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Module - 1 Lecture - 10 Methods of Writing Equation of Motion

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So, in the last lecture, we discussed about the concept of single degree of freedom system model. We have also discussed about the essential characteristics or essential features of a single degree freedom system model and we said this model is an idealized model, because we are allowing the mast move only in one specific or desired direction of displacement. Single degree is not called a single degree of freedom system, because the mass is lumped only at one point. It is not because of this factor. It is because of this factor that the mass is allowed to move only in one specific direction. Remaining all directions are restrained. We impose an artificial restrain in all other directions.

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Then, when we have a model like this, we say the model essentially will have the elastic restoring force. It may be a matrix; may be single value. It will have a mass component, which will represent the inertia force in a given system. It will also represent some frictional component and material loss, which will have to be identified by damping and of course you will have the external force, which is function of time, which is essential characteristic of dynamic loading, otherwise, in the analysis loading.

We also said in the last lecture that, in a given analysis format in dynamics, you can also eliminate c, eliminate f of t, but k and m should be always present. You can subdivide them into 4 categories and saying that, free vibration analysis, undamped forced vibration analysis, undamped, then damped free vibration and damped force vibration depending upon, why we cannot eliminate K and eliminate M in our whole analysis? We discussed this. Then we had a question that, in how many formats we can actually have an idealized single degree of freedom system model? Essentially, the first model was, as we discussed yesterday, a spring mass system. I can have a damper here or may not have a damper. I can apply f of t. May not apply f of t. This is what I call as spring mass system.

So, the system now will have only one direction of displacement, which is x of t. So, it is obviously, single degree of freedom system. Not because essentially one mass point, but there is only one direction of freedom or the degree of freedom, where we are allowing

the displacement for the mass. The second could be a Torsional pendulum. Let the pendulum have some thickness and let say, the diameter of the pendulum is a. This is theta, which will vibrate. Therefore, theta is going to be degree of freedom in this case. This is what we call as a torsional pendulum. So, what does it mean by this example is that, the displacement may be translational; may be rotational also.

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The third idealized model could be a spring mass pulley system, as you have here. I hinge the pulley here at this point and allow the rope to fall through this and have a mass suspended here. If I say the radius of the pulley is 'a' and the angle of rotation is theta, then the mass will also move as, let us say for example, y of t. Then from this problem, you may wonder there are two mass points concentrated. First is the mass of the pulley itself and second is the suspended mass m. So, I can call this as capital M for our understanding. Does it not mean that capital M denotes a matrix and small m denotes an element of a matrix? Capital M denotes mass of the pulley and small m denotes the mass of the object that is suspended from the pulley by a string or a rope. 'k' of course, denotes the elastic spring force, what we are having here. And there are two masses, there are two degrees of freedom, like theta and y. Why do we call this as a single degree idealized model?

Now, there is a combination between this y and theta. I can simply say, y can be expressed as a sin theta. So, if you look back the definition of classically for degrees of

freedom, we already said it should be number of independent displacement coordinates. I must not be able to explain the connectivity between the two degrees. As long as the connectivity exists, either I call y as my degree of displacement or freedom or theta as degree of freedom, so either one of them, because unfortunately, if we know one, we can get the other automatically. So, they are not independent each other. So therefore, even though this has two mass components, it appears there are two degrees of freedom as rotational translational. But, this is an idealized model of a single degree, because they are not independent. They can be expressed when one is known.

The fourth model is a vibration isolator. I have a mass, let say m1. Series of springs, I am just showing an example of a spring and I am showing a dash part also here and I have another mass, which is fixed to the base here. So, there is a mass component here. I can call this as, let say m2. There is a mass component here. Of course, there are stiffness components and there is a damping component here and there is a possibility that, this mass can also vibrate and this mass as well as can also vibrate independently. Why? Because these motions cannot be exactly same because they are connected and resting on a spring, which is restoring the action on mass m1, with respect to m2 or m2 with respect to m1.

On other hand, it is very simple for you to realize, if m1 and m2 rest on each one of them solidly like this, then if I mark this as x1, this as x2, then you will not agree because m1 and m2 will have a rigid body movement. But, I am separating m1 and m2 and putting a spring in between, which is having an action of restoring. Therefore, the relative displacement, the displacement of mass m1 can be different from the tau mass m2. But still, why we call this as single degree of freedom because I am interested in expressing the relative displacement? Now, the question comes, relative displacement of what? With respect to what? Displacement of mass m1 with respect to m2.

So, I have only one degree of freedom. So, these examples are standard idealist models of single degree of freedom system. Mathematical models, which clearly tells me that each one of them have independent degrees of freedom or displacement coordinates, which will help us to significantly identify the inertia contribution on this models. Here, as well as here and this gives me another crude understanding as far a single degree of freedom system, generally you should mark the coordinates of displacement like here, like here, like here, like here, at the points where mass is concentrated. On the other hand, I would not like to measure the displacement with respect to any value from here. It means, coordinates are generally grouped or lumped at the points, where mass is concentrated.

So, obviously, if we have a system, where there are two mass points, there will be two coordinates. So, this is a correlation between where do you want to measure and how do you want to measure. How do you want to measure is an idealized constraint imposed by the model. It is a single degree. Where do you want to measure is an understanding to make the problem easy. Now, you may ask me, if I do not mark the coordinates of displacement at the mass points, what would be the difficulty? We will answer this slightly later, which will have a coupling effect on the mass matrix itself and when talk about two degrees of freedom, we will talk about this.

If you do not measure the coordinates of displacement at the points of mass lumping, then the mass matrix will have a different indication in your analysis. In general, by and large, it has got great advantage of marking the displacement coordinates at the point where the masses are lumped. That is what we understand from this discussion. So, all these four are standard idealized mathematical models of single degree of freedom system. It qualifies to the force is a function of time in all the cases and the body do vibrate in all the cases. When the body vibrates, the body has a specific frequency. We would like to estimate that frequency as one of the essential outcome of this vibration of the body and of course, mass has to be present. Otherwise, I need not have to do a dynamic analysis at all. We all agree that this displacement components function of time in all the cases is significant. Otherwise, we could even do a compensative static analysis for this that is what we have understood. Is this clear?

Now, the fundamental question comes is, after we idealize a mathematical model in the format of 1 2 3 and 4, what I am looking from this model? What would be the first output from this model? The first output what I do is, I must write equation of motion for this model. You may wonder, why I should write an equation of motion because I am interested in displacement of this model at any given point of time? So, in this equation of emotion, the variable will be obviously, with respect to time. It is a time variant value, what we will write? So, let us try to write down the equations of emotion. There are many methods by which you can write the equation of motion for the single degree and multi degree as well.

We will start with, we will explain all the methods quickly and then we will select few methods amongst this and keep on applying them in the problems and try to write the equation of emotion. So, the first step in solving a dynamic analysis problem is that, you must idealize or pickup or choose a specific idealized model, may be single degree or may be multi degree. Pick up the model and then for that model, write down the equation of motion. Then solve the equation of motion. That is a first outcome what I will get from this analysis.

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So, there are many methods by which you can write equation of motion. You can use simple harmonic motion method, what we call SHM method to write the equation of motion. Then use Newton's law of motion. You can use energy principle or I will put this as energy method. You can use Rayleigh's method. You can use what we call D'alembert's principle. There are five methods. So, write down equation of motion for any mathematical model. I am going to apply this for a single degree of freedom system model to start with.

In all these models, as we just now saw, these are all discrete mass points. They are all continuous. So, what does it mean is, by variable field will be only with respect to time. Not with respect to span because the mass is not distributed uniformly along its span, along its length etcetera. We are lumping the mass at any specific desired point of our interest. Obviously, which will be the so called point of interest in this given mass? It

will be that point, which will be the center of gravity or the mass center of this body. It is as simple as that.

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So, if you look at the simple harmonic motion, simple harmonic motion has got two rules, which we all studied earlier in physics. The simple harmonic motion has a basic characteristic. The acceleration will be always proportional to the distance or the displacement of the practical. I put it as body measured along its path of motion. That is the first characteristic of SHM itself. The second characteristic is, the body will be always directed towards the equilibrium position or towards the fixed point and hence, this direction generally will be opposite to its motion. I use this characteristic and write down the equation.

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So, we already said acceleration is proportional. So, x double dot is proportional to minus of x. There are two reasons why I am putting this equation. x double dot is proportional to x. That is what one of the important characteristic of simple harmonic motion. I put negative sign, because it is always directed to the force opposite to its motion. So, I should say this is equal to rho, some constant of x. So, I can say x double dot plus rho x is 0. People can advise this constant to be omega also. We are not worried about that. It is a simple proportionality constant. So, what we infer from these equations are the following.

It is a second order ordinary differential equation in a time down. So, as we understand, if x is the displacement, then dx by dt, which I call as x dot is the velocity and d square x by dt square is again dx dot by dt, which I call as the x double dot, is the acceleration. So, these are all differential equations in time. So, I can simple write equation of motion using a simple harmonic principle, using these two characteristics of the motion otherwise defined also. Let us look at the Newton's law of motion. Any question here? Which method we will apply, we will see, but we will like to see all the problems in a simple manner first. Then with a variety of problems, try to apply any one of these methods and see can I write the equation of motion using this method. I will pick up this method.

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Let us do for the second method using Newton's law. We will demonstrate this method using a very simple example. Let say, I have a spring. No weight is attached to the spring. The moment I attach a weight to the spring, the spring will elongate. I am attaching some weight. I call this initial elongation of the spring, let say, as x naught and what will the resisting force in this case. The resisting force in this case will be the elastic restoring force offered by the spring. If I then apply a force here, pull the spring and leave it, at any instant of time, this will be my displaced position of the mass, which is again (()). So, I want to now draw a free body diagram of this at this instant and then apply Newton's law of motion to this. Can you try to do this? Draw a free body diagram of this.

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Write down all the forces of this because this is acting down and the restoring component is basically, k of x 0 plus x in this case. So, if I apply taking this as my positive sign, and this as my positive direction, I can say plus w minus of k of x naught plus x is a net force as per the Newton's law of motion. This should be equal to mass into acceleration. We already know from the first case, that the extension cause by their load on the spring, which is x naught should be compensated by the stiffness of the spring multiplied by x naught itself.

It means, as w will be equal to k x naught from the initial case, I will get this equation as minus k x plus is equal to m x double dot. So, m x double dot plus k x is 0 is my equation of motion. So, we infer important information from this. If you want to write equation of motion using Newton's law, what you should do is, ideally represent all forces; is a vector quantity right; we have to mark the directions also; not only the magnitude, but the direction also; all forces, then sum them up in vertical, horizontal, any desired direction and write equation of motion.

So, when you are trying to represent all the forces vectorially, this what we call as free body diagram. Draw a free body diagram and represent all the forces internally as well as externally acting on the body. Then sum them vectorially, may be vertically, horizontally etcetera, use a specific sign and symbol; add them up and then you will get what we call equation of motion. Again, you will see there is a comparison between this equation of motion and what we got in the previous example. This is also a second order ordinary differential equation in time domain. Let us look at the third case, where we will write equation of motion using energy principle.

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Kinetic Grergy com - fri of Vel. Potential Grergy component - fri & dispi

The energy method can be applied only to a conservative system. It can be applied only to a conservative system. What do we understand by a conservative system? Energy can neither leave nor enter the system, when the system is under operation. So, we will not consider any loss in the system. So, let us look into only two components. One is the kinetic energy component, which is of course function of velocity. Other is the potential energy component, which is of course the function of displacement.

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So, I can write kinetic energy expression as half m v square or half m x dot square. I can write the expression of potential energy as half k x square. Why we are using by the way half here? Have it. Because, you know wherever kinetic energy is maximum, potential energy is minimum or 0. On the other hand vice versa. I am looking for an average. Is that clear? That is right. It varies from 0 to maximum looking for the average value. So, half m b square or m s square and half k a square. Now, the total energy is, since I do not have any other contribution and it cannot allow me to enter and exit because the conservative system, the total energy I have is a sum of kinetic energy plus potential energy.

Since, the total energy cannot vary, I can say the first derivative of this with respect to time can be set to 0. Is it not? To have it maximum, then the second derivative should be negative or to have it minimum, second derivative should be positive. That is the second issue, but the first derivative should be set to 0. Let us do that here. I will differentiate these two with respect to time again and see what happens.

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So, d by dt of half m x dot square plus half k x square should be set to 0. So, which will give me half m 2 x dot x double dot plus half k 2 x x dot set to 0. Half goes away, x dot is common. So, x dot half m x double dot plus k x is 0. x dot cannot be 0 because the displacement is happening. So, velocity will exist. So, m x double dot plus k x is 0 is my equation of motion from the energy principle. So, what we learn from this case is, this can be applied only to conservative systems. It works on totally on energy principle. So, you should be in a position to compute different components of energy in the given system.

In my case, I have taken example of only kinetic and potential. There can be dissipation energy also, which is a function of damping and velocity which will add later. So, I keep on adding them. I apply the same concept and can write the equation of motion. What I have got to do is, find these components; add them, take a first derivative with respect to time; sum them up; you will get equation of motion.

So, it is an easy principle. So, this is a very common principle applied for systems in advanced dynamic analysis and applied mechanics schemes, where we focus on energy methods. There are many, let us say counter arguments of using energy methods for dynamic analysis. If you are looking forward for any response control of motion based on dynamic characteristics of a structure, you must do the dynamic analysis on energy principles. For example, I am looking for a damper design in a given building. I have a

building subjected to earthquake forces or any lateral force, may be, even wind. I want to control the response characteristic of the structure in time domain. When I am looking for any kind of control of energy, I must use my dynamic analysis in energy method. If I am looking for identifying the response not the control, I must use force method, which is Newton's law.

So, people divide themselves in two different groups and start writing equations of motion in their own manner. Can always cross them and verify, because they have been getting equations of the same order, whatever method you apply. But, it depends upon preferentially which method you want to employee for a specific type of a problem. That is why you will see, generally, we look at research paper which talk about response control of structures, talk about emphasis on relative motion of the body based on viscous dampers etcetera. You will find the equation of emotion generally written using energy principles.

There is software, which helps you to write the equation of motion and solve the equation of motion for a multi degree, which is a freeware available, developed by University of California at Berkley. I will link that software as we proceed further. It is a free ware available. You can also down load the software easily and can also easily find out the frequency of vibration of a multi degree freedom system model up to 3 instantaneously. You can also see the animation of the model. That is a very interesting part. I will run that software here for you. You can also download. I will give you the link. You can download the software. It is actually patented software developed by University of California, Berkley, civil engineering department on earthquake forces. So, we will use that.

So, you will have both the methods. You can develop the equation of motion and solve it using Newton's method as well as using energy methods. Then you will see the difference. That is, where, in which case you will use what kind of method. The question is, we are talking about not the solution of this. We are talking about how to write the question. But, I am giving one step ahead and saying if a solution is interesting based on some response control, you must write the equation of motion using energy methods. There is no set hard and fast rule you have to follow a specific method for a specific application. It will be comfortable for you to do the problem or analyze the system, if you follow the following techniques. That is what we are trying to say. Any questions here?

We are not considering the loss of energy in the given system. Dissipation is basically loss. It is a loss of material loss of energy, right? Alright? In this particular example, in fact in all the three examples, I have not taken the frictional component at all. Sea component, I am not modeling at all, I am taking only m n k, basic model. We will also do problems, where c is present. As I showed you c is present. We can write the equation of motion. We will do that. But, to start with, let us first understand the basic model. We all agree that k and m has got to be essentially present. Otherwise, singular freedom cannot be modeled. So, we are using that first to understand. Then we will include the dissipation energy also and we will try tomorrow. No problem. We can do that. In this example, I have ignored.

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So, we will pick up now the Rayleigh method. Rayleigh method is an extension of energy principle. We all know that, whenever or wherever in a given system, kinetic energy is maximum,; potential energy is minimum or 0. On the contrary, wherever potential energy is maximum; kinetic energy is either minimum or 0 in a given system. So, the Rayleigh method simply says, maximum potential energy should be equal to maximum kinetic energy in a given system. Use this algorithm and try to write equation of motion. Potential energy maximum equate to the kinetic energy maximum in a given system, which will be, of course equal to the total energy of the system. Using this principle, you can write equation of motion. I will demonstrate the problem later subsequently using energy method. Other time, I will extend this study using Rayleigh's method also for given examples.

So, nothing special about this method, except that, this is following the same algorithm, max that of an energy method. The only clue here is, do not look at the average there. Look at the maximum value here. That is all. Let us look at D'alembert's principle. It is a very interesting example or a method defined by scientist D'alembert's. See, people felt that the dynamic analysis is getting complicated, because of time variance in nature of the response and because of complications introduced at different levels or different kinds writing equations of emotion.

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What D'amlebert's thought is, can I convert the dynamic problem into an equivalent static problem. So, D'alembert's actually did it. He converted the dynamic problem to an equivalent static problem. Therefore, this is imaginary. It cannot be applied to real systems. It is a concept. Let us see what is this concept quickly. This will also give you an equation of emotion, but let us see how this concept can be understood.

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Let say, I have a spring mass system like this, which is having, let say x of t. I take this as positive. So, initially I will remove this. The spring is or the spring mass system is at rest. Initially the spring mass system is at rest.

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So, it is at rest. Apply a force, may be F of t and the mass is getting displaced by, let say a value x naught. So, apply F of t. The mass or the spring mass system gets displaced by $x \ 0$ as show in the figure. Then these are all spring's thickness k and mass m. I release this force and the mass will vibrate. Mass vibrates at any instant of time, let that be x.

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Let us quickly draw the static condition of this system at any instant of time. I have a mass, which is represented as m. When I try to apply the mass or pull the mass and release it, the only force which is counteracting or helping me to restore the mass moment is k of x. In the whole system, I have nothing except this stiffness attached here. The spring is attached here. If we try to pull the mass and release the mass, where an external force f of t for instantaneous time, the only remedy or rescue force what I have to bring the mass, or to attempt to bring the mass back to the normal position is restoring component coming from the spring. So, I am marking it in the opposite direction to that of motion of the mass.

So, you have seen in this example that, initially I moved it and I got in displacement x naught. I released the force. At any instant of time t, it is not coming back to 0. It is displaced by a value of x. Now, imaginatively if you want to bring this back to 0, I must apply an additional force in the same direction as that of the elastic restoring force to bring it back. That force will be mass propositional acceleration of this. So, I must say, apply an imaginative force, which will be mass propositional acceleration of this because that is what Newton law says. That is the force. Apply it in the same direction as that of the body, to bring the body to the normal condition.

Now, I write equation of motion for this applying a static condition because it is now static. At any time instant, I have this value. Is that clear? So, I simply say minus k x minus m x double dot is 0. Why 0 because the body has been brought to rest. So, on the other hand, m x double dot plus k x 0. So, I get equation of motion from D'alembert's principle, which is a concept and cannot be applied to reality. Because, at any instant of time, m x double dot will be keep on varying. If we really want to bring this mass to rest, you must have a program controller, which will be keep on machining the displacement any instant of time and provide a force opposite to the motion of the body, which is propositional to the acceleration of the mass at that time and this will be keep on varying. Is it not?

So, you can use this technique for devising or designing control algorithms. What we call as intelligence structures. You can design actuators, which will apply force, which is mass propositional to acceleration of the body at that instant of time. Develop a time history and keep on, try to bring the mass to the normal position. So, what do you understand by bringing the mass to normal position? It means, the mass is brought to rest. The moment the mass is brought to rest, dynamics does not apply.

So, D'alembert's converted that the dynamic problem into an equivalent static problem, using this technique. It is very very easy to apply this concept for a single degree of freedom system model. It is very difficult to apply the concept of multi degree because you do not know which mass you want to tune. If there are more than one mass point, if there are more than one coordinates of individual displacement, you cannot tune which mass and which coordinates you want to. So, this concept cannot be blindly applied more than a single degree freedom system model.

So, in that way, one can say this method is not popular. There is nothing drawback in this method. Not popular because of this constraint in applying this method for higher order problems. Any questions on any one of these methods? So now, I am capable of writing equation of motion at least for single degree freedom system models of any example given to me by applying any one of these methods and try to apply and write down the equation of motion. Five methods.

So, choice is yours. You can do. I will give you a tutorial. I will print the tutorial in next class; circulate it to all of you; I will give about 40 problems on single degree freedom

systems, different models; you have got to solve them. I mean, you have to write the equation of motion and what I mean by solving it is what I am going to discuss now. You have to solve them and show me. Need not submit, but you have difficulty, we will solve the problem here. You do not have difficulty; we will just go ahead with higher complication systems.

All these problems will be single degree. Remember that, I have idealized them in my own method based on the four mathematical idealized models, which I showed you in the beginning of the lecture. So, it will be one among these models. Identify the model; pickup any specific method of writing equation of motion; write the equation of motion and solve it. So, what do you understand by solving this. Any questions here?

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So, let us pick up a general equation of motion for a single degree, which is written like this, for an un damped free (()). Why free? Because f of t is 0. Why undamped? Because c is 0. Here, the damping content is not present in the equation of motion. I can rewrite this equation by dividing by mass, the whole equation x double dot plus k by m of x as 0, omega square is k by m, where omega is called the frequency of vibration of the system. Since, it is free vibration, people call this as natural frequency and people put a suffix omega n.

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So, the unit for omega n is radians per second, which we call as circular frequency or angular velocity in radians per second. Can also have f as cyclic frequency, where the relationship omega is given as 2 pi f and f is 1 over T. So, the solution what I mean by single degree of freedom system is, I am interested to know by frequency of vibration or period of vibration. So, the problem is, you must draw the given idealized freedom, single degree of freedom system model. Write down the equation of motion; identify the coefficients of x and x double dot in the given equation; bring it to this normalized form and get this value and that will be equal to your omega a. That is we mean by solving this. So, for omega n to be obtained for a single degree is as simple as that.

I must try to get the coefficient of this particular value of the displacement, whereas this is going to be unity and that will give me omega n directly. As you all understand why we are bother about omega n because I am interested to know whether this omega n, by any chance, will get tuned with the forcing function on the right hand side, which will act on the system. This forcing function, which is the function of time or frequency can be a wind load, can be a wave load, can be a current load, can be load additional due to marine growth, can be an earthquake load etcetera, which we already seen in the last lectures.

So, my principle objective is, whether this omega n will match in the band width of omega of my excited force. Suppose, if we know that, what will be the great thing which

I will get in an advantage of this. The greatest advantage we will get is, first I will know whether my body will resonate or not. Because, when these two frequencies get tuned, there is a possibility of getting resonated. The response can shoot up to infinity; can shoot up. How do I control? There are two methods of controlling it. One is, push omega beyond omega n or push omega n beyond omega, which control can you do? Omega is a natural phenomenon, which is coming from the wind, wave etcetera. Therefore, we have no control. We cannot put a ban on the structure saying, the structure should not be exerted by omega. So, the following band widths.

So, good please note that it is not possible. So, we can control omega n. Controlling omega n means, I have a full liberty to design a structure whose mass and system stiffness can be tuned. Mass comes from the geometry and stiffness comes from the arrangement of members, which is actually the design. That is why we said offshore structures are form emerged design. Not function. Form means, cross sectional dimension and material, which is giving me mass. Also arrangement of the members may be three leg, four leg, eight legged etcetera, which gives me the stiffness.

So, if I know my omega bands, which is exciting my structure, if I know my omega n being a single degree freedom system, can easily push these two bands apart, so that, my structure will not get resonated. Now, the question comes, very interestingly you may ask, sir, omega is not in our control. There is a possibility, omega with all these so called intelligent exercises, may still land up in matching omega n. Then what will happen? We will answer this further later.