Strength and Vibration of Marine Structures
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Lecture - 3
Deflection of Structure Beam – I

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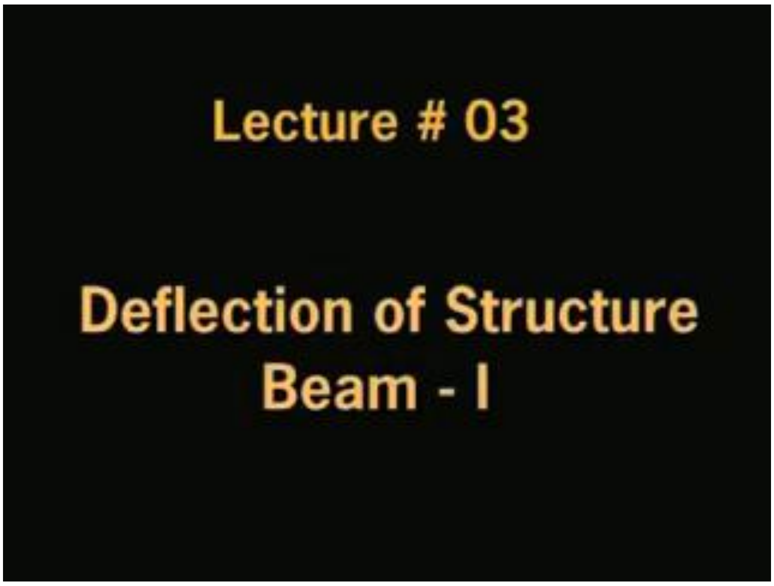

So, today we will be discussing deflection of beam.

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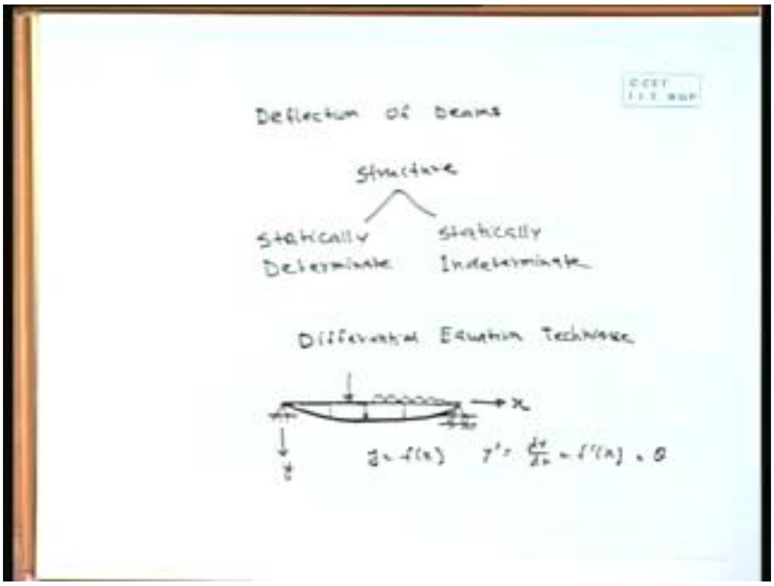

So, that is deflection of beams; that is the general topic of today's discussion. In our previous class, we are talking about stresses, and we were tried to get the idea that stress is a very important quantity. All the time we have to determine, calculate stress, to have some idea regarding the structural safety, because that stress should be within certain limit, and that limit, we have defined as allowable stress.

Now, deflection is not that important like stress, but there are some situations where we may require getting the information about deflection. In that case, the knowledge of calculating deflection at some point is necessary; so, that is one aspect. Apart from that, there is another requirement, which I should say it is much more important. In that context, I can explain in that manner, say, a structure, we divide the type of structure into two categories: one is called statically determinate problem; and another type of problem, we say statically indeterminate problem.

So, if we say a structure in general - structure: so one type is statically determinate type; another is indeterminate type. Now, what is statically determinate type and statically indeterminate type? Last class, we were talking about equation of statics and we have seen there are three equations, if we handle a two-dimensional problem or a plane problem. Summation of all the forces along x direction we have taken 0; summation of all the forces along y direction equal to 0; plus we can take moment at any point of all the forces it will be equal to 0. So, these are basically equation of statics if we think in terms of plane problem.

Now, there are some structural problems, where the unknowns, which are basically the reactions which we are getting from the supports; those quantities we can determine with the help of equation of statics. So, it can be determined by the equation of statics, which is statically determinate. Already we have taken a problem - two supports; one is hinge; another is roller; we got three unknown reactions; and there are three equations we have applied; we obtained the support reactions. So that problem was absolutely a statically determinate problem.

Now, all the structures will not be statically determinate. Say, there might be more supports, may be both the supports hinge. So, you will get four reactions; you may get five reactions; you may get more number of reactions; or not necessarily you cannot force that support should be such the reaction number will be always within this number

3 lk. Depending on the practical solution, it may be any number. So, it cannot be solved by only the equation of statics. So, that category of problem we say statically indeterminate problem. So, it cannot be determined with the equation of statics. So, that is why we say it is statically indeterminate problems.

Now, the question is - how we will solve a statically indeterminate problem, but real structure may be statically indeterminate; it can be solved. So, we will require more number of equations. So, we have three equation of statics. So far, the problem is within a plane. Say our unknowns are five. So we will require another two equations; and those two additional equations, it will come from the information regarding the deformation of the structure. So, structural deformation is nothing but the deflection of the structure. So, deflection of structure, we will try to utilize to get some more information; that more information will be reflected in the form of more equations.

So, structural deformation or structural deflection those relationships will be utilized in the form of additional equation in addition with the equation of statics. And that will help to solve the entire problem. So, ultimately what will be our problem? Our problem will be, if we are interested only bending moment, shear force, or stresses, and problem is not determinate, we have to go for some calculation of deflection of the structure, because that will help to give some equation.

So, that is, I think, one of the major requirement for having some understanding regarding the deformation of a beam or deformation of any structure. So we can start with a beam, because it is one of the very, very elementary structural element. So, that concept can be extended to some other complex type of structure like, say, frame problem or other type of structural problem.

So, here our starting point is deflection of the beam. And we will start with statically determinate problem. Everything will be known; at least the forces, reactions. So, how to find out deflection, that part we will study. Once that understanding will be developed, with that we can go ahead for a statically indeterminate type of problem for finding out the unknowns, so that we can determine internal forces, internal moments; simultaneously we can find out the deflection of the structures also. So, it will be a integrated process; deformation calculation, stress calculation - both will be carried out simultaneously.

Now, if we try to find out the deflection of beam, rather there are many methods available we can use, but here my attention is to cover only very, very basic type of methods. At least I will try to cover two, three, maximum four types of method; there is no point of covering all the type of methods available in the market. What are very basic type, very fundamental type, which can be used without much problem we can go ahead with that.

Now, I will start with a method; it is called deferential equation technique. So, this is defined in a different manner; sometimes it is defined differential equation method, differential equation technique or integration of differential equation, integration of differential equation technique. So, basically it is dealing with some differential equation of the beam problem. Here, when we will be handling with differential equation, we have to start from the deformation of that beam, with some bending moment, with shear force, with load, and all those quantities.

So, before coming to that exact treatment what is done in this particular method, let us try to find out the relationship between the different quantities. The different quantities just now I have mentioned: one is the load, shear force, bending moment, slope, deflection. So, they are not separate quantities; they are closely associated with each other; and some dependencies are there in between. So, let us try to get the idea how they are related.

Now, we can take a beam like this; can take any beam; there might be some loading here. So, it is just I have drawn a simply supported beam; means beam one end is hinge support, another is a roller support, and combination of that it will give a stable structure, determinate structure, and some loading. Here hinge will provide two reactions; roller will provide one reaction. So, three reactions. It can be determined with the equation of statics.

Now, if we try to make the hinge or the roller the left support, what will happen? Number of unknown will be less; two your equation will be more; in that case structure will not be stable. So, all the time we cannot make it as minimum as possible. So, a minimum number of destiny is also necessary, because any structure, basically, we want to fixed in its place; so, it will require some support. So, at least three direction it should be resting; it should not move this way or that way; it should not be stable, but rotate. So, all the possible movements we have to restrain again.

So, these are the minimum thing, and if we go beyond minimum, then it will be statically indeterminate. Now, here, I have drawn a line to represent the beam. So, basically, a beam has a very small dimension along the depth and width compared to the length. So, length is the predominant dimension. And normally, all the breadth and depth quantity, we more or less represent in a compact form; we take high value. So, everything is incorporated there for representing the bending.

So, the variable here is only along x and this line is basically the neutral axis or neutral line of the beam. Now, under the load it will undergo some bending. So, we are trying to plot the deformation of the beam, with its neutral axis; we sometimes we say it is a elastic line, because whole thing is elastic analysis and it is a line; we say it is a elastic line. It will undergo some deformation in an arbitrary manner depending on the type of load. Now, if I put here some axis system, say x along the axis of the member, and y is perpendicular to that. So, we can say the deformation of the beam or deflection of the beam, it can be represented as, say, y equal to function of x. So, y at any point, say, at a distance x, so this is x that will be the y. Now, it is different x, it will be different y; and it is another x, it will be another y. So, x is a variable quantity; depending on x, you will get the value of y. So, y is a function of x. Now what is y? y is the deflection.

Now, if I take the derivative, say, y dash it is nothing but your dy by dx or f dash x. If we have the expression of the function, we can take derivative. So, it will give the slope of the line. Slope of the elastic curve that we say it is sometimes, we write as theta; theta is the slope of the elastic curve. Now, the y is the deflection; its derivative is theta. Now, if we take further derivative, we will get d 2 y dx square or y double dash x or f double dash x; that quantity will have some relationship with moment.

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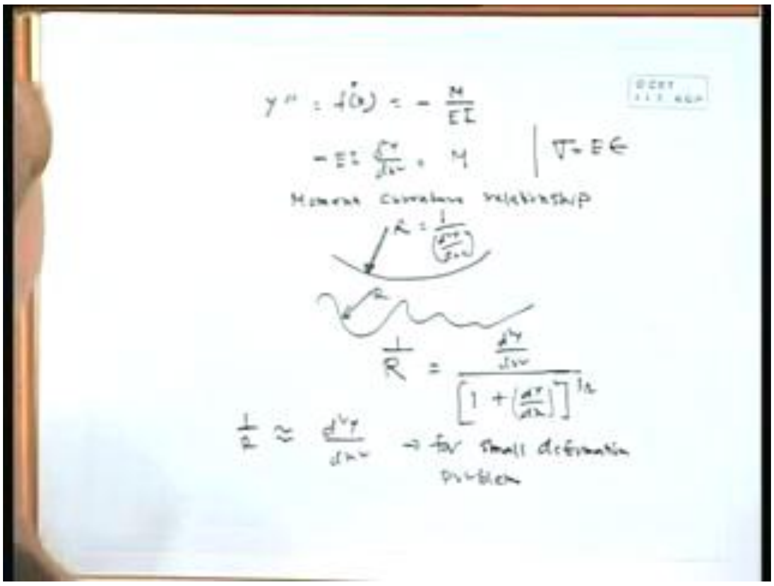

So, if we take y double dash or this one; this quantity will be equal to M by EI . So, for slope in first derivative, second derivative is, this term, basically the curvature and that curvature will be equal to the bending moment divided by EI. EI term already we have defined, which is basically the flexile rigidity of the beam. So, M is the moment. And that expression is very, very important. Sometimes we write as minus EI d 2 y dx square equal to M. So, that equation in a different fashion we have written.

So, this part d 2 y by dx square - this part - is called basically curvature. It is basically curvature. Curvature is inverse of radius of curvature. The curve will have some radius of curvature. So, inverse of that will be curvature. So, if we take a straight line, radius of curvature will be infinite and curvature will be 0. So, straight line no curvature, but if we have some curvature, so we will have some finite value of radius of curvature. So, this d 2 y by dx square will be the curvature and M is the bending moment. A is the flexile rigidity, and that relationship is called moment curvature relationship.

Now, just now, I have mentioned d 2 y by dx square is curvature; curvature of the deformed shape of the beam or the elastic line. So, if we say it is a deformed shape of the beam, now anywhere if that is the radius R. So, d 2 y by dx square it will be 1 by d 2 y by dx square. So, this is the radius of curvature and this is d 2 y by dx square is curvature. So, the reverse term of that is your radius of curvature.

In that context, I should say the expression of radius of curvature and curvature is not exactly the same. For a very arbitrary curve, say, any arbitrary curve, the R if we try to represent, so R is basically your d 2 y dx square divided by 1 plus dy dx. So, that is the general expression for that; that will be 1 by R. So, 1 by R is the curvature, it is d 2 y by dx square 1 plus dy dx whole square whole to the power 3 by 2 or 1.5 whatever you can say. But here, the denominator part we are not taking; that reason very simple, because we will be dealing with structural problem, and structural problem deformation will not be very, very large; it will be very small deformations; and when deformation is small, its rate of change is basically slope dy by dx. Slope will be much more small and square of that, it will be much more small. So, this quantity will be practically going to be 0; we can neglect that.

So, this part if you draw, it will be 1 to dy 3 by 2, it will be basically your 1. So, approximately, we can write 1 by R is your d 2 y dx square for small deformation problem; and in our case it will be a small deformation problem. So, we can simply write 1 by R equal to d 2 y by dx square; and this d 2 y by dx square, they are related to basically bending moment, with a factor of EI, and some minus sign will come.

Here I want to tell you we can compare that equation with your normal stress-strain relationship. Say normal stress-strain relationship is, say, here I am writing in the side sigma equal to your E into epsilon. Now, sigma you can compare with M, and strain you can compare with your curvature, and E you can compare with EI. So, it is basically something similar or analogous to your stress-strain relationship. So, it is not stress; it is moment; moment is nothing but one of the stress resultant, because throughout the depth of the beam, your bending stress will vary; it will be maximum, 0, minimum. So, total effect if you take, that is basically the bending moment. So, M is the resultant of the stresses. So, it is stress; one of the stress resultant. So, that stress resultant we can compare with stress. And here strain, we can compare with the curvature; and E here something related to EI. Now, we have written minus. Why we are writing minus here? It is basically a matter of sign convention.

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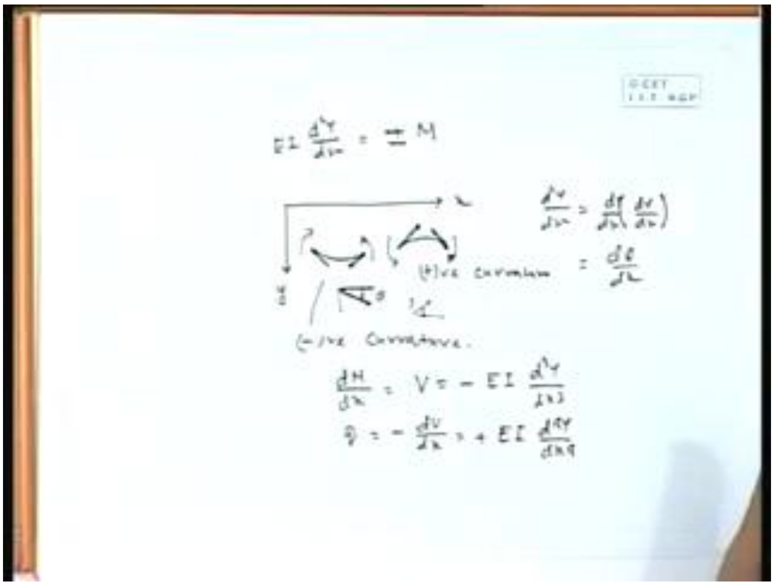

Sometimes we write here, say, d 2 y dx square EI. We say somewhere you will find the relation is something like this. So, you can take plus; you can take minus; where we will take plus, where we will take minus, that will depend absolutely on the sign convention. Say, I am taking a axis system, whatever we have taken earlier. So, it is x; it is y. Now, there might be a bending phenomena like this; there might be a bending phenomena like that; so, earlier we are trying to define a sagging/hogging type. Now, here there will be a type of moment, and here there will be a type of moment. Now, xy d 2 y by dx square is what? Your d 2 y dx square we can say it is d dy dx of dy by dx; and dy by dx is what? It is basically theta. So, we can write it is d theta by bx.

Now, what is theta? Theta is basically if we draw a line here that will be the theta. So, it should be theta. And theta will be positive when the way we have written or drawn it is basically theta. If we increase x, your y value is increasing, but if you draw like this with the increase of x, y is decreasing; y it is going towards the negative side. So, that will be a negative slope; that will be a positive slope.

Now, if I take that type of problem, here slope will be positive and here slope will be negative. So, d 2 y by dx square is rate of change of slope. So, it will be… it is changing from your positive slope to negative slope. So, curvature will be in that case negative, because here slope is positive, here slope is negative. So, whatever negative value minus that positive quantity will be negative divided by dx. So, here curvature is a negative curvature. But here this is a negative curvature, negative slope, and that is a positive slope; so, positive slope means with increase of x the value is increasing in the positive direction of y, and here if you increase x, y part is decreasing. So, this is a negative slope; that is a positive slope. So, rate of change of slope - this minus this divided by length dx - that will be basically d theta by dx or d 2 by dx square. So, this will be a positive curvature.

So, this should be a positive curvature, and this should be a negative curvature, because if it is a sagging one in terms of xy system, your curvature is becoming negative - d 2 y dx square is becoming negative - because a slope we have defined, which is plus and minus, and change of - rate of change of slope - that is basically d 2 y by dx square. That is becoming negative here; this is becoming positive here for the case of hogging.

Now, it is a matter of sign convention. If we take sagging moment is positive, then here our this side sign will be minus. If we take as hogging is positive, we will take the sign of M will be plus. So, it is again a sign conventional. So, normally, last day we are trying to take sagging as positive; we can go on with that - sagging as positive. So, here at that level we will write moment with a minus sign. So, we have started y, its first derivative is slope, second derivative with EI will be minus moment.

Now, next part is your dM by dx. If you take the derivative of moment, you will get shear force or if we put here, here it will be minus EI d 3 y dx cube. So, shear force is what? Derivative of moment. What is moment? Moment is your minus EI d 2 y by dx square. So, further derivative of that and minus sign we have to incorporate.

And the last term is the loading Q is equal to minus dv by dx or minus minus it will be plus; it will be EI d 4 y dx to the power of 4. So, what is Q? Q is basically the intensity of the load acting on the member; member is here; every problem it is a beam member. So, we are putting a load; we are getting a displacement. In between, your slope is there, moment is there, shear is shear is there, and they are related like this.

So, if we know the equation of the deflected curve, we take the first derivative, we will get slope; second derivative multiplied by minus shear, you will get moment; an another derivative you will get shear force; another derivative you will get how much is the intensity of the load. So, all the three, four quantities: deflection, slope, moment, shear, and load - rather five quantities; they are related like this.

Now, sometimes we use some of the equations for finding some of the important quantities, say, moment and shear force - relationship is just one order derivative difference. Say we want to get the maximum bending moment within a particular region. We have the equation and equation is not very symmetric type. So, from the physical look, you can get the idea it will be at the center, it may be somewhere intermediate, but you have to determine what will be the maximum value and where it will be located.

You can find out the value of moment, because if you substitute x equal to how much, from the equation you can get it, but you require the information where will be the location - means what will be the value of x? So, in that case, that will be very useful where moment will be maximum, so shear force would be 0. So, any maximum or minimum quantity it is derived. So, it will go to a peak and come, come to the lower side; means there the slope means derivative will be equal to 0. So, shear force equation if we equate 0, you will get the location, the value of x; at what x value shear force is becoming 0; corresponding to that value, your bending moment will be maximum. So, that x value, you can substitute in the expression of bending moment and get the maximum value of the bending moment there.

So, this information sometimes you will be using for finding out some specific problem. So, these are very, very important relationship: deflection, slope, bending moment, shear force, and the load distribution. So, they are related with that.

Now, in connection with our primary theme of today's discussion is - deflection of beam. The first line on this page, that is moment curvature relation EI d 2 y dx square equal to plus or minus M, with that we will try to solve deflection of beam using differential equation technique, because this is the differential equation, and the differential equation we will utilize for finding out the deflection of the beam. Now, we shall take some simple cases and try to explain how the method works.

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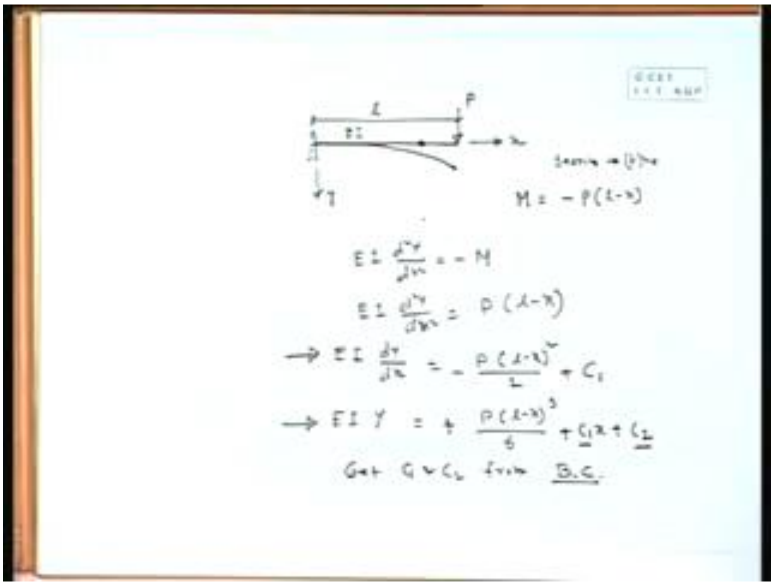

I can take a cantilever beam problem, which is quite simple type of problem. Say, load is acting at the free end, so it is P, and it has some length, some E values, some I value. So, this is a beam, left support is fixed or clamped; right side is free; at the free end there is a vertical load P; the total length of the beam is L; it has a material property E; cross sectional property is I, I means it is second moment of area or moment of inertia of the cross section. It is a uniform cross section; everywhere it is EI.

Now under the load, the deformation will take place like this. At at this support, there will be no deflection; there will be no slope. So, it will go; gradually it will deform; at the tip there will be a maximum, deflection maximum slope. And here, if we take some axis system like this x, and it is y. We can start with our differential equation; differential equation is nothing but your moment-curvature relationship. So, we can write EI d 2 y dx square equal to minus m.

If we say sagging is positive, it will be minus M. Now, what will be the expression of M? So, M part; M is bending moment at any cross section; say this is xy. So, if we take any value of x, what will be the moment? If it is x, this part will… be total length is L; L minus x. So, P into L minus x. And P L minus x will be your hogging moment. And sagging moment will be minus of that. So, it will be P L minus x minus. If sagging is considered to be positive, say, here, any, any station. So, P is the force and distance is L minus x. So, P L minus x.

What will be the type of moment? The moment will be in that direction. So, that moment will generate stress - tensile stress - at the top. So, it is something like this. So, hogging moment, tensile stress will develop at the top. So, if we take sagging as positive, we have to get a moment. So, the moment we are getting, it is basically negative moment. So, it is minus; minus is, it is not sagging; it is hogging; and the value will be PL minus x.

Now, here, if I substitute that value, this will be equal to EI d 2 y dx square; this minus minus, it will be plus; L minus x; we could take hogging as positive, in that case we could write this is plus, and this we could write plus. If hogging is positive, this would be plus. And here also it should be… it is basically hogging moment, so it will be plus. And if we take sagging also, in both the case it is negative.

In any case, if you start sagging as positive or hogging as positive, this equation will be automatically coming to the same equation. Now, this is our differential equation. So, EI d 2 y by dx square equal to PL minus x. So, this equation we have to integrate. So, if you integrate, this is a very straight forward equation; it will be dy by dx. And it will be P L minus x whole square divided by 2, and there will bea minus sign this side, because with x there is a minus quantity, plus there will be some constant, say C 1.

Now, next step we can take a further derivative. It will be EI y; it will be minus Pl minus x. So, 2 will be now 3, and below 3 will come. So, 3 into 2 it will be 6, and from x another minus will come, and this minus and minus will be plus, and here it will be C 1 x plus another constant you will get, say C 2.

So, integration of the differential equation already obtained. In first integration, we have got first derivative; from second derivative, in the second level, we have got without derivative, direct y we got. So, the last line is the expression for y deflection; previous line is the slope; previous to previous, it is basically the curvature relations here.

Now, in that process we have got one C 1 and C 2. These are the constants. And these constants are unknown here. These constants we have to determine, in terms of, say, L E P, something, whatever information we have related to the structure. So, here it is a beam problem. So, EI, l, P these are the quantities; so, in terms of those we have to determine. And those quantities can be determined using the boundary conditions, where the two ends there are some conditions, and those condition will be utilized, those are, we say, it is boundary conditions.

Say, here, this is a fixed support. So, y or the deflection is 0; plus this point itself, slope is equal to 0; not necessarily it will be all the time at the same point. If we have a simple supported beam, deflection will be 0 here, deflection will be 0 here. Now, we have to get C 1 and C 2 from your boundary conditions. So, BC is boundary conditions. Now, we have the equation.

Let us put the condition, say, first of all, at this level we put say at x equal to 0, your d 2 y by dx equal to 0; and the second level we will put at x equal to 0, y is equal to 0. So, first we will get C 1. C 1 value you can substitute here. And the second level, once you will put that requirement for deflection is equal to 0, we will get C 2.

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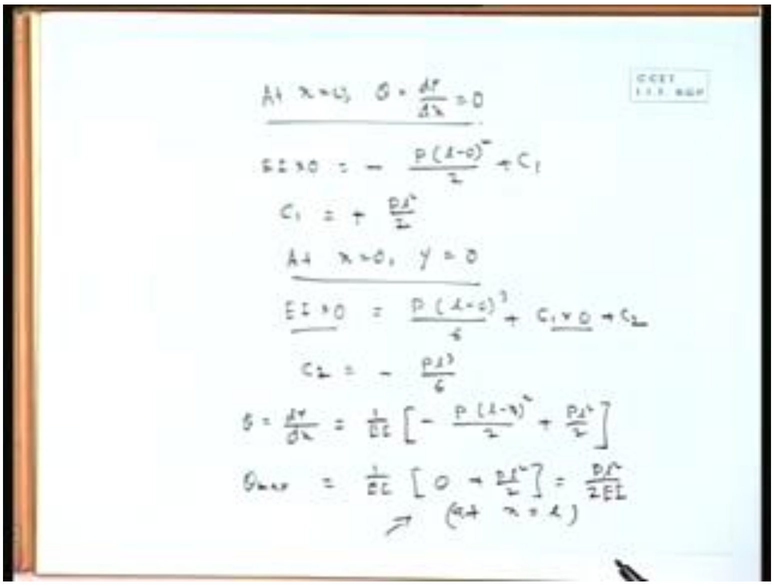

Now, if I take at x equal to 0, theta equal to dy dx 0. So, that condition will give your EI into… it will be 0, because dy dx 0, minus P l minus 0 square divided by 2 plus C 1. So, left part will be entirely 0; and C 1 we can write in terms of minus Pl square divided by 2. Once C 1 is determined, we will take the last equation. So, at x equal to 0, your y equal to 0. So, that condition will give your EI into 0; that will be equal to your P l minus 0 cube divided by 6 plus, C 1 part you can substitute or you can keep it C 1 multiplied by x, x will be equal to 0 plus C 2. So, these two terms are now 0.

So, C 2 will be minus Pl cube divided 6. C 1 will be positive. C 1 will be positive, because this is a minus sign it will be plus. C 2 will be negative. C 2 will be negative.

Now, once C 1 and C 2 we have, we can write the expression of slope; we can write the expression of deflection. Say if we write the expression of slope, it will be theta equal to dy by dx; that will be equal to 1 by EI minus P l minus x square divided by 2 plus C 1; just now we have determined it is Pl square divided by 2. So, in last page we have made minus P l minus x square divided by 2 plus C 1 is PL square by 2. EI part we have put right side with a one-way form. So, that will be the expression of that. Anyhow, this equation can be rearranged, because if you break Pl square by 2, that part will cancel. So, you can put it like this or in a different form. Even different book, you may get expression in a different manner, but ultimately it will be the basic terms; it can be generated from there.

Now, if we want to find the maximum slope, maximum slope will occur at the free end. So, we can get theta max; theta max will be 1 by EI. We can put x is equal to L. So, this part will become 0, and that will be Pl square divided by 2, or we can say Pl square divided by 2 EI, and this will be at x equal to l. So, at the free end we will get the theta will be maximum and that value will be P into l square divided by 2 EI . So, this term is becoming 0, say x. Theta at x equal to L. N slope, maximum slope that will be Pl square by 2 EI.

Now, this is one quantity Pl square by 2 EI; that is a cantilever subjected to a load at the free endc we are getting the maximum slope there, Pl square by 2 EI. This is one of the very important terms. There are some very standard. This is one of the standard cases and the value is one of the standard quantity.

So, if we try to solve a relatively difficult problem, some of the quantities like this PL square by 2 EI - that type of quantity, we will derive some of the quantities here. Those quantities it will be utilized later on. Say, a beam subjected to fully uniform redistributed load - what will be maximum moment at the center? Omega l square by 8 and that part frequently we use that. We need not take a piece of paper, try to derive what will be that maximum. So, it is just instantaneously we say it is omega L square by 8.

So, if we try to evaluate deformation of a structure, deflection of beam, some of the quantities at least we have to keep into our mind; otherwise, all the time we have to keep a book, and take the help of that - what is the value of maximum slope here, maximum deflection here.

Now, in that process, I am trying to find out some standard relationship plus the procedure I have explained, by that time I think you got the feeling.

So, the starting point is the differential equation, and differential equation is nothing but your moment curvature relationship; that is EI d 2 y dx square equal to M; and M that will be the expression of the bending moment diagram - the expression of the bending moment at any arbitrary section. So, it is a function of x. Once you get that expression, you have to integrate twice, you will get two constants; and these two constants you have determine with the help of boundary condition. Say slope 0, deflection 0, some information you will have for a particular structure.

Now a slope expression already we have obtained on the maximum slope. So, next step should be the deflection. C 1, C 2 we can put in the equation of your expression of y. It will give the equation of the deflection of the beam and we can find out the maximum value. Normally that will occur at the same point of free end of the beam. So, let us come to that part.

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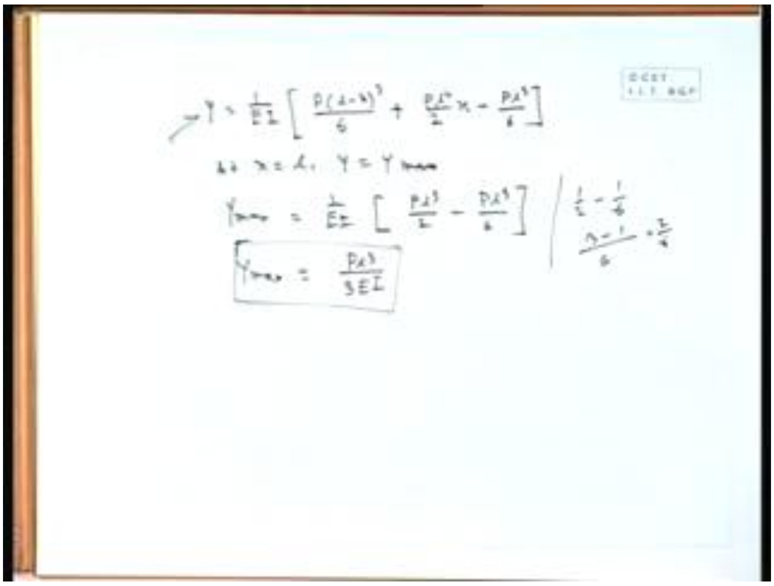

So, it will be y equal to 1 by EI. And here we got Pl minus x cube divided by 6 plus C 1. We got Pl square divided by 2 into x and C 2 we got Pl cube divided by 6. So, we have just substitute C 1 and C 2 in the expression of y. There EI part was in the left side, that part we have put on the right side; in the one-way form. So, that will be the expression.

This can be arranged in a different form, in a way you like; it may be given in different form in different places.

But if we are interested for finding out the maximum value of y, so at x equal to l y equal to y max. So, y max it will be 1 by EI. The first term will be equal to 0, because x will be l, l minus l, it will be 0. So, there it will be Pl cube divided by 2 minus Pl cube divided by 6. So, it is basically both are Pl cube. So, it is a half minus 1 by 6. So, half 1 by 6. So, if I just make a small calculation. So, it will be 6, means 3 minus 1. So, it will be 2 by 6 or it we can say one-third. So, we can write Pl cube divided by 3 EI; that will be the y max. So, this is another important expression. So, we got the expression for the elastic line. So, that is the line; that is the expression for that elastic line, and maximum value we got Pl cube by 3 EI.

Sometimes, we get confused whether it is l square or l cube, l 4; that form is the all the time you will find EI below, here P, l, the unit you can always check at least. So, EI means unit will be in SI unit Newton per meter square and I will be meter to the power 4. So, below it will be Newton meter square. So, here P will be Newton, it will cancel. So, Newton meter square means it will be L cube. So, meter cube by meter square it will be meter.

Say slope you got Pl square. So, it should be a dimensionless quantity. There should be no unit there. So, we can apply those ideas to check the order of the l, l cube, or l square, or l 4. If you are little bit confused, try to recall from your memory.

So, it is basically a simple cantilever beam problem. We have tried to find out the deflection using differential equation technique. And tried to get some maximum values of bending moment and shear force. Now, up to this we can keep in this class. In the next class, we will go ahead with similar type of problem and try to find out some standard relationships.

Preview of Next Lecture

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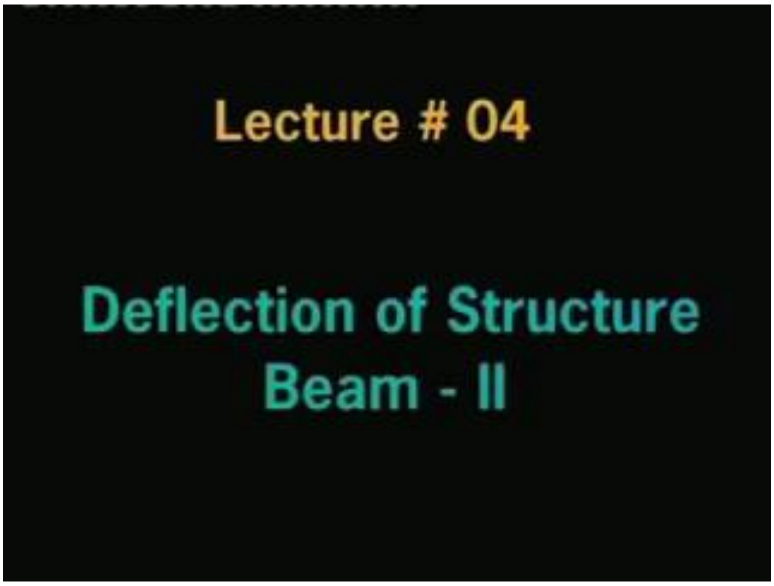

So, we shall continue whatever we are trying to cover in the last class, that is deflection of beam, and we are trying to solve the problem with differential equation technique.

We have taken a simple beam problem. It was a cantilever, with some point loaded at the end, with that I have tried to explain you how to find out the final equations, how to find out the constants through boundary conditions, and from there, we try to obtain some standard values of maximum deflection, maximum slope at one of the end.

Now, we will try some other standard cases, and try to get some important relationship. Technique is same; problem will be little different. Now, I can take, again, cantilever problem with a little different type of loader.

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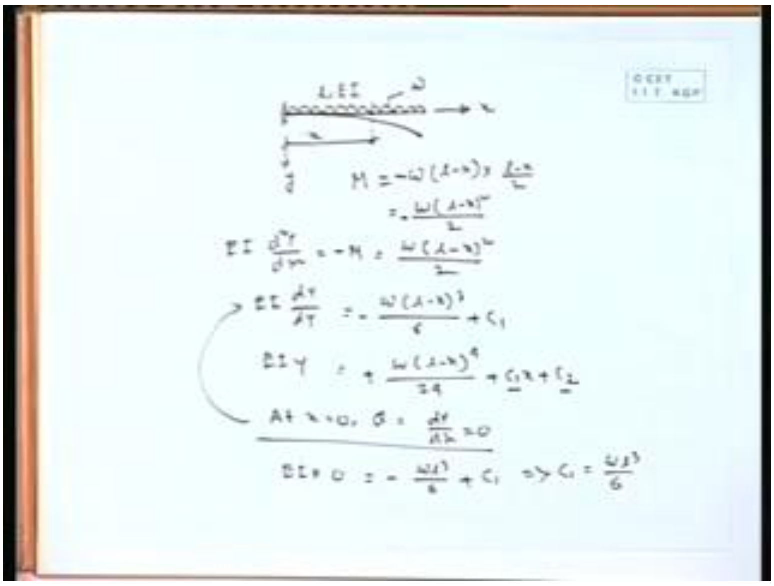

So, structure is same; it has a length l, EI, support identical, it is fixed, that is free, only the loading is different. The idea is to handle a different type of loader. Also, I think, before changing the type of structure; next step we can change the type of structure.

So, our first job is we have to find out the expression for the bending moment. Now, at any distance x, so there is one cross section. What will be the expression of bending moment? We can calculate from the right end, because that is a free end it is easier to calculate. So, beyond that section right side, the loading is first of all the length of that will be L minus x. And the loading is say w omega; that is intensity of the distributed load; the load per unit length. So, length is l minus x and w is per unit length - that is the load. So, total load will be W into l minus x and that is any distributed manner.

So, moment will be, it is a uniform uniformly distributed load, means we can assume that load is acting at the centroid of that part again. So, it is it will be acting half distance of lyx. So, we can write omega is the intensity, l minus x is the length, that multiplied by l minus x divided by 2. So, that is one of the very familiar type of expression. If we start from this side, well at omega into x that will be the load into x by 2, or straightaway we will write omega x square by 2; in that case x is not x, it is l minus x. So, it will be omega l minus x square divided by 2.

Now, we have to put some sign here again. At this moment, we have to decide which sign convention you will take positively. If you take sign as positive, it should be negative, because it will give a falling type of moment. So, if we follow the same thing, it will be minus and minus.

I think this part is clear - bending moment expression. Once that expression is ready, we can use that moment-curvature relationship; that directly we will put, the moment expression. So, there it will be EI d 2 y dx square equal to minus M. So, it will be, we can say it is minus M, and we can put here omega l minus x square divided by 2. So, minus, minus; it will be plus. We are sagging; sagging we have taken positive, but here moment is hogging, so we have put minus. And if it sagging as the positive moment, expression will be with a minus M. So, this minus, minus it will be plus.

So, in a similar manner, we will integrate as we did in our previous problem. Iit will be omega l minus x cube divided by 6 minus C 1. And EIY equal to omega l minus x 4. So, 4 into 6 24, and minus and minus it will be plus, C 1 x plus C 2. Absolutely same. Only the right side part is different. Once you have that expression, our job is to determine C 1 and C 2 - these unknowns - through boundary condition. So, it is in a same manner at x is equal to 0, so theta or dy by dx is equal to 0. So, this is the first expression or first condition. So, if we put here, I think. So, it will be EI into 0. It will be minus omega l cube divided by 6 plus C 1 or we can say C 1 equal to omega l cube divided by 6. So, your C 1 part is available.

Next part is the C 2. So, next requirement is to find out C 2. And that we can find out with the second boundary condition. That is deflection at this point - at x is equal to 0, it will be equal to 0.

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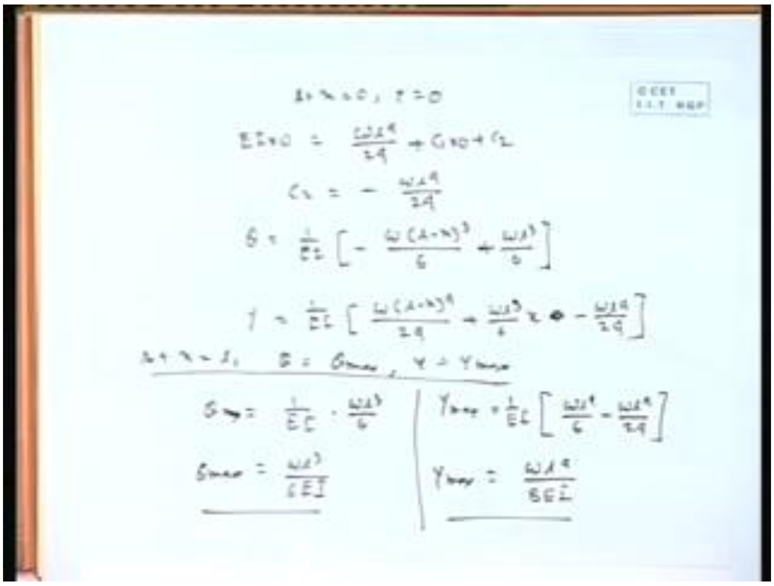

So, substitute that boundary condition - at x equal to 0, y equal to 0. So, we will get EI into 0. That will be omega l 4 24 plus C 1 into 0 plus C 2 or we can get from here C 2 equal to minus omega l 4 24 or these values we can put in the expression of slope and deflection. So it will be 1 by EI minus.

So, C 1 and C 2 we have substituted the values in the expressions we obtained earlier for slope and deflection, and it will be something like this - that is the expression of theta; that is the expression of your deflection. Now, we will get the maximum deflection, maximum slope. So, you here at x equal to l, theta will be theta max, and Y will be Y max. So that part if we find out, so theta x it will be 1 by EI, and here, this term will be 0; it will be omega l cube by 6 or we can say omega l cube divided by 6 EI.

Now, if I find out Y max, 1 by EI; this part - it will be 0; and this part will be omega l to the power 4 divided by 6, omega l to the power 4 divided by 24. So it is omega l to the power 4. So, 1 by 6 minus 1 by 24. So, if you operate, it will be 24; this side it will be 4 minus 1. So 3. So, 3 24 means it will be 8. So, it will be omega l to the power 4 divided by 8 EI. So, this is Y max; this is theta max. So, this is one of the quantity; that is another important quantity. So, same problem with a distributed load; we have tried to repeat that process with a different moment expression, and got the values theta max, Y max.

Now, I will take a problem, where your type of structure will be different. So, beam will not be a cantilever beam, say, we will take a simply supported beam. So, two supports at the two end, and take some load, say, we can take uniformly distributed load, whatever we have taken, at least load will be same like this, but support condition will be different.

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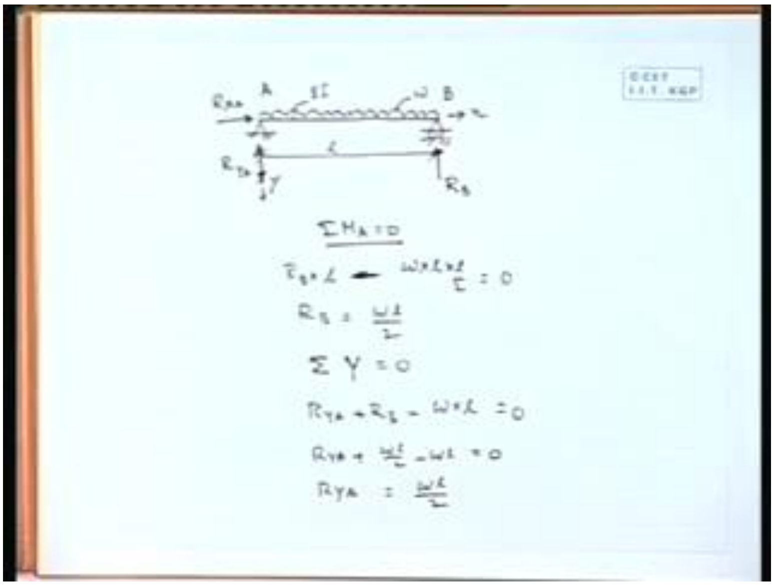

So, this is a distributed load of intensity omega or w; and the member is some EI; the flexible rigidity; this is the length; this is x this is Y. So, our first job is to determine the expression of bending moment. Now, if we start from this end or the other end, both the sides supports are there. So, we have to know the value of those reactions.

Earlier case, support was on the left side and one free end was there. We were trying to proceed from the free end; we need not bother how much will be the reaction at the supports, but here we cannot start from this end or other end; both end are supported. So, first job is we have to find out the reaction. And that will be utilized for finding out the bending moment expression. And that will go to the differential equation. Now, here there will be one reaction and here. So, this part we can say y; and this part we can say Ry; it is say Rx; let us say A, that A; this is B; say left end if we say A, right end if we say B; it is R along x at A; Ry y at A; this is RB.