Strength and Vibration of Marine Structures
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Lecture - 25
Ship Vibration III

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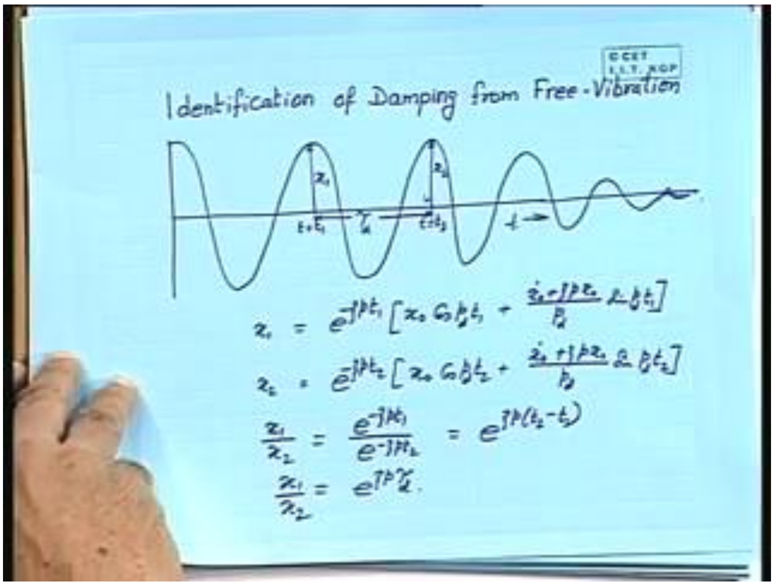

Now, next we will try to see identification of damping from free vibration record. Now, one thing we have realized that structures do have some sort of damping in it and therefore, the response will decay down. Now, how much of damping is there, how do you measure it? Once we have made one gross assumption that we are replacing it by a dashpot, all the system does not follow the same, but we have assumed that.

So, now let us try to take the response. So, some response is there. It is decaying down. So, how do we try to find out what is the system damping coefficient here? So, now let me assume that this is the maximum amplitude at t is equal to t 1, and we say that this amplitude is x 1 and at t is equal to t 2, the amplitude is x 2. So, I can write an expression for x 1, x 1 is equal to e to the power minus eta pt 1, and then x 0 cos pdt 1 plus x 0 dot plus zeta Px 0 by Pd sin pdt 1. This is for x 1.

Similarly, x 2 mind here that one of these terms will be present and the other part may not be because this maximum value, either you will get as cos is equal to unity or sin is equal to unity. So, at the same time cos and sin, two of them cannot be present. One will be there, another will not be there. So, to be on easy side, let us assume that this part is there, but let me write it in the full form.

Now, let me take the ratio of x 1 by x 2. The bracketed term if for simplicity I eliminate the second term here, what is cos pdt 1 unity? What is cos pdt 2 unity? So, what do you get here? It is x 0 by x 0 and therefore, the bracketed term will cancel out while taking the ratio, right. So, this is what you will get x 1 by x 2 or in other words, t 2 minus t 1 is this. What is the difference between this? Tau d is after one cycle and therefore, now this way handling the problem is a problem and therefore, we take the logarithm on both sides.

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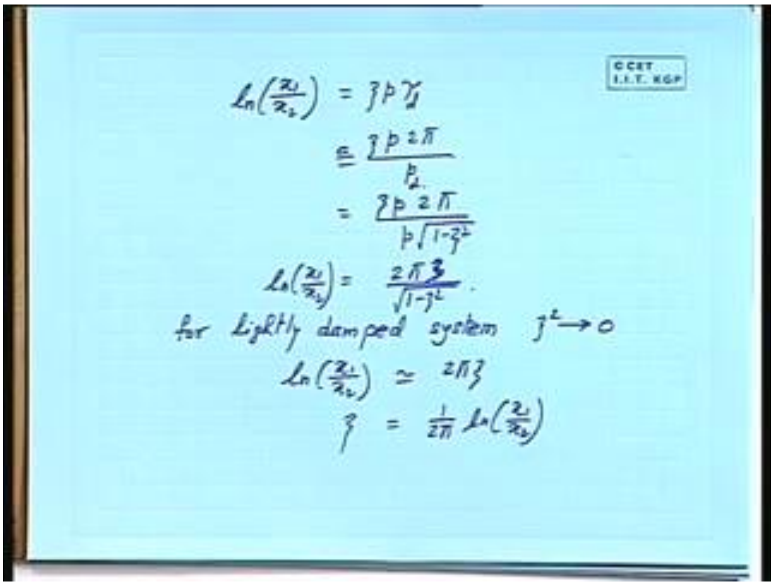

What is the value of tau d? It is 1 tau d 2 pi. Then its frequency is equal to p by 2 pi and time period is equal to 1 by f. So, if I write the reverse order, so p will cancel out and I can write 2 pi zeta by 1 minus zeta square. How do I get zeta from here? This is a non-linear expression and therefore, we have already said that zeta has to be between 0 and 1. So, we will make one assumption for lightly damped system. We will say zeta square tends to 0.

Suppose we take 0.3 as zeta. So, zeta square will be 0.09 and 0.91 minus 0.09 is equal to 0.91. Root over of that will work out to be some 0.99 and therefore, we can say zeta square tending to 0. So, with this assumption we get the approximate value and from here zeta is equal to 0. So, we get the damping ratio was as 1 by 2 pi log of x 1 by x 2.

Come back to this diagram, how precisely can you measure this? This is decreasing, but how precisely can you measure it? This value and this value hardly any difference. So, when you try to take a physical measurement, you would not find any difference, but as you go down, you will find that this comes down because of the exponential decay here. So, the best will be, do not consider on the two cycles, but consider n cycles.

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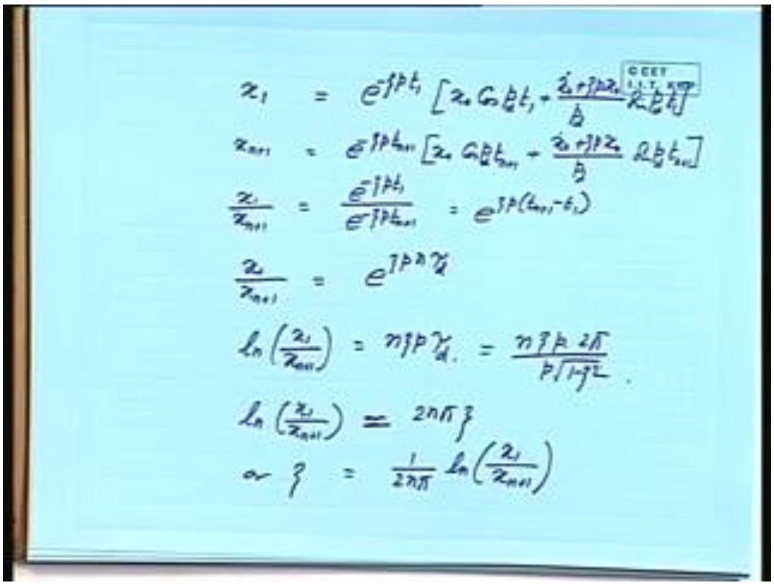

Therefore, the best will be you write say x 1 is equal to and you consider xn plus 1 here, and now you take the ratio of x 1 by xn plus 1, which will be e to the power zeta pt 1 which will be equal to and then we can say x 1 by xn plus 1 n cycles will be there between 1 and n plus 1. Now, if you take the logarithm, substitute the value here and what you get? P P cancels out. You again make the same assumption. Zeta is very small. So, this tends to 0. So, let me say it is equivalent to 2 n pi zeta. So, this is how we try to identify the damping in the system, where the damping coefficient is a small value of. Any questions you have here because we are interacting more. You can ask question. We can answer it also. No. Yes sir. So, we will proceed further.

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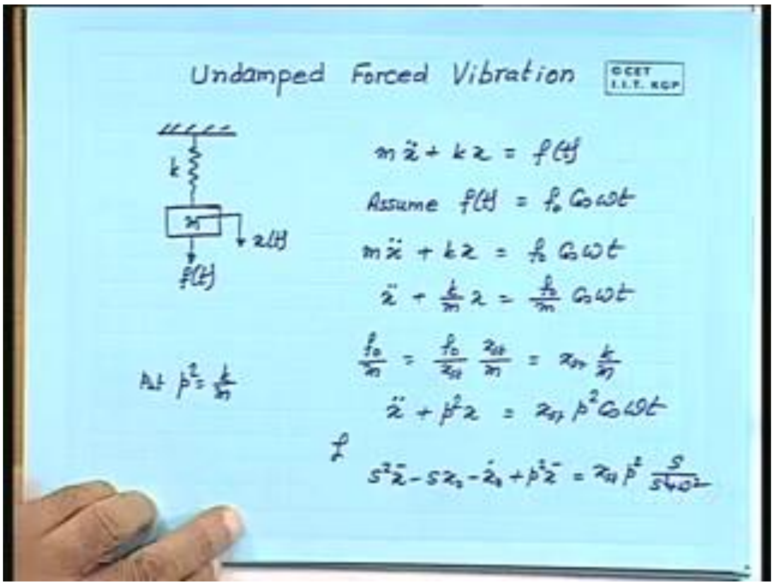

Still have a lot of time know undamped forced vibration. See these are all simple mechanical vibration of single degree freedom system. What we are discussing is I will just discuss few cases only. We have taken undamped free vibration of single degree freedom system; we have taken damped free vibration of single degree. Now, let us take damped forced vibration again of single degree freedom system.

Now, these are important to understand why we say that in a particular system what we should do. It is the same diagram which we have used earlier and we had written the equation also. I am not drawing the free body diagram. We had earlier drawn it and simply reproducing the expression, and this time I am not eliminating this f of t is equal to 0. It is not a free vibration, it is a forced vibration. So, assume the forcing function f of t is given by a harmonic force whose function is given like this f 0 cos omega t.

So, let us assume that now we are taking a single degree freedom system and say have a machine, just a rotating machine which has got some extrenticity. So, it is giving a maximum force of f 0, and moving at a frequency omega. Omega is the rpm you can say. So, if this is the expression, then the differential equation which governs the motion, we follow the same procedure divide all through by m. Now, f 0 by xst, xst by m. Now, f by x is equal to k and therefore, this I can write as f 0 by xst is equal to k. So, k by m into xst, right. Now, I substitute k by m is equal to p square.

So, what happens x double dot f 0 by m, I am writing like this. Xst k by m substituting k by m for k by m p square. So, this is xst p square. Now, you take the Laplace transform. So, s by s square plus p square is the transform of cos pt. So, s by s square plus omega square is the transform of cos omega t.

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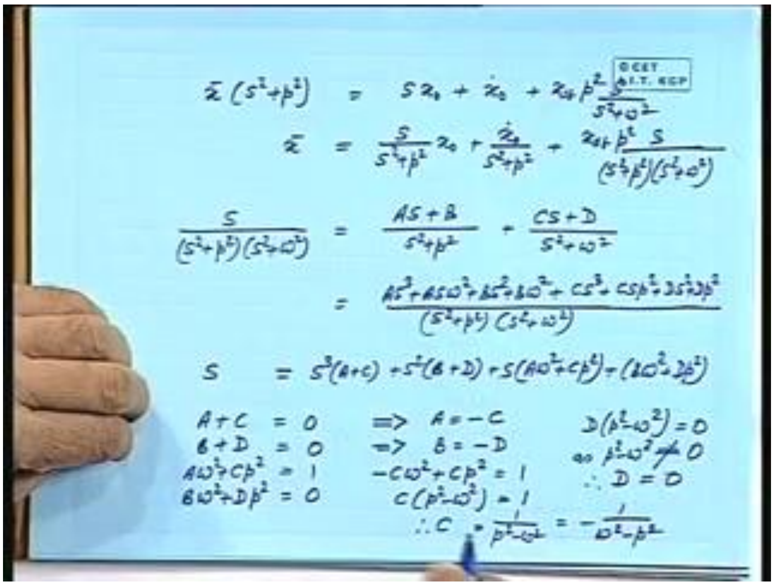

Putting x bar here I get. Now, the expression for x bar dividing it by this, these two terms. No problem, we have done it. Now, the third term is going to create some problem for me. So, let us first try to solve this transform part. Suppose that you have forgotten algebra which you have done in school time. We will try to recollect that.

So, this we will try to break it up and get the denominator one in the form of a square plus b square, another a square plus omega square. So, partial fraction if we say, so let us assume that this is equal to some sort of a quantity, where ABCD are some constant to be evaluated. So, take the LCM on the other side. So, this into this, we have to write here, this into this cross multiplication. Both sides the denominator is same. They can be canceled out and therefore, just a numerator. I should write S is equal to (()). Now, let me collect the coefficients of S to the power in the descending order.

So, let S cube, let me take S cube. So, we have A here, and C there. Then let me take S square B is here and D is here. Then let me have S A omega square and you have Cp square from here, and then without S we are having B omega square and then you have Dp square. I think I have collected all the terms. So, left hand side is equal to right hand side. That mean the coefficient of equal power of S must be same. So, what is S to the power 3 here 0. So, my first equation is A plus C equal to 0. What is S square? Here it is again 0.

So, second expression is B plus D is equal to 0 and then the third expression gives me A omega square plus Cp square is equal to 1 from here, and the last one D omega square plus Dp square gives me 0 once again. Now, from the very first equation, you get that A is equal to minus C. The second equation gives you B is equal to minus D. Substitute these values here. So, A is equal to minus C means substituting the first one in this is minus C omega square plus Cp square is equal to 1 or Cp square minus omega square is equal to 1. Therefore, C is equal to 1 by p square minus omega square, or one can say 1 by 1 by this is also equal to this.

So, C is equal to this, A is equal to that. What about B and D? Substitute minus D for B here. What will you get? You will get here minus D omega square plus Dp square. That mean D omega square D into p square minus omega square. So, let us write D into p square minus omega square is equal to 0. Now, as p square minus omega square is not equal to 0.

There are two distinct frequencies. One is the natural frequency, another is the forcing frequency. So, they cannot be equal to 0. This difference I mean they are not equal. That may be a special case when if they are equal, but in general, they are not equal and therefore, this is not equal to 0. Therefore, D is equal to 0. If D is equal to 0, B is also equal to 0. So, now, we have evaluated the constants.

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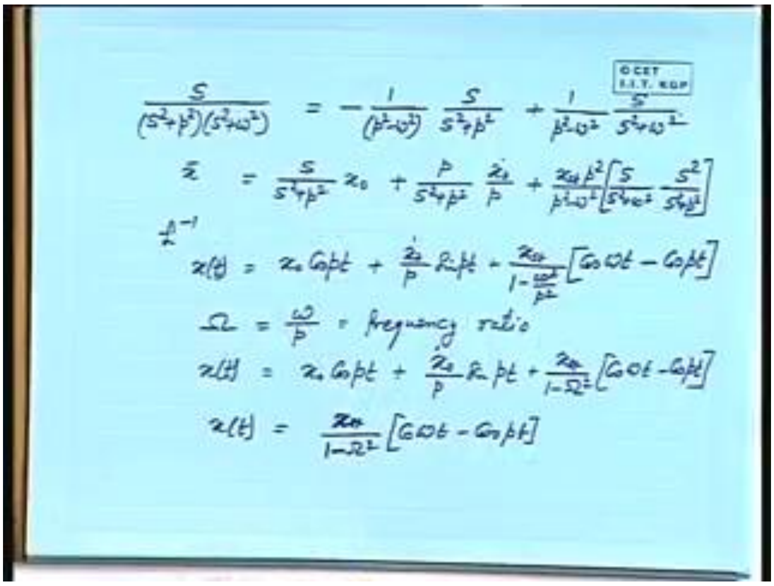

We can write that S by S square plus p square into S square plus omega square is equal to A is equal to minus C and minus C is equal to 1 by this. A is equal to minus C. So, let me write minus 1 by p square minus omega square into and either plus or minus C is equal to this. So, it is broken up. Now, let me go back to this response expression.

Now, which one should I write? One is negative, one is positive. It does not matter. Let me write. Now, we will see that this p square minus omega square is common. So, I take it out and then let me first write this and then with a negative sign I put this here. If any mistake is there, please point out otherwise some disaster may take place. I suppose it is.

Now, let me take the inverse of it. To get the response is a very simple thing. I divide with this p square, this term here. So, I write it in this fashion. Now, this is cos omega t and this is cos pt. So, this is the full response. Now, we may introduce another. This omega by p, this is the forcing function and frequency and this is the natural frequency. So, usually the notation capital omega is used for small omega by this which is the frequency ratio introducing this. We can write it in this fashion. Right up to this is now here you see in this response, we are getting the initial conditions response, which is for the free vibration, and then the forcing function part of it.

Now, here we put an argument. The argument goes like this that every structure has got inherent damping in it and therefore, the terms associated with the initial conditions are totally dependent on natural frequency. It will die down with time as we have seen for a free vibration case, where x 0 cos pt x 0 dot by p sin pt. The whole thing dies down after some time. After that what will happen here? So, the first two terms here will vanish after some time, but this term will continue because this is the forcing function term here. In that response, x of t you find that xst by 1 minus omega square omega square capital is the frequency ratio cos omega t minus cos pt.

So, this response contains or there is a presence of natural frequency in the response of the forced frequency. That means after sometime, you can say that x of t will be given by this. Now, from here we try to figure out what will happen. You just concentrate on this expression here, and let us see that the natural frequency of the system which is p and the forcing function which is omega, they are very close to each other. If they are very close to each other, then what happens, say when a motor starts, you are starting from 0.

Now, suppose the natural frequency of this table is p and I am running motor here which can have an operating speed of omega. So, many rpm, but when I start it, it starts from 0. So, when it starts from 0 and I take a condition where I say that the operating speed of this motor which is omega a straightly less than p. It has not crossed p. It is reaching p, then what happens? So, omega pi p which is the ratio here, this is slightly less than 1 and when you square it and deduct it from 1, you will get a small quantity less than 1.

Now, what is this xst? The static displacement under that particular force f 0 because the forcing function I have assumed is f 0 cos omega t. So, if there is no frequency and I assume that f 0 is the maximum force which is static equivalent. So, under that I am getting this x static. So, this x static divided by this quantity which is less than 1 is what quantity is much more than 1 a x static. So, if suppose I take the ratio of the frequency as 0.9, what do you get here? We get 0.9. If I take this is xst by 0.2. That means this part will be 5 times this value. Now, here I assume that this will become unity or whatever it is. So, I can expect this response to the tune of 5 times or 10 times if the case may be of the static deflection.

Now, when I have designed my structure to withstand that static load may be with certain factor of safety, I do not say is over loading. It is the un-force in-forces. So, actually speaking factor of safety, it should not be taken as a load factor sort of thing and with that loading if I have assumed that the structure is safe, it does not cross the elastic limit. What happens if this particular forcing function is applied to the structure, where the displacement goes 5 times and again if you consider it to be say elastic, the stress will increase by 5 times, but what happens that you are not having that much of margin. It will just cross and go to the non-linear range. It goes to the plastic limit and the structure may fail. So, that is the bad effect which we are bothered about, and we say that when it is approaching the forcing function is the forcing frequency is approaching the natural frequency. This is what happens.

If it is the same, then we can see here that this will tend to 0. If omega is tending to unity, this will tend to 0 and once this tends to 0, this entire thing will tend to infinity and there will be a catastrophic failure. Fortunately system is having some damping. We are considering here undamped system, but the system is having some damping. So, it will try to limit that and shift a little bit. Now, in this condition when it is approaching, then what will happen?

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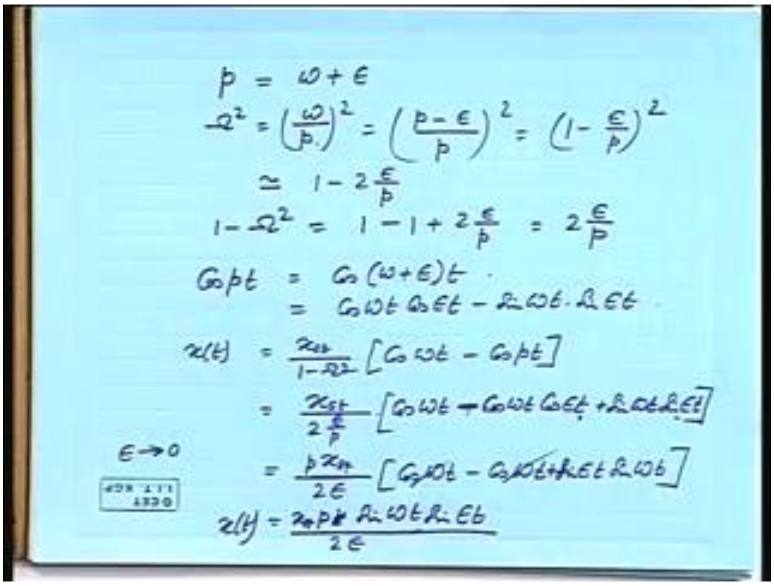

So, let us say that the difference between p and omega is some small quantity epsilon. Omega is approaching p. So, p is equal to omega plus epsilon. So, omega square which is omega by p is equal to this omega. I will write as plus epsilon square by p. Square term will come epsilon square is a very small quantity. So, I will try to neglect that. So, it is approximately equal to this quantity that I am calculating for this part here, and 1 minus omega square, it is now cos of pt which is again in this expression here.

So, let me put instead of p, this value and this is your cos omega t cos epsilon t minus sin omega t sin epsilon t. So, let me take this expression x of t xst by 1 minus omega square cos omega t and substitute these values here minus. So, when epsilon is tending to 0, this cos epsilon t will tend to unity and sin epsilon t will tend to epsilon t. So, as epsilon tends to 0 in the limiting condition, let me take this p up. This cancels off and this E will also cancel out. Pt sin omega t xst, this is what is the response.

Let me keep it here. That will be nice. Instead of epsilon cutting, let me put both sign terms here. I do not want to remove them. So, what I am going to get is xt into p which is supposed to be constant, and I take this small e. So, what is happening here is a multiplication of two harmonic terms. One is sin omega t, another is sin epsilon t.

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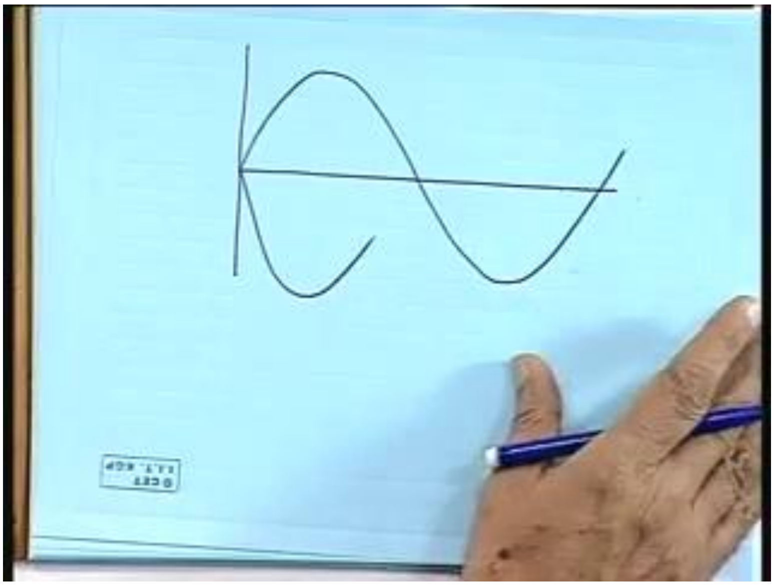

Now, if you try to plot this here I have drawn.

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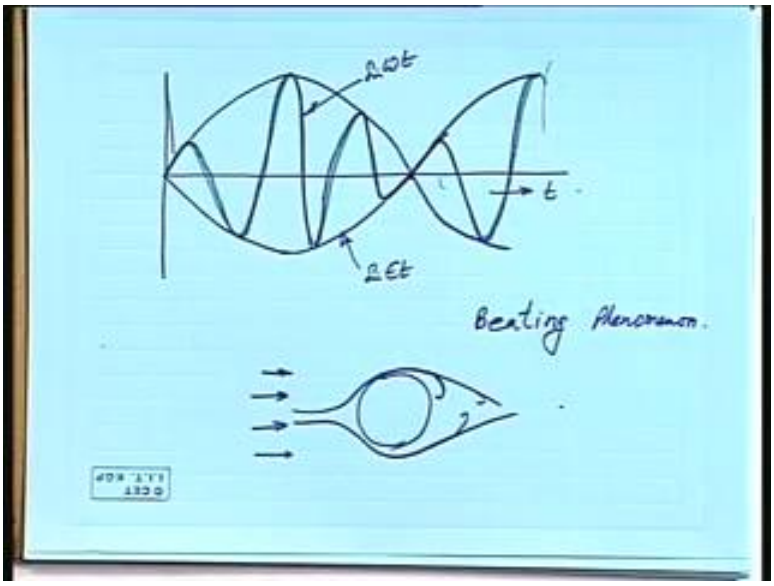

This is the term. You see these are the two terms here sin omega t sin epsilon t. So, epsilon is smaller than omega. So, obviously this will have a larger time period, this will have a small time period. So, what I am trying to draw actually is your motion will go like this which is product of two terms. So, this is what happens now.

Example of this I will try to give you. I do not know you may laugh at that many times when you are school children. If you happen to walk down to the school and you will find that telephone poles are hollow poles, and during the rainy season, fine evening wind is blowing or some such things. You are coming back from the school and pass by that particular hollow telephone pole and you hear some noise, some sound is coming and it goes up and then in a musical tone, it comes down and again it builds up and then again it comes down. At that time, nobody knew what that was, but today after understanding this part, we will find that it is this phenomena which is giving you that sound.

Now, what happens is that particular air column inside that pole has got a particular frequency. Now, when the wind is blowing, you have the wire, the telephone, cable or telephone wire which is connected to the two poles. Wind is passed about that. You now consider a circular cylinder in a uniform field of fluid flow. So, what happens is a real fluid and once it is a real fluid, you have a cylinder and there is a flow pass this cylinder. The stream line will go like this because it is a real fluid. There is some boundary layer separation will take place and some sort of a eddies will be formed, and these eddies are known as one common vortex, and they come in an alternate manner.

So, they try to generate some sort of (( )) in the cable. That frequency gets because it is connected to the pole gets transmitted to the pole there. Now, the pole is subjected to this forcing function. It has got its own natural frequency and the forcing function generated by the wind blow is coming to the telephone pole. Now, if that is p and this is omega, and the difference between them is epsilon, then the phenomenon is something like this.

So, when it builds up, you hear that noise coming up. When it is going down, then it tapers down, then again it builds up and then again it tapers down. So, this phenomena you have experienced the same thing in a diesel engine also. When it is idling, mostly when the buses they stop at a level crossing or a red light, then you will find the glass shutter which is very loose. There it vibrates like this or the gear changing lever. It is static and then suddenly it vibrates like this and then again it comes back. So, what happen is that the natural frequency of that particular item and the idling frequency or the idling rpm of the engine, they are very close to each other. So, this amplitude increases and then it goes down and then again it builds up. So, this type of phenomenon is known as the beating phenomenon.

Now, this is a very usual thing which we keep on experiencing in many items. We try to ponder our self that why it happens. Now, the answer is here why it happens. It happens because the two frequencies are very close to each other, and if you have some sort of an absorber there. Then that absorber say for example, the glass is rattling and then if you are having a nice channel there made of good rubber, then that will absorb that noise. It is bound to be there because any continuous material is going to have a continuous frequency contained within it. Some of them build up, some of them are not that important.

So, which of them will be excited will depend on the forcing function, and once it is excited, then it becomes a very difficult part to cool it down. Therefore, such packing or whatever you call it channel in your car glasses, sometimes you know you will find say for example we had a problem of Harshavardhan. Harshavardhan the inner bottom, no sorry it was the outer bottom within the engine room used to break after every (( )). It is to crack. The reason was not known, but the ship's captain and chief engineer, they found out one very nice system. They say that when we are sailing out, there is no problem, but when we are coming in the ballast condition, then only it cracks on the return path and not while going.

The reason was that when they are leaving, then the double bottom is pressed up. The two plates are acting as a sandwich construction in between is the liquid mass. After then the fluid starts getting consumed, but still it does not come to a particular level. So, there is a decrease in the mass of that. First of all the packing has gone. Second, the mass starts depleting because of the fuel consumption and at a particular mass level, by a bridge and the vibration is such as it is being disturbed by the machinery inside.

Now, which was the machinery that he could not tell us whether it was the genset, or some pump set, or the main machinery, or the thrust block, or whatever it is, but what happened that once the mass is reduced to a particular level, then the natural frequency property of the structure changes. Now, it is continuously changing at a particular level. It gets very close to this and what happens that the displacement goes up like that, and then it cracks. So, these are certain realistic thing. Anyway, we will continue further. I think today I have given a slightly heavier dose of mathematics.