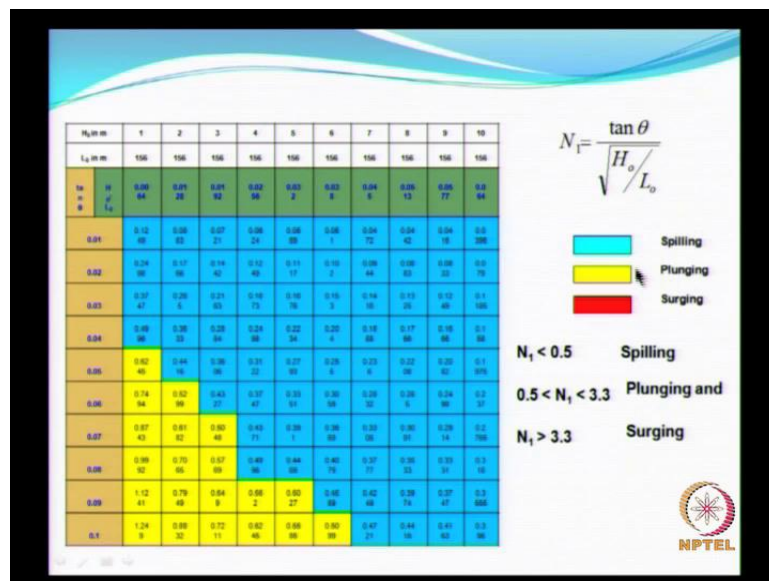


Coastal Engineering
Prof. V. Sundar
Department of Ocean Engineering
Indian Institute of Technology, Madras

Module - 1
Wave Deformation
Lecture - 3
Wave Deformation (Problems –I)

I hope now it is a bit clear about the different types of breaking. So, we have seen this in detail, having done that we will just examine what kind of a breakers you could see under different conditions. For example, of Bay of Bengal may be a south east coast of India, the kind of breakers you would see may be completely different from the type of breakers, you would see somewhere near of Bombay. So, this depends as I have earlier explained basically on three parameters, I would say that is the beach slope and then wave height and the wave period or wave length.

(Refer Slide Time: 01:15)



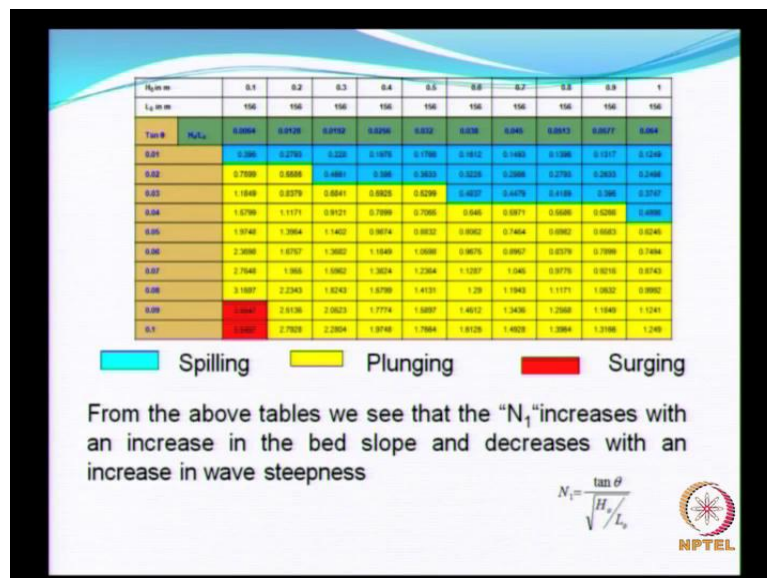
So, the parameter which we had defined earlier is N_1 , and N_1 is the ratio of the beach slope and the square root of the wave steepness. So if you look at this table, what we have done here is, we have assumed the deep water wave height as 1 to 10 meters in intervals of one meter, and we have assumed the beach slope here $\tan \beta$ in orange color. As you can see here from 0.01 to 0.1 in steps of 0.1, 0.01, and then here the green color the values

given here is the variation that we have the values that we have adopted for the deep-water wave steepness.

So from this we are trying to obtain the value of N_1 , as you can see in this. So you see this blue color spread all over gives that the waves are going to be the breaking of waves would be mostly the spilling type, and try to recollect the definition of spilling type of breakers. So you see that there is a wide range of wave heights and beach slopes for which the breaking will be spilling. So in general, you can say spilling is most frequently occurring compare to the other and even plunging to some extent.

So plunging the breakers, if you see that is marked in yellow, and this is more or less the transition between the spilling and plunging type of breakers. So for a given site, if you have the wave characteristics, you can very easily examine what kind of breakers you can anticipate that particular site. So you here, we have varied a deep-water wave height from 1 to 10 meters, but you can also have deep water wave height less than one meter. So let us examine what is the how the variation would look like for wave heights less than, I mean the wave steepness less than 0.1 meter.

(Refer Slide Time: 04:15)



So for example here, in the next, in this slide, you see that we have varied wave deep water wave height from 0.1 meters, 0.1 meter to 1 meter; in intervals of 0.1 meter. And then you look at the variation of your N value that categorizes the different types of breaking of waves. Now you see from this table that we do have a few points, the conditions under

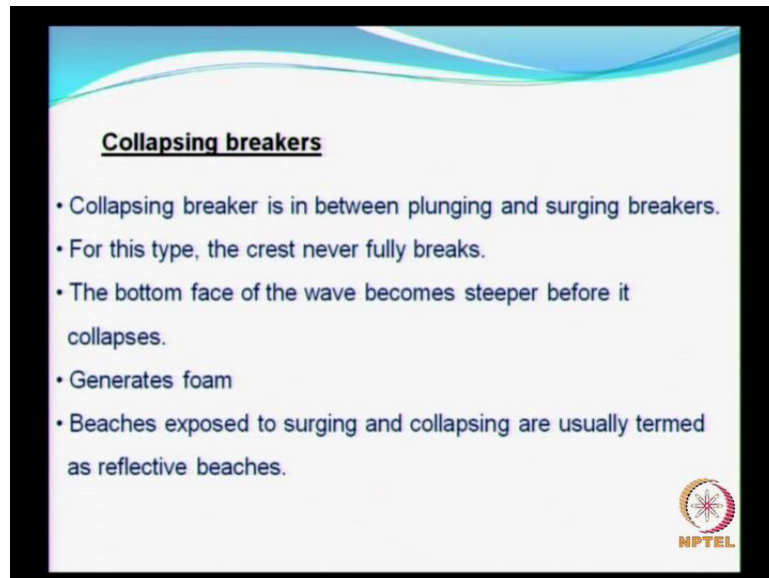
which you can have the surging type of breakers. Surging type of breakers you should I am sure you will be in a position to recollect, surging break of it type of breakers will occur over steep slope, very seldom you would not see them, because normally the tendency is we normally go to beaches where the beach slope is quite flat.

So you for example, if you go to Goa, where the beach is quite flat, then you can walk quite a long distance inside the ocean. And you would be experiencing kind of spilling type of breakers, and of south east coast of India, for example, Chennai where we have the harbor etcetera, in that vicinity you will be able to see a plunging type of breakers occurring more frequently. So here for certain conditions as we have shown here 0.01 and point one for the beach slope and or very low of wave steep wave height you see that you can have your surging type of breakers.

What does the results indicate, the results indicate from the above table you can see that the parameter N_1 that categorize the types of breakers, increases with an increase in the beach slope or the sea bed slope, and decreases with an increase in the steepness. So this certainly matches with what we have earlier seen that for flat beaches, the type of breakers you will experience is the spilling type of breakers and for the steep slope. Slope, you would have surging type of breakers and in between you should have plunging type of breaker.


So this table helps us to understand more clearly these two tables; so I suggest you have a critical look at these tables try to understand what could be the wave characteristics. Suppose if you are located along a beach (()) the beach slope is of certain order of magnitude, which is indicated. Try to look at the table and try to go and examine yourself, whether what kind of breakers you have. Now apart from three widely classified breakers, there is yet another breaker which is not that popular and that is called as collapsing type of breakers. All these breakers, actually they have been examined through laboratory investigation, the characteristics of these kind of breakers or have been verified or have been understood or studied only through critical laboratory investigation - the detailed one.

(Refer Slide Time: 08:20)



Collapsing breakers


- Collapsing breaker is in between plunging and surging breakers.
- For this type, the crest never fully breaks.
- The bottom face of the wave becomes steeper before it collapses.
- Generates foam
- Beaches exposed to surging and collapsing are usually termed as reflective beaches.



So they have examined while doing so, they have also found out another type of breakers and this collapsing breaker is found to be in between plunging and surging breakers. For this type, the crest never fully breaks. So the crest will be more or less looking stable. The bottom face of the wave becomes steeper that is the bottom portion of the wave becomes steeper and that is before it really collapses, and when it collapse it generates lot of foam.

So, now surging and collapsing as we have seen or mostly due to steep waves, propagating over steep slopes so in such case we can term or we can call such kind of beaches, where you have this kind of a two kinds of a breakers namely the surging and collapsing as reflective beaches. So I hope that you have been exposed now on the details of the wave deformation, and also on wave breaking, the conditions of wave breaking etceteras and also categorization of your different wave breakers.

(Refer Slide Time: 10:01)



Parameter	Spilling	Plunging	Surging
$(H_o/L_o)/(\tan^2\beta)$		0.09	4.8
$(H_o/L_o)/(\tan\beta)$		0.00048	0.011

These limits are only approximate. They were obtained for slopes of 0.05, 0.1 and 0.2

Another kind of a classification that has been used widely in literature. Apart from the parameter N 1 and N 2 are shown here in this slide. So, we have the parameter here in terms of the deep-water wave steepness and tan two beta and tan beta. And the classification is given here as spilling, plunging and surging. And the range are shown here 0.09 for this parameter, and in this case it should be 0.00048. And this are all obtained from based on only experimental investigations. So naturally, it cannot be holding, it will not hold good for all kinds of beaches etcetera. For the slopes which have been adopted that is 0.05, 0.1 and 0.2; for few other slopes also you do have some information at this should be good enough as a broad guideline, so that shows the that explains the complete description of the spilling plunging and surging type of breakers.

(Refer Slide Time: 11:32)


Formulae for prediction of Breaker Depth/ height

The Majority of the existing formulas represent a relationship between the breaking wave height (H_b) and the variables at the breaking or deepwater conditions, i.e., water depth at breaking (h_b), wavelength at breaking (L_b), bottom slope (m), deepwater wavelength (L_o), and deepwater wave height (H_o).

McCowan (1894) $H_b = 0.78h_b$ (1)

Miche (1944) $H_b = 0.142 L_b \tanh\left(\frac{2\pi h_b}{L_b}\right)$ (2)

LeMehaute and Koh (1967) $H_b = 0.76H_o \left(\frac{H_o}{L_o}\right)^{-1/4} m^{1/7}$ (3)



We had earlier looked at the different criteria for the waves to break. And we also saw that the waves can break not only in shallow waters, but also in deep-waters. Waves break in shallow waters, because we see every time we go to the shallow water we see the waves breaking. And there are certain conditions or criteria for a wave to break, which I had earlier explained as the depth limited criteria which will takes place in the shallower waters or the steepness limited criteria in the case of deep-waters. There are several formulas available in literature, apart from the criteria which we have seen, for example, McCowan he has he is relationship is widely used what is a breaking wave height you ask it can be easily said that its equal to approximately 0.78 times the water depth. This 0.78 is a kind of an empirical coefficient and some people use it as 0.8.

Then Miche he has, so if you look at McCowan's criteria, do you get the breaker wave height as a function of just the breaker depth or nothing else. For example, if you are knowing a certain location, this is a location where the wave is breaking I am talking about the shallower waters. From that depth, you can easily find out what could be the wave height at that point which will be governed by this relationship. Miche's criteria as early as 1944, he has given a relationship between wave height, water depth and of course, breaking wave length. So this is another criteria which people use.

The third one is LeMehaute and Koh in 67, they obtained an expression in terms of deep-water steepness, deep water wave height and beach slope. So which formula you want to

use, apart from this, you have several other formulas. Now it depends on the kind of problem you are working on. So if you want to have just personal information like for a first approximation, you can use the first one without any problem. But, if you want to account for the effect of beach slope, and in case you have the deep water characteristics like your deep-water wave height and the wave steepness, wave deep water wave height and wave the period then probably you can use the third one, because this takes care of your beach slope that is m into m rise to 1 by 7.

So one where you use the breaker depth on breaker a height all this information is one either you use it for evaluation of the wave forces, you do breaking waves or may be it can be in the costal engineering practice to estimate sediment transport etcetera. So that is different subject by itself, but you should know that there are formulae's available; apart from this there are several other formulae, but I am restricting my discussion only in this only to this some of this three formulae's.

(Refer Slide Time: 15:56)

Problem 1


A 10 second period wave having a height of 5m propagates from deep to shallow waters. Assuming that the bottom contours are parallel to each other, compute the wave height at a water depth of 10m.

Solution

$$L_o = 1.56 T^2 = 1.56 * 100 = 156 \text{ m}$$

Corresponding $\frac{d}{L_o} = 0.0642$ (from wave table)

$$\frac{d}{L} = 0.1082, C_o = \frac{L_o}{T} = 15.6 \text{ m/sec.}$$



Having seen all this things, now let us understand some of what we have gone through on the topic wave deformation with a few problems. I will start with the most fundamental problem, and try to show you the application of the different formulae and that should give you a broader understanding of the subject what we are discussing about.

Problem number one - 10 second wave period, period wave having a height of about 5 meters here, propagates from deep to shallow waters. The assumption here is the bottoms

depths are parallel to each other something like a ramp, so this could be the most easiest problem you can think of. So this is only an assumption, so there can be loss or gain. If the bottom sea bed is not even; if it is uneven, you can either have increase in energy or decrease in energy. So here we want to estimate the wave height in a water depth of 10 meters. So you already know how to calculate your wave length, so calculate your deep water wave length. It is come to d by L naught then calculate using from the tables you can calculate d by L. And then calculate your wave length and also the deep water celerity in this case 15.6 meters per second.

(Refer Slide Time: 18:02)


$$L = 92.4\text{m}, C = \frac{92.4}{10} = 9.24\text{m/sec.}$$

$$K = \frac{2\pi}{L} = 0.065, \quad 2kd = 2 * 0.065 * 10 = 1.31$$

$$n = \frac{1}{2} \left[1 + \frac{2kd}{\sinh 2kd} \right] = 0.88$$

$$\frac{H}{H_o} = \sqrt{\frac{C_o}{C} \frac{1}{2n}} = \sqrt{\frac{15.6}{9.24} \frac{1}{2 * 0.88}} = 0.9828$$

Therefore wave height, H at d = 10m is 0.9828 * Ho = 4.91 < Ho



And the celerity from the earlier, earliest value, you can because the water depth is known, so you can from this relationship you can get the wave length and the celerity is l by t as we have seen earlier. Now we have the wavelength and you can calculate the other parameters and this is the, these is all this parameters are needed in order to evaluate the factor n which can also be taken from the tables directly, from the wave tables. Once this is known, you know what is the relationship that is since the bottom depths are parallel to each other, we are now considering the only the phenomena of shoaling. So the less of a botheration, straight away use this equation which we have already proved by equating the power in the deep water to the power in the shallow waters. So from this, you get the ratio, the value for the ratio of H and deep water wave height is that clear.

So here it is 0.98, so therefore, the wave height in a water depth of 10 meters is now working out to 4.91 meters. What was the deep water wave height, 5 meters. Now this is what you get for this kind of a wave period and this kind of assumption of parallel water depth contours.

(Refer Slide Time: 19:54)

Problem 2

A deep water wave of height 3.5m and period 10 seconds is refracted so that the distance between the orthogonal is reduced by fifty percent at the depth of 10m and additional 30% at 5m water depth. What will be the height of the wave here assuming no energy losses.

Solution

$$\frac{H}{H_o} = K_s K_r$$


$$K_s = \sqrt{\frac{1}{2n} \frac{C_o}{C}}, K_r = \sqrt{\frac{b_o}{b}} = \sqrt{2}$$

$$L_o = 1.56 T^2 = 156m$$

$$\frac{d}{L_o} = 0.064, \frac{d}{L} = 0.1082$$

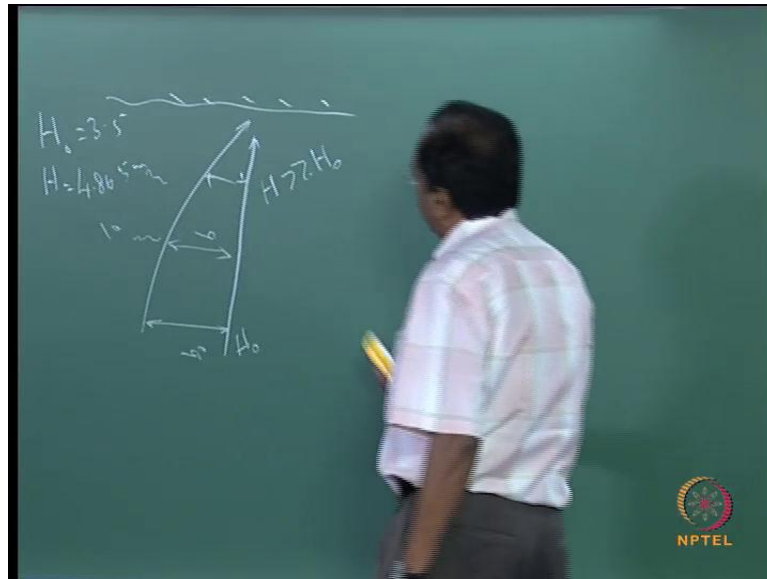
$$\sin \alpha, \frac{b_o}{b} = 2$$

$$K_s = \sqrt{\frac{1}{2n} \frac{C_o}{C}} = 0.9828$$

$$\frac{H}{H_o} = 0.9828 * \sqrt{2} = 1.389$$


Let us look at this problem. Here in the deep water wave height is 3.5 meters, the wave period is 10 seconds and it is refracted so that the distance between the orthogonal is reduced by 50 percent in a water depth of 10 meters and reduced by 20 percent in 5 meters water depth.

(Refer Slide Time: 20:32)



So what does this mean, this means that when the wave is this is the shore, so when the wave is moving like this. So you see that this is the deep water, distance between the orthogonal, here if you have about something like 10 meters and something 5 meters. So this is the b , the reduction is said to be 50 percent something like. It is just an assumption, so here it is percent reduction percent means 80 percent of the this will be 80 percent of the b naught, this will be 50 percent of b naught; b naught is the distance between the orthogonal in the deep waters. Recollect again, what we have seen under the topic wave refraction.

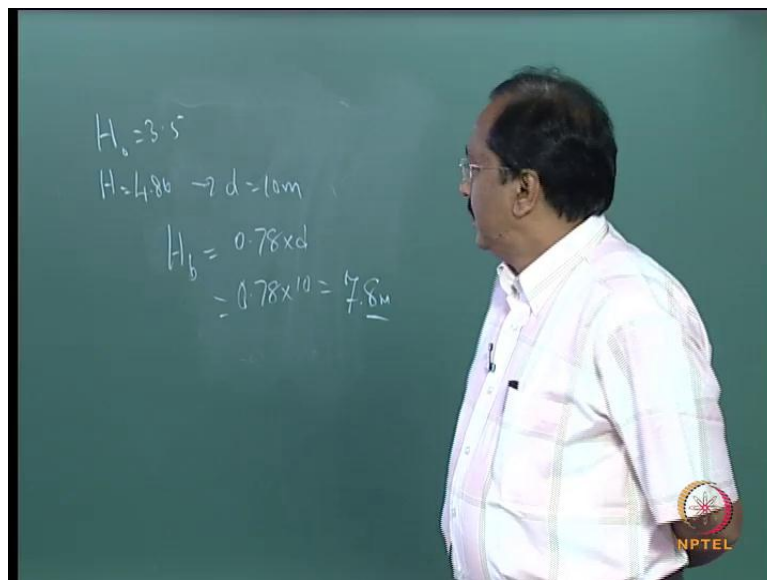
So as we have seen, we can change the problem also like you can have divergence. If you practically do all this things, you will for a practical problem if you apply the phenomena of refraction. As we have seen earlier, you can have either the orthogonal converging or orthogonal diverging. So here we have taken the case of convergence of orthogonal, what does that means? Convergence of orthogonal means, the wave energy should increase is that clear? So based on this what is the solution for the so we what will be the wave height assuming no energy losses. So energy loss can be due to so many other factors, we are not considering diffraction here. There can be a some amount of energy which is escaping in the lateral direction. So in this case, what is the solution, what is the expression h is equal to H by H naught equal to K_s (the product of shoaling coefficient) and the refraction coefficient. And the shoaling coefficient you have already have the expression as shown here, whereas refraction coefficient is the ratio of square root of the ratio of the spacing

between orthogonal in deep waters to the spacing between the orthogonal in the shallow waters any given water depth.

So now as per the problem, it is clearly said if this is b_{naught} . There is reduction of b by about 50 percent, so b_{naught} by b is going to be two. So your refraction coefficient is square root of two. So when b_{naught} is then you calculate your other parameters L_{naught} then d by L then you know how to calculate your case as we have seen in the earlier problem is that clear? So now you have both your shoaling coefficient and the refraction coefficient, substitute and kept the ratio between H and H_{naught} .

So, you see that if this is the deep-water wave height, if there is a convergence of the orthogonal, the wave height is going to increase, increase compare to H_{naught} , is that clear? What was the deep-water wave height? The deep-water wave height was given as 3.5 meters what is the wave height in now in 10 meters water depth, it is 4.86 meters, so you have H_{naught} as H_{naught} as 3.5 meters. Now H is equal to 4.86 meters in the water depth of how much, in a water depth of in a water depth of 10 meters is that clear?

(Refer Slide Time: 25:06)



The other things what you have to check is, do we sometimes you have a numbers, but you should also verify whether that numbers what you get is correct or not. So in this case, because all those values which we have taken for the problem, it is all assumed. I have assumed cretin value for the deep-water wave height. I have assumed that the ratio will be the ratio between b_{naught} and b will be of this value. So later, we have to make sure that

the assumptions are correct or not. It should not give some wrong results. There may be a situation that the orthogonal will never come close, this depends on so many other factors, for example, the bathymetry etcetera.

So in a water depth of ten meters, we have a wave height of 4.86 meter what is the maximum wave height that can occur that is equal to 0.78 times water depth, which is nothing but your breaker wave height, breaker wave height is 0.78 times water depth, and here the this water depth is shallow a 10 meters. So you can definitely have wave height of up to about 7.8 meters. So there can be situation where the orthogonal can converge even less than even what is indicated here. Suppose by having an assumption and if you have a value much greater than this that means this kind of a situation will not occur is that clear?

(Refer Slide Time: 27:25)


Where as breaking wave height $H_b > 0.78d$, which for 10m depth will be 7.8m.

For the above problem, if refraction is not considered,
 $H / H_o = K_s = 0.9828$,
Hence $H = 0.9828 * H_o = 3.43m < H_o$ of 3.5m

b) H at d=5m

$$\frac{H}{3.5} = \sqrt{\frac{1}{2 * 0.9349 * 6.8 * \sqrt{\frac{1}{0.8}}}}$$

$$H = 3.5 * 1.24 = 4.34m > 0.78d (=3.9m) \text{ for } d=5m$$



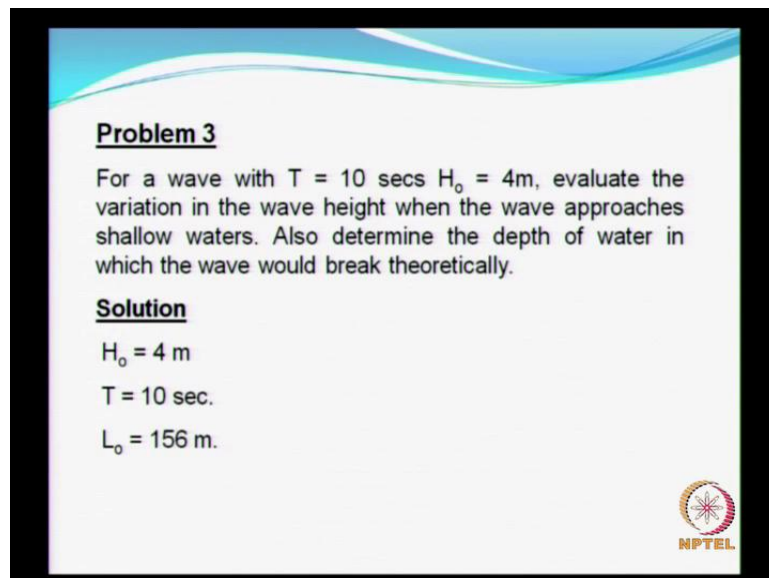
Now we go in to the next, so this is what is explained here. So this is, what I have explained here, whereas the breaking wave height of H_b of approximately 7.78 times water depth will be 7.8 meters, this what I have explained here. For the above problem, suppose if the refraction is not considered, if the refraction is considered the wave height is this much; if the refraction is not considered then you see that your shoaling coefficient will be just less than one, which means your wave height in 10 meters water depth will be less than the deep water wavelength.

Now is it clear, wave can increase or decrease in shallow waters compare to the its value in deep waters. It depends on the variation in the bathymetry. So earlier what we did, we did

not consider the bathymetry variation, we just consider that it is the a slope. Here also if we do not consider the variation in the bathymetry, you see that the wave height in 10 meters water depth is less than the deep-water wave height. Now the second subdivision, the next subdivision in the same problem is wave height is equal to 5 meters, I mean in water depth of five meters. It is said that the ratio of b naught to b is 1 by 0.8. It is said twenty percent reduction. Is that clear? So that is what I explained the beginning of the problem, now you calculate this is your sorry that is your shoaling coefficient, this is your shoaling coefficient and this is your refraction coefficient.

Now you look at the value, the value is something like 4.34 meters. So will this happen 4.34 meters, what is in that particular water depth, in this particular water depth of 5 meters, what would be the wave height, what could be the maximum wave height 0.78 times water depth that is equal to only 3.9 meters. But, what are you getting here, you are getting a value 4.34 which is greater than 3.9 meters, so this cannot occur, so sometimes it is better to know the negative aspects, so that you understand that the subject more clearly, is that clear? Now you look that this effect.

(Refer Slide Time: 30:36)




Problem 3

For a wave with $T = 10$ secs $H_o = 4$ m, evaluate the variation in the wave height when the wave approaches shallow waters. Also determine the depth of water in which the wave would break theoretically.

Solution

$H_o = 4$ m
 $T = 10$ sec.
 $L_o = 156$ m.



So next, so the previous problem says that in the situation what we have seen, when you have a reduction by about 50 percent in 10 meters, there may not be a possibility of having further reduction within 5 meters that is what it says. So now, the next problem is for a wave of 10 seconds period, H naught equal to 4 meter. Now we will try to find out the


variation of wave height, when the wave approaches shallow waters from deep waters, and also determine the depth at which the wave would break theoretically. So L naught can be calculated.

(Refer Slide Time: 31:41)

The depth in which the wave would break is computed from the Miche steepness equation i.e.,

$$\frac{H}{L} \geq 0.142 \tanh kd.$$

The wave would break at $d \approx 5$ m.




Now all these things we are trying to use this definition which is the Miche's criteria which we have seen earlier. So latter, you will see when we examine this, we will see that wave would break something around 5 meters, but the problem is, in the problem you have been asked to look at the variation of the wave height.

(Refer Slide Time: 32:10)

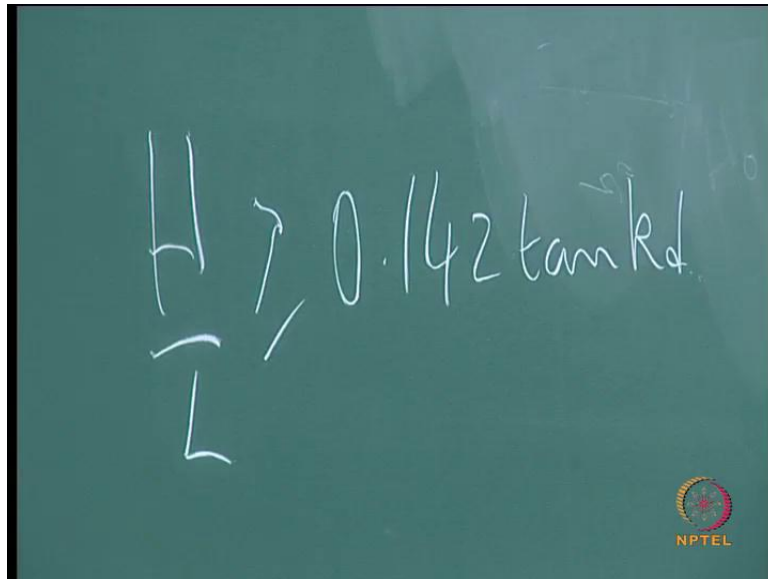
Calculation of variation of wave height

d(m)	d/L ₀	d/L	L	H/H ₀	H	H/L	0.142 tanh kd
70	0.45	0.4531	154.49 1	0.9847	3.938	0.0254 9	0.1390
60	0.385	0.3907	153.57	9728	3.891	0.2534	0.1379
50	.32	.3302	151.42 3	9553	3.821	0.2524	0.1357
45	.288	.3014	149.30 3	9449	3.779	0.2531	0.1338
35	.224	.2455	142.56 6	.9242	3.697	.02593	.1278
25	.160	.1917	130.41 2	.9130	3.652	.0280	.1169
15	.0964	.1380	108.69 6	.9358	3.743	0.3444	.09786
6	.0386	.08175	73.395	1.072	4.228	.05842	.06608
4	.0257	.06613	60.487	1.159	4.636	.07664	.05504



So now I take a water depth of 70 meters, I start from 70 meters.

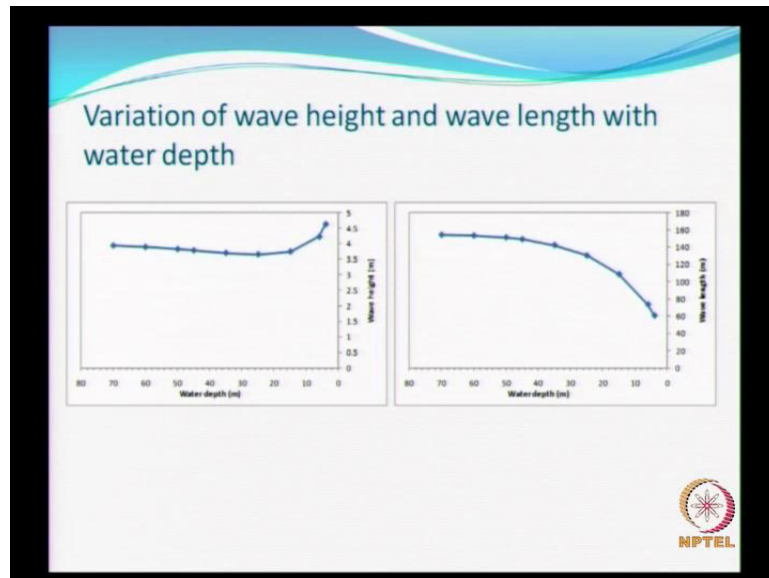
(Refer Slide Time: 32:27)


$$\frac{H}{L} = 0.142 \tan h k d$$

What is the criteria the criteria is H by L , H by L is $0.1472 \tan h k d$. This is the well-known criteria. Now what does this indicates, so we will use this, now try to arrive at the relationship on the right hand side and the left hand side with the help of this problem. Then the water depth starts from 70 meters. And for 70 meters, I have calculated d by L naught, d by L wave length is calculated then H by H naught is calculated from which I can obtain H , and then H by L , and then this parameter which is on the right hand side which are you suppose to which are the two which we are suppose to monitor we are suppose to look at the variation of these two.

So all through you see that h by l is less than $0.412 \tan h k d$. So this is less than this, all the way. Until somewhere you see that between 6 and 4, you see that this is less than this, but then this value increases that so that means the wave is going to break in between 4 and 6 meters. So this is a problem which explains you that depth limited breaking, the criteria of depth limited breaking depth limited as well as your steepness also. Suppose, in case, you have only the deep water then your $\tan h k d$ will not come into picture at all; only H by L will come into picture. Now since that water depth is also included, I am calling it as depth limited as well as the steepness limited.

(Refer Slide Time: 34:38)



Now for the same problem, what I have shown here is, the variation of your wave height from a water depth of 70 meters, and you see that there is a gradual decrease. And as you proceed towards the coast, you see that there is a slight increase in the wave height and somewhere close to the breaking condition the wave will break. And this is a figure, the right hand one right side the variation of the wave length is shown, for the above problem that is as you go towards the shore the wave length is going to decrease. So this two phenomena, we have examined in this.

(Refer Slide Time: 35:29)

Problem 4
For a wave with period $T = 10\text{sec}$, deep water wave height $H_0 = 3.5\text{m}$ and beach slope $m = 1/15$ calculate the breaking wave height using various formulae.

So, we will look at final problem for today. So, for a wave of about 10 seconds, for a wave period 10 seconds, deep-water wave height is given, and beach slope is 1 is to 15. Calculate the breaking wave height using various formulas. Earlier we have seen about three formulae, now I will list some few formulae, I will not try to solve, because it is all just application of some simple formulae. But, you will get the variation in the wave height predicted by the some of this formulae.

(Refer Slide Time: 36:15)

Sl.No	Formulae for Breaking wave height	Breaking wave height (H _b) m
1	Le Mehaute and Koh(1967) $H_b = 0.76 \cdot H_0 \cdot (H_0/L_0)^{-0.25} \cdot m^{1/7}$	4.668
2	Komar and Gauhgen(1972) $H_b = 0.56 \cdot H_0 \cdot (H_0/L_0)^{-0.2}$	4.189
3	Sumura and Horikowa(1974) $H_b = H_0 \cdot m^{0.2} \cdot (H_0/L_0)^{-0.25}$	5.262
4	Singamsetti and Wurd(1980) $H_b = 0.575 \cdot H_0 \cdot m^{0.031} \cdot (H_0/L_0)^{-0.254}$	4.854
5	Ogawa and Shuto(1984) $H_b = 0.68 \cdot H_0 \cdot m^{0.09} \cdot (H_0/L_0)^{-0.25}$	4.819

So the first one is le Mehaute and Koh problem, which accounts for the variation in the beach slope as we can see here. And this predicts a value of 4.7 approximately for a breaking wave height, whereas the next below one that is Komar and Gauhgen gives a value of about 4.2. And the other one is the third one is something like a 5.3, then fourth one is 4.85. And finally, the fifth one Ogawa and Shuto gives you a value of about 4.8.

If you see there are several other formulas as stated earlier, mostly the values is around 4.8 to 5 something like that. So for example, the second one looks slightly underestimating the a breaking wave height. So depending on the kind of wave you have, there are several other kinds of formulas for the breaking of estimating the breaking of waves. So I think, we have seen enough on the breaking of waves and then in case you have any questions you can ask me right now. So, if not, I and I assume that it is all clear for today.

Thanks.