

**Design of Offshore Structures**  
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**Module - 1**  
**Lecture - 7**  
**Loads on offshore structures 7**

Today, we are going to see the structures loaded due to large amplitude waves. Of course, the waves is consider to be linear wave, non-linear wave depending on the amplitude and wavelength. Now in this particular case, basically we have a structure which is slightly bigger in size. So what really happens is the first assumption made by Morrison earlier that we were thinking about the disturbance is too small can be ignored that is why Morrison formula was applicable and then there was correction factors. Now in some cases, in we know you may have the size of the structure is bigger, typically in the order of twenty percent are higher than the wavelength.

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**Wave loads on Offshore Structures**

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**Diffraction Forces**

- Assumption of "No disturbance" is not valid if  $D/L > 0.2$
- Part of Wave reflected once the wave touches the structure and part of it pass around
- This phenomenon is called diffraction
- These forces also can be measured experimentally
- Many research papers exist for different types and shapes of structures

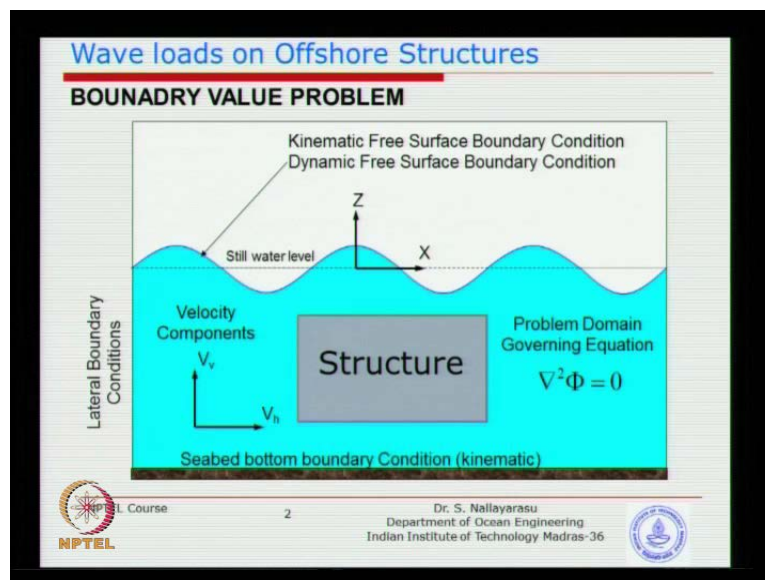
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So that means the D by L ratio what do you see here is greater than twenty percent of the wavelength. You may see that the presence of the structure alters the water particle velocity and the acceleration. I think you can easily visualize, we make a structure and you go to our wave plume or to a river, you just place it in the middle of the flow conditions, you could see that the

the flow get modified. So, what really happens, if there is a complete obstruction, for example, you construct the dam across the river, what will happen? The velocity goes, and just get obstructed and then probably retarded, reflected if there is a dynamic condition, waves come and reach that solid wall gets reflected.

So things get modified that is what you need to understand that means the velocity or the acceleration gets added or reduced. So in this particular case, if the dimension of the structure is so large, that it modify the velocity and acceleration. You could not or you will not be able to use the wave theory that we have derived so far. You have linear wave theory, you have stoke wave theory and so many wave theories you have derived, mathematically describing the flow within the domain with the boundary condition of the sea bed and the sites on the surface everything is nice.

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Unfortunately the obstruction within the domain is so large that the can't anymore apply the assumption of the structure is invisible to the wave that is what the assumption we made earlier, because all the jacket tubular are so small compared to the wavelength. Still error exists, you may not actually at this stage, you may not actually agree that the assumption is right. You may have it in your mind back off in your mind thinking that I have such a big jacket structure, but still we have assumed that the structure is not being influencing the whole water particle

dramatics and acceleration. But there will be error, but that the error is so small, but we could possibly accept it to reduce the laborious work that we require to do the design that is why historically we have taken a standard, we will take that small error acceptable. As long as the calculation method is so simplified that we can do with ((Refer Time: 03:22)) equation.

Whereas, when you come to a structure of this kind very large and if you still assume the same idea, for example, if it is too big, but the error induced is so large, that there is no meaning in doing this calculation based on the ((Refer Time: 03:38)) formula, because the difference could be as much as fifty percent. In such a case, it will not be possible to accept. So that is the situation where when the structure is so large that we would like to look for a alternative solution and that solution is basically a problem associated with going that to the basics.

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**Wave loads on Offshore Structures**

**Governing Equation**

The governing equation for the diffracted spatial potential can be expressed as follows.

$$\frac{\partial^2 \phi^s}{\partial x^2} + \frac{\partial^2 \phi^s}{\partial y^2} = 0 \quad \text{in } \Omega$$

The velocity potential due to an incident monochromatic wave traveling in the positive direction can be expressed as follows:

$$\phi = -\frac{iga}{\omega} \frac{\cosh k(y+d)}{\cosh kd} \exp^{(ikx)}$$

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You know, we did define the basics flow equations when we started the course class on two or three sessions back. We will looking at the Laplace equations and then the boundary conditions at the surface, boundary conditions on the sides and boundary conditions at the sea bed. And also right now what we are going to introduce is a boundary conditions along the periphery of the structure or body which is submerged which can be submerged or it need not to be always submerged. You may have surface bed shrink structure like ships or other floating bodies, so you can have so what is the additional boundary conditions we are going to apply is only one on the

structure basically the surface of the structure will not observed any fluid that means there is no flow across the structure within this domain.

So basically, the same boundary conditions what you have applied earlier on the sea bed, there you assumed as no flow conditions perpendicular to the sea bed. Here what we are going to assume is no flow condition on all sides of the structure, for example, this is very nice rectangular structure; so left side, right side, top and then bottom. But you can have a arbitrary shape, not necessary that you should have something nicely like this. You can have any shape within the domain, and then that means the flow normal to the surface is zero, so that is the conditions, which we already have applied to the seabed, normal to the seabed, because nothing is going in. Whereas the tangential to the sea bed, the flow is going because along the sea bed bottom some flow will be allowed, so that is why we always have to have normal to the solid body, there is no flow going inside and nothing coming outside from a...

So if you apply this boundary condition, we could derive or we could find out what is the velocity potential of the incoming wave and what is the velocity potential of the disturbed wave in either travelling path the body or reflected backwards towards the starting point. So basically, we are interested in so called the scattered velocity potential which is nothing but because of the presence of the structure, the water particles are scattered away either in this direction or in the back direction and that velocity potential is noted as  $\phi_s$ . You see the subscript because that is what we are interested, because we already know what is the incoming velocity potential due to the incident wave, the wave is coming from left side that is already known to us, we have the wave theory to solve. What is of our interest is the wave velocity potential, which is scattered due to the structure of the structure or the body. So you could actually derive the same principle, if you have the structure is very simple like this.

For example, a rectangular body, applying a boundary condition on the top surface in here, or on the sides or on the bottom it is very easy because it is a rectangular body. I can say boundary condition at minus  $y$  or minus  $z$  equal to some depth, I can apply boundary condition on that length.

But the same thing cannot be done if you have a arbitrary shape. You can have very different shapes, so you cannot apply that boundary condition, because it is continuously changing means

we cannot do an analytical solution using the equation, you may have to try and find out a numerical solution by several methods either finite element methods or finite difference method. So this methodology of solving the wave scattering problem due to presence of large structure by analytical means is very laborious, of course, can be done and several people have tried in the past try to do solution for simple submerged rectangular cylinder, circular cylinder, half circular cylinder, ellipse. So if you go back to as earlier nineteen sixties, several research papers have been published by simply trying to derive equation for scattered potential in mathematical form, so that people can use it in design. That is before, all these numerical methods were evolved, you know those days no big computers, no finite element method. So those days people were using only mathematical formulations try and find out the solution for that.

You will find in textbooks, some of them may be presented. Nowadays nobody is reading that, because you got big computers and software. So nobody want to read those equations because it is so laborious to, first of all read and understand and then do a computer programming. And also the equations for arbitrary ((Refer Time: 09:01)) so complicated, that you may not even get a solution by solving these, so that is why they were always looking for software which could solve complex problem easily.

So this is the incident velocity potential which we have already; only thing is you see the difference instead of writing down in terms of cosine or sin function, I have just expressed in terms of exponential function which is easy to do integration and differentiation later on, because you will see multiple integral. So basically, I don't know whether you will be covering in hydrodynamics course, this derivation is very lengthy. So the boundary condition what we saw earlier for without the structure inside the domain is exactly the same.

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**Wave loads on Offshore Structures**

**Free surface boundary condition**

The kinematic boundary condition on  $y=\eta$  can be written as

$$\frac{\partial \eta}{\partial t} + \frac{\partial \Phi}{\partial x} \frac{\partial \eta}{\partial x} + \frac{\partial \Phi}{\partial y} = 0$$

which after linearization by neglecting the nonlinear terms in equation

$$\frac{\partial \eta}{\partial t} = 0 \quad \text{at } y=0$$

The dynamic boundary condition on the free surface where the atmospheric pressure and surface tension are taken as zero can be written as

$$-\frac{\partial \phi}{\partial t} + \frac{1}{2} \nabla \Phi \cdot \nabla \Phi + g\eta = 0 \quad \text{at } y = \eta$$

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This is the pre surface boundary condition, which we are applying instead of the adulated surfaces we are supplying at the  $y$  is equal to zero that means we assumed that amplitude of wave. So this again linear diffraction whatever we are trying to do using Aries wave theory do a diffraction analysis or scattering analysis.

This is the kinematic boundary condition and then the body boundary condition basically is same as what we applied as sea bed boundary condition. You know no flow across and then basically if there is a body across the wave, it could come and get reflected hundred percent, if there is solid boundary. If it is basically a porous boundary, some amount of flow can go that is basically defined here. But of ((Refer Time: 10:39)) there is no flow across the sea bed which is zero. And this is the radiation boundary condition at the two extreme ends, that means we have absorbing beaches. For example, you go to our wave plume, we have a small beach at the end that means the waves coming scattered by the presence of body reflects back. But some of the wave will reach across the body and go to the other side doesn't come back from the beach because if that comes, it will increase the reflected wave, which is not our interest; our interest is see the reflection from only the structure that is we accounted.

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**Wave loads on Offshore Structures**

**Diffraction Forces**

- Solution of Boundary Value Problem for Velocity Potential
- Calculation of Pressures
- Integration of Pressures to obtain Forces

Pressure 
$$p = -\rho \frac{\partial \Phi}{\partial t} = -\rho \frac{\partial}{\partial t} (\Phi_i + \Phi_d)$$
$$= -i\omega\rho (\phi_i + \phi_d) \exp(i\omega t)$$

Horizontal Force 
$$F_x = \int_{\Gamma_B} p n_x ds$$

Vertical Force 
$$F_y = \int_{\Gamma_B} p n_y ds$$

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So these are four boundary conditions, so how do we get the solution to our problem. Our interest is not solving this; we are interested in find out what is the horizontal and vertical forces induced by the wave, because the structure is to big. So very simple, what we require is velocity potential of incident wave which is already known; the velocity potential of the scattered wave or reflected wave which is possible to calculate. Now the combined effect, if you know the velocity potential, you can calculate the pressures all around the body. So you go back to this pictures, if you know the boundary coordinates, all along you know the coordinates, because this is your structures now potential is known, so you can differentiate with respect to time to get the pressure distribution all along the body, is not it. And then integrate the pressure to get the forces.

Now the same thing would have happened even if the if you look at the earlier formulation where the structure is small quite small and that is why you assumed that  $d$  by  $l$  is less than point two, we can ignore it. But if you actually look at it, you just use the same velocity potential, for example, incident velocity potential say at the top of the surface and bottom of the surface, there will be a difference in pressure, because of your depth itself. So you could see that when you do the pressure difference using the Morrison formula itself, you will see a pressure component coming, but fortunately that pressure difference will be very small because the structure is quite small across the dimension whether it is diameter or depth whichever. The reason here the

dynamic pressure difference between the top surface and the bottom surface will be large enough because the body size is bigger. For example, you go to this rectangular quantum left side versus right side, you just super impose wave like at this point the water particle velocity and the acceleration and the pressure will be some amount. And the right hand side there could be possibly opposite. So when you look at the difference in this horizontal pressures, the unbalanced pressure will introduced the horizontal force. Basically what we are interested is integrate the pressure difference across the body in vertical and horizontal directions. So simple numerical integration or manual calculation as long as you could find out what is the velocity potential derivation.

Of course, we have easy equation for incident potential which is very well known; only what we are looking at is, the diffracted potential or scattered potential, which is the question mark. How do we derive it. And for most of the application in offshore, for example, you take the oil and gas applications or ship applications, we have very very pure defined shapes. For example, ship shape, you might have as well start with the barge which is a rectangular body very easy to do integration. If you go to ship shape body, may be a little bit difficult because you profile is not uniform. But you come to oil and gas application, we have very few class of structures like spars, I think we have seen pictures of spars know, simple vertical circular cylinder, is not it. So derivation of such thing if you go back to some of the text books, they do have solution for diffracted potential for a simple vertical cylinder partly submerged in water, which is exactly like a spar. So you go to ((Refer Time: 15:01)) book, you will find a close form solution, but it is so lengthy even to program that you look at considerable amount of time, because you have used greens function and second identity to come up with a mathematical equation which will describe this diffracted potential. Once you know that then doing horizontal and vertical force is very simple numerical integration can bring out your forces.



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**Wave loads on Offshore Structures**

**Methods and Software**

- Diffraction forces can be calculated manually for simple shapes
- For Complex shapes Numerical Methods shall be used such as
  - Finite Difference Method
  - Finite element method
  - Source and Sink Method
  - Boundary Element Method
- Many commercial codes including SACS, SESAM and others can be used

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What are the methods available basically simple shapes that is what I am trying to explain for quite some times. Complex shapes we have finite difference, finite element, boundary element and source and sink, these are some of the methods if you take the numerical methods course – next semester I think. Ye, next semester you will have, in fact all the methods taught including training in some of the computer software, so you will be able to get the idea what is the difference between each of the methods, how do they operate. Basically again an approximation to the reality, finite element is actually trying to bring the problem into smaller geometry, solve the problem within the geometry and make sure the continuity of the variables is followed between the broken down elements.

So finite element method is very just common sense solving the problem in pieces, trying to assemble apply the global boundary condition. Whereas if you look at the analytical formulation, we try to keep the size of the problem same and try to solve there itself. Whereas finite element and finite difference or other methods like boundary element, you solve trying to solve the problem in smaller element, because the geometry become one line and solve it and then go back apply the global boundary condition between the interconnecting elements, so that you don't lose the basic equation of requirement.

So these methods is not new today, they have found their place in engineering mechanics probably like fifty years back, but only application to various types of problem has been growing and growing. And in the recent time like last twenty five to thirty years, a lots and lots of research gone into application of F E M to hydrodynamic problem, and basically you will find lot of software, so we got quite a number of software which offers, solution to the this fluid structure interaction problem. So starting with ((Refer Time: 17:38)), I think you might have heard of this, this is one of the hydrodynamic software where it could do solution to arbitrary shape for wave structure, but unfortunately that program can only do a solution to a body which is not having mooring, basically free motion. There are other software now you know basically like ANSYS have come of with Aqua software where you can solve body free floating as well as moored system. And there are few other small small software which could actually even do body in motion like for example ships. Ships are not in one place know, so you can have forward speed, because most of the application for cargo handling is like that. But offshore application normally our structures are standing in one place which is suitable.



And then they are few other codes in the recent times have come up with even integrating these problem with structures. For example, SACS and SESAM have come up with it can even do solution to the structural analysis part. Whereas the ((Refer Time: 18:47)) Aqua or other software cannot do such portion, can do only hydrodynamic loading, stops there, because then you have to do manual calculation to do that the structure is safe.

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### Wave loads on Offshore Structures

Calculate the wave force on a pile structure installed at a water depth of 100m subjected to regular wave of amplitude 1m with a wave period of 12 sec. The drag and inertia coefficients are 0.6 and 2.0 for respectively. The wind driven current at the surface is 2m/sec and the thickness of the marine growth is 100mm. The wave force needs to be computed at 2.5 sec from start of the wave.

Wave height	$H_w = 2 \text{ m}$
Water depth	$d = 100 \text{ m}$
Wave period	$T_w = 12 \text{ sec}$
Diameter of cylinder	$D_c = 1 \text{ m}$
Thickness of marine growth	$T_{mg} = 100 \text{ mm}$
Hydrodynamic Coefficients	$C_D = 0.6$ $C_M = 2$
Density of water	$\rho_w = 1030 \frac{\text{kg}}{\text{m}^3}$
Wind riven Current Velocity at surface	$V_{cs} = 2 \frac{\text{m}}{\text{sec}}$

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So these methods could be used basically to solve, we will just look at this problem and you will solve it and bring the answer, we can stop.