Strength and Vibration of Marine Structures
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Lecture - 18
Theory of Column – III

So, we were talking about theory of column. Last class we have completed the short column part; you must have remembered we have divided the column in two groups, one is short column, another is long column. Short column part we have more or less completed. Long column just what is the mode of failure, how it will behave under the load, what is buckling; all these things we were trying to explain. So, it was just the basic structural behavior of a long column.

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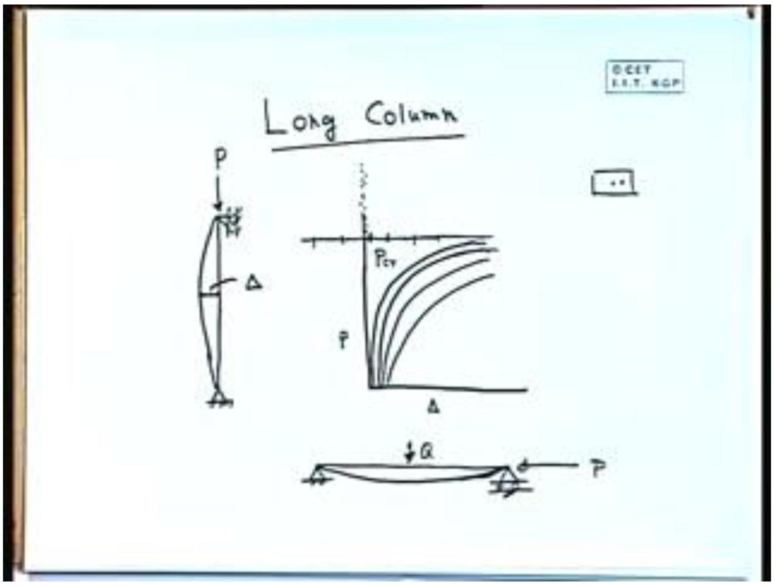

So, we continue with that long column. Now if we draw the diagram, what we drew in the last class, say, there is a column like this both end supported. Now the load deflection curve you must have remembered delta this was P. Now if it is a perfect system. So, if you apply the load, it will directly take that load in the axial manner; there will be no lateral displacement. So, this delta is lateral displacement, right. So, if I draw the lateral displacement here, say, maximum displacement if we try to define as delta. So, normally if the load at a lower range there will be no delta.

Now it will come to a point P equal to P critical, then we will get the equilibrium as the neutral equilibrium stage. So, earlier it was a stable equilibrium. Now at this level if we give any displacement delta. So, it will be there. Now this value may be any value we can put here; it may be 1 millimeter, 2 millimeter, 3 millimeter, minus 5 millimeter. And if we cross that load level disturbance is not that it will take care, but if there is any disturbance, naturally there will be an excessive deformation and the whole thing will fail.

Now this is that load deflection curve for absolutely a perfect column. Now if I say the load is not acting at this point. So, if I draw the column section here. So, it is not at the centroid; it is little bit away. So, what will be happening? Load if we shift, there will be load; there will be bending moment. Now this bending moment will try to bend that member plus the load, load part is there

So, automatically from the beginning if we apply P equal to very small value, there will be a small moment; there will be a small deflection. So, if we increase the load, our moment is basically P into E. So, when we increase the load moment part will increase, deformational will increase. So, if we start from some imperfection here or we got some curve like that, the level of eccentricity or the level of imperfection if we increase, the curve will be much more flat type; if it is 0 it will try to take a shape like this or eventually we assumed the deformation is very large; they are very small, we are not taking it as large deformation problem.

Now here when the deformation is large where we can get a very small part of the curve. So, if we try to get because at this level, curve will try to become flat. So, at this range if you want to go, there will be a huge deformation. So, if we want to go to that range, we have to take that to a large deformation, right. So, whole problem will be nonlinear problem. And solution of nonlinear problem is quite difficult. Now that here if we just follow that curve, naturally you can take any small deformation; so, this curve is valid for a small deformation as well as large deformation.

So, with our small deformation that type of problem we can handle, right. So, up to this level we are in a position to handle a small deformation problem, and this curve there is no deformation, certainly, there is a deformation. So, here we can put small or large and we have the capability; generally, we can handle this type of problem not the other type of problem. Other type of problem definitely we have to have the understanding of large deformation problem which is basically a nonlinear problem.

Now if we try to study that problem absolutely perfect case, then what will be the value of P critical? That we can determine from our calculation, right. Now before going for calculation of P critical force, certain type of beams or certain type of rather a column not beam; I should mention one thing, say, there is a member; let me draw a member here, say, if we apply some load. So, it will be under P some compressive force. Now the force can be applied in the form of with some eccentricity automatically moment will be there or we can put some lateral load.

Now here due to Q we will get some deflection, right; say P is not there due to Q you will get some deflection, now if we add P. So, what will be happening? Because P into that distance that will give some additional moment; due to the additional moment there will be a further change in the deflection, right. Now due to the additional change P into that additional change, it will generate second level of moment. So, that will cause further deformation of the structure.

So, if we try to follow what is that? We are not getting a direct unit solution; the reason is very simple. The problem is not a linear problem. So, earlier we have applied the method of superposition; say, beam it is subjected to a number of load; the effect of all the load we have just simply algebraically added together to get the response, and we define the technique is method of superposition.

Now here actually the problem all the structural element whatever I have drawn this member is called beam column because here there is a load; if P is not there, it is a beam; if Q is not there, P is there. So, it is called column. When PQ both is there, we say it is a beam column problem. Now beam column problem, the deflection estimation is absolutely nonlinear. So, it is not linearly depended on Q and P; just I can give one example, say, Q equal to 1 Newton, P equal to 1 Newton.

So, if we get some deflection; if we make PQ both double, deflection will not be double, it will be more than double, right. So, here the phenomenon is nonlinear for a beam column problem. Now beam column at this moment we are not interested; you may be interested little later. But here let us take a pure column problem under a compressive force, and we will assume everything is perfect; means cross section throughout the section is uniform, material property everywhere is same; there is no distortion or deformation within the member.

So, it is absolutely perfect load is acting at the centroid of the cross section of the member; the supports are absolutely perfect, it is properly placed at the two ends. So, whole thing is in a perfect condition. We want to give the load P when we get the critical load. Now this problem we can take. This is one of the fundamental cases a column it is hinged at both the end, right.

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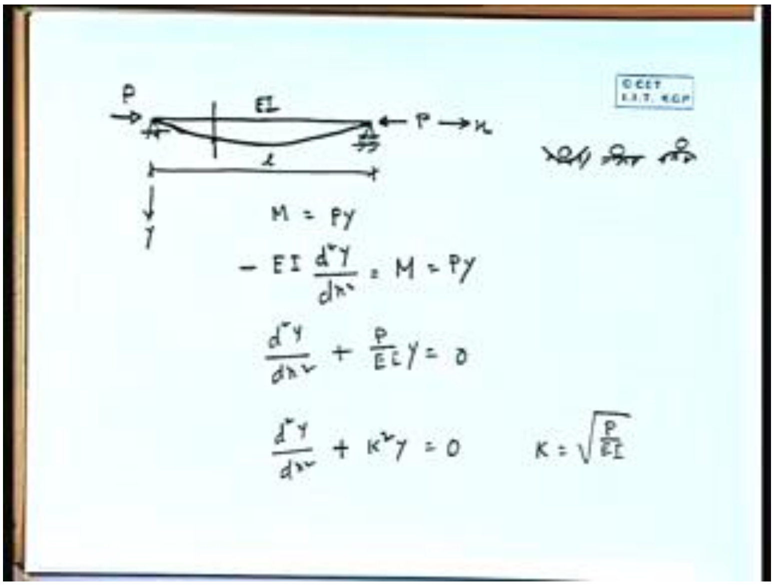

Now we can draw in a vertical manner anyway or we can draw in a horizontal manner also. So, if we draw in a horizontal manner it is material the way your; now, say, the column is drawn in a horizontal manner. There is a load P acting or from this side this is hinge; automatically there is a reaction force P will be there. So, entire member the P will be transmitted. So, everywhere it will be under P. Now the length we can take it is l, right, and we can take the member cross sectional area is EI.

We can pick up some axis system, say, it is a y, this is a x. Now if we take the bending, it will be something like this, okay. So, L is the length, EI is the flexural rigidity. Now this is hinge that is roller P is acting. So, bending is shown, x y is the coordinate. Now when we will get bending? When P it is below P critical that bending will not be there, because there is no transverse load. So, it should be either critical condition?

Student: Or above

Professor: Or above, say, critical part we are interested; above it will not be stable, right.

So, we are talking about the three form of stability, right. So, there is no disturbance here. So, it will be here, but if it is in neutral, from here it can go to some other place. But here you cannot keep it, right, say, it is something critical condition. So, if we give some displacement, it will be there. Now if we apply load P, in this end there will be a reaction P, right. So, the reaction you can say the reaction will also be P; any vertical reaction you will get in that case? There will be no vertical reaction, because both the ends are simply supported. So, there will be no moment there.

Now if there is a reaction here, it should be balanced by the opposite reaction, because there is no applied load. So, if it is some R, it will be minus R. So, force part will be balanced, but they will create a moment. So, there is no applied moment. So, automatically those forces should be equal to zero. So, from there we can get the idea that there will be no vertical reaction here. Now if we take any section, say, at a distance x, what will be the bending moment? Not it will be 0; it will be P into y, right.

Say, this is a typical section; it has a distance x. So, here the deflection is y. So, x y we have shown the axis system the deflected curve, say, it is represented by y is a function of x but value is not known. But there will be some displacement here. So, this P into that distance and that will be the moment.

Student: This is after the deflection takes place

Professor: After the deflection when it is under deformed condition.

So, here what will be the moment? So, there is no reaction here. So, that reaction into that distance we are not getting, but this force into that distance we are getting some moment, or we can just cut that section and take the left piece. Left piece if there is a P there should be a P, but there is a gap of P, the distance is y; P into y that moment will be coming there I think so.

So, in any form you try to investigate the problem you will get a moment that is simply P into y, then what will be the type of moment? Here it will be sagging type of moment, alright. Now we have studied our moment curvature relationship where we have seen it is minus EI d 2 y d x square; that is equal to your sign moment or we can it is P into y, right. So, EI the flexural rigidity d 2 y by d x square is a curvature. So, that should be equal to M and we know if the moment is sagging type of moment with that type of sign convention.

So, here the curvature is negative curvature, right. So, negative curvature means there will be a minus sign; if the moment is a reverse manner hogging type with that sign convention, the curvature should be positive. So, here this should be positive. So, that part we have discussed earlier. Now this equation if we try to solve. So, here we can write your d 2 y d x square plus P by EI y is equal to zero. So, EI part if we put here and both the term if you take on the same side, it will be like this or it is defined in a much more it will be K square y is equal to 0; that K is what? It is P by EI, right.

So, P by EI square root is K. So, P by EI will be K square, right. So, that entire term was defined by a single quantity, but we have taken a square. So, it is a matter of convenience, because later on when we solve we will find the square part will help us to represent some expression in a convenient manner. Now this is a very standard form of differential equation d 2 y by d x square plus K square y equal to 0, right. Now we have to solve this equation, and from the solution we will try to get some idea about the critical value of the load. Now what will be the solution of that equation?

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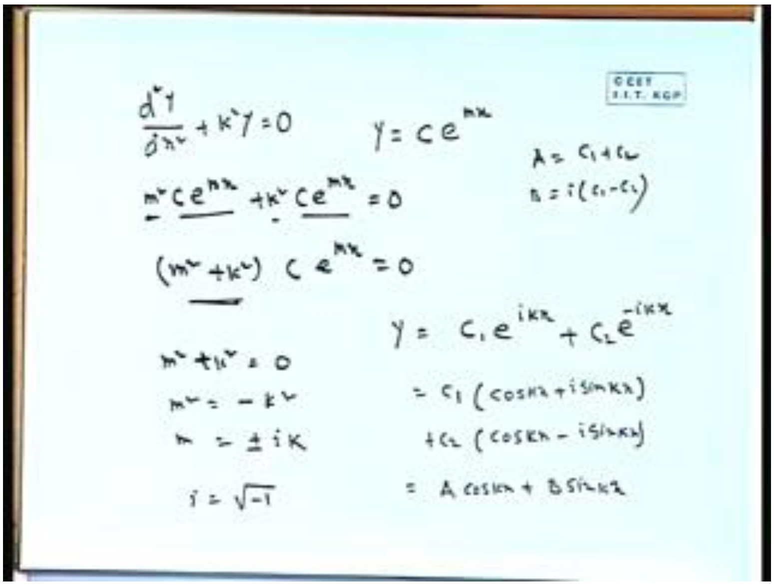

So, if we write the equation again. So, it will be d 2 y by d x square plus K square y that is equal to 0. Now it is solution will be now this is what first term is a derivative of y twice, and second term is y itself multiplied with a factor. So, the solution will be such if we take derivative twice, the type of expression will be same thing only some constant part. It will be coming out and constant part we can adjust plus minus, and whole thing should be equal to 0, right.

So, say, if we take y equal to some, say, sin x, right. So, if you take derivative twice, sin will be cos sin or if you take y equal to cos, cos will be sin cos. So, if it is a sin cos type of thing, it will come back after taking derivative or sin cos it has much more elegant form of representation is e to the power something, say e to the power x. E to the power x if you take that derivative it will be e 2 to the power x; any number of derivative you take it will be e to the power x

And e to the power x if you expand, we can get sin cos sin hyperbolic cos hyperbolic. So, for that type of equation, the solution is y should be some constant e to the power m x, right. So, here this m and C they are the constant quantity. So, main idea is if we put here our e to the power m x will be e to the power m x after derivative. So, here you will get C m. So, that constant part we have to manipulate and get the actual solution, right

So, that is the basic mathematical tool, and the reason is coming from there, because if you take derivative you should get this term. So, if you substitute here. So, what you will get if you take. So, it will be m square C e to the power m x, right. So, C e to the power m x will be there; if you take derivative C is the constant e to the power m x form m will come first. Second derivative, another m will come; it will be basically m square plus k square and C e to the power m x, right.

So, this is the part with that it is multiplied. So, this is the part after taking derivative m square is coming or we can say m square plus k square C e to the power m x that is equal to 0, right. So, m square plus k square C e to the power m x equal to 0. Now from here, this quantity should be equal to 0; either C will be 0, or this will be 0; this will not be always 0, right. So, x may vary. So, somewhere e to the power something if it is 0 also you will get 1. So, either C will be 0 or this will be 0; if C equal to 0, then we are taking y equal to 0.

Now that solution we know I think; if it is the load is within your P critical, there will be no y no deflection. So, problem is solved, but we can say that is one of the solution. So, that solution where we are not interested; we are interested in other solution, other alternative; other alternative is this will be zero. So, if that is 0 your m square plus k square equal to 0. Now I was talking about why it was taken k square not k because here there is a m square; otherwise, m should be square root of this. So, that part is just simply avoided.

So, we can write m square equal to unfortunately it will be minus K square and m should be plus minus; it is I k, I is what square root of minus 1, right. So, here I is square root of minus 1; now here minus came. So, it will be under square we have to take one imaginary parameter i. So, that will be the thing. So, there are two possibility of m 1 should be plus I k, another will be minus I k. So, what will be the solution? Y equal to C e to the power m x; so, y will be, say, one of the C value is, say, C 1.

So, e to the power I k x; another value is C 2 e to the power minus I k x, right. So, it is e to the power m x; m is I k minus I k. So, if we put I k, it will work with minus I k also it will be working. So, we can say both will be there with C 1 and C 2, right. Now if we expand that that should be C 1 e to the power I k x can be written as cos k x plus I sin k x. And the C 2 part we can write cos k x minus I sin k x or we can write A cos k x B sin k x.

So, we are getting two values of m; one is plus k x, another is minus k x; we have substituted here, both may be there. So, C might have a value C 1 with plus C 2 with a minus; e to the power I k x if we expand this trigonometric expression is cos k x plus I sin x, if it is minus cos k x minus I sin x. So, this cos k x part if you take common; so, it will be C 1 plus C 2 that we are writing as a different constant it is A. And sin k x part, say, C 1 I minus C 2 I that part we can write as B, right.

So, here we can write, say, A equal to C 1 plus C 2 and B equal to C 1 minus C 2. So, we can write in this form. So, that is one of the standard form or we are talking about the solution of this equation earlier that y may be a cos x or y may be a sin x, right. So, there also it will satisfy the equation. Now if cos x can satisfy that equation, sin x can also satisfy. So, the total expression is in a general form, say, A cos k x plus B sin k x, right.

Now there are two constant quantities, and these two constant quantities we have to find out from the boundary condition.

So, a deflection of beam more or else you did the same thing; you obtain d 2 y by d x square is some moment integrated got some constant and substitute some boundary condition and that constant you obtained. Now here what are the boundary conditions? So, deflection is 0 at both the end. So, at x equal to 0 your y equal to 0; at x equal to l your y equal to 0, right. So, we come here, so that I can show you the beam. So, we have started with this beam. So, at x equal to 0 and x equal to l deflection will be 0.

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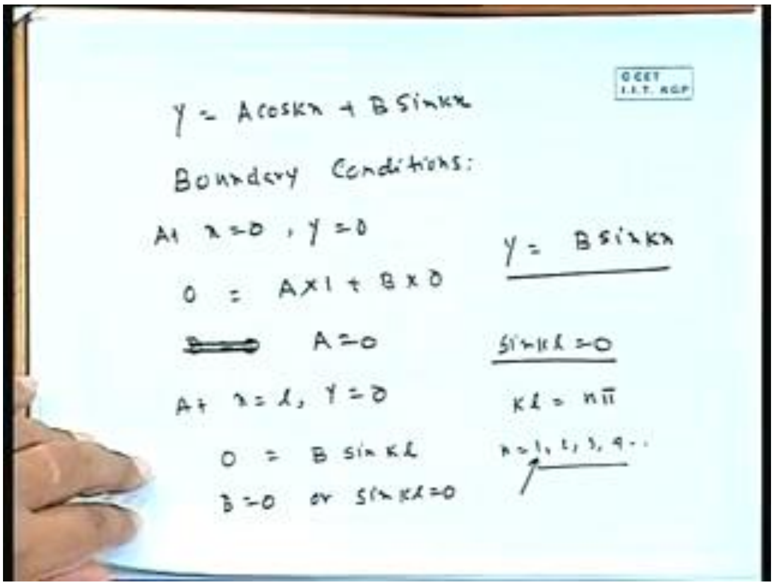

So, we got y equal to A cos k x B sin k x. Now if we put boundary condition. So, at x equal to 0 y should be equal to 0. So, if we put 0 will be A, cos 0 will be how much? One and B sin 0 will be zero. So, if we put x equal to 0 it will be sin. So, sin 0 equal to 0 and cos 0 is equal to 1. So, form here we can write B is equal to 0, not B equal to zero; A equal to 0 because with B we are multiplying 0. Now if A equal to 0, your expression of y will be B sin k x, right. So, the other boundary condition if we put, say, at x equal to l y equal to 0.

So, y equal to 0 A already became 0. So, there is no point of taking it. So, it will be B sin k l, right. Now with other boundary condition we are getting B sin k l equal to 0 or after this we got y equal to B sin k x because A became 0; second level we are getting it. So, at x equal to l it became B sin k l y equal to 0. Now there are two possibilities. One is B is equal to 0 or sin k l equal to 0. Now if B equal to 0. So, your equation will be y equal to B sin k x.

So, B equal to 0 means y will be 0. So, it is again that case means y is basically your deflection of the beam. So, deflection of the beam equal to 0. So, that is the condition when load is within the critical range. So, then y will be equal to 0. So, that solution we will get. Now other option if B equal to something, it is nonzero quantity. Now sin k l should be equal to 0 if B equal to nonzero if B equal to 0. So, entire solution will be 0. So, that is the prebuckling condition.

Now we want to get something else what is happening at least when the equilibrium is neutral equilibrium and the load is at the critical load level. Now there, say, B is not equal to 0 or y equal to B sin k x is there. So, equation of the curve is B sin k x. So, it will follow a sinusoidal curve having an amplitude B and it will vary x equal to 0 0, and other end it will be 0; it will vary something. So, at critical range if we give some displacement, it will take a sinusoidal form. So, B if we give 1 centimeter, so with a 1 centimeter amplitude, we will get the deflected curve.

Now during that condition if B is not 0, then sin k l must be equal to 0 because that equation should be satisfied rather boundary condition. So, if sin k l equal to 0, k l will be how much? K l will be your n pi. So, if n equal to 0 it will be 0. Say, sin k l equal to 0, say, k l equal to some theta sin theta equal to 0 when theta will be zero. It will be at 180 degree at pi; it will be at 2 pi; it will be at 3 pi; it will be at 4 pi. So, n will be m multiplier; it may be 0 1 2 3 4, say, if n equal to 0 again it is that basic case.

So, it is entirely 0, right, but here we are interested n should be at least 1 2 3 4 some value. Now n it may be 1, it may be 2, it may be 3, it may be 4, anything; it may be 0 also, 0 definitely we are not interested, because 0 again it will give that few buckling case, but normally this one is a very sensitive value. Now we got k l equal to n pi.

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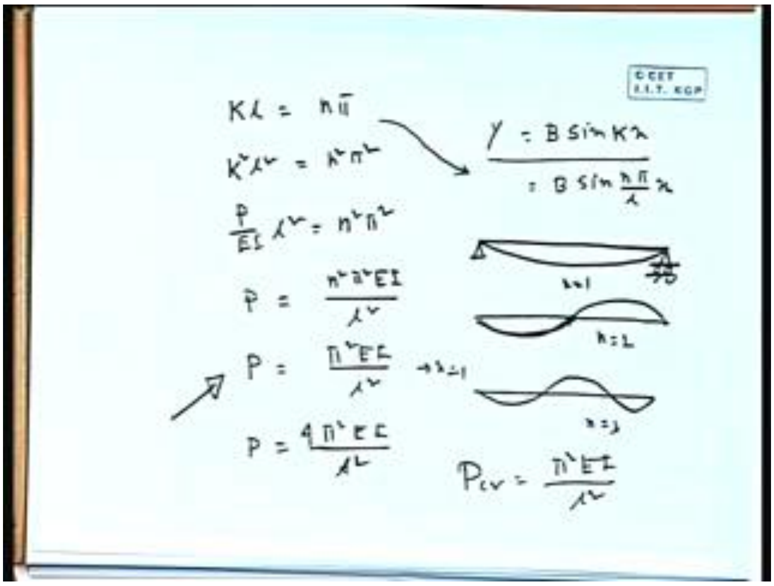

Now if we write your k l equal to n pi, what is k l? If you take square of that, it will be k square l square equal to n square pi square; what is k square? P by EI. So, we can write P equal to n square pi square EI divided by l square, right. So, we got one deflected y equal to B sin k x and k l for that case when B is not equal to 0, k l equal to n pi. So, from there we are getting P equal to n square pi square EI by l square. Now if n equal to zero, so what will be the value of the load? Zero; 0 means when there is note it will be straightened, but if n equal to 1. So, P will be?

Student: Y square EI by l square

Professor: Y square EI by l square, right.

Now if I draw the, say, beam part if you draw again, right. Now it will be deforming like this, right. So, y equal to B sin k x, now your this k the k is your n pi by l. Now if n equal to 1, so k will be your pi by l. So, if you put here y equal to, say, B sin say from here or in general form if you put n pi by l into x, right. So, from here this k if we substitute. So, it will be B sin n pi by l into x. So, if n equal to 1; so, it will be B sin pi x by l. So, pi x by l its curve is like this.

So, when x equal to 0 it is 0; when x equal to l it will be pi, when sin pi means zero and when it is midway. So, it is pi by 290, it will be maximum, and the value will be P equal to pi square EI by l square. Now if this is for n equal to 1. Now if n equal to 2, your P will be pi square EI by l square, and here n square means there will be 4 coming here, and here n equal to 2 means, it will be sin 2 pi l by x. So, if you put l by 2, then this value will be l by 2, two two cancel, it will be 0.

So, in that case if I put the deflection equation, it will be 0 and it will be like this. So, if n equal to 3, it will be like this, right. So, here n equal to 2, n equal to 3, n equal to 1; so, it is half of a sin curve; it is a full sin curve; it is we can say this is 1 and half of a sin curve. So, if it deforms like this, we are going to get a value P equal to pi square EI by l square. If you want to deform in that manner, it will be pi square EI by l square into 4. If you deform like this, it will be pi square EI by l square into 9, right. So, like that it will increase. So, what will be the minimum value? So, this is the minimum value, right

So, this value is what? This is basically your critical load of buckling and the minimum critical load of buckling and the different form of deflected shape whatever I have shown here, we can say these are the mode of buckling in which mode it will buckle. So, this is we say it is a first mode or the fundamental mode; this is the second mode; this is the third mode like this. Now in normal case if it is a perfect case, it will try to deflect or fail in the fundamental mode or first mode, because it will require less force, right.

Now if you try to prevent here in fundamental mode, the middle part will undergo last deformation, say, there is a column; centrally, there are some resistance, okay. There is a column, and due to the load it will try to buckle, and centrally, somehow some member is passing. So, it is preventing, right. So, there might be little gap not it is fully ensured, but anyway due to some physical constrain, it cannot undergo moment. So, what will be happening? It will start buckle in the second mode because here it cannot.

So, automatically it will take that step; in that case will get four times of buckling load. So, it is n square. So, n if it is 2, it will be 4. So, buckling load will be increased. So, we can say this part is represented like this. So, this will be l by 2. So, if I took l by 2 square, automatically 4 will go there. So, you can think in that manner; it is hinge hinge here or it is second mode these are support. So, in anyway if you try to explain, you will get the value of the critical load. So, sometimes this P we write P critical.

So, you can write P critical equal to pi square EI by l square. Now if there is no resistance if it undergone a failure mode according to the fundamental mode, right. So, for fundamental mode that is the value, and these are the shape of your buckling modes, fine. Now this case we consider as one of the basic case of fundamental case, because it is a symmetrical case; both the end are hinged, and we were getting a nice expression pi square EI by l square that is the critical load. Now we shall try to take different boundary condition, right. So, not necessarily all the cases we will get a beam having both the ends simply supported. Now we will rather get different types of boundary condition.

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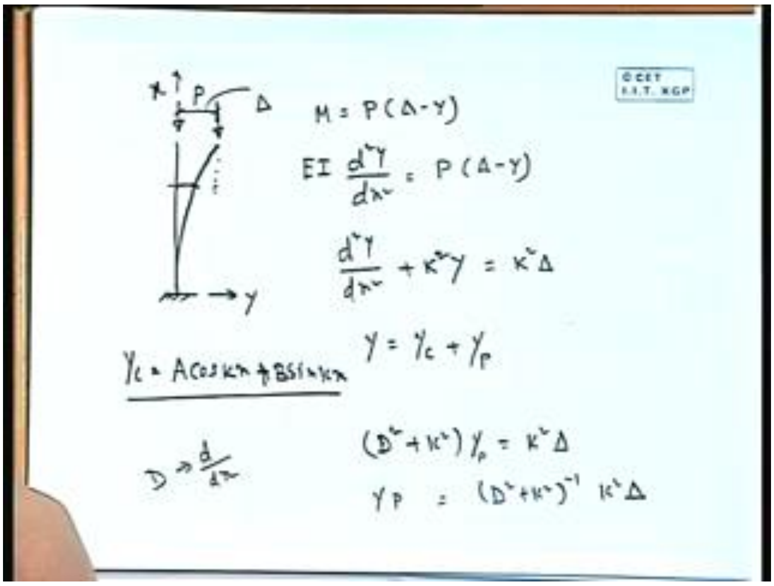

Now if I take a second category in the form of a cantilever type of column, say, if I draw like this alright. So, we could draw in a horizontal way also anyway; here just I have drawn in a vertical manner, say, this side is your x, and this side is Y. We could keep the page in that manner also, no problem. Now due to that load, there should be a deformation like this, right. Now here this part is, say, delta. Now the load is applied; if it is beyond critical value, it will be straight.

Now if it is a neutral value if it is beyond neutral, definitely it will be entire failure; at least we can think the neutral case in a very theoretical manner. So, if you apply the load. So, it will be straight, but we can give little deflection and release it will be there. So, say, that is the situation. So, delta is the tip deflection, and it will follow a cantilever mode of deformation. Now we have x y. So, at any point there will be bending moment. Now P will be shifted from here to here now.

So, initially it was straight load was here; now say if it is different load is here. So, at any station, so from x, what will be the moment? Say, this is x. So, this value will be y. So, moment will be not P into Y; here P into that distance will be the moment, right. So, it will be P delta minus y, right. So, your moment in that case it will be P delta minus Y. So, here to here it is delta, this is Y. So, this part will be delta minus Y. So, P delta minus Y that part will be the moment here.

Now that moment is giving a, say, if we look from this side, it is giving a hogging type of moment, right, or it will give a positive curvature. So, slope is positive that is increasing with x. So, it will give a positive curvature. So, we can write EI d 2 Y d x square equal to your P delta minus Y, right. Now this expression we can write, say, EI part we can put below P. So, P by EI we can express in the form of k square. So, d 2 Y d x square plus k square Y is equal to your k square delta where the EI part you just put on the right side below P.

So, P by EI will be k square. So, k square Y minus that part if you take this side, it will be plus k square y. And here k square delta will be on the right side. Now if we try to compare with your earlier equation, we got d 2 Y d x square plus k square Y equal to 0, but here this side right side is not 0. There is one quantity; it is k square delta, and it is a constant quantity; it is not a variable function of x or Y.

Now this is one of the standard form of differential equation. We say if the right part there is nothing if it is 0; we say it is a homogeneous equation, right, and here the left part is identical with this, but there is something on the right. Now the Y it will have two component of solution; one is called a complementary solution, and one is called your particular integral.

Now what is complementary solution? Say, if we make this part is equal to 0. So, we will say it is a homogeneous part of that equation, right. So, if we make it 0. So, it will be the earlier equation. So, what will be the solution? Y will be A cos k x plus B sin x. So, if we put here. So, what will be happening? This part will be 0 because 0 will be 0. Now there will be Y particular part; if we put here that part should be equal to Y square delta, right. So, it is just like integration constant.

So, if you integrate we get some constant; that constant may have some value, it may have 0 depending on the boundary condition. Here also this complementary condition it may be 0 or some value because this is the part; if we put this side it is not going to affect the value on the other side, right, because Y c is coming by making it 0 and we have the solution Y c will be your A cos plus B sin x. So, here if we put A cos x will be A cos x with the minus and k square, and B cos x will be also B cos x with the minus and k square, and this side will be Y square with the k square. So, automatically it will cancel out and cancel out means 0 will be equal to 0 this side because equation will be satisfied.

So, this is one part of the equation plus there should be if this is the part, left side will equal to 0; it will not match with the right side. So, there should be another part, and that part if we substitute that will yield a value k square into delta, right. So, right part we can say k square delta plus 0; for the 0 part there will be complementary solution; for k square delta there will be another part. Now we can say why we are taking it. I mean there is a possibility; the value may be entirely 0 or it may be the other.

Now these are all mathematical you can say we can follow strictly mathematics or being a engineer sometimes I try to explain from physical side that, okay, this part may be 0 part; if it is 0, then there should be a solution. If there is something if that part is there, at least that is not going to hamper. And Y particular that should be there to get some values which should be on the right side. Now that is called Y particular. Now here this Y c we are talking about it is A cos k x plus B sin k x.

So, this part already we have derived for the earlier problem. So, if you make right side equal to zero. So, we will get the homogeneous component of that equation and for that the solution is this. Now for finding out the Y p that particular integral that treatment made is in this form. So, we can write, say, this one this is D square plus k square into Y into k square delta. Now this Y part we are taking common. So, Y if you take common it will be k square; if we take Y common it will be D 2 by d x square. So, this capital D is an operator; it is basically your d by d x.

Student: So, that equation we will write in this form.

Professor: So, that equation is written in this form.

So, Y if you take common it will be this operator square plus k square Y equal to k square delta. Now here we can write here Y particular, right, it is particular integral or you can solve. Finally, we can take that as a continuation for the Y p. Now what will be Y p? So, this part we can put on the other side. So, Y p will be your D square plus k square minus 1 k square delta. So, D square plus k square whole to the power minus 1 into k square delta, okay. Now if I go to the next page.

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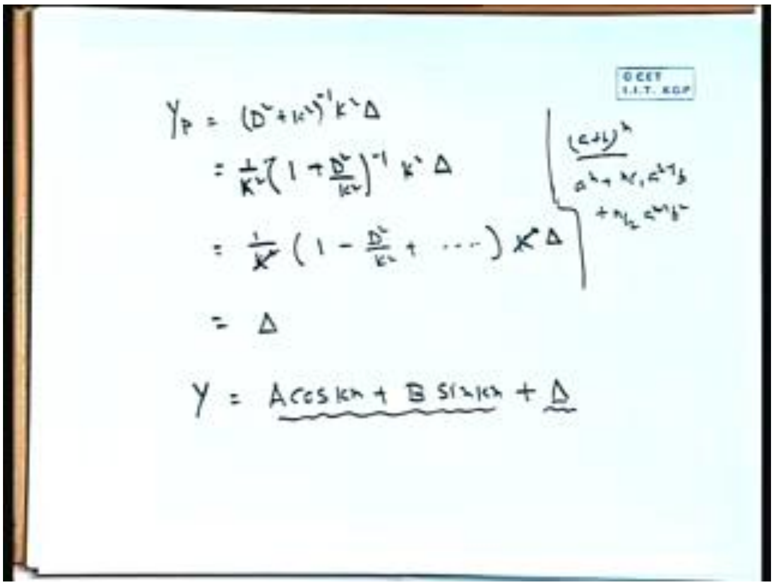

So, just now we have written Y p equal to D square plus k square k square delta; now this to the power minus 1, right. Now here this is written in a convenient form; this k square part will take out. So, it will be k square will be out and it will be minus 1 means k square will be there and this side will be 1 D square plus k square minus 1, and it will be k square delta. And if we expand in a series this form, it will be 1 minus D square k square.

Now what will be happening, say, we know a plus b to the power n. So, it is a to the power n plus n c 1 a to the power n minus 1 b; we write in this form. So, here a to the power n n c 1 n minus 1 b n c 2 a n minus 2 b square. So, it will go like this. So, it will start with a to the power n, next a n minus 1 b to the power 1. So, next gradually it will change. So, 1 to the power something is always 1 and here first this will not occur; second time it will be n c 1 is n, n is here minus 1, and this part will be there.

So, if you take this term, second term it will be D to the power 4 by k to the power 4; after that D 2 the power 6 k to the power 6. Now what is D? D is the differential operator. Now this side there is a constant quantity, right. So, if we multiply first of all this k square k square will cancel. Now it will be basically delta into 1, second term if you take. So, delta if you take derivative twice, first derivative itself it will be 0; second derivative there is no way of getting any value.

And if you take further terms, order of derivative is much higher. So, all the term will contribute 0. So, ultimately you will get Y particular is equal to delta. So, we can write the Y equal to A cos k x B sin k x plus delta. So, this part was complementary part, this part is the particular integral part. Now this is the solution of the cantilever beam. Now next we will put some boundary condition, find out AB, from there will try to find out the critical load, right So, in this class I am closing with that equation. In next class, we will try to utilize try to find AB critical load and all those.