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Module No. \#05
Wave Loads on Structures
Lecture No. \#05
Waves loads on Large Bodies
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We have earlier seen the wave loads acting on slender bodies. So as we talk, as we have seen earlier, when we talk about the structures, we normally talk about slender piles first, slender piles when D by L is less than 0.2 ; where in, we had used the Morison equation or the equation proposed by morisonetal in 1950. And, that is well entrance in literature, there is absolutely no ambiguity in using it, but although there are some certain uncertainties which I have already discussed.
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Now, we move on to the cases of a wave loads larger on bodies. That is, we refer to D by L greater than greater than 0.2 .
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So, this is normally said to be in the diffraction regime, and if you look at slide here; so when the relative characteristic dimension D by L is greater than 0.2 , the instant wave after striking the subject object it get scattered.

Application of this of the Morison equation to this regime is invalid. So, however it may be used provided the force is predominantly inertial. That means, the drag component
can be dropped at certain for certain conditions, we can drop for certain flow regions. So please refer to my earlier lectures on the force regimes as given as a function of scattering parameter and a keulegan carpenter number.
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So, as a guide line you can say that, general guide line if H by D is less than 1.0, we could safely assume that the forces is inertia or the discussed force discussed forces can be neglected. In a specific case, we are talking about a vertical cylinder; linear theory gives the keulegan carpenter number at the still water level as indicated there. This can be easily be proved and it has been already been done, while defining the keulegan carpenter number.
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Some of these are basic information's are necessary. The maximum wave steepness is given by that expression 0.147 into tan hd and then over a range in depth but in shallow waters the H by D about 0.83 . This implies that, the K is limited approximately to that expression given in 29 equation 29.

So thus, when D by L is greater than 0.2 the wave diffraction is expected to be very important is and then K will not exceed 2.2 and will usually be less than about 1 . So that is, for this kind of force regime.
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So the purpose of dimensional analysis you should be knowing. So mean, there are several problems where you can solve forces on structures using numerical model.
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Where in, you may have the prototype structure itself directly. Put in your numerical model and you can solve for the problems. And there are certain problems, where you have to resort to only experimental investigation or may be an experimental investigation is used to validate a numerical model.

In the earlier case, that is in the former case as if you want find out an intake well for example, I am just giving it an example given here. So now, I want to find out the forces on this structure. So you know, what are all the different parameters applicable here.

So you have the water depth; you have the wave. I means the water depth, diameter, the wave height, wave period. These are the most important parameters L or T. Assume that, you have a performed the experiments and you have determent the you have measured the total force. Now, you need to before that and for example, if it is for site particular site condition.

So, you normally adopt a particular scale so to shoot your experimental facility which you have because, you are experiential facility can vary. There are experimental facilities where in, the range of wave heights and the range of wave periods will be rather limited. There are facilities large size, large scale facilities where you can use bigger models.

So, if you use bigger models the kind of accuracy involved in extrapolation is going to be much more close to reality. So this we call it as scale effects. So when you adopt a particular scale and then you have assumed that you have adopted a particular model scale and fabricated this model and it is been subjected to the wave characteristics.

One way of doing it is, you have a particular a given wave height for which you need to have the wave force. Then, you adopt using the model scale and you run the model and you get the force. For example, if your sea state, the random sea state is known to you. You adopt a model scale and as per that model scale you simulate the wave elevation, measure the force and then, that force using the model scale you can get the force that is expected in the proto. The other thing is, if in case you are doing carrying out research work.
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If you want to for instance, you have some kind of a floating structure where in, you are interested how is going to perform this floating structure in attending the waves. Then, there are so many parameters.

Now you have the, you are also measuring the mooring force for example, so you have again the wave characteristics, water depth and then the characteristics of the material, the size of the material.

So you will be dealing with the host of parameter. And now, if you want to make use of this study then, you have to resort to some kind of presentation of your results in a dimensionless form. Because simply having this, I means subjecting this model and then.
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So what you will be doing is, you will be may be performing up to about 0.25 meters in the lab; may be your wave period is say 3 seconds. So and water depth may be around what you have adopted in the lab. After performing a series of test and this one you have done up to 0.25 in steps of say 0.3 meters for example, and here you for each wave height you are adopting a wave period of 1 to 3 seconds in steps of 0.1 second.

So you imagine now, for a single set up you have a number of wave conditions for which, the structures is going to be subjected. May be you are going to also look into the effect of water depth, we do not know.

It depends on the kind of research you are going to undertake. So in that case, when you have you are dealing with all centimeters and meters such a small value of being adapted in the lab. Now, these result have to be adopted to the field conditions. Then that is, why you are asked to; then in this case, what are you going to do? You have to go in for dimensional analysis.

A topic which I am sure you have undergone in your under graduate programmer. So I use the Buckingham pi theorem which is more common. So, include all the variables and
then arrive at the dimensionless parameter. So as you can see for example, if I include all the values, all the variables and it is convenient to represent the force in dimensionless form.

So for our case where, they are talking about a force exerted on a large body; the force now, it can be expressed in terms of in dimensionless form as can be indicated as shown in the slide. It can slightly model, it can slightly vary also, it can be different for different kinds of a structure. For example, here I have put circular structure then, may be it may be different for a rectangular I mean rectangular or a square a case on a pile.
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So you see that, the force which is presented as. So this representing the wave height diameter can be represented as been a can be a presented as a function of some of the other dimensionless parameters as indicated. This is, please refer to Sarpkaya and Isacson. So, the book by Sarpkaya and Isacson discusses more in detail. So you see that, the parameters that are being consider here is, the related water depth. So, this is going to be a function of a relative water depth that is small d by l ; H by L is nothing but the wave steepness; then capital $D$ by $L$ is refer to as characteristic dimension; and Re is going to be the usual Reynolds's number. So considering the flow separation effects; so naturally, when you are talking about flow separation effects, the Reynolds's number can be conveniently omitted as, and we are talking about diffraction being more important. So naturally, you can leave the I mean you can leave the Reynolds's number.


And then, that will boil down to three parameters. In addition, if we are considering that wave steepness is small then, linearizing approximation may be may success the force will vary linearly with wave height.

So, when you are considering only the linear variation, you can you can remove your H by L also. And then, you will be having only the relative characteristic dimension and the relative water depth.
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And the force can be force can be you can have for different $d$ by $L$, you can have the variation of the force as for different sizes of the bodies; or for capital D by L, you can have the variation of the force as a function of water depths. For different water depth conditions, how the force is going to vary. So, it can also be rewritten as shown here; then, that it is note that the assumption of the small wave steepness it is in where will be made in the diffraction analysis and that corresponds to the linear theory, which we have already discussed.

Equation 31 can again be rewritten in an alternate form as indicated down. That is why equation 32, which has the advantage of presenting or reporting the force coefficient as a function of D by L for a given structure located in a particular water depth having a brim after having brief exposure.
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Now, we will go into and although the; in order to solve this kind of problem large body problems, you the diffraction problem this usually referred to as linear diffraction problems. In this cross, I will be just presenting analytical solution and I will stop with that. But, if people are more interested you can use the same book which I just now measured mentioned or there are so many other book wherein, the numerical methods you can either go in for fe m or boundary integral method or finite volume methods. It depends at the kind of problems and so and the kind of a expertise you have. You can use
any of the numerical model models for determining the forces on structure forces on structures of arbitrary shape.
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So, now let us look into only the basic things. So we consider, the linear diffraction theory which arises when, the wave height is assumed to be sufficiently small. So that is, we are basically, we are we say that the linear theory is applicable.

Now, the problem is solving the Laplace equation as we have seen in deriving for this small amplitude wave theory. So, this Laplace equation is subjected to the boundary conditions which we have already seen. The 34 is nothing but the phase of boundary condition and then apart from the phase of boundary condition,
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we do have the other boundary conditions which you have already seen. That is, dou phi by dou is equal to 0 ; that is nothing but the kinematic bottom boundary condition. But in addition to this, since we are talking about large body, we also have body surface that is the dou phi by dou $n$ is going to be equal to 0 on the; that is velocity normal to the body is 0 ; body surface is 0 . So here, n is a denoting the distance in a direction normal to the body surface. So you have a normal to the body surface. See the radiation condition may then be introduce after initially representing the velocity potentials as submission of.
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So here in when the, when the body when there is a large body and the wave is going to come and hit the structure, then you have the scattering from the body taking place.

So now, the total velocity potential is now defined as the incident velocity potential for which you already have or you know what it is. That is nothing but the initial velocity potential. Plus you have to estimate the, you have to calculate the scattered velocity potential under scattered velocity will depend on so many other parameters including the size of the body etcetera.

Now, this requires the use what is called as a sommerfield radiation condition. That is given by this expression wherein, $r$ is the distance from the body surface. So, at a distance r equal to infinity at a large distance it only implies that the total potential can be equal to the incident potential wave. That is scattered potential can be neglected.

So that is the; in order to account for this scattering, we have this boundary condition. In order to satisfy this boundary condition to satisfy the Laplace equation.
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The body surface boundary condition may be written as

$$
\begin{equation*}
\frac{\partial \phi_{s}}{\partial n}=\frac{-\partial \phi_{i}}{\partial n} \quad \text { at the body surface } \tag{40}
\end{equation*}
$$

and provides for the dependence of $\phi_{\mathrm{s}}$ and $\phi_{\mathrm{i}}$. Thus equations (33) - (37) applied to $\phi$ s together with eq. 39 and 40 define the problem in terms of $\phi_{s}$. Once $\phi_{s}$ and consequently $\phi$ are determined, the pressure throughout the fluid may be evaluated by the linearized Bernoulli equation
$p=-\rho g z-\rho \frac{\partial \phi}{\partial t}$
Appropriate integration of the pressure acting on the body surface may then be carried out to obtain sectional forces and overturning moments as required. Of the available theories only MacCamy and Fuch's linear diffraction theory, is discussed here.

So, the body surface boundary condition can also be written at a body surface as a mentioned in equation 40 . Now this 40 equation now, this is going give you the dependence of the scattered potential.

You can estimate the scattered potential as the function of incident velocity potential. So now, all these equations which we have seen from 33 to 37 applied together to F s to
equation together with equation 40 , define the problem of velocity and in the scattered velocity potential. You have to use all those boundary conditions and the method which we have just now seen to estimate the variation of the velocity.

I mean, the scattered potential. Once the scattered potential phis and consequently the once you estimate phi s then naturally, you can estimate the total potential. Once you estimate the total potential, then your job is quite straight forward and it is simply you by using the linear Bernoulli equation as given in the equation 41 wherein, you can get the pressure distribution and integrating the pressure over the body surface can easily give you the sectional force. So sectional force and once you get the sectional force for the whole structure.
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So, for the so initial integrate get the force acting over a particular section. Like that, you do you do force entire structure and then integrate, you will get the total force acting on the pile, so we are talking only about a pile here.
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So, there has been number of I think of all the available theories, the one which is by Mac Camy and Fuch's. So MacCamy and Fuch's MacCamy and Fuch’s its also mention called as MacCamy and Fuch's linear diffraction theory. Because, he is talking only about a linear diffraction problem.
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So I will just present this method which is analt analytical solution and its quite straight forward in order to obtain the values or the forces on large bodies, in the formulation of the problem the coordination access originate at the still water as per our usual definition
then the instant velocity potential is taken as cosine, in this case we have taken as cosine you please refer to our wave mechanics equations, I think probably we would have considered sin theta for velocity potential.
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So, the velocity potential can be taken as either sin or cosine but only thing is once you take a particular phase you stick on to that till you solve that particular I mean problem.
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So here we are consider he has consider a cosine and the Bernoulli's equation by after solving the for the total potential and also solving the Bernoulli equation gives you the
pressure and then the horizontal force on the pile per unit length he obtained a solution analytical solution as indicated here, wherein all the parameters are known to us except that A of ka, ka is nothing but the scattering parameter and that is the argument here and that is going to be A of ka is defined by this expression as shown here, wherein J one prime of ka is nothing but the differential value of Bessel J function of first kind and first order, the argument is k ka then y 1 prime of ka is nothing but the differential value of Bessel y function of the second kind and the first order.

Now of course, the argument is ka; so a is nothing but the radius of the cylinder radius of the pile, so this alpha you have this alpha can be obtained as shown here, so once you evaluate a of ka and alpha then it is just a simple equation from which you can arrive the wave force acting on a pile, so at a every elevation you can at each of the elevations you can that is at every e z you can get the wave force and then that is nothing but the sectional force and then you integrate the sectional forces over the depth you can get the total force acting.

So, recollect what we have already done for Morison equation, so we derived the sectional force and also found out the moments and then integrated right, so this theory is confined to large circular cylinders resting on the ocean bed and piercing the free surface and for waves of smaller steepness, so there is because we are talking about small steepness waves then this possess a some serious or practical application so for more vigorous calculation you have to get into the numerical modeling aspects, which is not covered under this subject.


So, coming back to the coming back to the Morison equation so far we have just now I have introduce you to the last diameter last diameter piles how to calculate the wave forces, now just one step backward you when we just did about when we discussed about the Morison equation that is D by L for less than 0.2 then we had the problem of the one of the uncertainty is this the evaluation of $C D$ and $C M$.

So, these coefficients are determined or measured or obtained from measurements they use different techniques for measuring the for evaluating the C D and C M, I will not go into the complete details but one of the basic steps given here is usually the C D and C M are obtained from flume or basin test.

So, you have a cylinder resting on the seabed on the flume bed so that matter and then you measure the total force and you also have the eta, so when you have a and you can also measure if you want the $u$ or sometimes it is a obtained from linear theory, so you have the measured force you have the eta, so now and one thing is very important is when such experiments are being done so in plan so you need to have the wave gauge installed here so as so that you are getting the phase, so if you keep the wave gauge here if this is the wave direction everything is the wave elevation and the force is going to be out of phase, so we need to measure the wave and the force in when they are in phase so if you have like this, so for example, this is your eta of T and this is your F of T then from this I can obtained the so if for example, if you are considering a sin curve so if you
are considering a sin curve then eta is a sin curve and refreshed again u max into sin theta right and u dot will be something into Cos theta, so actually u and u dot will be out of phase by 90 degrees, so now when I say at this at the crest at the crest what is going to be its going to be $u$ is going to be maximum because $\sin 90$ is $\sin 90$ is one so you will have the entire force now when I drop into this will be the force corresponding to face angle of 90 degree.

So, which means at that particular this is the total force which is the summation of drag force and the inertia force, this is going to be something into or into sin theta plus Q into Cos theta, when you have this at this particular point you have this force equal to what this force will be nothing but R into sin into sin and at that particular point $\sin 90$ is Cos 90 is going to be 0 Cos 90 is going to be 0 , so at this particular point this is going to be 0 and the total force is going to be only the drag force and now for which you know the expression half into C D into rho into A P into equal to this force, so, all are known to you only C D can be evaluated, similarly at this particular location Cos of 180 is going to be $\operatorname{Cos} \cos 0$ is equal to one so and $\sin 0$ is going to be 0 so at this particular point whatever magnitude you are getting that is going to be the entirely the inertial force, is that clear.

So, and if you look at the wave and force traces from that itself looking at traces recording it selves you can easily say whether it is a vicious dominant or drag dominant I means or the inertia dominant etcetera, so from this so usually this is the procedure which is adopted these are available in a number of books and I just given you the so the basic methodology is that the you measure the total force correlate with the force with that the either the measured or the theory.


I means the linear theory, and then you can determine the hydrodynamic coefficients as I have just mentioned, this is one of the methods, but there are other methods to determine C d and Cm . So, this is from the force, and from the from the measured elevation you obtained the U U dot, etcetera from the measured force. You you correlate these three to get the hydrodynamic coefficients; that is what this is explained in this slide.
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Now, we again go back to the un separated flow model, what is meant by un separated flow model ? the un separated flow model means that you your are vicious effects are
negligible which I have already mention that for H by D is less than one you can safely consider the total force to be almost close to equal to the inertial force that drag force being neglected, Ok that means we have so far consider the drag I mean a coming back to the wave forces on a non circular case on and when we talk about the un separated flow model then we are talking only about the inertial force component.
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So, it was now Mogridge who came up with a mathematical model, Mogridge and I think there was Mogridge and Jamieson, so they are basically from Canada so they came up with equation when we are talking about I means inertial force we he came up with they came up with an equation as indicated here because since the vicious is neglected you see that your effects is going to be C m into rho into B square for a square region, only the area of cross section right; that is the inertial force into dou $u$ by dou T so here in C ms is defined as defined as the coefficient of mass for the square section.


According to the assumption this effects will be equal to the can be assumed to be equal to the inertial force acting on a circular case on of equivalent diameter, understand so instead of square he is considering equivalent diameter and then he equates the forces due to on the on the right hand side of equation 56 you see that you have the assumption of D that is equivalent diameter now this can be equal to the equal to the this is an assumption based on which we get a relationship between relationship an expression for the diameter in terms of the size of the case on the square case on square cross section.

So, we also can assume it is it was further assumed that C ms equal to Cm Cm and hence he came up with an expression that is a relationship for the equivalent diameter as a function of the size of the square cylinder, but you know we know that the MacCamy and the Fuch's theory already we have seen.
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So, he basically they basically use this MacCamy and Fuch's theory and equated at it to equated to the inertial force I am from which so he equated you see this is a force on a square on a circular cylinder based on it is an analytical solution from MacCamy MacCamy Fuch, so the other is the un separated flow model for a square case on, so this can be equated together and then they came up with modified coefficient of mass as indicated as shown here, so it is just a derivation and then once you have the C ms you can you also have the C m star.
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So, once you have this you can evaluate the total force by using this equation, this equation all the parameters are known to you A a of ke is coming from the MacCamy Fuch theory and then you can estimate this, once you know this you can evaluate the total force and at sigma T equal to alpha you have the maximum force obviously because Cos 0 is equal to 1 and then the maximum force can be represented as a simple equation for the square pile is that clear.
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So this is what they have done Mogridge and Jamieson where they have compared their results with hogben and standing some of the experimental results and this is the kind of the information we have so this can be used as a design chart for calculating the wave forces on large bodies particularly in the diffraction region.
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So this has also been extended for angle of orientation of this of the square region with respect to the wave direction, so here you have you are having 0 angle and then for 45 degrees, having seen the details on the wave forces on slender piles as well as on large structures, we now move on to wave slamming; wave slamming is because when due to the sudden emersion of a structure in water when a wave is in is propagating that location so this kind of sudden so you expect a the impact force the force will be similar to an impact force and it is an impact force.
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The forces on the member will increase due to buoyancy drag and as well as virtual inertia. So, in addition to these forces predicted by the Morison equation there is a kind of a transient peak load on the immersed body, so this the it is unlike like a sinusoidal variation it is some kind of a sudden transient kind of a phenomena so which needs to be taken into account well designing this members, so cylindrical elements are common in offshore platforms and hence slamming forces on horizontal tubular members, so that is so this tubular members only will be having a the slam force. So, will be due to that is a slamming force due to waves needs to be estimated.
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And it is often it is similar to nothing but the velocity dependent drag force, in the case of drag force you know that the drag force is a function of square of velocities right, so you need to use that same thing the slam force is also be similar to a drag force the resultant slam force on a member is maximum when the axis of the cylinder is parallel to the water surface.

Slam force become more important in relation to other loads when particularly when the Froude number that is Froude number calculated based on the impact velocity to the member that is if that exceeds a 0.6 then it is extremely important to consider the impact loads or the slamming forces.
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So, we have an empirical relationship to get that we slamming force and the horizontal members particularly in the splash zone will be subjected to such slamming forces and that is given to given as a similar to what we have seen the drag force so you have a slamming force here that is the only thing which is not known, and slamming provision can be approximate said to be equal to 3 to 3 and a half 3.6 for tubular numbers.

So, the with this we have covered forces on small bodies, large size bodies that is in the diffraction region so we have we have seen a the linear diffraction methods I mean linear diffraction theory, and then we went on also we also examined the Froude krylov forces and now the slamming forces, and we have considered the most common types of structures that is ah horizontal members incline members as well as the vertical members. So, this wave forces on a members wave forces on structures is still a gray area where lot of work need to be done and research is in progress


And some of the lectures, most of the lectures are available in a many of the text books, that are that are in in fact many of them are also you can find it as E books, I am sure that the students will get full benefit of this video lecture.

