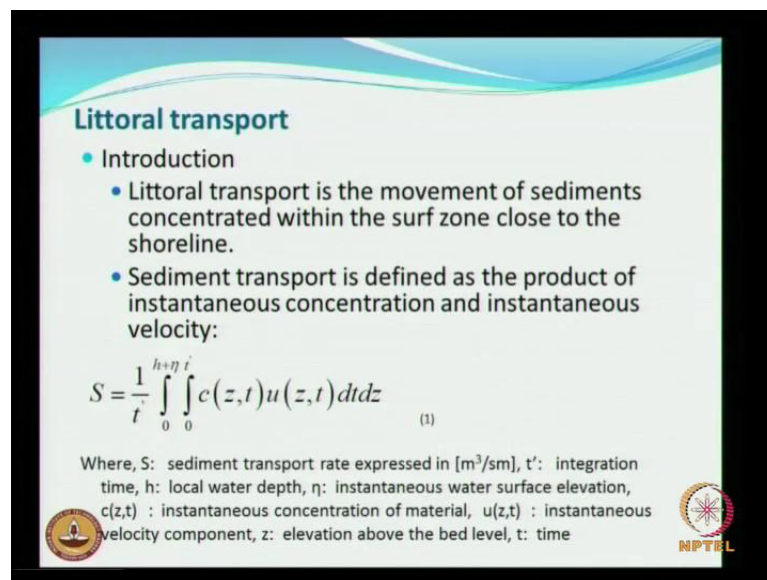


Coastal Engineering
Prof. V. Sundar
Department of Ocean Engineering
Indian Institute of Technology, Madras

Module - 2
Sediment Characteristics and Longshore Sediment Transport
Lecture - 3
Radiation Stresses - I

I have already explained it to you to certain extent, how this sand moves in the ocean, but when the sand moves, the physical process we have seen to some extent, but unless there is a kind of driving force, the sand will not move. There should be something which has to push the sand. So, the main driving force behind the sand movement are the radiation stresses. What are these radiation stresses? This is very important as it is the only driving, main driving force and then, they are wave induced and this is what we will try to see in the present lecture.

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Littoral transport

- Introduction
 - Littoral transport is the movement of sediments concentrated within the surf zone close to the shoreline.
 - Sediment transport is defined as the product of instantaneous concentration and instantaneous velocity:

$$S = \frac{1}{t'} \int_0^{h+\eta} \int_0^i c(z,t) u(z,t) dt dz \quad (1)$$

Where, S: sediment transport rate expressed in [m³/sm], t': integration time, h: local water depth, η: instantaneous water surface elevation, c(z,t) : instantaneous concentration of material, u(z,t) : instantaneous velocity component, z: elevation above the bed level, t: time

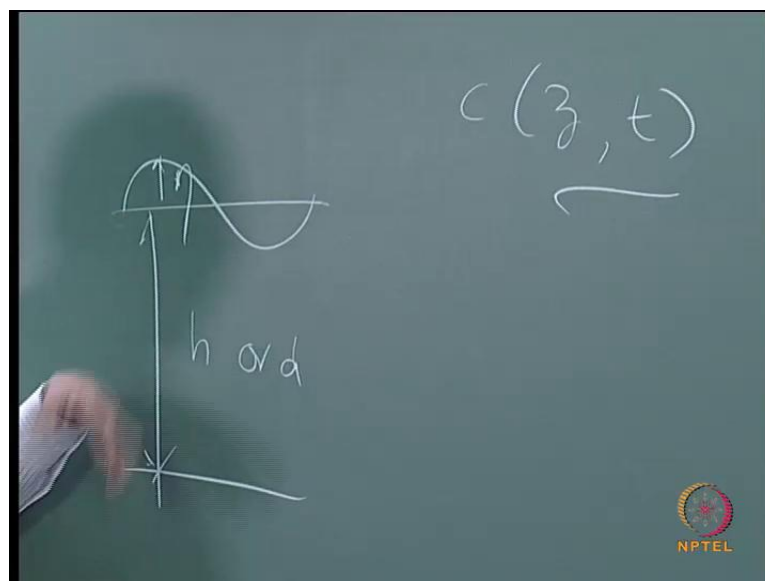
 

Littoral transport or the movement of sand or the movement of sediments concentrated within the shore line, and it takes sense within the surf zone which we have already seen.

Once the wave breaks and that is why, you have the stresses set in the longshore currents set in and the movement. If it is in the oblique direction, you have two components; one along the shore and one normal to the shore. All these things we have already seen.

Now, sediment transport is defined as the product of instantaneous concentration and the instantaneous velocity. So, this can be average. As we have seen earlier, how do you get the total energy per unit surface area? We integrate. Remember, you try to recollect when we were trying to get the potential energy or the kinetic energy. In the case of a kinetic energy, we used to have triple integral and in the case of potential energy, you used to have double integral. So, one is here. This is integrated form from 0 to h place η over the entire depth including the η , including the wave elevation above the water surface or above the still water.

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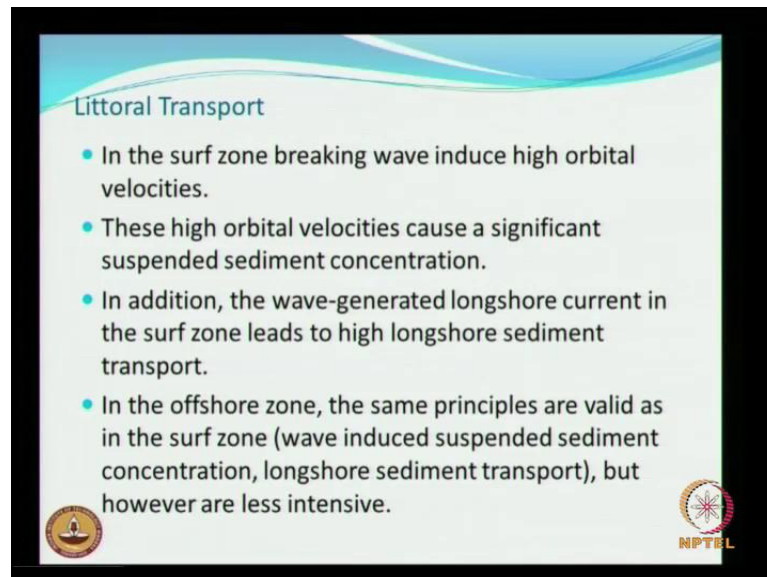


So, this is η and this is your h or d . So, here S , the sediment transport rate expressed in meter cube, per meter cube per meter, then you have the t dash which is the integration time over the entire time. It is averaged out and then, h is h or d is the local water depth and instantaneous η is the instantaneous water surface elevation c , z t is the concentration, instantaneous concentration of the material. This we have already seen earlier.

Remember, as one of the, I mean sediment under the sediment characteristics, one of the parameter that is the concentration. Remember? Yes or no. Under the sediment

characteristics and then, u is the instantaneous velocity component and z is the elevation above the sea bed. So, in this particular claim, it is above the sea bed. It can be, you can always have a change, the coordinate axis. You can have this as minus t and this as η . So, this can also be 0 or this can be 0. Some people take this as 0.

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Littoral Transport

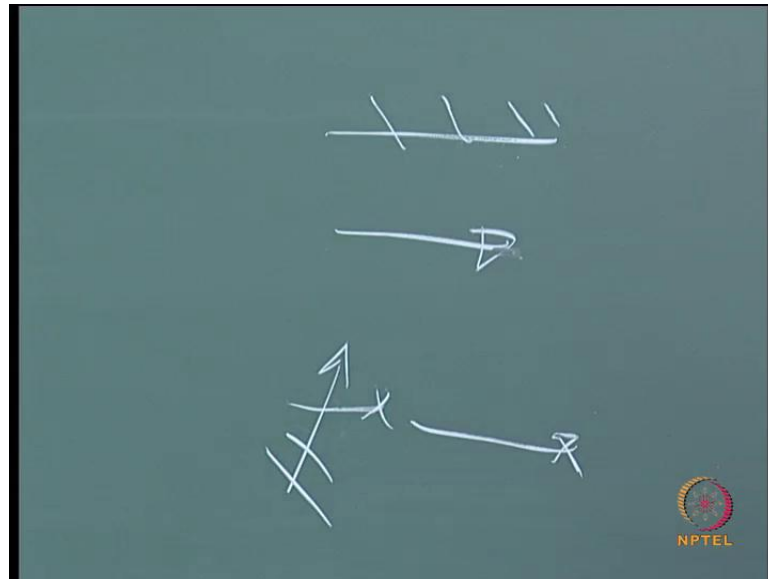
- In the surf zone breaking wave induce high orbital velocities.
- These high orbital velocities cause a significant suspended sediment concentration.
- In addition, the wave-generated longshore current in the surf zone leads to high longshore sediment transport.
- In the offshore zone, the same principles are valid as in the surf zone (wave induced suspended sediment concentration, longshore sediment transport), but however are less intensive.

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So, this is just to refresh whatever we have already seen. I will just go through it. In the surf zone breaking wave induce high orbital velocities. This we know. These high orbital velocities cause a significant suspended sediment concentration. This also I have explained. In addition, the wave-generated longshore current in the surf zone leads to high longshore sediment transport because this also we have seen. If you have the shoreline, the breaking takes place and then, you have longshore sediment transport. Once you have a wave approaching at an angle, thus that will break and that induces the longshore sediment, I mean currents and which is going to drive the longshore sediment transport.

In the offshore zone, the same principles are valid as in the surf zone. Wave induced suspended sediment concentration, longshore sediment transport, but however, they are a bit less intensive. So, this is about your sediment transport.



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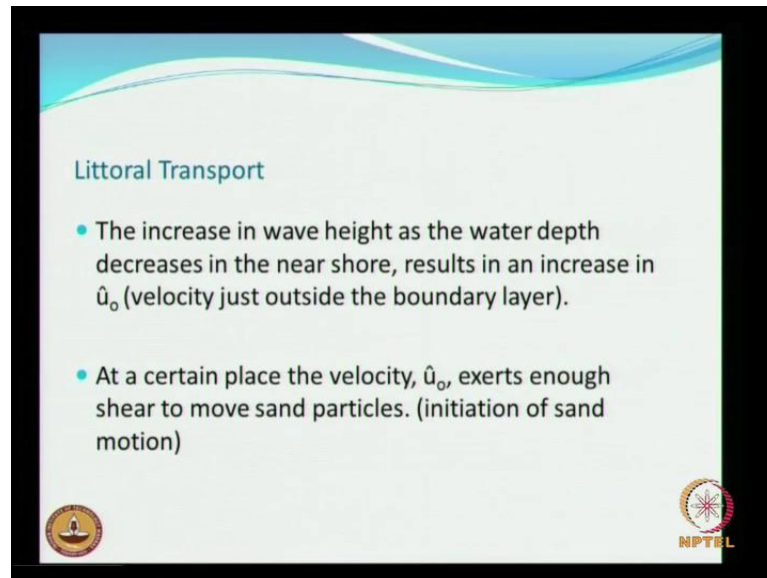
Littoral Transport

- Wave travelling towards shallow water depth reaches a depth, where, the water motion near the bottom begins to initiate the motion of the sediment on the sea bed.
- At first, only low-density material (such as seaweed and other organic matter) moves.
- This material oscillates back and forth with the orbital motion of the waves.



Then, we will move further. Wave travelling towards the shallow water depth reaches a depth, where the water motion near the bottom fields. So, what we are, initially we spoke about the wave breaking, the setting of the currents which is nothing, but the driving force and now, we are talking about the initiation of the schedule motion. This also we have seen. Remember and we have also worked out a problem on the initial initiation of the, so this material oscillates back and forth because of the orbital motion of the waves.

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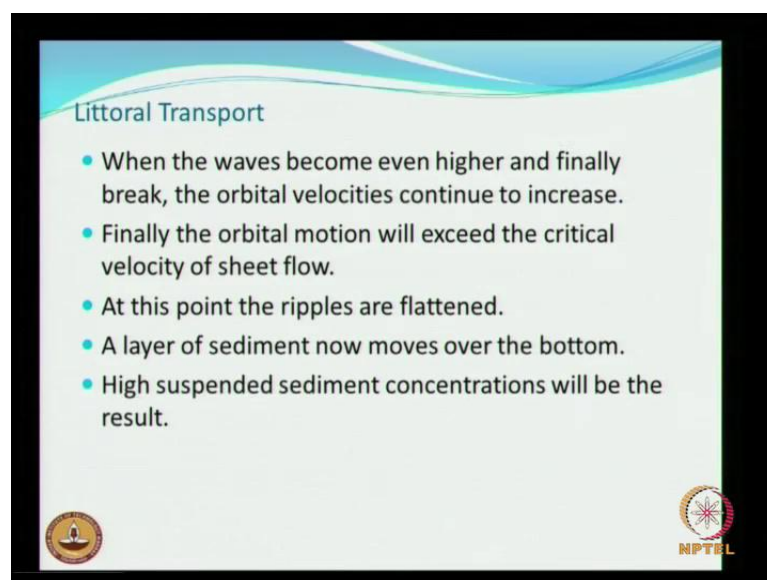


Littoral Transport

- The increase in wave height as the water depth decreases in the near shore, results in an increase in u_0 (velocity just outside the boundary layer).
- At a certain place the velocity, u_0 , exerts enough shear to move sand particles. (initiation of sand motion)

This also we have seen. We are just refreshing the whole thing. The increase in wave height as the water decreases in the near shore, results in an increase in the horizontal velocities. Then, at certain place, the velocity exerts enough shear to move the sand particles. This is nothing, but the initiation of the sediment in motion that is the velocity that is required to move the sediments.

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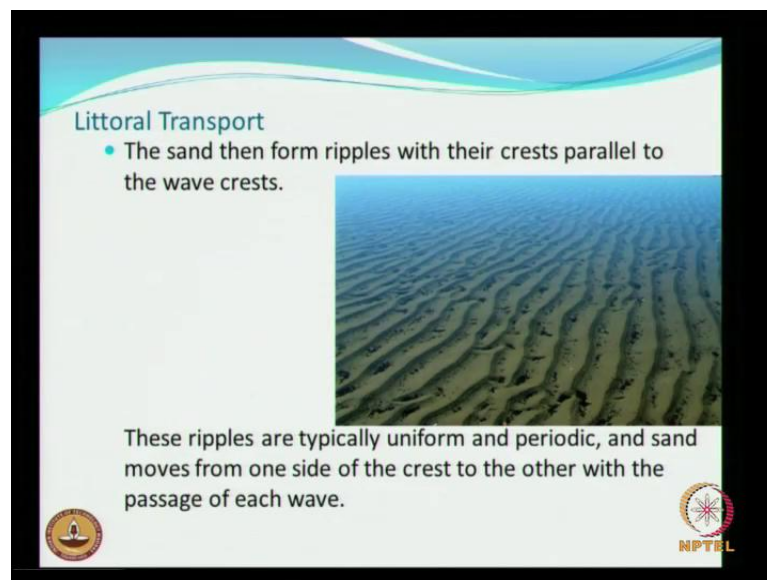
Littoral Transport

- When the waves become even higher and finally break, the orbital velocities continue to increase.
- Finally the orbital motion will exceed the critical velocity of sheet flow.
- At this point the ripples are flattened.
- A layer of sediment now moves over the bottom.
- High suspended sediment concentrations will be the result.

So, this again is the same thing that is when the waves break, when the waves become even higher and finally breaks, the orbital velocity continue to increase. Finally, the

orbital motion will exceed the critical velocity of the sheet flow. At this point, the ripples are flattened. I think there is a layer of sediment now moves over the bottom. So, what will happen is the group idea? So, the sand then forms ripples with their crest parallel to the wave crest. So, as you can see here, initially this is how the ripples are formed.

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These ripples are typically uniform and periodic and the sand moves from one side of the crest to the other with the passage of each wave. Then, when the waves become even higher and finally break, the orbital velocities continue to increase. Finally, the orbital motion, they will exceed the critical velocity of the sheet flow. At this point, the ripple gets flattened to some extent. A layer of sediment then moves over the bottom. So, it moves over the bottom and high suspended sediment concentration will be the result. Some of these sediment will get picked up depending on the sediment characteristics as well as the wave characteristics. It will get picked up and the characteristics also allow them to get distributed over the depth.

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Littoral Transport

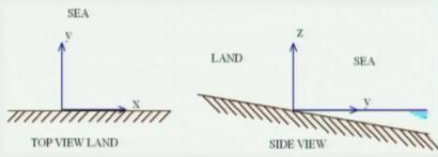
- The processes which are involved in the sediment motion can be divided into three categories:
 - the forces that are responsible for the generation of the longshore current,
 - the sediment sources,
 - the transport mechanism.



So, the process which is involved in the sediment motion can be divided into the following three categories. One is the forces that are responsible for the generation of longshore current, and this is what we are going to look into in this today's lecture. Then, the sediment sources of course can easily be determined whether there are any rivers and mostly, it is rivers or may be some from hills or some sand dunes etcetera; then finally, the transport mechanic. See these are the important categories under which the sediment motion can take place.



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Littoral Transport



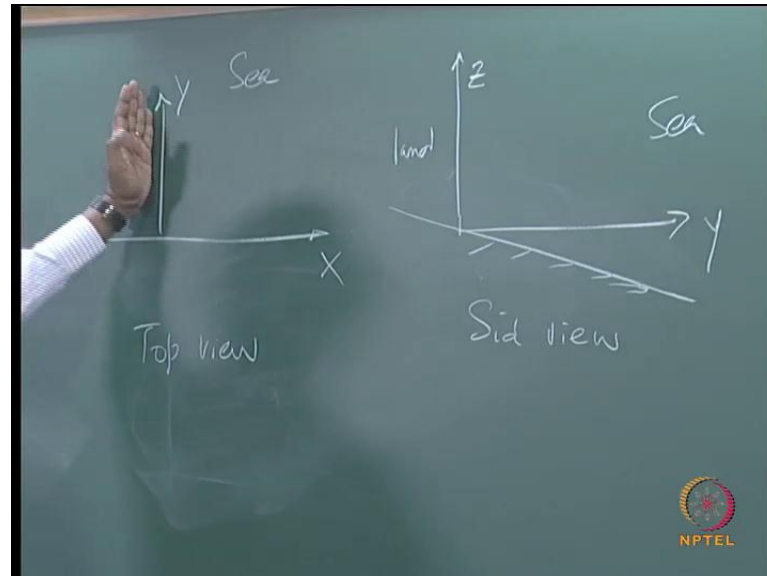
Axis system used

- The x-axis is horizontal and parallel to the shoreline.
- The y-axis is also horizontal, normal to the shoreline and positive in the direction of the sea.
- Waves approaching with crests parallel to the coast are therefore travelling along the y-axis in the negative direction.
- The x-axis therefore coincides with the shoreline.



Now, when we are trying to derive the expressions for the radiation stresses, we assume the following here in the top view.

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So, here you have Y and this is the sea and here, this is X. So, this is the side view indicating this is the beach slope and this case, this will be Z and this will be your Y. So, this will be the sea and this will be the land. Have this clear in your mind what we are talking about. So, there is X and Y and these are Z and Y. The X axis is horizontal and parallel to the shore line. That is clear from this picture. The Y axis is also horizontal, normal to the shoreline and positive towards the offshore. Waves approaching with the crests parallel to the coast are therefore travelling along the Y axis negative and X axis. Therefore, coincides with the shoreline. That also is clear.

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Now, what is meant by radiation stress? This radiation stress, there is a classical paper by Longuet and Higgins. Longuet Higgins is the single name. As early as I think, it is 1958 and the paper quite runs into a number of pages, several pages wherein he discusses very clearly about all these aspects.

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RADIATION STRESS

- It is known surface waves possess momentum which is directed parallel to the direction of propagation and is proportional to the square of the wave amplitude.
- Now if a wave train is reflected from an obstacle, its momentum must be reversed. Conservation of momentum then requires that there be a force exerted on the obstacle, equal to the change of a wave momentum.
- This force is a manifestation of the radiation stress.
- For sediment transport considerations the x-y plane is placed on the sediment bed in the water. The z-axis is then vertically upwards from the bottom. Herein the stress is by definition equivalent to a flow of momentum.

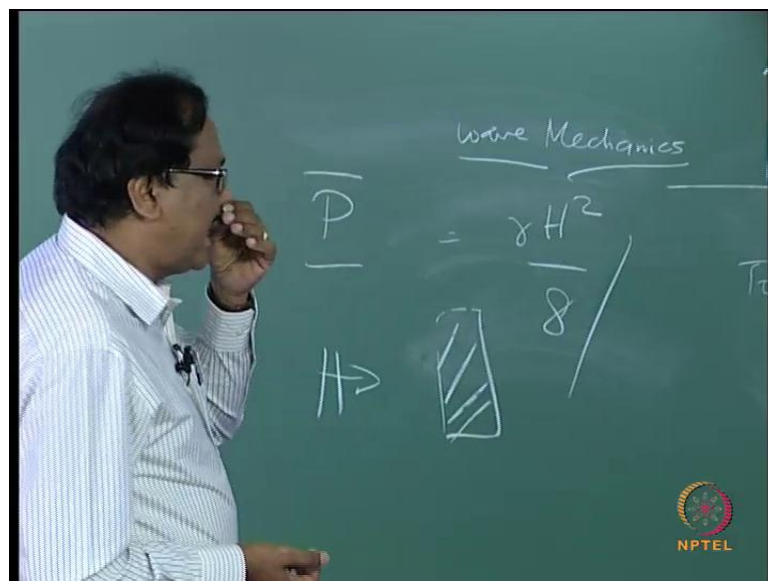
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So, what does this radiation stress say? I have given, there are some points. Just we will go through these points. It is known that surface waves possess momentum which is

directed parallel to the direction of wave propagation and is proportional to the square of wave amplitude. Am I right?

Wave power. What is wave power? Definition of wave power. It is the rate of change of momentum in the direction of flow, extending over the entire plane, extending from the sea bed and piercing the water surface.

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Do you remember how we got the power? We derived in the same class. So, look at the lecture material on basics of wave mechanics, wherein we have seen about all these. The first point is the rate of change of momentum over a vertical plane extending from the sea bed piercing the water surface and this is what is called as the wave power, right and wave power is what γ into H square by 8. That is what is being said here.

So, now if a wave train is reflected from an obstacle, so you have an obstacle. Now, its momentum is then reversed because of the presence of the obstacle, but consideration of momentum is then required that there will be a force exerted on the obstacle equal to the change in the wave momentum. So, now, this force is nothing, but a manifestation of the radiation stress. Is that clear?

Now, whenever you have a, again let us repeat. If a wave train is reflected, you have an obstacle. The wave is moving in this direction. You have a obstacle. The rate of change of momentum is going to be changed. The rate of change of momentum is not going to

be the same when the obstacle is not there. Definitely there is going to be change in the wave height. All those things are going to happen and that is a phenomena which is coming here because which is going to affect these changes.

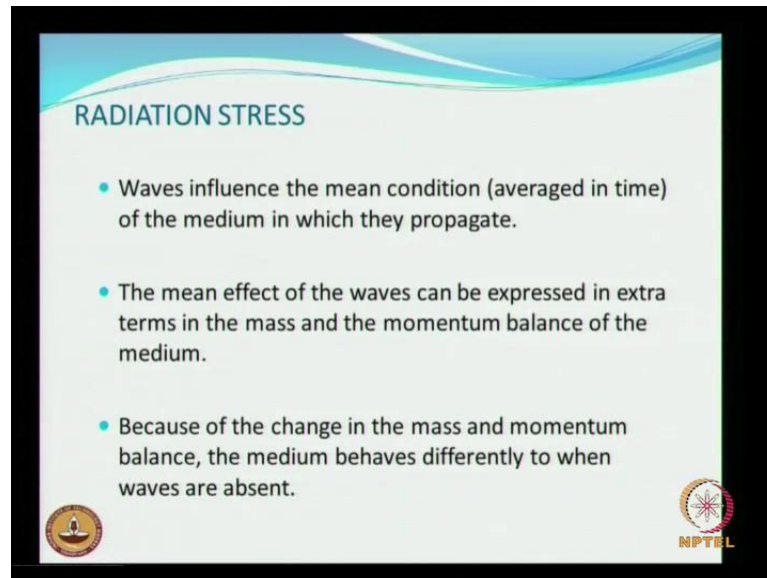
What are the phenomena? Reflection, diffraction and refraction, because of which the entire process will change. So, that is what it says, but consideration of momentum, so this is going to be definitely reverse the momentum, but consideration, what does consideration say?

In order to have the consideration going, so that would require, the consideration of momentum will require that there will be a force exerted on the obstacle equal to the change of the wave momentum. So, it has to compensate. You understood at that force is the manifestation of the radiation stress. You understood? If there is no obstruction and these waves are simply moving without undergoing any kind of changes, then this does not arise at all, but in the open ocean, you know that there is always the phenomena of refraction, diffraction taking place. At least if not diffraction, refraction will be taking place because of the variation in the bathymetry variation. Bathymetry is common.

So, for sediment transport consideration, the X Y plane is placed in the sediment bed in water. The X axis Z axis is now vertically upward, herein the stress is the, definition of the stress by definition is equal to the flow of the momentum in both horizontal plane. Is that all right?



Now, you have an horizontal plane. When you have an horizontal plane, when you have an obstruction, the change in the momentum can take place either in this direction or in this direction. Is that clear? How does the change take place? The change will take place over the depth also. So, you have a beach slope. Now, there is a horizontal plane. One is the X plane and the other is Y. Both axis you will have change. I mean the radiation stresses, you will have radiation stress and that will be varying and that is what it is indicated here.

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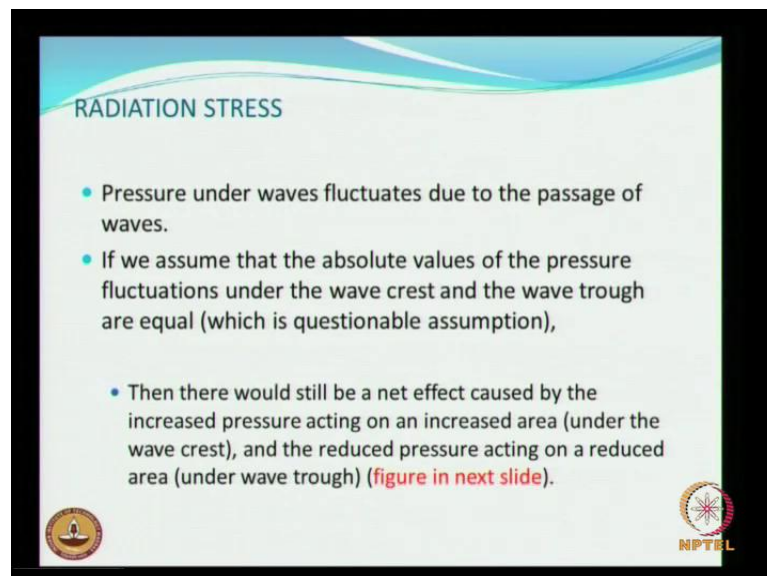
RADIATION STRESS

- Waves influence the mean condition (averaged in time) of the medium in which they propagate.
- The mean effect of the waves can be expressed in extra terms in the mass and the momentum balance of the medium.
- Because of the change in the mass and momentum balance, the medium behaves differently to when waves are absent.



Waves influence the mean condition averaged in time of the medium in which they propagate. We will see, look into it quickly. The mean effect of the waves can be expressed in extra terms in the mass and momentum balance because you have a crest and you have a trough. There will be some kind of residual effect which will be seeing. So, because of the change in the mass and the momentum balance, the medium behaves differently when there is presence or absence of the structure. You understand?

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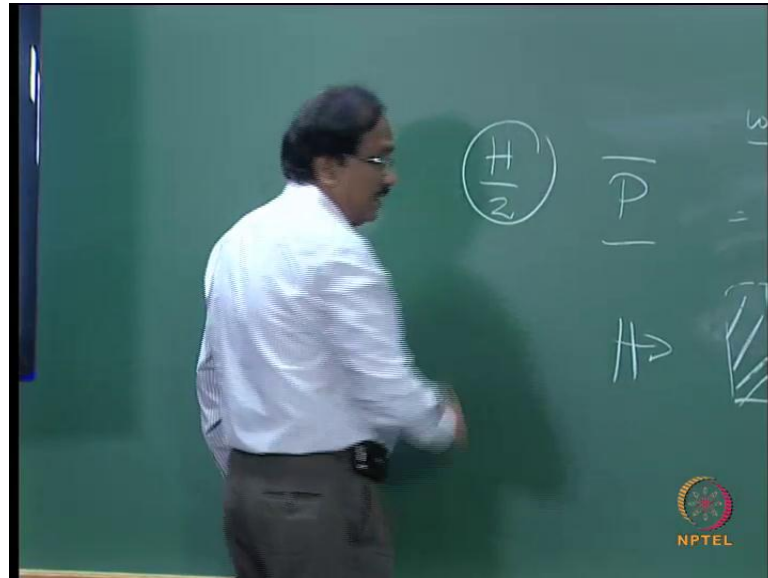
RADIATION STRESS

- Pressure under waves fluctuates due to the passage of waves.
- If we assume that the absolute values of the pressure fluctuations under the wave crest and the wave trough are equal (which is questionable assumption),
- Then there would still be a net effect caused by the increased pressure acting on an increased area (under the wave crest), and the reduced pressure acting on a reduced area (under wave trough) (figure in next slide).

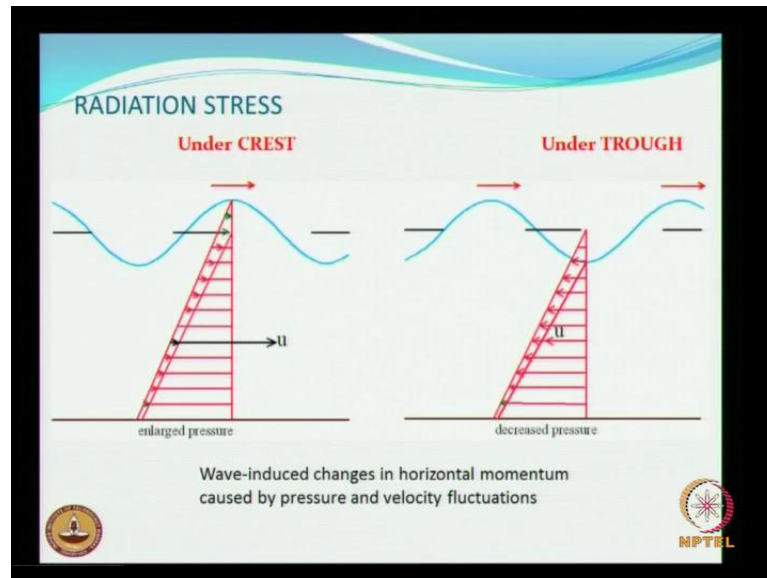
So, now here pressure under waves fluctuates due to the passage of waves. This we are clear. Now, if you assume that the absolute values of the pressure fluctuations under the wave crest and the wave trough are equal which is of course questionable.

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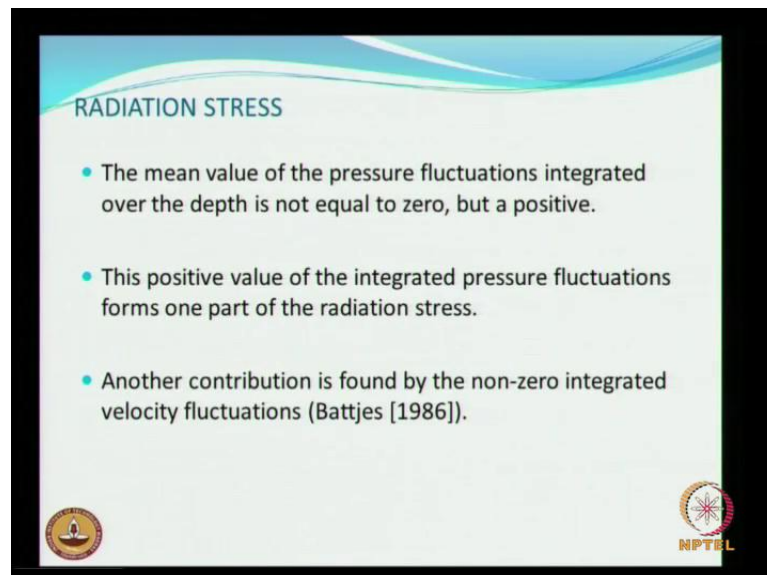
Can you have exactly in the sense; it has to be H by 2 . 100 percent sinusoidal. That is very difficult to get. There is always a kind of a difference in the pressure when a trough is there and when a crest is there. So, now that is what is being discussed here. There will be still a net effect caused by the increased pressure acting on the increased area under a trough, under a crest and the reduced pressure acting on a rest reduced area under the trough. Now, this is clearly explained with the help of this picture.

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So, when you have the pressure, yeah this is the hydrostatic pressure and this is the increased pressure due to the presence of the crest and when a trough goes, this is the difference in the pressure decreased. So, this is on the left side, you have under the crest and on the right side, you have under the trough.

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So, the mean value of the pressure fluctuations integrated over the volume, over the entire depth will not be 0. Understood? It will have a positive value because there is an increased pressure when crest is moving. Is that clear?

So, the positive value of the integrated pressure fluctuations forms one part of the radiation stress. So, when you integrate the pressure fluctuations over the entire depth that will be one part of the radiation stress. That is integration of the pressure fluctuations. Another contribution is said to be by the non-zero integrated velocity fluctuations. So, this was reported by one Battjes in 86 and most of these information which I am discussing is from Netherlands that is from Technical University Delft, where they have done a lot of work on the variation of radiation stresses etcetera.

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RADIATION STRESS

- In reality, the radiation stress is neither a true stress (force per area, N/m^2), nor a true force (N), but a force (force per unit length, N/m) over the entire depth (resulting from the integration of the force per unit area over the water depth).
- Unlike hydrostatic pressure, the radiation stress is not isotropic; indeed, just as with stresses, it is associated with a given direction or plane.
- This will yield the principal stresses; S_{xx} and S_{yy} . (Note: when the radiation stress is related to the longshore current, the axes will become parallel to the coastline.)

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Now, with this background, let us see what is meant by radiation stress. In reality, the radiation stress is neither a true stress that is force by unit area nor a true force, but a force per unit length over the entire depth. You understood because normally we say stresses per unit area. Now, we are talking about entire depth. You have the stress. The variation of pressure is over the entire depth. You have an obstacle, you have a crest passing and you have a difference. So, this residual is called as the radiation stress. Unlike the hydrostatic pressure, the radiation stress is not isotropic. Indeed, just as with the stresses, it is associated with a direction or a plane.

So, when you have an obstruction like this, because of this, there is a change in the pressure fluctuations and that is going to create a stress in this direction as well as in this direction. You understood? This is called as the radiation stresses and this will yield actually your S_{xx} and S_{yy} . Suffix X and S suffix Y y. So, when the radiation stresses relate to

the longshore currents that is along the shore, you call it as the radiation stresses along the shore.

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RADIATION STRESS

- The instantaneous, total rate of transfer of X-momentum through a plane ($X = \text{constant}$) with a width of 1m is given by:

$$\int_0^{h+\eta} (p + \rho V^2) dz \quad (2)$$

where

- h : water depth
- η : instantaneous water level
- p : pressure
- ρ : water density
- V : velocity

Now, this can easily be obtained as integration from, remember that this is the zero in this case of a definition and you have the pressure and the velocity term. The instantaneous total rate of transfer of X momentum that is through a plane X is equal to constant overall width 1 meter is given by this.

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RADIATION STRESS

- This integral is equivalent to the X-component of the force acting in this plane; its unit, in the SI-system, is kgm/s^2 , or N (Newton).
- The wave-induced contribution to the time average value of the integral per unit width is, by definition, the XX-component of the radiation stress, written as S_{xx} :

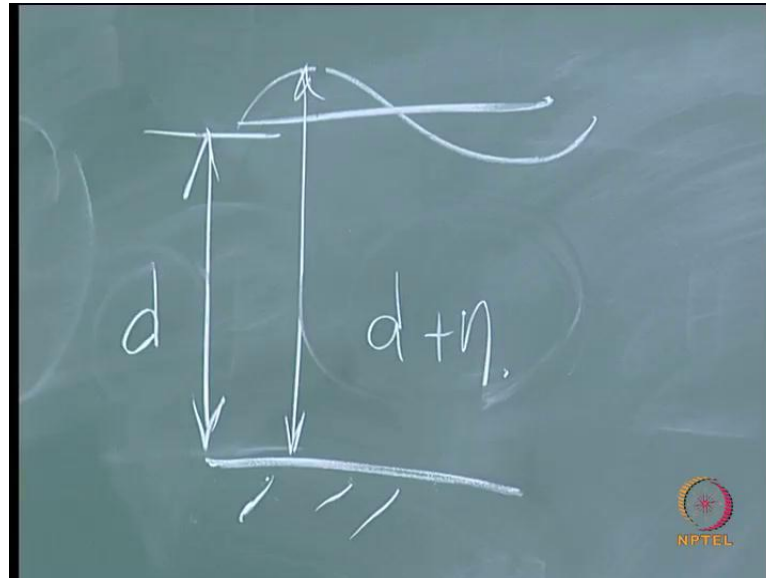
$$S_{xx} = \int_0^{h+\eta} (p + \rho V^2) dz - \int_0^h P_0 dz \quad (3)$$

In which the hydrostatic contribution is equal to:

$$\int_0^h P_0 dz = \int_0^h \rho g z dz = (1/2) \rho g h^2 \quad (4)$$

So, this integral is equivalent to the X component of the force acting in the plane. The unit will be in Newton's. Now, the wave induced contribution. There are two things as we have seen in the energy, potential energy. Remember?

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What did you do in the case of deriving the potential energy due to the waves alone? You looked at the potential energy due to wave and η and then, you took the potential energy due to the static head alone and then, you subtracted the potential energy due to the water depth from that due to η plus d in order to get the potential energy due to the wave form alone.

In the same way, we are trying to do here wherein you have one as your p plus ρv , that is the dynamic component which is going to vary and then, you also have P naught which is going to be the static component and that static component can be expressed as shown here. Is that clear?

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RADIATION STRESS

- In the notation S_{XX} , one subscript (X) stands for the direction of transfer (through a plane $X = \text{constant}$) and the other for the component of momentum being transferred (X)

Principal radiation stresses

Now, in the notation S of X S suffix X , one subscript stands for the direction of transfer. That is true. A plane X equal to constant and other for the component of momentum being transferred. That is also X . So, both is in the X direction. So, if there is a wave moving in this direction and this is the kind of an element which we are talking about, there will be X component here. X X in this direction. The transfer direction will be called as the S Y Y . So, these are the principle radiation stresses.

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- The value of S_{XX} , defined in Eq.3, can, in principle, be calculated from any wave theory.
- When we carry out this integration – it is a considerable task – over the depth on a plane perpendicular to the X -axis, the result is then (Battjes [1986]):

$$S_{XX} = \left[\frac{2kd}{\sinh 2kd} + \frac{1}{2} \right] E \quad (5)$$

where

- S_{XX} : the principal radiation stress component in the direction of wave propagation
- d : water depth
- k : wave number ($2\pi/L$)
- L : wave length

Here, we have seen the expression what we have for this S X X . So, in which case you derive the expression for pressure substitute and you can derive what is S X X in terms

of your other well known parameters like your energy, $k d$, $k E c$, wave number etcetera.
Is that clear?

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RADIATION STRESS

- E : wave energy given by:

$$E = (1/8) \rho g H^2 \quad \dots\dots\dots 6$$
 Where, g : acceleration due to gravity, H : wave height and ρ : mass density of the water
- $n = \frac{kd}{\sinh 2kd} + \frac{1}{2} \quad \dots\dots\dots 7$
 n : ratio of wave group velocity to wave celerity
- $n = c_g / c \quad \dots\dots\dots 8$
- S_{xx} can therefore be denoted by:

$$S_{xx} = (2n - 1/2) E \quad \dots\dots\dots 9$$
 Where, E : wave energy

So, you look at here. This is a well known, all these things are well known to us. So, energy is this much and n is also known to us and n is nothing, but the ratio between the group celerity and the celerity. So, you can modify this or simplify this as S_x equal to this expression given by equation 9, straight forward solving that expression.

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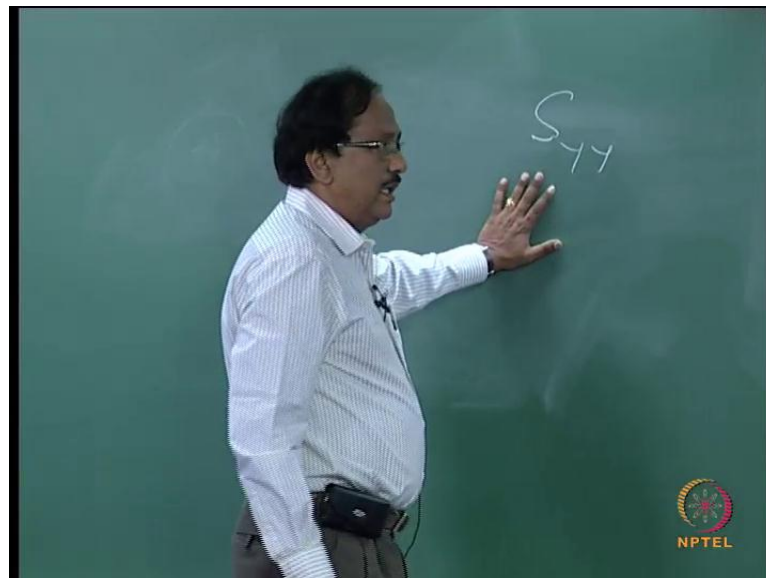
RADIATION STRESS

- Note: although S_{xx} is proportional to the wave energy E , it is not meaningful to think of S_{xx} as an energy per unit area.
- Its physical meaning is that of a rate of momentum transfer per unit width, or a force per unit width (SI-unit: N/m).
- This latter form is often more convenient in practical use.
- Computation of the second principal radiation stress component acting on a vertical plane perpendicular to the wave crests (earlier Fig) yields:

$$S_{rr} = \frac{kd}{\sinh 2kd} E \quad \dots\dots\dots 10$$

Now, note that S_X is proportional, one proportional to the energy. It is not meaningful to think of S_X as an energy per unit surface area. When we looked at the energy, energy is what per unit surface area. How do you get the total energy? You integrate over one cycle both with respect to length and the time and we said that it is energy per unit surface area and that is what it is being recalled here. Its physical meaning is that a rate of change of momentum transfer per unit width or a force per unit width that is newton per meter because you are having at different elevations, right. So, the later form is often convenient and it is quite useful for practical purpose. So, computation of the second principle, the first one is in the X direction. You do the same type of analysis. This way you will have S_{YY} .

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If some of you are very much interested in knowing these details, it is not so easy, but it is not so difficult also. So, you can refer to some of the references and the material which I have taken for the radiation stress which is from a lecture material which is given as a reference at the end of this chapter, and this entire chapter will be made available to you.

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RADIATION STRESS

- For sinusoidal, progressive free gravity surface waves expressed in terms of n ,


$$S_{YY} = (n - 1/2)E \quad \dots\dots\dots 11$$
- Application of the usual approximations for deep water ($n = 1/2$) yields:

$$S_{XX} = (1/2)E \quad \dots\dots\dots 12$$

$$S_{YY} = 0 \quad \dots\dots\dots 13$$
- In shallow water ($n = 1$) these stresses become:

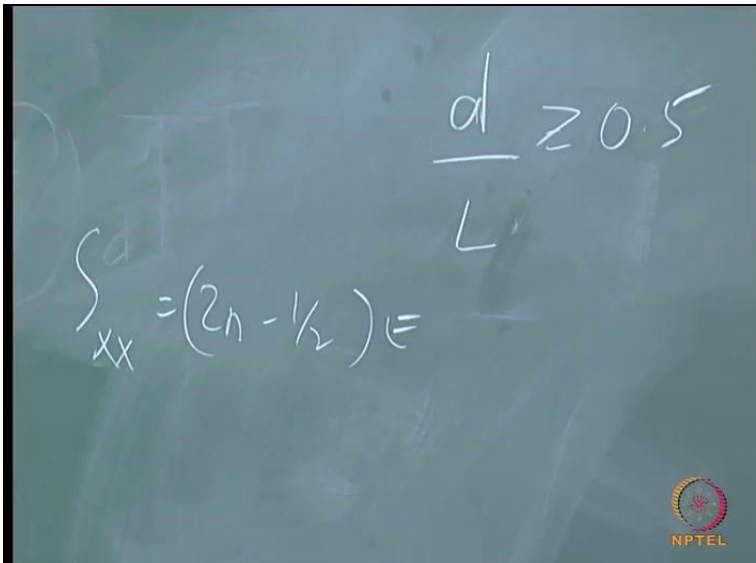
$$S_{XX} = (3/2)E \quad \dots\dots\dots 14$$

$$S_{YY} = (1/2)E \quad \dots\dots\dots 15$$




So, we can certainly prove that for a sinusoidal wave S_{YY} can again be proved to be equal to that expression given by 11, and you have the usual approximation that is for deep water wave, n equal to half.

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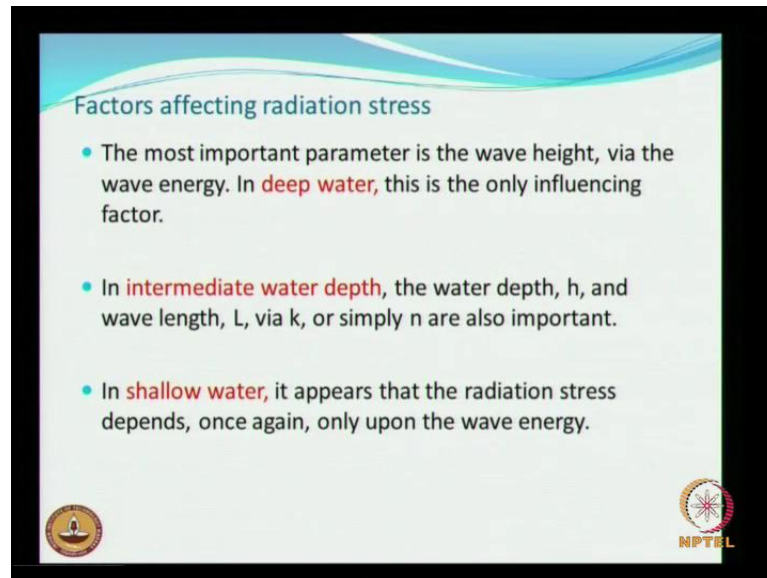
$\frac{d}{L} \geq 0.5$

$$S_{XX} = (2n - 1/2)E$$


So, S_{XX} will become half into total energy. So, now, you see that once you put n equal to half, then S_{XX} becomes half into energy and S_{YY} becomes 0, and in the case of shallow water, you will have the stresses being as shown here. So, what it applies in the deeper waters, you have only the stresses in the direction of the wave propagation and when it comes to the shallow water, you have both the stresses and this contributes to

both the onshore and offshore. This again depends on so many other factors like you break your angle etcetera.

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The slide is titled "Factors affecting radiation stress" and contains three bullet points. The first bullet point states that the most important parameter is wave height, which is the only influencing factor in deep water. The second bullet point states that in intermediate water depth, water depth (h), wave length (L), wave number (k), and wave number (n) are also important. The third bullet point states that in shallow water, radiation stress depends only upon wave energy. The slide includes a logo in the bottom left corner and the NPTEL logo in the bottom right corner.

Factors affecting radiation stress

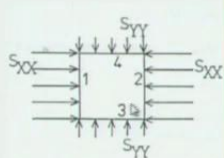
- The most important parameter is the wave height, via the wave energy. In **deep water**, this is the only influencing factor.
- In **intermediate water depth**, the water depth, h , and wave length, L , via k , or simply n are also important.
- In **shallow water**, it appears that the radiation stress depends, once again, only upon the wave energy.

So, factors affecting the radiation stress. The most important factor is the wave height; via wave energy. In deep water, this is the only influencing factor that is true, and in the intermediate water depth, the wave height and water depth and wave length, that is via wave number or simply n are also important as we have seen even through the expressions. In shallow water, it appears that radiation stress depends once again only upon the wave energy.



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Factors affecting radiation stress

- If we now consider a square column of water enclosed by four vertical principal planes shown in plan here,
 - Then, if the wave conditions and depth at all four planes 1,2,3,4 are identical, the radiation stress component on opposite sides of the 'block' shown in the figure are identical and there is no resulting force.



- Only if the wave conditions vary between planes 1 and 2 or 3 and 4, will there be a resultant force.
- Thus, we can expect the radiation stress to influence physical processes only in areas where wave conditions change. Such areas would, therefore, be at locations where wave refraction, diffraction, shoaling, or breaking occurs.

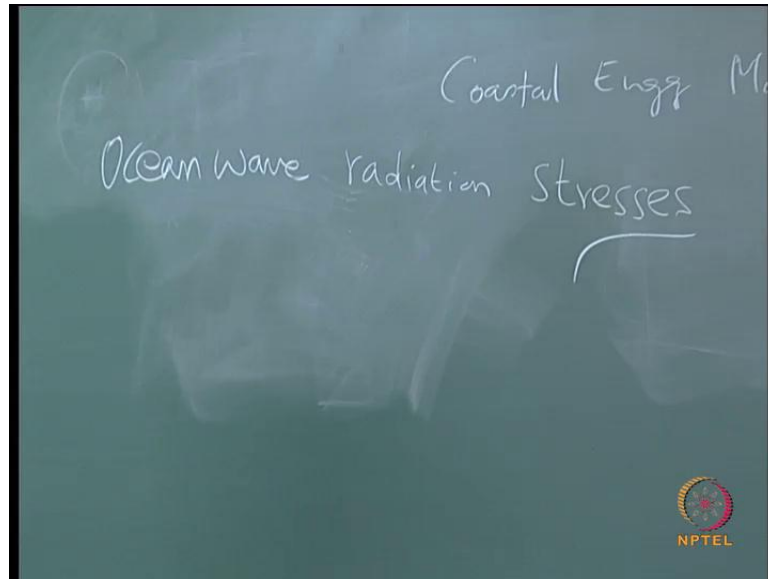
 

So, you have to go through the lessons again continuing if you consider small area as indicated here. If the wave conditions on the depth at all four planes are identical, then you have a radiation stress this side and countered by this side and similarly, countered by this side for this side. So, if wave conditions and depth at all four planes, one, two, three, four are identical, the stress, radiation stress component on opposite sides of the block shown in this picture are identical, then there will not be any net resulting force. That is if this is equal to this and if, this is equal to this, then there is no net force.

Only if the wave conditions vary between the plane one, between this plane and this plane, you will have a net stress and similarly, in the transverse direction, only if one exceeds the other, you will have an excess and this is nothing, but the excess flow of momentum. So, in other words, it is also said that the radiation stresses is nothing, but the excess flow of momentum. If there is no excess flow of momentum, then this does not arise at all. You understood?

So, thus we can expect the radiation stress to influence the physical process only in areas where this excess momentum will really take place whenever there are changes that is what we saw that is due to the phenomena of refraction or diffraction. So, in equality of the stresses on both the direction to take place and there will be net effect on one particular direction. So, if you still have any doubts on the radiation stress, I suggest you have complete discussion is available in many of the text books on coastal engineering. You refer to several books. See coastal engineering manual. Also, there are so many other books as listed in several of my lectures.

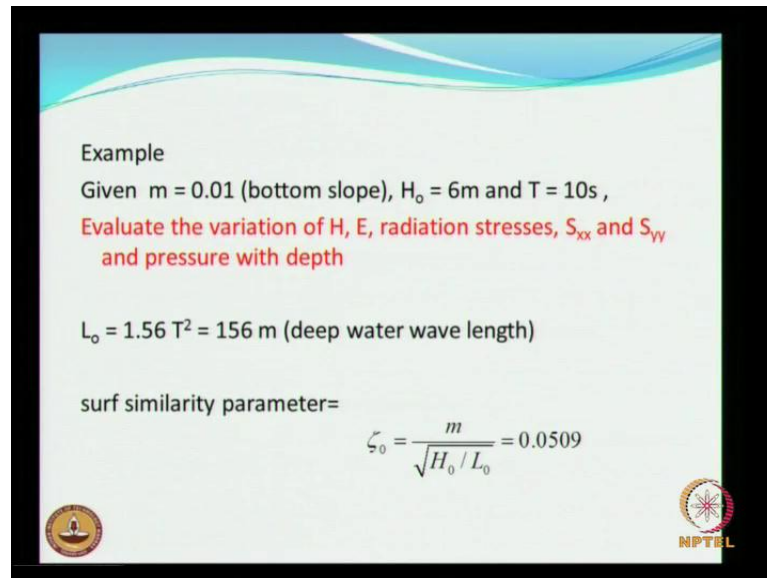
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So, you can refer to any standard book and if you want to have a complete description starting from real fundamental going up to the complete understating, the best paper is the older; the first paper that has come that is Longuett Higgins paper on radiation. Just you Google it as radiation stresses by Longuett Higgins and you are bound to get a number or even just put ocean wave radiation stresses. You should be able to get a number of articles through which you can get some additional information because I cannot devote the entire lecture on entire subject just on radiation stresses.

So, I have just introduced you some salient features of the radiation stresses. Have you copied, but I suggest additional reading.



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Example
Given $m = 0.01$ (bottom slope), $H_0 = 6\text{m}$ and $T = 10\text{s}$,
Evaluate the variation of H , E , radiation stresses, S_{xx} and S_{yy}
and pressure with depth

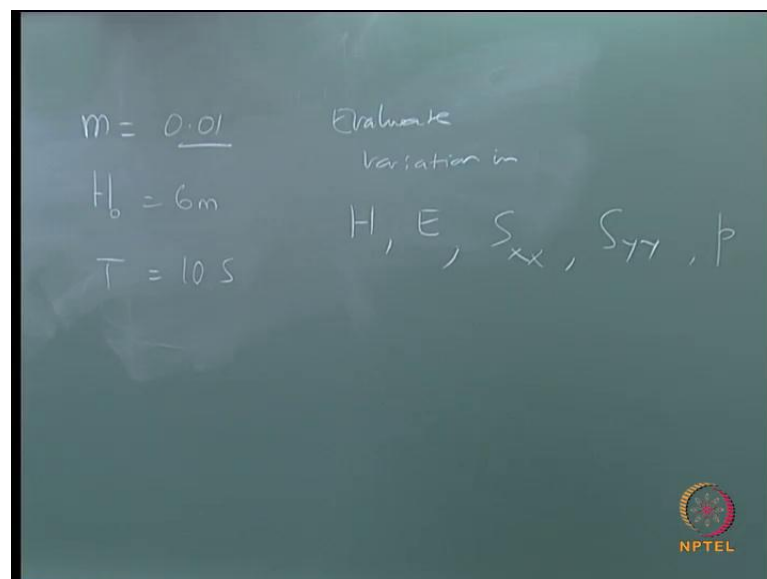
$L_0 = 1.56 T^2 = 156\text{ m}$ (deep water wave length)

surf similarity parameter=
$$\zeta_0 = \frac{m}{\sqrt{H_0 / L_0}} = 0.0509$$




So, now let us look at this problem. So, there is given beach slope.

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$m = 0.01$
 $H_0 = 6\text{m}$
 $T = 10\text{s}$

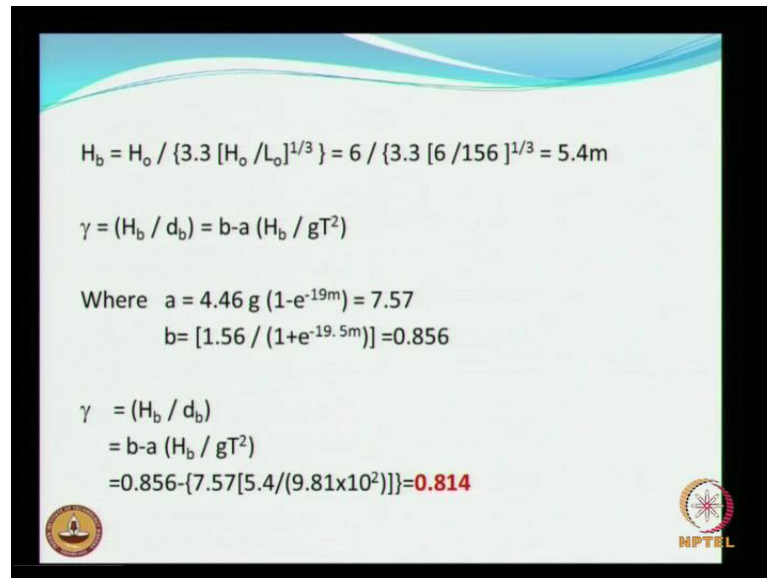
Evaluate
variation in
 H, E, S_{xx}, S_{yy}, p



So, m is 0.01, H_0 is 6 meters, then T is 10 seconds. Now, what we want to get is evaluate variation in H , E both and pressure. So, this variation of wave height already we have done. Do you remember under wave deformation?

So, first you calculate the surf similarity. Parameter has been given here. M is equal to m divided by square root of the deep water wave steepness. So, that is going to be so much.

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$$H_b = H_o / \{3.3 [H_o / L_o]^{1/3}\} = 6 / \{3.3 [6 / 156]^{1/3}\} = 5.4\text{m}$$
$$\gamma = (H_b / d_b) = b - a (H_b / gT^2)$$

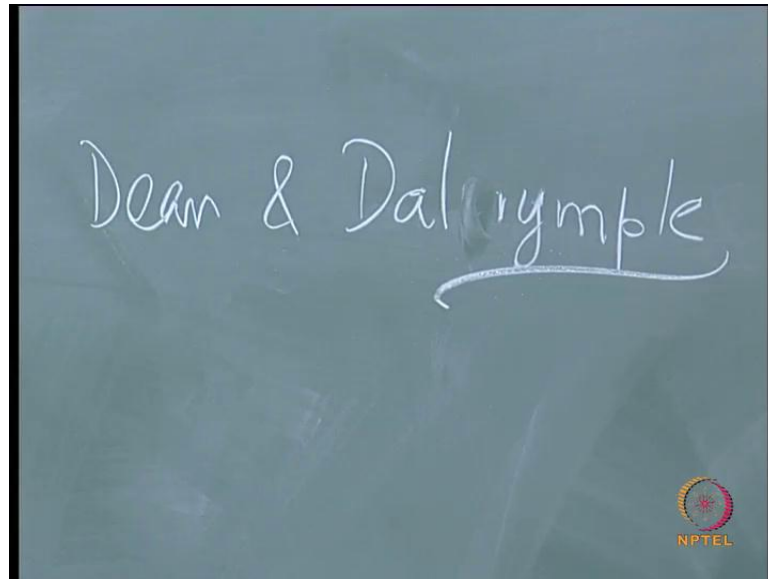
Where $a = 4.46 g (1 - e^{-19m}) = 7.57$
 $b = [1.56 / (1 + e^{-19.5m})] = 0.856$

$$\begin{aligned} \gamma &= (H_b / d_b) \\ &= b - a (H_b / gT^2) \\ &= 0.856 - \{7.57 [5.4 / (9.81 \times 10^2)]\} = \mathbf{0.814} \end{aligned}$$

Then, you know the formula for calculation of the breaker wave height. So, this is the formula for breaker wave height, where H naught is equal to 6 meters, L naught is equal to 156 meters because T equal to 10 seconds. So, L naught equal to, so remember under the deformation, under the breaking, how you take the beach slope into account while calculating the breaker depth of the breaker wave height.

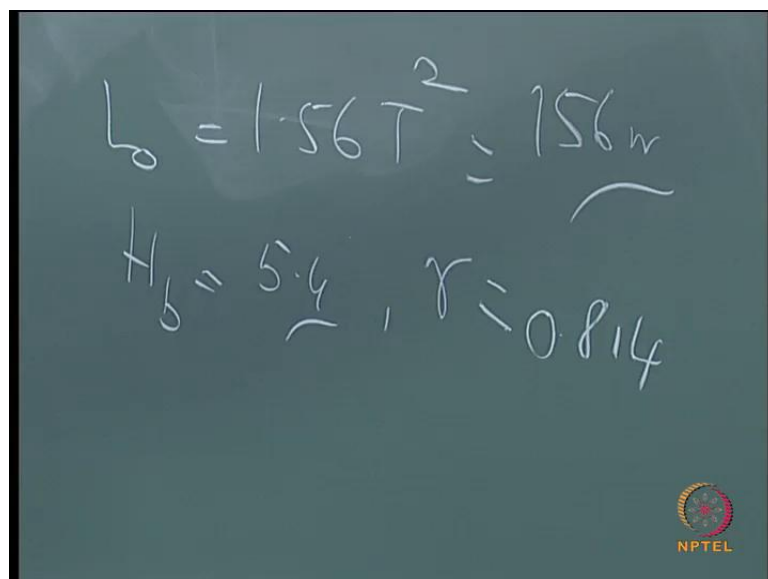
We have two parameters as indicated here. Gamma, that is the breaker index which is the ratio between the deep breaking wave height and the breaker depth is approximately is set to be b minus this factor, where a and b are given here in terms of slope.

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Do you remember this equation? It is given clearly in deal in the book, Dean and Dalymple. In this book, this formula has been detailed and even in our which is also given in my own lecture material, of course taken from dean and Dalymple. So, you look at these values because m is known to us. So, you can calculate these too. So, this factor works out to 0.814.

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So, H b is already known to you. H b is now computed as 5.4 and now, you have gamma as 0.814. We use simply gamma as an empirical coefficient as 0.78. Do you remember?

Now, you see that there you also have a formula which gives this breaking index in terms of beach slope.

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y = d/m (distance from the coast)

$$n = \frac{kd}{\sinh 2kd} + \frac{1}{2}$$

$$\frac{H}{H_0} = \sqrt{\frac{1}{2} \left(\frac{1}{n} \right) \left(\frac{C_0}{C} \right) \sqrt{\frac{b_0}{b}}} \quad \text{Considering parallel bottom contours, } \sqrt{\frac{b_0}{b}} = 1$$

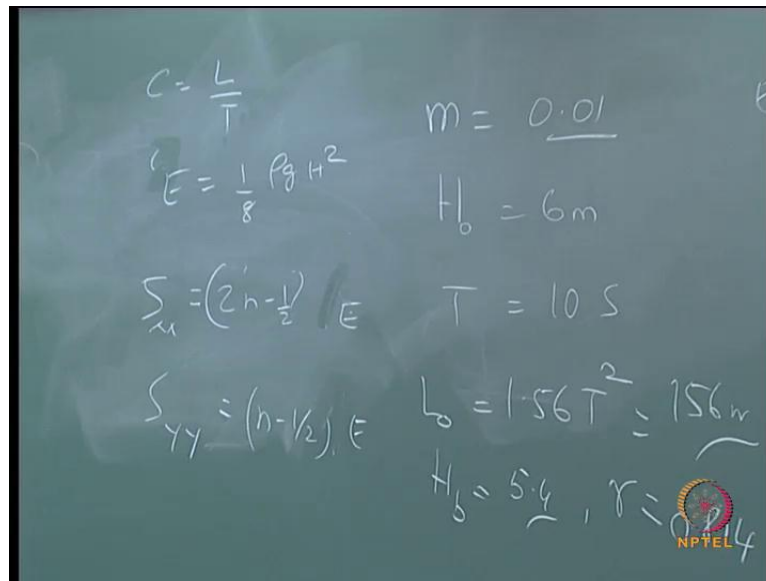
$$C = L/T; C = L_0/T \quad \& \quad E = (1/8) \rho g H^2$$

$$S_{xx} = (2n - 1/2) E \quad \quad S_{yy} = (n - 1/2) E$$

Pressure = (1/2) $\rho g d^2$

So, y is the distance from the coast. That is nothing, but water depth calculated divided by the beach slope, that is what is written there. So, this factor you can, you know how to get this. So, by considering both the refraction as well as the (()), this is the expression you have giving the ratio of the wave height in given water depth to the deep water wave height, and we consider here for simplicity that the bottom depth contours are parallel to each other, then this refraction coefficient will become 1. So, then you know all these variables, all these expressions are given. So, for example, I will show when we try to get the values, you can easily get it.

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So, C is L by T and then, E is a radiation stresses x. So, you have all those equation there.

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Table shows the computations as a function of depths.

Y (m) (1)	d (m) (2)	d/L ₀ (3)	d/H ₀ (4)	kd (5)	L(m) (6)	n (7)	H/H ₀ (8)	H (m) (9)	H/L (10)	0.142 tanh(kd) (11)	τ (N/m) (12)	S _x (N/m) (13)	S _y (N/m) (14)	Pressure (N/m) (15)
10000	100	0.641	0.641	4.03	155.90	0.503	0.9977	3.99	0.0384	0.1419	45043	22751	115	50021000
9000	90	0.577	0.578	3.63	155.78	0.505	0.9956	3.97	0.0383	0.1418	44851	22884	229	40525110
8000	80	0.513	0.514	3.23	155.51	0.510	0.9915	3.95	0.0383	0.1416	44483	23137	448	32019840
7000	70	0.449	0.452	2.84	154.94	0.519	0.9845	3.91	0.0381	0.1410	43801	23654	852	24515190
6000	60	0.385	0.390	2.45	153.71	0.536	0.9727	3.84	0.0380	0.1399	42814	24518	1556	18011160
5000	50	0.321	0.331	2.08	151.18	0.560	0.9550	3.97	0.0390	0.1376	44838	28264	2923	12507750
4000	40	0.256	0.273	1.72	146.27	0.611	0.9344	5.61	0.0383	0.1331	39509	38501	4373	8004960
3000	30	0.192	0.219	1.37	137.21	0.677	0.9166	5.50	0.0401	0.1249	38013	32447	6721	4502790
2000	20	0.128	0.165	1.04	121.17	0.785	0.9174	5.50	0.0454	0.1103	38085	39214	10085	2001240
1000	10	0.064	0.108	0.68	92.33	0.874	0.9835	5.90	0.0639	0.0940	43765	34578	16348	500310
900	9	0.058	0.102	0.64	88.21	0.885	0.9953	6.00	0.0680	0.0903	45183	37423	17416	405251.1
800	8	0.051	0.095	0.60	83.78	0.897	1.0182	6.11	0.0729	0.0763	46906	40744	18643	320198.4
700	7	0.045	0.088	0.56	78.94	0.910	1.0425	5.70	0.0722	0.0718	40608	53845	18720	245151.9
600	6	0.038	0.082	0.51	73.59	0.922	1.0722	4.88	0.0664	0.0670	29982	40305	12657	180111.6
500	5	0.032	0.074	0.46	67.65	0.935	1.1106	4.07	0.0602	0.0616	20821	38513	9052	125077.5
400	4	0.026	0.066	0.41	60.93	0.947	1.1622	3.26	0.0534	0.0555	13325	38588	5963	80049.6
300	3	0.021	0.056	0.35	53.13	0.960	1.2368	2.44	0.0460	0.0484	7495	38649	3451	45027.9
200	2	0.013	0.046	0.29	43.72	0.973	1.3545	1.63	0.0372	0.0398	3331	4820	1577	20012.4
100	1	0.006	0.032	0.20	31.09	0.987	1.5944	0.81	0.0262	0.0283	838	1227	405	5003.1

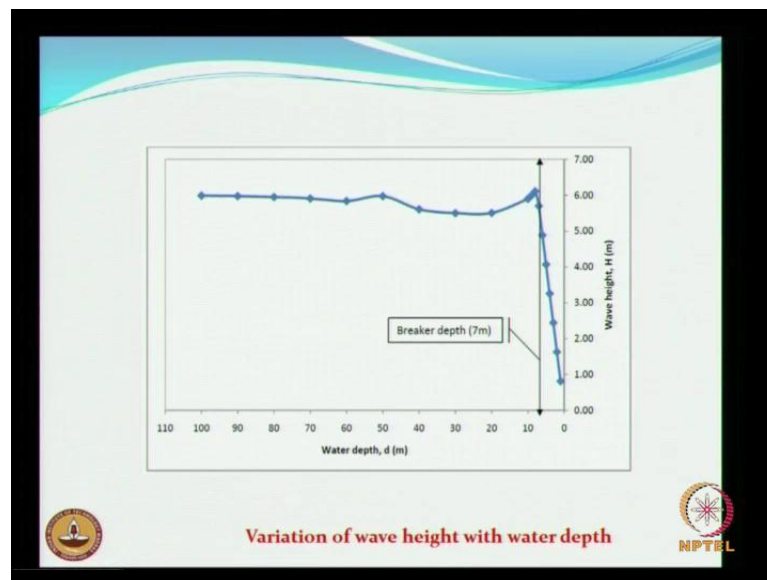
Note: At d=7m, the wave breaks as the value of H/L > 0.142 tanh(kd). Hence for d < 7m, we adopt depth limited wave breaking criteria for obtaining H, (H = d)

So, when you use all these corresponding expressions here in this table, I have put everything there. So, let me explain here. So, we have taken the distance Y from this expression which I have indicated earlier. So, the water depth is given here in the second row d by l naught calculated d by l, then k d wave length is calculated. Here, n is also calculated. You know the formula H by H naught prime. H naught is given here, then H

can be calculated for corresponding to each of the water depth given here and then, H by L and then, this also has been done up to this. We have done earlier in our wave deformation class. Remember?

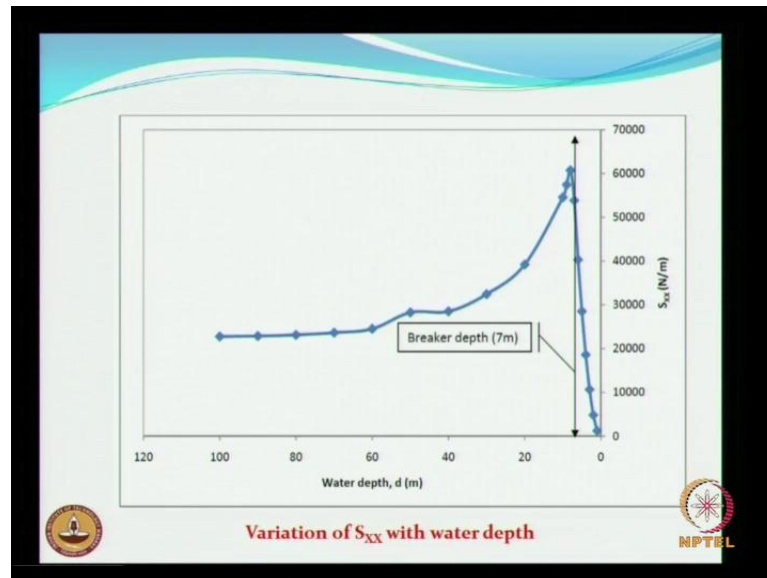
So, only thing which is coming into additional as the $S X$ and your $S Y Y$ and the pressure and all these things are given here. Now, this is the location, wherein you see that d equal to 7 wave breaks as the value H by L is greater than, H by L is greater than this one. That is the point which we have already seen.

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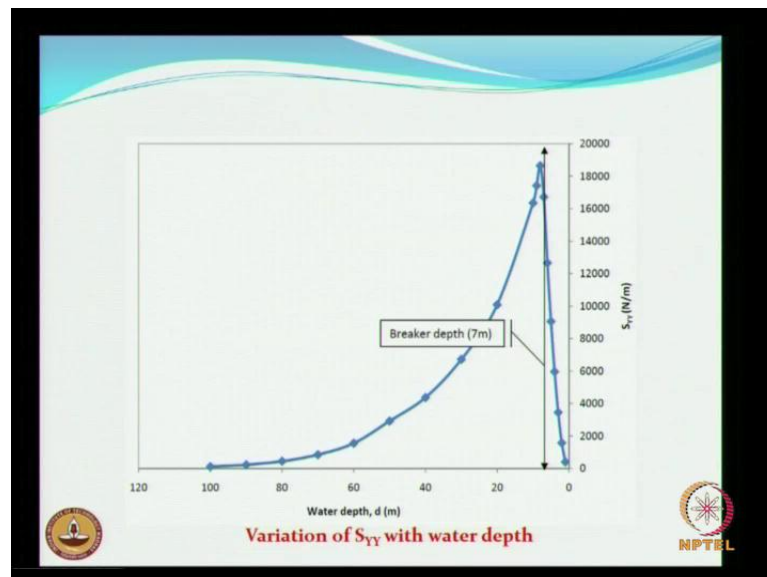
Now, when we plot the values, this is the variation of the wave height with the water depth. So, look at the variation and anyway, in the lecture, material will be available to you. So, you can have a detailed inspection of the results.

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This is the variation of your S_x as a function of water depth. So, you see as it reaches the shallow waters, the S_x increases drastically and the breaker depth is indicated here and near the breaker depth, the stress is really very high and similarly, your S_y that is the other in the X in the Y Y direction.

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So, you see that these are two stresses which are going to serve as the driving force for the generation of the currents within the mostly concentrated within the surf zone; so in order to why we are calling them as driving currents? Because see they are the stresses

which are very close, which are maximum near the breaker line and once the waves break, all these stresses set in and try to generate the currents. You understood? This is the variation of pressures.

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So, this is the lecture material. This is a classical because I referred to so many materials. I thought this is the material, which has been very nicely dealt with on explaining the radiation stresses. So, I will try to send this material as quick as possible, so you can verify all these things. Any doubts? So, we have looked at the radiation stresses which are the main factor or the main driving forces for the longshore currents. So, after this, we have to get into how the longshore currents are determined, how the longshore sediment transports are estimated and how they are computed, calculated etcetera and why we are doing this, already we have seen through the past lectures.