

Ocean Structures and Materials
Dr. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Module - 2
Lecture - 7
Design adequacy- Example 1

Ladies and gentlemen , we will continue to discuss with the uncertainties involved in analysis and design which we continue in the last lecture.

(Refer Slide Time: 00:23)

Ocean Structures and materials.
Lecture 7 - Module 2.
- Life Time Estimate
- Design adequacies.
$$D(T) = \int_0^{\infty} \frac{E[\tilde{N}(a)]}{N(a)} da \quad (4)$$


Where $E[\tilde{N}(a)] da$ will be the expected
NPTEL

(Refer Slide Time: 00:56)

of cycles (stress cycles) with amplitude between a and $(a+da)$; during the time T .

$$D(T) = v_x^+ \omega T \int_0^{\infty} \frac{f_{xp}(a) da}{N(a)} \quad (5)$$

$$N(a) = \frac{(S)^m}{k} = \frac{(2a)^m}{k}$$

$$D(T) = v_x^+ \omega T \int_0^{\infty} \frac{(2a)^m}{k} f_{xp}(a) da \quad (6)$$


In the last lecture we said that the cumulative damage accumulated is given by equation four where the expected value of N tilde a d a will be the expected number of cycles, in fact I should say number of stress cycles with amplitude between a and a plus d a , of course during the time T . Therefore, D of T can be expressed as v x plus 0 of T f x p a d a by N of a , which we said equation number five or we can say equation number five a and so on.

We already know the value of N a is given by the transformation from the first equation. Stress range to the power m by k , in my case the stress range is 2 a to the power m by k substituting for N a in equation five I get D of T as v x plus $naught$ t integral 0 to infinity 2 a m by k f x p a d a , let me call this as equation six. Now, having said this let us look at this equation six and substitute for these variables and try to evaluate this function for a variable range from 0 to infinity in the integral domain.

(Refer Slide Time: 04:02)

For a narrowband process (Gaussian process),
$$f_x p(a) = \frac{a}{\sigma_x^2} \cdot \exp\left\{-\frac{a^2}{2\sigma_x^2}\right\} \quad (7)$$

Substituting (7) in (6), we get
$$D(t) = \frac{V_x^2(t) \cdot T}{(2)^m \cdot k \cdot \sigma_x^2} \int_0^{\infty} a^{m+1} \exp\left\{-\frac{a^2}{\sigma_x^2}\right\} da$$

For a narrow band process which is Gaussian, $f_x p a$ that is this value can be given by $\frac{a}{\sigma_x^2} \exp\left\{-\frac{a^2}{2\sigma_x^2}\right\}$. I call this equation number seven. So, I know the value of $f_x p a$ given by this function if it is a narrow band process which is having Gaussian distribution, substitute this value of $f_x p a$ in equation six for the variable here and then try to find out the cumulative damage $D T$ from expression six.

So, I should say substituting seven in equation six we get D of $T v x$ plus naught T , I am rewriting this slightly in a different manner. I have $2 a$ raise to the power m here. I have a also here, I rearrange them slightly and write it in this form here as I see in this expression, 2 to the power of minus m that is 2 power m 2 power minus m k , I already have k here which is a material constant and σ_x^2 I already have it here. Now, I have a power m and a , I put it inside. I can remove this all, exponential minus a square by σ_x^2 of d .

(Refer Slide Time: 07:25)

$$D(T) = U_x^+ T \frac{(2\sqrt{2} \sigma_x)^m}{k} \Gamma(1+m/2) - (8)$$

$\Gamma(1+m/2)$ is gamma function
- value of these fns are available in the std tables.
 $\Gamma(n+1) = n!$
where $n = 0, 1, 2, \dots$

Now, integrating this equation from the domain 0 to infinity, I can rewrite this equation as D of T v x 0 plus T 2 root 2 sigma x m. It should be rather the power of the whole variable by k gamma function of 1 plus m by 2. Now, where this is a gamma function, the value of these functions are available in the standard tables, but for the sake of viewers let us say gamma of n plus 1 is simply given by n factorial, where n can take any value from 0 1 2 etcetera. So, let me call this equation as equation eight.

Now, in this equation I have the value of T which is nothing but the lifetime estimate of the structure based on the cumulative damage caused because of exceeding sub stress values in the stress range of $2 a$. So, it is fatigue damage. So, I am interested in finding out the life estimate of the structure based on the cumulative damage caused because of exceedence beyond the threshold value in a range $2 a$ in a period T .

(Refer Slide Time: 10:15)

From Eq (8), Life estimate can be computed as:

$$T = \frac{K [v_x + 0]^{-1}}{(2\sqrt{2}\sigma_x)^m \Gamma(1+m/2)} \quad \text{--- (9)}$$

$[v_x + 0]^{-1} = T_z = \text{Zero mean crossing period}$

$$T = \frac{K T_z}{(2\sqrt{2}\sigma_x)^m \Gamma(1+m/2)}$$

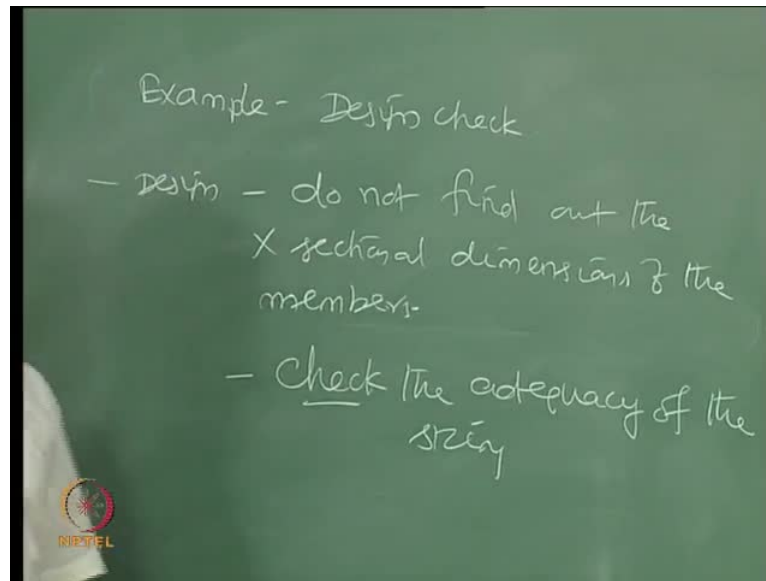
$D(T)$
 $\Gamma(1+m/2)$
 $- \text{val}$
 S
 Γ

NPTEL

So, from equation eight, life estimate can be computed as $k v_x + 0$ inverse. I have interest in finding out T divided by $2\sqrt{2}\sigma_x m \Gamma(1+m/2)$ that is in this equation I substitute D of T as unit T that is the damage, I call as equation number nine. $v_x + 0$ inverse is otherwise addressed the literature as T_z where T_z is called 0 mean crossing period.

Therefore, T can be given as $k T_z / (2\sqrt{2}\sigma_x)^m \Gamma(1+m/2)$ where m and k are material properties. So, this is how one can estimate the life estimate of a given system when subjected to an accumulated damage caused because of exceeding sub stresses or stress cycles which we called as fatigue damage. So, this is the uncertainty which we are talking about in the lecture in the last presentation. Having said this, we also said when we discuss about the analysis and design of offshore platforms there are uncertainties associated in calculating even the design checks.

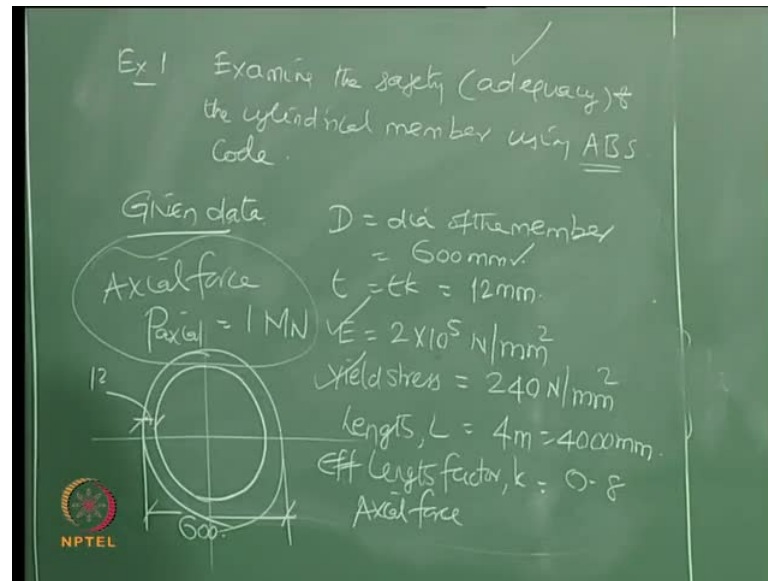
(Refer Slide Time: 13:23)



So, let us take an example of design check done on off shore members. As I said, in design we actually do not find out the cross sectional dimensions of the members, we do not find; what we do is, we check the adequacy of the size. So, the member dimensions are really known to me. I only just check whether they are adequate or inadequate. So, we will take up an example using a specific international code.

We will solve this example and show you how to calculate the check for design adequacy. This is just only an introduction for people understand this course. If you want really look at the detailed design procedures you must follow additional courses available in NPTEL on design of offshore structures.

(Refer Slide Time: 14:49)



So, I have example one. The question says examine the safety or I should say adequacy of the cylindrical member using ABS code. Given data are the following. D which is diameter of the member we say it is 600 mm, t which is thickness of the member which we say 12 millimeters, Young's modulus of the material, yield stress of the material, length of the member that is 4000 millimeters, effective length factor which I call as k is 0.8 and subjected to an axial force which I say the axial force P_{axial} is 1 Mega Newton.

So, I have a cylindrical member whose thickness is 12 millimeter and the external diameter of the member is 600 millimeters. The member is having effective length factor as 0.8 and the overall length is 4000 millimeters. The material characteristics in terms of Young's modulus and yield strength are known to me. I want to check whether the member is adequate to carry the axial compressing force of 1 Mega Newton. So, ladies and gentlemen, you will agree that in design the member dimensions are not arrived, the member dimensions are already pre fixed. We only check the adequacy whether this is sufficient and we use a specific code which I will do now and demonstrate for you.

(Refer Slide Time: 18:59)

ABS - Guide for Buckling & ultimate strength assessment for offshore structures - ABS-2008.

Soln Step #1
Basic design parameters

$$\text{Area, } A = \frac{\pi}{4} (D_o^2 - D_i^2)$$
$$= \frac{\pi}{4} (600^2 - 576^2)$$
$$= 22167.08 \text{ mm}^2$$
$$\text{Axial Comp Stress} = \frac{P}{A} = \frac{1 \times 10^6}{22167.08} = 45.11 \text{ N/mm}^2$$

NPTEL

So, the code which you are referring in this example is ABS which is guide for buckling and ultimate strength assessment for offshore structures, ABS 2008. Now, we will solve this problem. Step number one; we know the data given to me are the following. I will work out basic design parameters. That is what I am going to do in step number one. So, I will find out the cross sectional area which nothing but pi D square by 4 D outer minus D inner.

So, let us say 600 square, the thickness of the member is 12 millimeter so both the sides 24 so I should say 576 square. So, 600 square minus 576 square into pi by 4 I get 22167.08 millimeter square. Now, I can find the axial compressive stress which is nothing but P by A. So, P is given to me as 1 Mega Newton. So, 10 power 6 Newton and 22167.8 which is 45.11 Newton per mm square.

(Refer Slide Time: 22:14)


Moment of Inertia, M_oI .

$$= \frac{\pi}{64} (D_o^4 - D_i^4)$$
$$= \frac{\pi}{64} (600^4 - 576^4)$$
$$= 0.96 \times 10^8 \text{ mm}^4$$

Radius of gyration, $r_{gy} = \sqrt{I/A} = \sqrt{\frac{0.96 \times 10^8}{22167.08}}$

$$= 207.93 \text{ mm}$$

Eff Lengths = 0.8×4000
 $= L = 3200 \text{ mm}$




Now, we find the moment of inertia which is pi by 64 D power 4 minus D i power 4. So, in my case pi by 64 600 4 minus 576 4 which is 0.96 into 10 power 8 mm to the power 4. Now, I want to find the radius of gyration, which is about the minimum axis being circular member is going to be uniform about x x and y y which is going to be I by A which is root of 0.96 10 power 8 by area is available here. So, 22167.08 the square root which comes to 207.93 millimeter. Now, the effective length of the member is going to be 0.8 of 4000 which I call l as 3200 millimeters.

(Refer Slide Time: 24:52)

Polar MoI, $I_p = \frac{\pi}{32} (D_o^4 - D_i^4)$
 $= \frac{\pi}{32} (600^4 - 576^4)$
 $= 1.92 \times 10^9 \text{ mm}^4$

Saint Venant's constant, for the tubular member, k .
vide ABS-2008, Table 1, page 7

$$k = \frac{\pi}{4} (D - t)^3 t$$
$$= \frac{\pi}{4} (600 - 12)^3 (12)$$
$$= 1.92 \times 10^9 \text{ mm}^4$$




We, also now work out polar moment of inertia, I call this as I naught which will be given by pi by 32 D outer minus D inner which is pi by 32 600 minus 576, 1.9 to 10 power 9 mm power 4. I also work out Saint Venant's constant, for the tubular member given by k, this available in ABS guide ABS 2008; table 1 on page 7, the equation is available, k is given by pi by 4 D minus t the whole cube of t. I substitute now, so pi by 4 600 minus 12 cube into 12 which is again 1.92 into 10 power 9 mm to the power 4.

(Refer Slide Time: 27:38)

Step #2
Euler's buckling stress, σ_{EN}
(vide ABS 2008, clause 3.3)
$$\frac{\pi^2 E}{\left(\frac{kL}{r_y}\right)^2} = \frac{\pi^2 \times 2 \times 10^5}{\left(\frac{0.8 \times 4000}{207.93}\right)^2} = 8334.2 \text{ N/mm}^2$$

Warping constant, $\Gamma = 0$ (for tubular member)
 d_{cs} = distance of centroid and shear center along the major axis = 0.




Step number two, I want to compute the Euler's buckling stress, σ_{EN} ; this is given by clause 3.3. Now, $\pi^2 \times 2 \times 10^5$ by kL already we said it is 0.8 of 4000 and r_y we already have 207.93 plus square value this. So, calculate this we get 8334.2. That is what we call as Euler's buckling stress, so many Newton per mm square.

We also compute the warping constant which is given as γ . Generally, this is given as 0 for tubular members. Also d_{cs} that is the distance of centroid and shear center along the major axis, which is also 0 for the present problem, because the section is symmetric.

(Refer Slide Time: 31:40)

Now, Compact limit of the section.
Compact limit is established, then the section will not buckle.
Yielding precedes buckling.
Non-compact sections "local buckling" should also be considered.



Now, we shall establish what we call the compact limit of the given member or the section. Now, you may wonder why one is interested in estimating the compact limit of the section. Suppose, if the compact limit is established then the section will not undergo buckling. So, yielding proceeds. So, yielding proceeds buckling. However, for non-compact sections local buckling should also be considered, that is the catch here.

So, it is important for us established whether the given dimension of the member is a compact section or a non- compact section. If the given member is a compact section then yielding will precede buckling, no local buckling will occur. If the given member is a non- compact section then I must include local buckling also while checking the design adequacy for the given member under the axial force. So, how to check the compact limit of a given member?

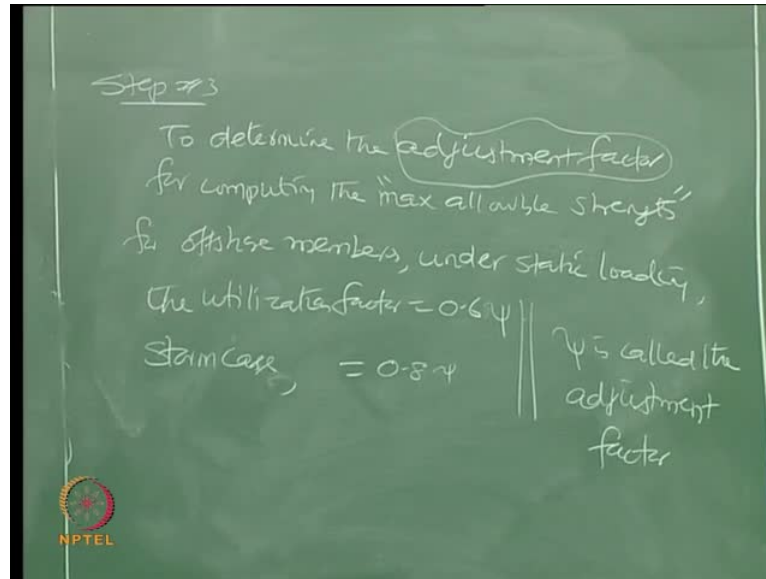
(Refer Slide Time: 34:08)

Referring to Table 1 of ABS-2008,
ratio,
 $\frac{D}{t} \leq \frac{E}{9\sigma_0}$
In my case, $\frac{D}{t} = \frac{600}{12} = 50$
 $\frac{E}{9\sigma_0} = \frac{2 \times 10^5}{9 \times 240} = 92.39$
 $\frac{D}{t} < \frac{E}{9\sigma_0}$ {compact section}

So, referring to table one of ABS 2008 we should calculate a ratio D by t to check whether this is less than or equal to E by 9 sigma naught. Let us say in my example for my case I say D by t is 600 by 12 which is 50 and E by 9 sigma naught 2 into 10 power 5 by 9 sigma. 9 is the axial stress which we calculated in the first step which is the permissible yield value which is 240 in my given problem, which is 92.39.

Now, D by t is less than E by 9 sigma naught. Hence, the section is compact. So, what is the advantage? Once, I say the section, given section is compact, yielding will precede buckling. Local buckling need not be considered in my design adequacy, is that clear? ...

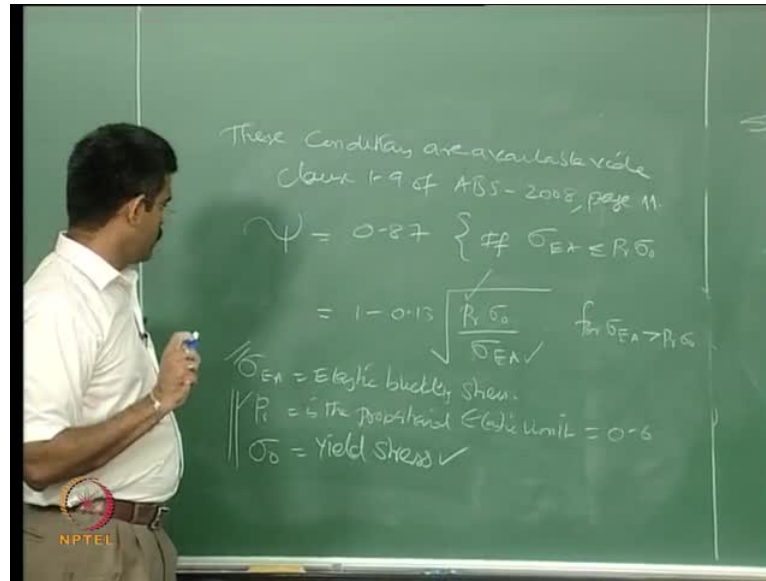
(Refer Slide Time: 36:21)



Step number three, I want to determine the adjustment factor for computing the maximum allowable strength. This is where we are trying to compute the design adequacy. The maximum allowable strength cannot be considered as it is; I must multiply this with what we call as adjustment factor. These all are the level of uncertainties introduced by different international course while executing the design checks for any offshore member.

For offshore members under static loading the utilization factor is given as 0.6 of psi. When we talk about the storm weather or the storm case, there is a rough storm case then this factor is increased to 0.8 psi where in this equation psi is called the adjustment factor.

(Refer Slide Time: 38:52)



So, these conditions are available. These conditions are available vide clause 1.9 of ABS 2008 on page 11. So, psi which is the adjustment factor is given by simply 0.87 if sigma E A is less than or equal to P r A, P r sigma naught if it is not satisfied then is given by 1 minus 0.13 square root of P r sigma naught by sigma E A, for sigma E A exceeds P r sigma. So, we must know the value of P r the value of sigma E A and of course, we already know the value of sigma naught for this given section. We know these two values and check whether this condition or this condition is satisfied in the given problem, substitute and find out psi which we call as adjustment factor. Sigma E A is called as the elastic buckling stress.

P is the proportional elastic limit which is a material property and sigma naught is the yield stress which I know for the given problem. You can see that the proportional elastic limit and sigma naught become material property, you can always find out this proportional elastic limit from the standard stress strained curve of the material. Due to unavailability of this data one can take this value as 0.6. So, I should be interested to know how to compute the elastic buckling stress for the given problem. Compare that with this ratio and check where do fall in these two category and accordingly I pick up the psi factor which is the adjustment factor given in the clause on page 11 of ABS 2008. Once, I know this I will proceed further to find out the storm case utilization factor and check the adequacy. Let us move to the next step.

(Refer Slide Time: 42:24)

Step No. 4 To compute Elastic buckling stress σ_{EA}
 (vide clause 2/3.3 of ABS-2008)

$$\frac{I_0}{\sqrt{A}} \left[(\sigma_{EA} - \sigma_{E\eta}) (\sigma_{EA} - \sigma_{ET}) \right] - (\sigma_{EA} d_{cs})^2 = 0$$

σ_{ET} = Elastic torsion buckling stress

$$\sigma_{ET} = \frac{EK}{2.6I_0} + \left(\frac{\pi}{KL}\right)^2 \frac{EI_0}{I_0}$$

$$= \frac{(2 \times 10^5)(1.916 \times 10^9)}{(2.6 \times 1.92 \times 10^9)} = 0.769 \times 10^5 \text{ N/mm}^2$$

$\sigma_{E\eta}$ = Euler's buckling stress = 8347.84 N/mm²

Step number four, where we would like to compute the elastic buckling stress σ_{EA} as given by the section 2 stroke 3.3 of ABS 2008. Let us see the equation here how we can compute σ_{EA} from the clause of ABS 2008 from this equation. So, this is the equation given in clause 2 stroke 3.3 where σ_{EA} will be a quadratic equation. We will solve this equation to find σ_{EA} which is nothing but the elastic buckling stress for the given problem.

So, σ_{EA} will be unknown in my equation and I_0 and A are already computed in step number one which we already know for the given problem and d_{cs} in my case is nothing but the center distance from the cg to the shear center for a circular symmetric section this will be actually 0. So, this term goes away. So, what I wanted to know in this will be $\sigma_{E\eta}$ and σ_{ET} . σ_{ET} is nothing but elastic torsion buckling stress which is given by the expression. So, the elastic buckling stress is given by this equation. In my problem as I already know the warping constant will remain 0 for a symmetric circular section. So, this term goes away and I have all other values with me E Young's modulus of the material, I_0 and k are values calculated in the first step which are basic properties of this section. So, let me substitute here which will give me 0.769×10^5 , $\sigma_{E\eta}$ is the Euler's buckling stress, which is Euler's buckling stress which we already computed in step number three and the value is... So, in the equation of elastic torsional buckling stress I know $\sigma_{E\eta}$, I know σ_{ET} , this term goes

away, I know I naught and I know A. So, I will solve this quadratic substituting these values and try to get sigma E A which is the elastic buckling stress.

(Refer Slide Time: 47:09)

Substituting,

$$\left\{ \frac{1.92 \times 10^9}{22167.08} \right\} \left[(\sigma_{EA} - 8347.84)(\sigma_{EA} - 0.769 \times 10^5) \right] = 0$$

Solving, $\sigma_{EA} = 8319.42 \text{ N/mm}^2$

Step # 5 Compute adjustment factor

Check whether, $\sigma_{EA} > P_r \sigma_0$?

$$8319.42 > (0.6) \times 240$$

adjustment factor, $\psi = 1 - 0.13 \sqrt{\frac{P_r \sigma_0}{\sigma_{EA}}}$

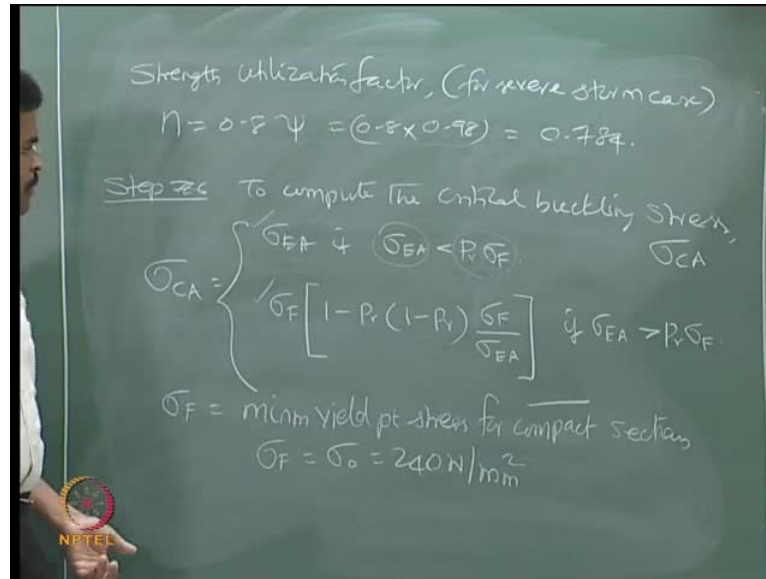
$$= 1 - 0.13 \sqrt{\frac{0.6 \times 240}{8319.42}} = 0.98$$

NPTEL

So, let me substituting solving I get sigma E A as the lowest possible value. So, in step number five let me compute the adjustment factor. Now, to compute the adjustment factor I want to check one condition whether sigma E A computed is greater than P r sigma naught. We have got to check whether this condition is satisfied. So, in my problem 8319.42 comparable with P r is 0.6 and sigma naught is 240.

You will obviously see this value is much greater than this. If sigma E A is higher than P r sigma naught then the adjustment factor psi is given by an expression 1 minus 0.13 root of sigma E A. So, I substitute them I get 0.98.

(Refer Slide Time: 51:30)



Therefore, the strength utilization factor for severe storm case because I am looking for the problem where the platform is located in severe storm condition. This is given by 0.8 of psi, in my case 0.8 of 0.98 which comes to 0.784.

So, step number six I want to compute the critical buckling stress sigma C A, that is the notation used in the code. Now, sigma C A is given by two expressions is directly equal to sigma E A if sigma E A is less than P r sigma naught sorry sigma F or sigma F multiplied by... So, depending upon the value of sigma E A in comparison with P r sigma F I will either select the upper equation to find sigma C A or I would select the lower equation to find sigma C A. So, in this equation I already know P r, I already know sigma E A, I have computed sigma E A in the earlier step. So, what I wanted to know is what would be the value of sigma F, if we know this value you can check the condition, select the appropriate equation and get sigma C A.

Sigma F is the minimum yield point stress for compact sections. Ladies and gentlemen you will remember that already in the earlier step we established that the given cylindrical member is a compact section. So, sigma F will be a minimum yield point stress for a compact section. So, sigma F can be taken as equivalent to sigma naught which is 240 Newton per mm square for my given problem.

(Refer Slide Time: 55:35)

$$\begin{aligned} \text{Now, } \sigma_{EA} &> P_r \sigma_F; \text{ hence} \\ \sigma_{CA} &= \sigma_F \left[1 - P_r (1 - P_r) \frac{\sigma_F}{\sigma_{EA}} \right] \\ &= 240 \left[1 - 0.6 (1 - 0.6) \frac{240}{8319.42} \right] \\ &= \underline{238.32 \text{ N/mm}^2} \end{aligned}$$

Now, you will know that σ_{EA} will be much greater than $P_r \sigma_F$ and hence σ_{CA} will be given by the second expression which is σ_F of $1 - P_r$ of $1 - P_r$ of σ_F by σ_{EA} . So, let us substitute this value back and compute. So, substituting I will get this value as 238 point, this is 238.32. So, obviously in the next step we will check the adequacy of the design of this member under the subjected axial compressive force.

(Refer Slide Time: 57:27)

Step 7.
"Buckling limit" (failure).

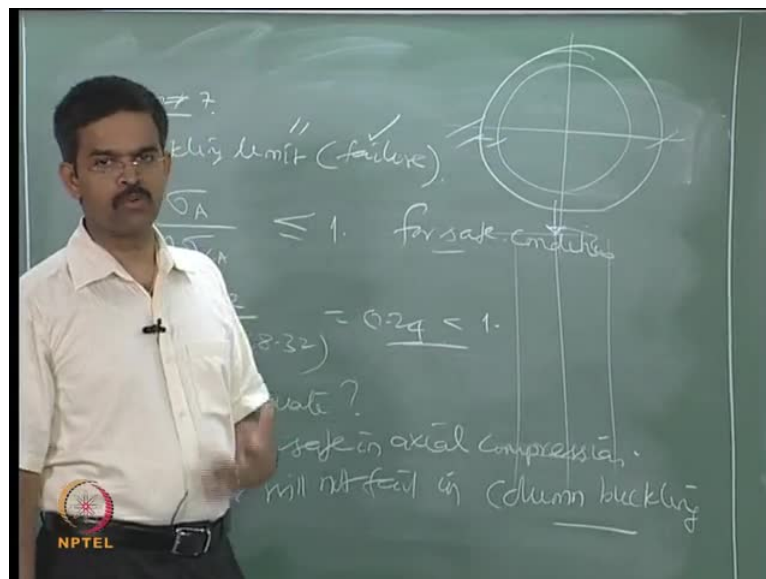
$$\frac{\sigma_A}{\phi \sigma_{CA}} \leq 1. \text{ for safe condition}$$
$$\frac{45.12}{(0.75 \times 238.32)} = \underline{0.24} < 1.$$

Safe & adequate?
The member is safe in axial compression.
The member will not fail in column buckling.

So, let us establish the failure due to the buckling limit which says σ_A by n σ_C A , is it less than or equal to 1 for safe conditions. So, let us see in my problem what this value is. σ_A we computed the direct compressive stress in the first step which is 45.12 and n I have just computed in the previous step which is 0.78 and σ_C A is 238.32 which gives me a value of 0.24 much less than 1. So, the section is safe and adequate.

Now, the question is, safe in what? What is the safety level? The member is safe in axial compression and the member will not fail in column buckling. That is why in this step we check the buckling limit of failure. When the buckling limit is satisfied the member is safe in axial compression and the member will not be failing in column buckling. So, ladies and gentlemen, this is one of the level of uncertainties what we have assessed and always in design it is only checking of the adequacy of the given member.

(Refer Slide Time: 59:44)



Just for summarizing you already have been given a circular member of a specific diameter and of a specific thickness for a specific load carrying capacity. I am only checking the adequacy of the member using a specific code using specific clause. So, there are many empirical relationships being used in working out the adequacy. So, these all are considered as uncertainties.

However, the level of uncertainties involved in checking these adequacies are far lower compared to experimental investigations carried on a members. So, by following the

detailed procedures as given by the code for checking the adequacy it is always said and understood and established in the practicing professional that the formulae and equations recommended by international codes used for checking the design are always being towards safe design and execution of the members.

Thank you very much.