limits.

I think with that we have to conclude today.

We can try to solve it in the next class.

Answers.

A number of answers are there: theta a, theta b, theta c.

Preview of Next Lecture

Particular type of energy method. So, the main concept behind that is, basically, in the energy principle. So, that we have defined in a particular fashion, and we defined that method as a unit-load method. That method, we have tried to demonstrate with a beam problem.

And at the end, we have tried to apply to a much more complicated type of problem.It was a portal frame, having absolutely arbitrary type condition, because the legs were different. We can go ahead with that to get the feeling about the applicability of method to this particular type of structure.

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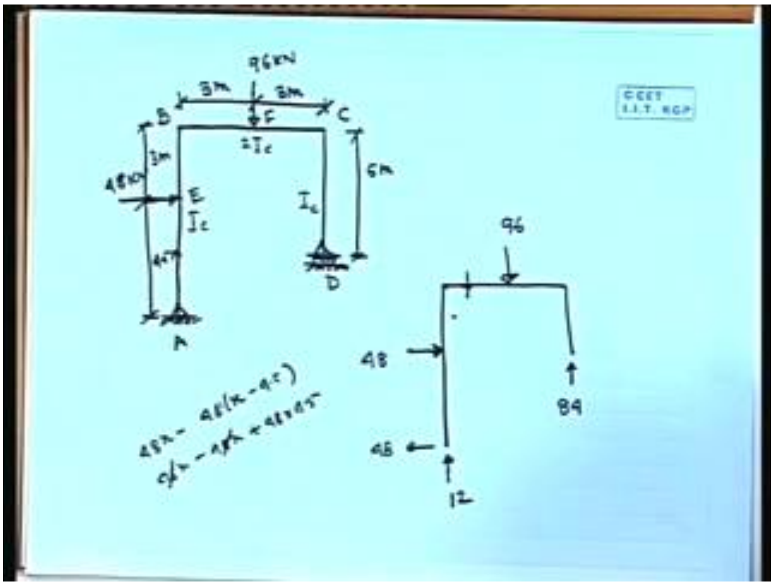

So, the frame you must have remembered it was something like that. So, this leg it was quite long compared to the other leg. That leg, the right side leg, it had a height of 5 meter; and left side above the load, it was 3; below that it is 4.5. So, total 7.5 and the horizontal.