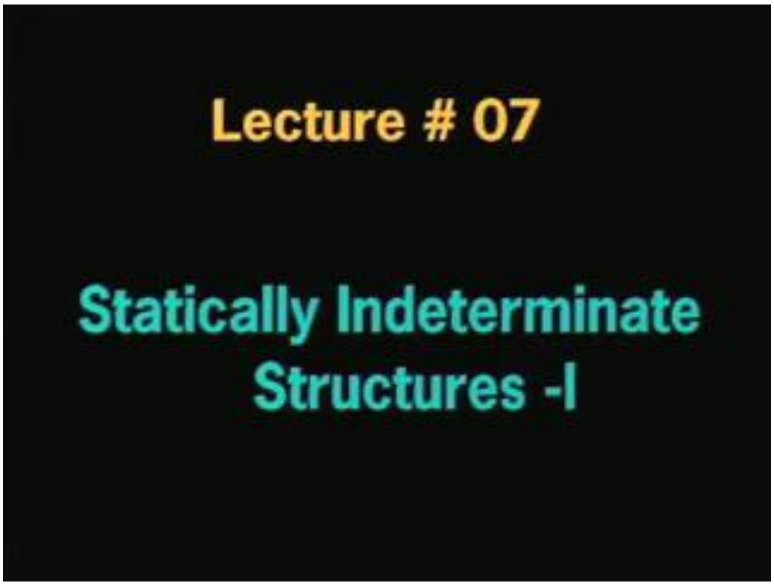
Strength and Vibration of Marine Structures

Prof. A. H. Sheikh and Prof. S. K. Satsongi
Department of Ocean Engineering and Naval Architecture
Indian Institute of Technology, Kharagpur

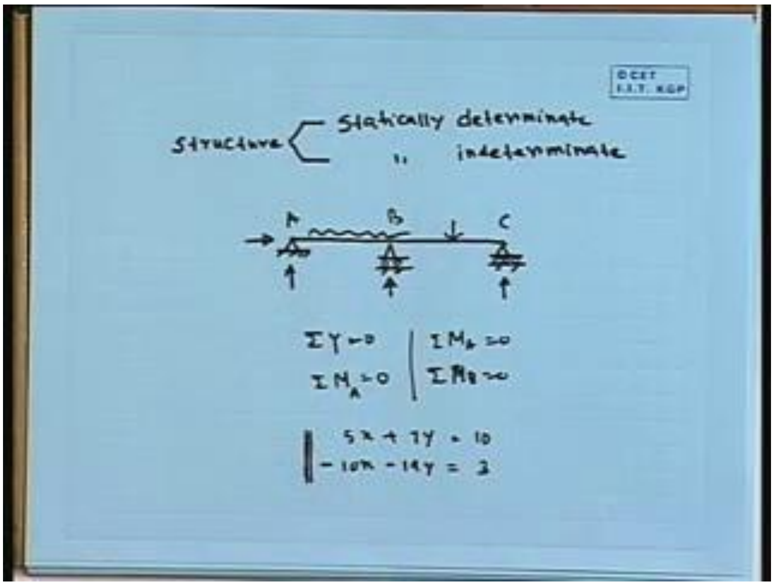
Lecture - 7
Statically Indeterminate Structures – I

(Refer Slide Time: 00:41)



Today, we will discuss statically indeterminate problem. Already we have discussed what is statically indeterminate problem, earlier. Now, here our job is how to handle that type of problem. Now, if I write structure in two forms, though we have written it earlier, still we write here a structural problem.

(Refer Slide Time: 01:33)



So, it is these two groups: statically determinate; statically indeterminate. The division is made basically on the idea of equation of statics. Any problem, there are some equations of statics. If we take a three-dimension problem, we will have six equations of statics. If we take a plane problem, it will have three equations of statics. In the space, summation of all the forces along xyz that will be 0; plus moment about xyz, it will be 0. In plane, summation of force along x will be 0, along y will be 0, plus moment about z means moment perpendicular to that plane - so, that will be 0. So, these are the three basic equations.

Now, if we have a problem, at the basic unknowns, which normally we find in the form of support reaction. If it is exactly equal to three, it is statically determinate; means that problem or those unknowns can be determined with the equation of statics. So, we define it as statically determinate. If the number of unknowns, it is more that number of equations of statics, so that problem cannot be solved. So, this problem is statically indeterminate. So, it cannot be determined with the equation of statics.

Say, if we take a beam problem like that; here there is a hinge support; here there is a roller support; there might be any load on that. Now, at the hinge there will be two reaction components; so, one vertical and another horizontal. In the roller there will be only one vertical reaction component. Now, here we can say the horizontal component, it will be 0. Normally, in beam problem, we take a loading all perpendicular to the axis of the beam; usually, we do not get any component of force along horizontal direction - along x. So, most of the cases we get it 0. But we cannot provide that hinge support as a roller support, because if we put roller on both the sides, loading, usually it is vertical. So, it will take care of both the problem, but if there is a small component of horizontal force due to accidental reason, so structure will not be stable in space; it will go on rolling on the two roller; so, it will not be placed, where you want to keep it. So, for any unknown or undesirable horizontal force, we have to prevent that, at least left hand or right hand. So, it may be 0 or any small value depending on the type of load. So, it should be there.

So, theoretically we can say for a wind problem, usually, we are not going to take any vertical load; all the time it will be 0. So, that horizontal component, all the force, that condition is automatically satisfied. So, from the three equations, normally, one equation is automatically satisfied. So, other two equations are there; from there we can find out the two reactions.

Now, here the two equations are: we can take summation of vertical force along y direction; and we can take moment about this point a; or we can take moment about the other point b. Now, if we have some third support here, definitely there will be three reaction components plus one horizontal component. Normally out of three already one equation we have taken care of, in order to handle the horizontal part. So, vertical and moment part - two reactions we can determine. So, if we have three unknowns and two equations we cannot find out. Normally the number of equations should be equal to the number of unknowns; if it is more than that, it cannot be determined.

Now, I was talking about taking moment. Now, there are three forces: one option is we can take moment at one point; and other option is, other part is we can take all the forces in the y direction equal to 0; and alternative of that, say, if I write summation of y equal to 0 and summation of moment at any point, say, if it is A, if it is B, if it is C, we can take MA equal to 0.

Other option we can take we can take MA equal to 0; MB equal to 0; that also you can do. Say, initially that support is not there. So, we can, if we take moment about this, that will get; if we take moment about that, the other reaction we will get. Now, this may give some idea, that instead of taking force - all the forces - along y direction, we can take moment about MA and MB, we were getting two equations. Now, we have three unknowns, but two equations we cannot solve. So, there might be a temptation that let us take moment about C or there might be a number of support, we can go on taking moment at different point and go on generating a number of equations.

So, it can be theoretically solved, but really if you try to generate that equation, you will find all the equations are not independent equations. Independent means, you will find one equation is identical to the other equation. It may not be exactly identical, but if we multiply with the factor minus 1 plus 1 plus 2 minus 2, you will get the other equations or two equations if you combine, you will get the third equation. Just I can give you one simple example. Say 5 x plus 7 y equal to 10. If I have minus 10 x minus 14 y equal to, say 3, anything actually; right side not necessary it will be equal. Now, the first equation if you multiply with 2 and add, so x will cancel; y will also cancel. So, in that process, you cannot find out neither x nor y; none of the quantities you can find out from this system.

So, what will be happening? There should be a tendency, initially, we can take moment at any number of point, generate any number of equation, and any number of unknown we can solve; but theoretically if you make an attempt, we will more or less getting that type of equations. So, we will basically get two independent equations here, I think, apart from the horizontal summation of the force. So, two independent equations you will get, if you try to generate more, so those equations will be basically coming out from these equations. May be, this plus this if you add or this multiplied by 10 minus this multiplied by 5 add you will get the third equation or the fourth equation. So, whole thing will be not a independent system and equation cannot be solved.

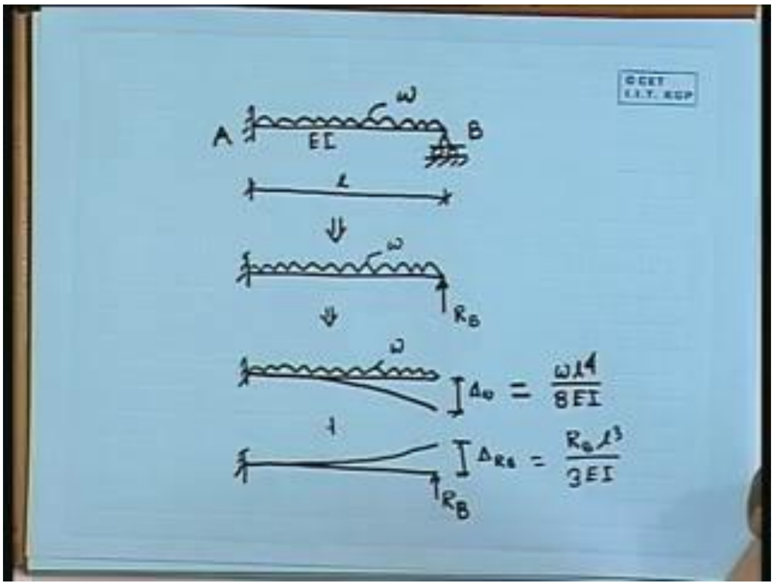
So, our main idea is three equations and we may take all in the form of moment, but the number of equations it will be just three, or horizontal part, if we just forget, it will be only two equations in a moment form or one moment, another just force form.

Now, this problem, what I have? So, this problem, what is drawn here? Here basically there three reactions and we can take equation in this form or in that form. So, the third reaction, we cannot find out, because total number is more.

Now, here other problem can be solved. The problem can be solved, in addition with that equation, we require some other equation, say, three is the reaction, and equation of statics is two. So, we require another one extra equation. And that equation will come from the deformation of the structure or deflection of the structure. So, more or less we are trying to handle deflection of the structure in last two, three classes. So, those ideas will be required here, and if we use those equation in the form of structural deformation, structural deflection, we will get some information in the form of additional equation.

So, apart from these two, we will get the third equation, from our structural deflection or structural deformation. And that will help to find out the reaction of the structure. Now, let us take one simple case, which is a very popular type of problem.

(Refer Slide Time: 12:04)



Say, this a cantilever, and here, this end is supported. We take any load, say, it is distributer load throughout. This part is l it is EI; you can take it is A; it is B. So, length is l; this end is a clamp support; this is a roller support; fully distributed load; uniform flexural rigidity EI. Now, how many components of reactions are there? First of all, this clamp support will have horizontal component, vertical component reaction; and here there is one reaction. So, 3 plus 1, 4; number of equation is w; so, one unknown is more. So, we say degree of indeterminacy. So, degree of indeterminacy is - how many extra? So, 3 is the normal number. So, here 1 extra is there. So, degree of indeterminacy here it is 1. Or if we just remove that support, it is a cantilever beam, There is a load and that problem already we have solved, because it was a determinate problem. Three reactions at one point; that is support A. Now due to putting A support here, at the right hand B, a problem becomes statically indeterminate one. And it cannot be solved with the equation of statics. We cannot determine the reaction. And if we cannot determine the reaction anywhere, we cannot go and find out the shear force and bending moment.

So, there is one set of force; that is applied load. Another set of force, it will come from your support, in the form of reactive force. Once all those forces will be known, then only we can come to a particular place and sum up all the force, we get shear force; or we can take summation of all the force in the form of moment at the station, we will get bending moment there. Now, the main job here is to determine the reaction. Once that will be obtained, we can find out anything inside.

Now, how the problem can be handled? Now this problem we can just write in this form. Say it is a cantilever problem, with that load plus the roller here, it will give a reaction RB. So, at B there is a roller; roller will give a reaction; reaction force is R at B; RB is the reaction. And this force, normally, this omega is the load per unit length intensity of distributed load, that value will be a known value, some numerical value will be given, because that is the external applied force.

Now, this RB, this force will be supplied by the support in the form of reaction, and that force is unknown here. Now, this problem can be solved in two steps. So, this may be divided as… Already we have talked about method of super position. If a structure is subjected to more number of load, we can get the effect of the individual R component of force, then we can combine their effect, through simple algebraic addition, that we say it is method of super position, because whole thing is a linear system.

Now, here there are two loads; one is omega. So, first step we have put this distributer load. And this RB we have removed; that we are putting in the second level. So, this RB, that force, and this force, they are applied separately. So, deformation of the beam subjected to w, and deformation of the beam subjected to RB, if we combine we are supposed to get the deformation of the actual structure. Now, due to omega, it will take a shape like this; and due to RB it will take a shape like that.

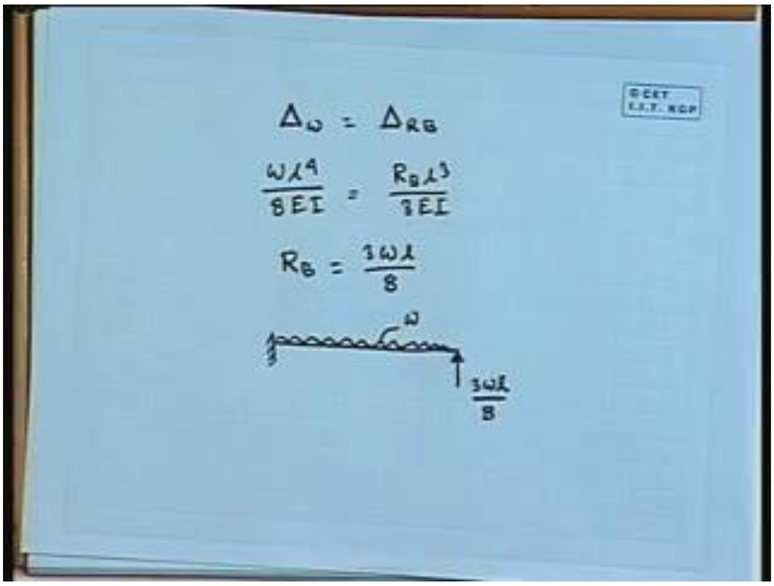
Now, here, if we say this is your delta due to w, and this is basically delta due to RB; these two values will be equal and opposite. Because in a real structure, joint B is connected to a support. So, it cannot move vertically upward or downward. So, under w, whatever w - delta w – we will get, that should be compensative by RB in the form of w

RB. So, if you would take it as positive, it should be negative; if it is 2 millimeter it should be minus 2 millimeter. Ultimately, whole thing if we combine, we should get back to the original situation, where deflection at B should be 0.

Now, this delta w delta RB, the deflection values already we have determined. If we take a cantilever, at the end if we apply a load, whatever deflection we will get or if it is fully distributed case whatever will be the standard value that already we have determined. Now, we can write this value here again, delta w. So, it is fully distributed load and that value was, already we have determined, it is omega l to the power 4 divided by 8 EI. So, it will be omega l to the power 4 8 EI.

Now, this force will be how much? It will be RB L cube divided by 3 EI. Now these two deflections should be equal and opposite. So, if we just equate these two equations, we can find out RB in the form of your W, l, EI; something we will get from there I think.

(Refer Slide Time: 20:01)

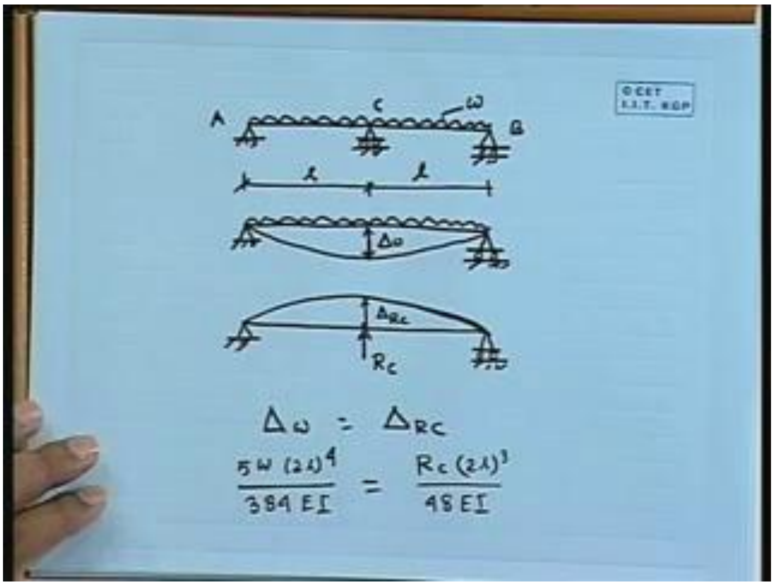


Now, next step if we just equate that. Your delta due to w should be equal to delta due to your RB and this we have written as… So, from there RB we will get, it will be this l this 3 divided by 8. So, EI will cancel; lq will cancel; it will be l remaining; omega will be there; 3 will go to that side, it is below. So, 3 by 8 omega l will be the reactive force there. Now, if I draw the beam again. So, actual structure, it can be represented by a simple beam without the support at the right end, in lieu of that, we will supply the reactive force supplied by the support. So, the reactions supplied by the support is RB and the value we have calculate this 3 omega l by 8. So, total load is omega l on the beam and it will be three-eighth of that, it will be coming in the form of reactive force there. Now, at any point, if we try to find out the shear force, it will be your 3 wl by 8 minus omega into x or moment if you want to take, so this 3 wl by 8 multiplied by x minus omega x square by 2.

So, this plus or minus, depending on the sign ,effect of that can be taken as plus, so effect of this will be minus or it may be reverse. So, or you can draw the bending moment diagram of a beam, you can draw the shear force bending movement of the beam. Finally, we may be interested for finding out the maximum stress. We will be interested for finding out the maximum shear stress. So, all this quantity we can find out from the structure.

So, the main bottleneck was number of reactive force was more and in a normal way we could not proceed; naturally we had to take the help of deformation of the structure. So, we have tried to match the deformation. So, this technique sometimes we say it is a consistent deformation technique; means structure will have a deformation. It will maintain some consistency, because there is a support. So, deformation should be equal to 0. Or somewhere there might be a support, left side, right side slope should be equal, or deflection should be equal. So some form of deformation compatibility we have to establish. So, basically your, this displacement part, we have to take care and we have to get a additional equation. Now, in the next case, if we take a problem like this: say, let us take a simply supported problem.

(Refer Slide Time: 24:04)



This is fully loaded omega and I am making it quite simple. So, l is the span; l is the span; AB. So, it is ABC, and this side l; and this is l; fully loaded omega. Now, already that problem was defining, because there are three vertical reactions plus 1 horizontal reaction, though that will be a 0, but we have to take. So, 4 is the total number of equations; 3 equation of statics. So our extra equations we require is that extra number of unknowns. So, extra number of unknowns, though that we were defining in terms of degree of statical indeterminacy. So, degree of statical indeterminacy, here it is one. So, at least one additional information, in the form of some equation, we should get for finding about these reactions.

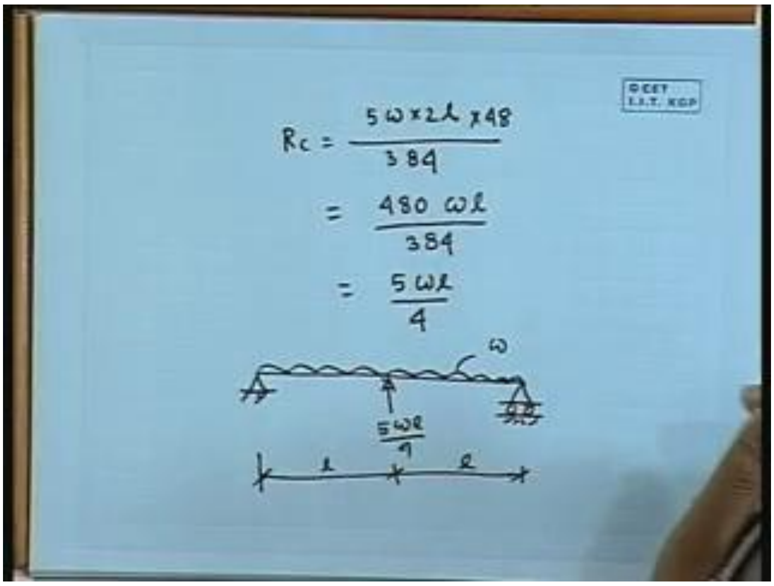
Now, this problem can be handled like this. Now say at this level we remove the central support. Now due to that we will get some deformation here. Now, we are not supposed to remove the support at the middle. To take care, we have to supply a reaction, say Rc. So, in actual case, this omega will be there plus there will be a reactive force that we see due to this roller support. So, if we want to remove that, we have to take the effect of that in the form of some reactive force. So, that is Rc. So, that we are putting in the second line. So, if we combine we are supposed to get the actual structure.

Now due to Rc, the beam should deform like this. Now, whatever deformation you will get here, if we say this is delta for omega, and this is the deformation whatever we will get delta for your Rc. Similar to your earlier case, this value and that value should match.

So, if we write your delta…so this value should be equal and opposite, and that opposite part that we have already drawn here. So, we need not bother about the sign, because sign part is reflected through drawing. Now, this delta omega, delta Rc, this values are… this is a simple determinant problem. A simply supported beam, fully loaded, the centre deflection we know, already we have the expression. Now, here also it is a simply supported beam with a centre load. And for that problem, also we have the standard expressions. Some of the cases we have derived; some of the cases, any time, we can derive with our basic understanding in the form of differential equation technique or moment-area theorem or we can just consult some standard table; from there we can pickup. So, what will be this value? Omega l; omega l will be your 5 omega l to the your 4 divided by 3 84 EI.

Only this l part here, it will be l and l, it will be 2 l. So, it will be 5 omega and 2 l 4 divided by 384 EI. And that should be equal to… this will be Rc 2 l cube divided by 48 EI. So, it is EL by 48 here and this will be 5 omega l divided 384 EI; only l part in this problem is equal to 2 l; just we have substituted that. Now, from here, this Rc can be easily determined; just we have to put the expressions. So, if we just simplify that equation.

(Refer Slide Time: 31:01)

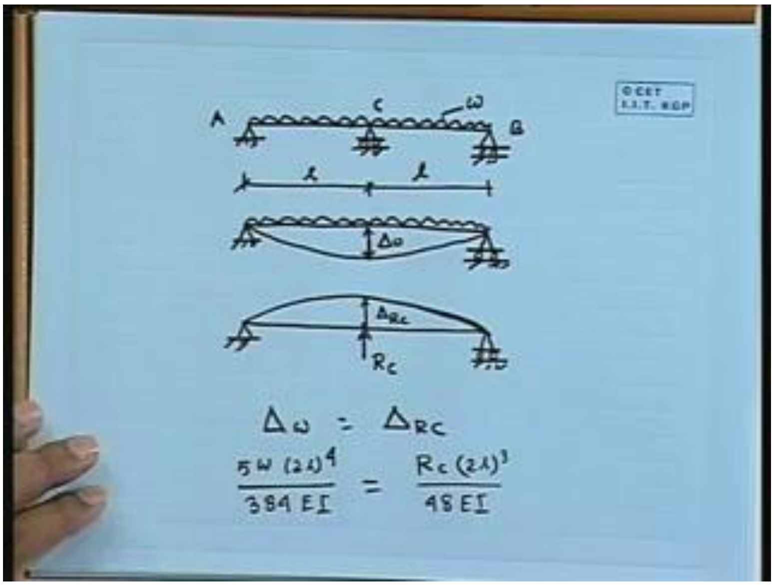


So, your Rc will be, it will be 5 omega, and 2 l part it will be q and 4 it will cancel; 2 l; and it will be 384, and this 48, that will go off. So, it will be multiplied by 48. So, it will be 480 on the numerator, because 5 into 2 10 48, it will omega l divided by 384. I am sure it can be further. It will be 5 by 4. So, that will be its reduced form. Now we can draw the beam again. It will be 5 omega l by 4 and total length is equal to your l and l. So, twice l.

So, the support, we have first replaced in the form of some reactive force unknown Rc. And that Rc we got by 5 w by l divided by 4. So, it is a simply supported beam, with a vertically downward applied fully distributed load, plus at the mid span there is a concentrated load, value is known, but that value is acting in the upward direction. So, this problem can be easily solved. The remaining reactions, and you can find out shear force, bending movement, shear stress, bending stress; any important information you want, you can find out from there.

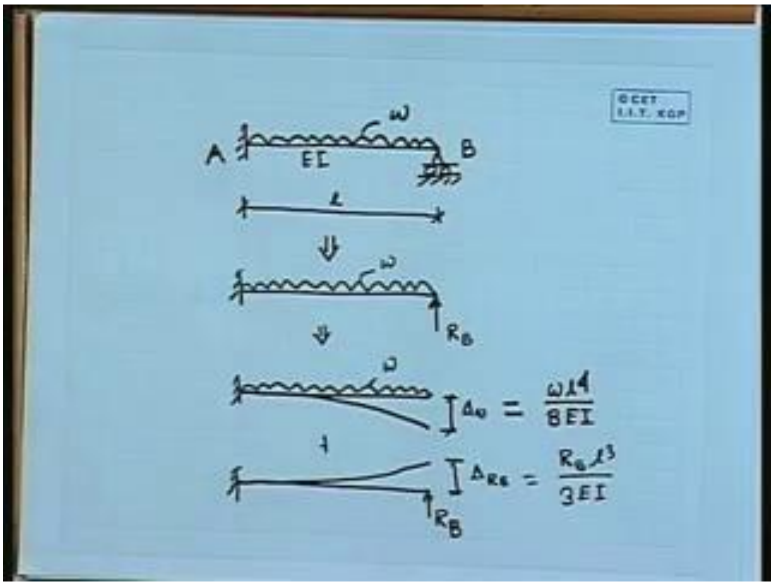
So far, we are handling a problem where the degree of your indeterminacy is 1. So, extra unknown is 1. So, cantilever beam, end reaction we have removed; here the central part we have removed. Here I want to mention, the central reaction we have removed, so not necessarily that is the only alternative.

(Refer Slide Time: 34:01)



So, if we come to the original problem. So, here this was removed by a force; we could keep the support; remove this one; and put a reaction here. That can be also done or we can keep the support, remove that, put a reaction there.

(Refer Slide Time: 34:27)

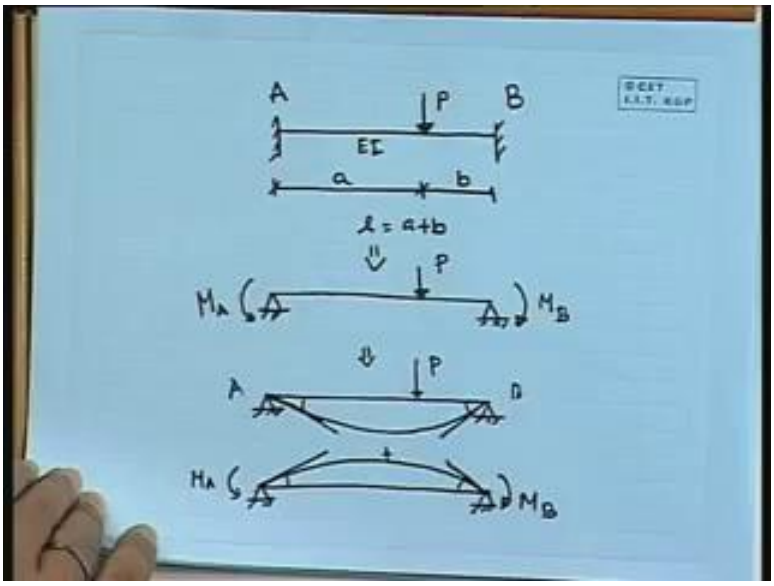


Or if I take the cantilever problem; cantilever problem we have removed that. Here, the restrain against rotation we can release. So, we can make it simply supported. So, we can make it a hinge support, with a applied load, and that load will be in the form of your bending movement. So, one bending movement will be acting at the end and that bending movement is a unknown quantity. So, it will be simply supported beam, fully loaded, plus a end moment. So, one extra quantity that.

So, there, due to the fully distributed load, there should be a slope. And due to that moment, slope will be in the reverse side. So, the vertical load, it will try to make a slope like this, and this moment it will give a slope in a reverse manner, and it should be equal and opposite, because it is a fixed support, so slope should be equal to 0. So, in that manner not necessarily, this is the only place where we can remove and put the load. There might be some other possibilities. So, same problem can be solved in different manner.

For all the cases, our extra-unknown degree of indeterminacy is one. So, we are writing only one equation; we are trying to match one deflection; and from there we are getting the result. Now, once we have solved two problems - two different types of problems. Now, I want to take a beam, where at least the number of unknowns will be more than one. So, your degree of indeterminacy will be two. We can take a very simple type of problem.

(Refer Slide Time: 36:28)



Say there is a beam; it is fixed at both the ends. Now, there is a load, and this load is, say, acting here. And this part, we can say, here to here is a; here to here it is b. It will have vi, l, everything and we can say l is nothing but a plus b. And it has some flexural rigidity EI. No, it will not stretch. So, the right end is fixed along with the left end. So, if we remove the right end, it will be a normal cantilever indeterminate. So, if we remove the right end, how many reactions you will just eliminate or you have to take in the form of some reactive force.

First of all, it will give a vertical reaction force; plus there will be a rotational part. Now, horizontally part we are not considering, because there is no horizontal force, and we say this horizontal force here and here, it will cancel. So, basically we will have two equations apart from the horizontal equation. So, two equations, we can take one vertical reaction; one moment. So, one vertical; another rotation, that part will be extra. So, two additional unknowns are there. So, your degree of indeterminacy is equal to 2.

There is another possibility; we can just make both the clamped support as in support. Take some reactive force and allow the rotation. So, it will be a simply supported problem with moment. So, there, if we say it is A and if we say it is B, so there will be a moment MA; there will be a moment MB - one option.

Another option is we straightaway remove that support, and take a vertical reaction, and a moment. So, in two ways we can handle the problem.

Now this problem we can, say, if I idealize in that manner. So, we can release the restraint imposed by support A and support B against rotation; and put some moment in lieu of that resistance or that restraint. So, restraint will basically give some moment. So, that moment, say, we have given in the form of MA, and moment here is MB. And just it was converted with in support, but our requirement is we have to find out ME and MB.

Now, this problem can be divided into component. So, if we put the load here P, plus…Now, due to that load, there will be a deformation like this. And due to the applied moment MA and MB, it will try to bend in the upward direction.

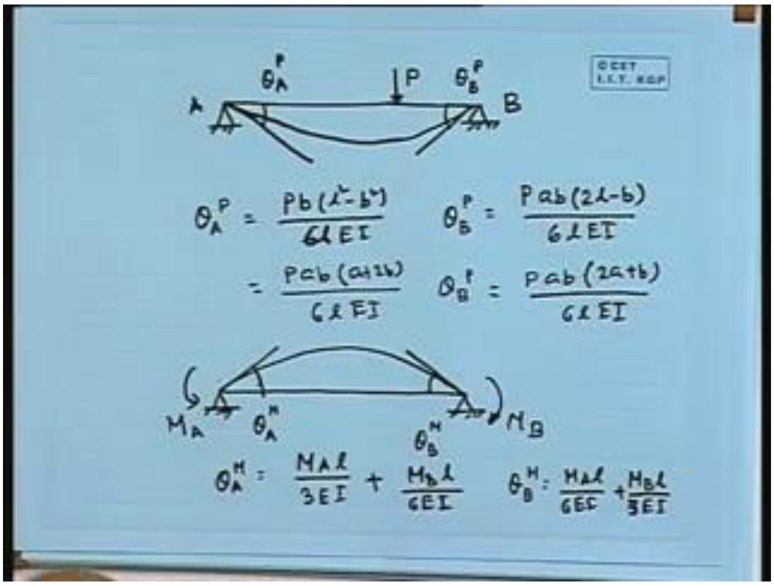
Now, the second part, we can put in two different steps also. We can put P, second step MA, third step MB or here MA, MB second, third we have more or less combined. So, more our maturity will come, we will try to combine the effect. We will just imagine this is happening, so we need not break the different steps. So, it is better to break in some steps at the beginning, to get a feeling what is happening.

Now, here due to that load P at A, we will get some slope; at B we will get some slope. Now this slope at A will be produced by the load P only. Now, due to MA and MB, we will get some slope here. So, this slope should be balanced by this slope, because in actual structure there will be a clamp support, slope will be equal to 0. Similarly, here slope and slope it should be balanced together.

Now two additional unknowns we have taken in the form of MA and MB. And these two unknowns can be evaluated from the compatibility of slope at A, matching of the slope at A, and similar way we just equate the slope at B, obtained from here and here. So, basically slope computability at A and B, we will try to impose. And from there, we will get two information in the form of two equations, and these two equations will help to evaluate MA and MB. So, two unknowns, at two places we have to look after the deformation of the structure.

Now, intentionally, I have put the load in a very arbitrary place. So, distance is A and it is B. In a special case, if I put B is equal to at the center of the beam, say, it is at l by 2. In that case, MA should be equal to MB. So, we could calculate only one quantity, only matching with the slope at A or B. But if we make it unsymmetrical, definitely your slope will not match this side and that side due to P. And your MA and MB will be different. But anyway, if we handle a general case, we can make A is equal to l by 2 for a specific case can be determined from there. And there should be a checking, MA should be equal to MB. Now, these two steps, let us find out. So, just draw it here again with some notations.

(Refer Slide Time: 45:02)



So, the first part… and there is a P. Now it will deform like this. So, we are talking about the this is A, this is B. So, this angle will be your, say, A due to say P. And here, it will be theta B due to P. So, there is a super script P to identify this is responsible for P only. Now, this theta A and theta B; that you can derive from with your earlier concept of moment area theorem or differential equation technique. Or here, the theta A and theta B it is available. So, theta A, if I put here, theta AP, it is given Pb l square minus b square divided by 6 l EI.

So, 6 l EI is the denominator; Pb l square minus b square is the rotation of the left end. This part is A; this is B; total length is L; already we have written that. Now, this equation, and theta B due to that P, that part is Pab 2 l minus b divided by 6 l EI. Now, this is the rotation, at this end. I want to change this expression little bit, because a plus b is equal to l.

Now, the numerator part, I want to keep entirely in terms of ab; denominator it will be l, because there some part is a, some part is b. And, if we express that, you will find expression will be more or less similar type. So, it will be identical expression. So, here we can write P. So, l square minus b square you can write as l plus b l minus b. So, l minus b is a; l minus b will be equal to a. So, this a if we take common, so it will be ab. So, here ab is there; I want to bring it there actually. So, l minus b already we have taken in the form of a and l plus b; l plus b; l is what? A plus b. A plus b. So, it will be a plus 2 b divided by 6 l EI. Here also, this l part will be ab; l is your 2 a plus 2 b. So, it will be, 2 a plus 2 b minus b will be b divided by 6 l EI.

So, what is happening? E ab divided by 6 l EI, that part is entirely identical; only this end it is a plus 2 b; and there it is 2 a plus b. So, a plus b, there is a factor 2. So, here this end load is away from a. If we take a is more compared to b. So, this slope will be little less. So, here it is twice b. So, b is small; it is twice b; and this side slope will be more, it is twice a.

So, if it is Pab a plus b; some of the quantity will be twice; which one will be twice, that also from the sense we can find out; plus the denominator will be 6 l EI that component. So, this part we can keep in that form, and the other part, we took in that manner. So, we can draw it here, and there is a MA, there is one MB . So, it should with theta A due to moment and theta B due to moment. So, theta A plus moment; so this is theta A due to moment; this is theta B due to the moment. Now this theta AM, theta BM - they are produced by MA and MB.

Now, if we take a beam, apply at a moment, the slope we will get - how much? It is Ml by 3 EI. So, if you take a simply supported beam, apply a moment, so where we are applying a moment it will be Ml by 3 EI; other end it will be half, Ml by 6 EI.

If we take a simply supported beam, apply a moment, so if it is not there, if we apply a moment, this angle will be Ml by 3 EI, and this angle will be Ml by 6 EI. Similarly, due to this moment, if that is not there, this end slope will be MB l by 3 EI, and this end slope will be MB l by 6 EI. So, both the actions are there. So, in that case our theta AM, it will be MA l divided by 3 EI plus MB l divided by 6 EI.

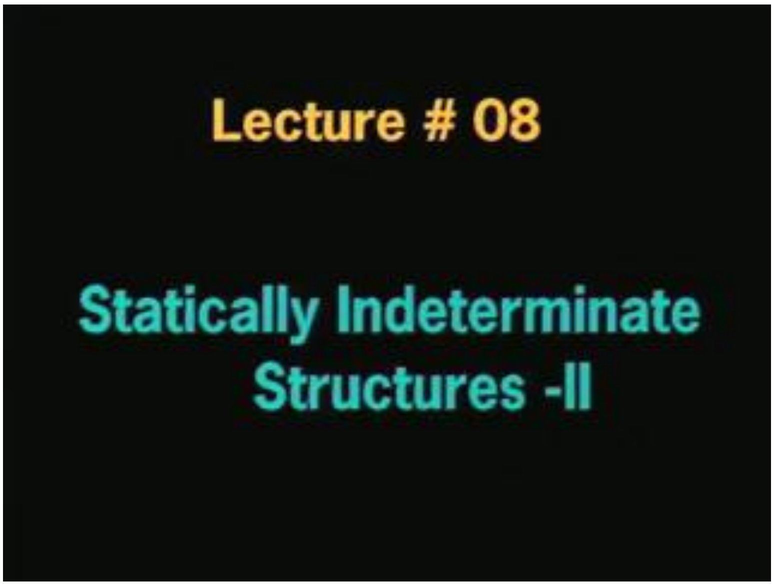
Similarly, theta BM; it will be MA l by 6 EI plus MB l divided by 3 EI. Can you see that? So, it will be MA l by 3 EI inherent and MB l by 6 EI. It is coming from the moment applied at the other end, because this will be this way, this will be this way; both side, it will be just additive.

Now, next part is - this slope and this slope should match; and this slope and this slope should match. So, if we match that, we will get the reactive force.

Now, up to this, I am keeping it for this class. So, next class; in the next class, we will continue from equating these slopes, and finally, get the reactive force. Gradually, we will switch over to other type of problem.

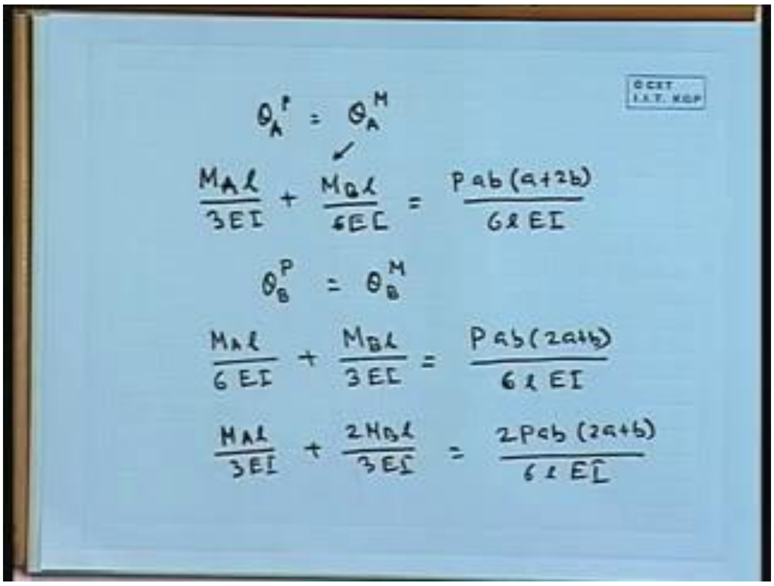
Preview of the Next Lecture

(Refer Slide Time: 54:11)



So, we were discussing this problem in our previous class. The starting of the problem was like this. So, fixed beam, load placed little bit away from the center. So, a load, bending moment, at the two ends force, bending moment separately we have applied. And due to the load, the slopes we have written, and due to your MA, MB, you wrote the expression of slope at the two ends. So, next part is just we have to merge the slope, because slope here and slope here; it should compensate, because it is a fixed support; and here also same thing.

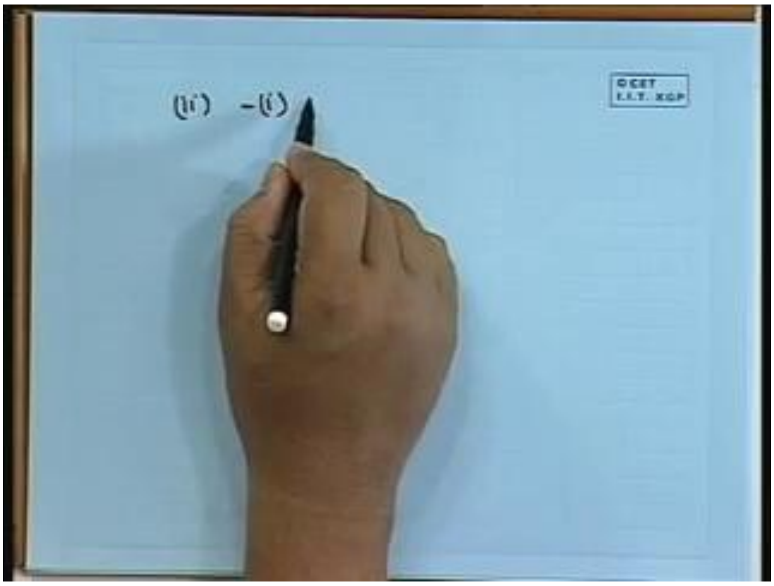
(Refer Slide Time: 55:08)



So, we can write theta A due to P should be theta A due to moment. And theta A due to P and theta A due to moment, the expressions whatever we have written, so if we just write it on the left side, it will be your MAl 3 EI MB 6 EI; that was Pab a 2 b divided by 6 lEI. So, definitely, this part on the left side, the other part on the left part here, this is on the right side.

So, similarly, at B, due to the load, and theta B due to the moment, if we equate, if we write here, so it will be MAl divided by 6 EI MBl by 3 EI; that is equal to Pab 2 a b 6 lEI. So, here it is a plus 2 b; here 2 b plus a; here it is 3; it is 6; it is 6; it is 3. So, these are the two equations and two unknowns: MA, MB; MA, MB. Now we have to just process these two equations and find out MA and MB Now the lower equation if we multiply with 2, so this 6 will be 3, and you may part it, we can cancel. So, this equation we can write, we can straightaway multiply or we can write in a separate line. So, it will be MAl by 3 EI; 2 MBl by 3 EI; it will be 2 Pab 2 ab divided by 6 lEI. Now, if we just subtract that equation or from this equation, this equation if we make it minus. So, your MAl by 3 EI part will be cancelled and here MB part will be remaining.

(Refer Slide Time: 58:47)



So, if I come to the next page, say, this is your, if I say this is your equation one; if we say it is equation two. So, if we make equation two minus equation one.