Dynamics of Ocean Structures
Prof. Dr. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Module - 1
Lecture - 16
Examples

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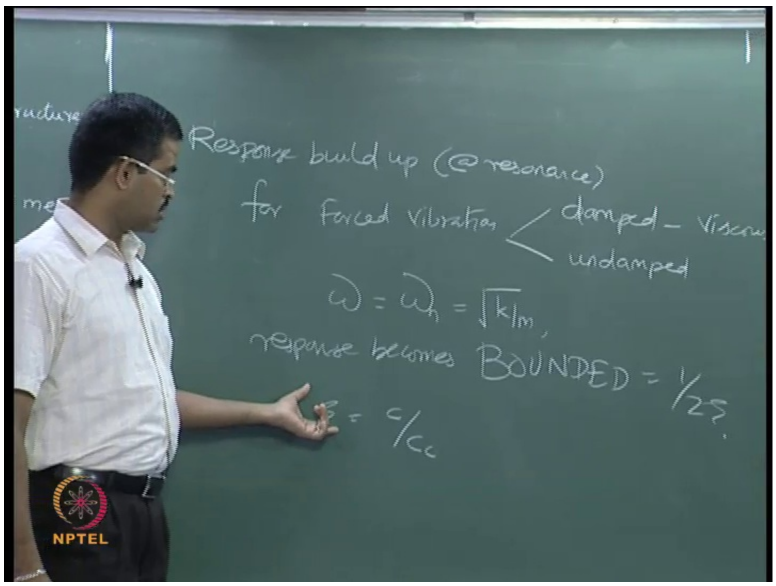

So, in the last lecture we discussed about the response build up at resonance state for forced vibration, which is damped and undamped. The moment I say damped, we discussed a viscous damping model. We already understood that when you have a damped system, even at resonance when the excitation frequency is equal to the natural frequency of vibration of a model, which is nothing but group of k by m, the response becomes bounded, which is where zeta is, what we call as the damping ratio, where zeta is c by cc. It is a ratio of critical damping is what the present damping, what we are using. So, in today's lecture we will talk about half power band width method, which will estimate the zeta value for a given system and we will also quickly see some of the interesting application problems in single degree.

Of course, we have three tutorials, already two, I have given you, but still I am projecting it today. One I will solve it here as an exercise sheet, then for further exercises I will print some more tutorials and give you can keep on solving them as a self exercise. So, do we have any questions till this stage, as we have discussed so far, from anybody, any doubt any questions? So, having said this, let us try to talk about estimation of damping. Damping is inherently present in ocean structural systems because of fluid structure interaction. So, we are worried only about the damped model and we are also more focused towards viscous damping. We have explained the reasons of all these things in the previous lectures.

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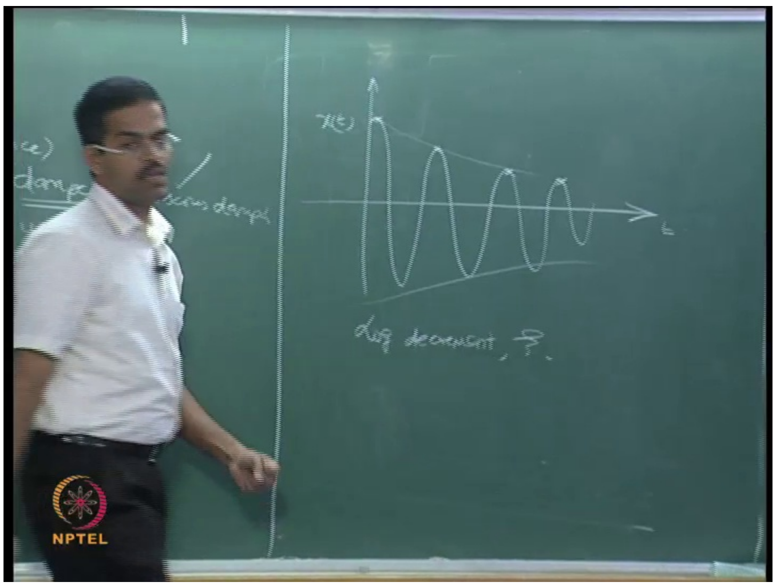

Now, estimating of damping for a free vibration analysis has been already done using an experimental data. If you have an experimental data where the response time is (( )) of anybody, which is vibrating is known to you, can pick up, the peaks may be successive, may be alternative, many n number you want and we have an expression for the logarithmic decrement based on which I can compute the damping ratio zeta. That is one way of estimating damping. There is another way of estimating damping, which we call as half power band width method. We will see this method quickly now.

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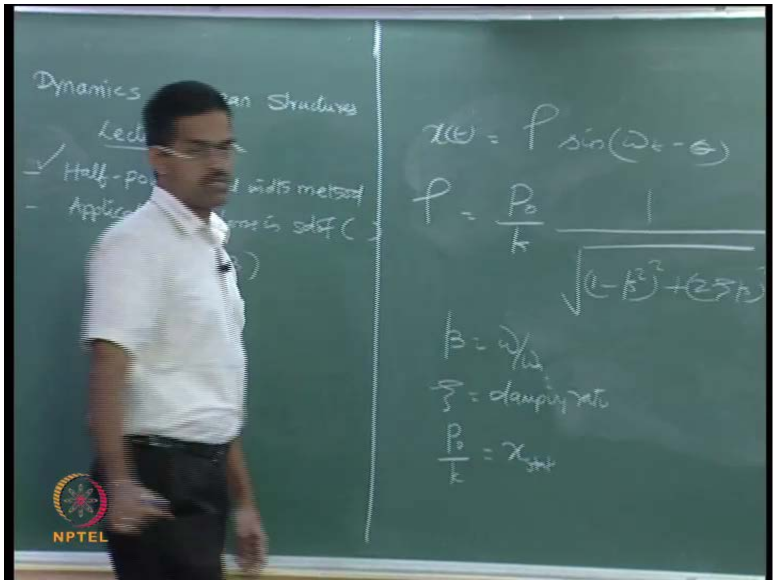

So, getting the feedback from what we had in the last lecture. We already said, x of t, which is rho sine omega t minus theta where rho is a resultant response, which is P 0 by k 1 by 1 minus beta square square plus 2 zeta beta the whole square, where beta is the ratio of frequencies and zeta is the damping ratio. Of course, P 0 by k is called x static that is a static defraction.

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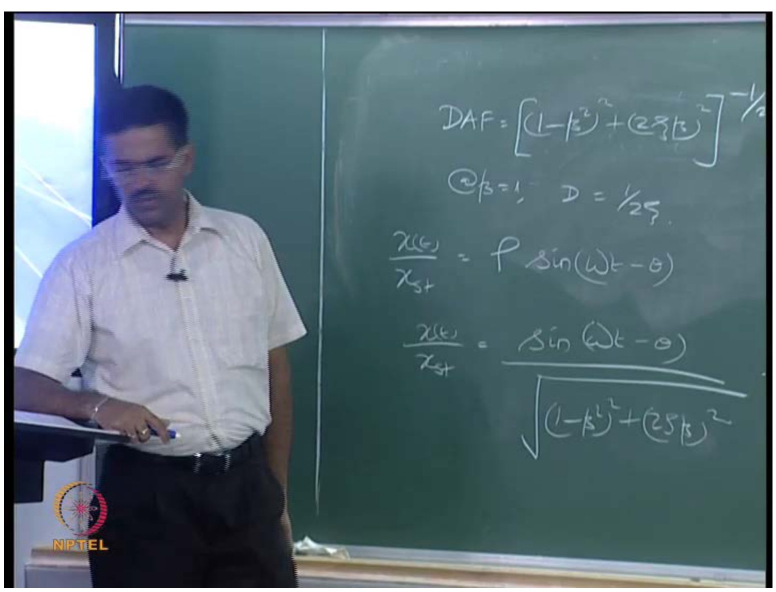

Based on this we already said my dynamic amplification factor amounts to DAF, which is 1 minus beta square the whole square plus 2 zeta beta the whole square in the power of minus half. So, at beta equals 1, that is, when the excitation frequency is as same as the natural frequency of vibration, at beta equals 1, my D becomes 1 by 2 zeta. Analytically, from this expression I also know, x of t by x static from this equation, x of t by x static P 0 by k will go away, I can write rho sine omega t minus theta.

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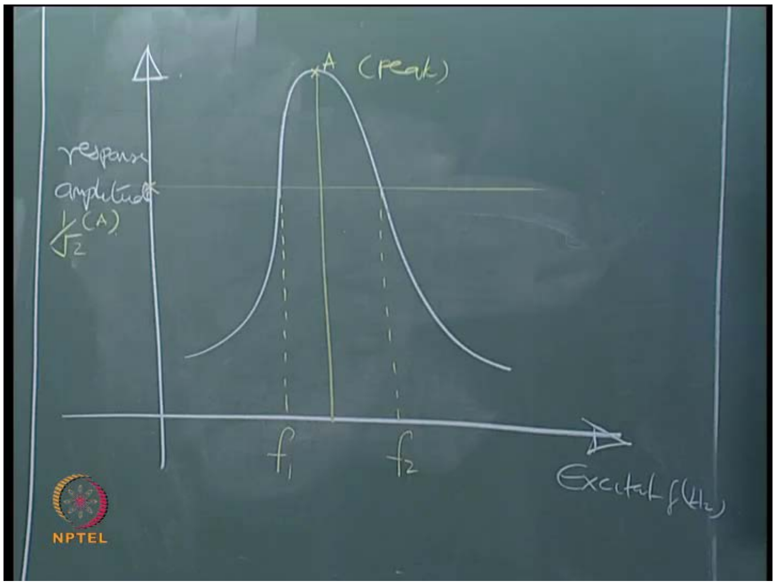

Now, I try to plot at beta, I vary only the excitation frequency because I know omega n value. So, for different excitation frequency in Hertz I try to plot the response amplitude. Let us say the plot, what I get from this by varying omega alone for a specific omega n appears like this. So, obviously, there will be a peak, which you will get when these values are equal, I call this peak as A, which is the peak of this response.

After knowing this value I draw a horizontal line at 1 by root 2 of this peak. This will now intersect the response curve at two frequencies; I call this as f 1 and f 2. These points are called half power points. So, I say, x t by x static substituting back in this equation, sine omega t minus theta by root of whole square. Let me call this as equation number 1 for our discussion. I will remove this.

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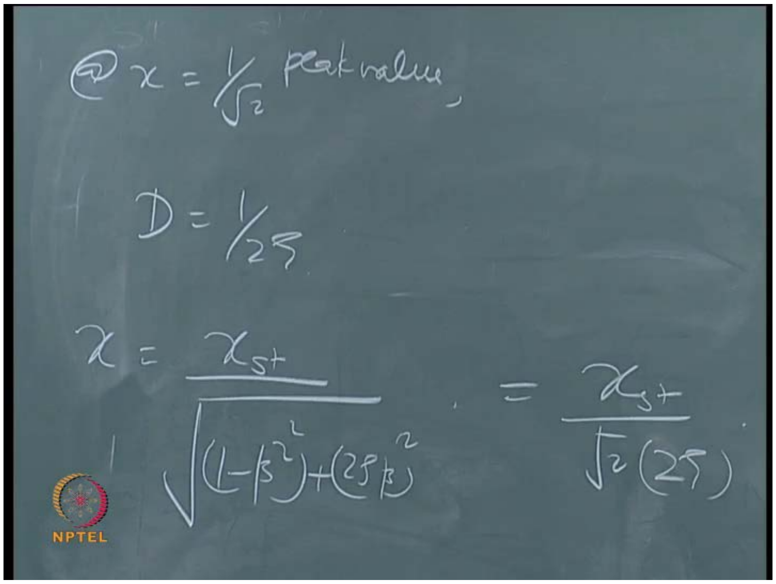

Now, at x equals 1 by root 2 of the peak value, I apply this equation. So, let us see what happens. So, we already know D is 1 by 2 zeta; we already know this value. So, we also know, x is x static by root of 1 minus beta square plus 2 zeta beta the whole square. I equate these two, x static of root 2 of zeta of 2 zeta, sorry, root 2 of 2 zeta. So, x static, anyway, in both equations goes away, rearrange them, square them and eliminate higher powers of zeta. Let us see what we get. Can we quickly do this, simplify this? It will be a quadratic equation, eliminate higher powers of zeta and simplify, the roots will be beta 1 and beta 2 beta 1 because they are quadratic in beta.

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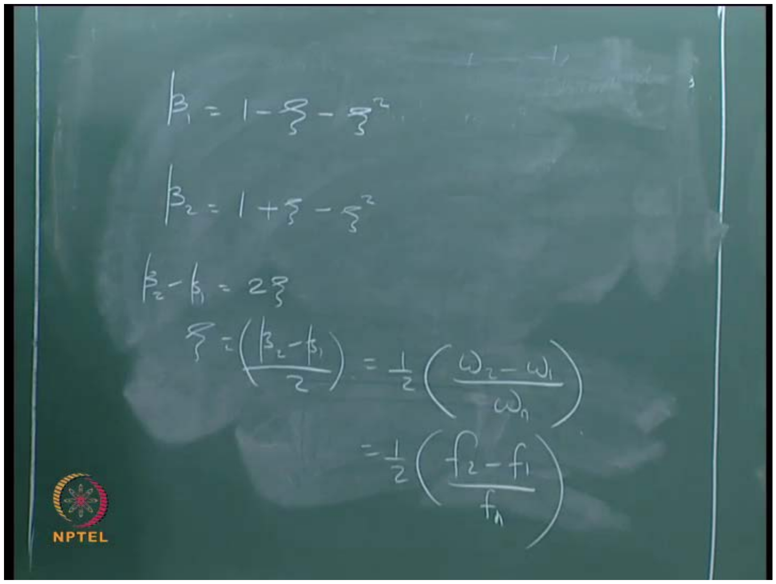

So, if we simplify, I get beta 1 and beta 2 as 1 minus zeta minus zeta square, 1 plus zeta minus zeta square. So, beta 2 minus beta 1 will be 2 zeta. This goes away, this goes away, gets added up, you get plus 2 zeta. So, on the other hand, zeta is beta 2 minus beta 1 by 2 and beta already we know is omega by omega n. Let me substitute back, so I will get this as half of omega 2 minus omega n by omega 2 by omega n, or corresponding to this figure I will get, half f 2 minus f 1 by f n and the peak will occur at the midpoint of these two obviously, because it is a symmetric curve.

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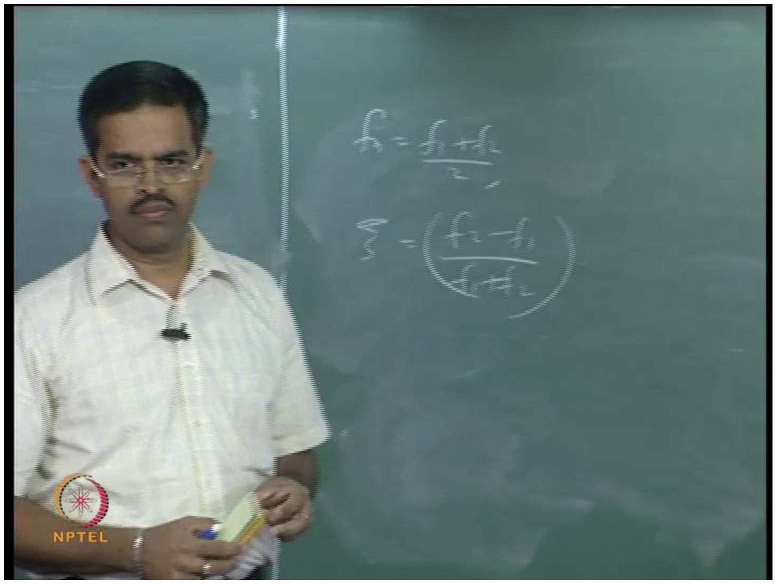

So, I can also say, f n is f 1 plus f 2 by 2. Therefore, zeta will be f 2 minus f 1 plus f 2. So, if you simply have a plot of response amplitude versus varying, only the excitation frequency for a known omega n we get a plot, get the peak, bounded peak, because zeta is not 0. For zeta not 0, any value other than 0, we get a bounded value. We have already seen this in the last lecture.

So, you get a bounded peak, pickup that value, take 1 by root 2 of that value, draw a horizontal line, it will intersect the curve in both places, get the corresponding frequencies. If you know these values you will get zeta at which this curve was drawn from this expression. So, it is one of the interesting ways of computing zeta. This method is called half power band width method. There is a bandwidth, is determined by a half power point, that is why it is called half power band width. This is very common method wherein you can estimate zeta if you have the plot.

We have discussed formally the single degree freedom system models completely. I will take up some application examples now, we will solve them here. There are four problems I will solve. Now, here I will give you some more problems for the exercise, you try to solve them. If you have difficulty, we can then discuss it here later. After I finish this exercise today, tomorrow I will move on to two-degrees and multi-degree freedom system models for another three or four more classes. That is the plan. So, we finish multi-degree discussions, we will have an examination quiz one. So, single degree, two-degree, multi-degree discussions we complete, will take up an exam on quiz one, then we will extend further discussion on multi-degree.

In the next module, I will talk about dynamic analysis of different kinds of offshore structures where I will also show you how dynamic analysis can be done for different class of structures with software support. So, we will make an online presentation, I will show you how this can be done with the screen shots of different software for different kinds of structures. We will discuss the results from the research papers directly. We will not be able to run it here, takes lot of time. I will discuss the results directly by the, give you the methodology how it was done, that we will discuss in module two after which we will take a quiz. Then in module three we will talk about stochastic dynamics; that is the plan. So, the first module, first degree, single degree freedom system will get completed today with the four example problems, which I will solve now. So, I will project the problems, let us try to solve one by one.

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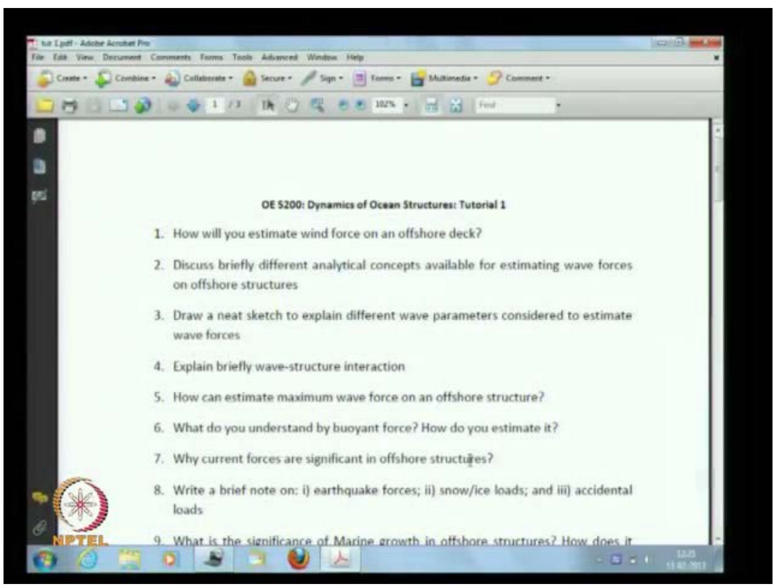

So, tutorial one, so already we have discussed this. How will you estimate the wind forces? I will give you the printout sheet of this, we have discussed this. It is a theory question.

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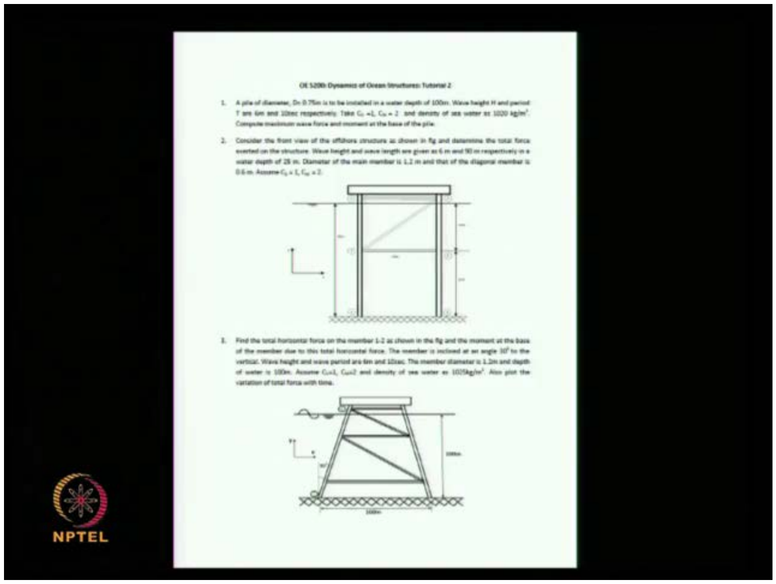

Tutorial two was again estimating the member forces, three problems.

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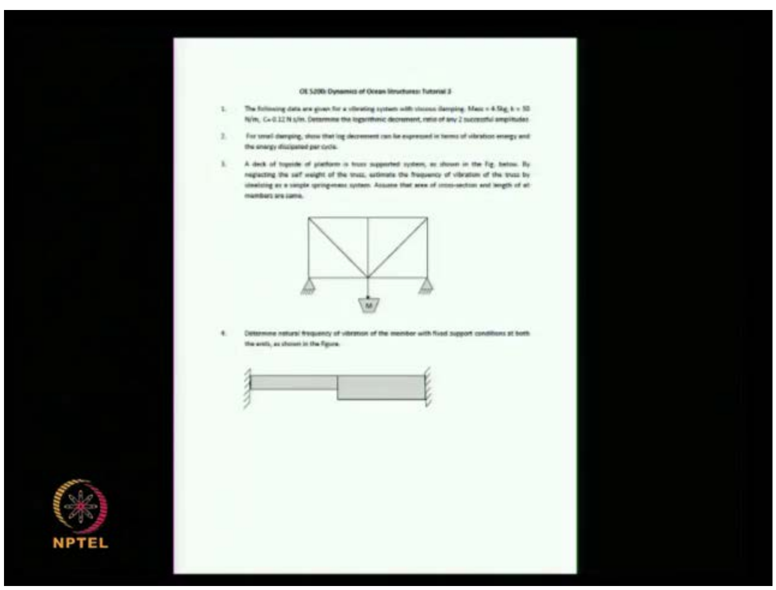

Tutorial three is what we will solve here as an exercise sheet. Are you able to read it?

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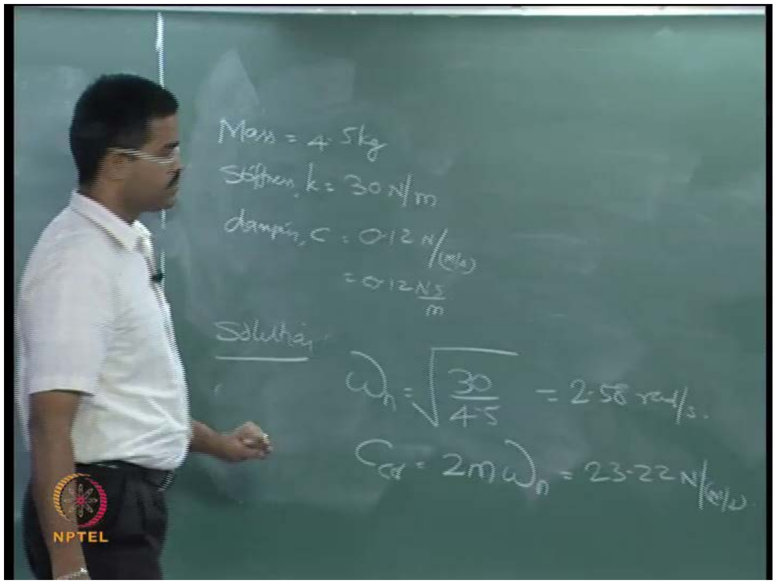

So, the following data are given for a vibrating system with viscous damping, 4.5 kg; stiffness, 30 Newton per meter; damping constant, c, 0.12 Newton per meter per second, because it is viscous damping, so it becomes 0.12 Newton second per meter. What we want is, find the logarithmic decrement of this. So, try to solve this problem. Let us see, are you able to do it? So, you can compute, omega n, k and m are known to me, so which is 30 by 4.5, which comes to how much? 2 point radiance per second. I can also estimate critical damping. What is the equation for this? 2 m omega n, I know the value of m and I know the value of omega n, this comes to 23.22.

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And zeta is the ratio of c by cc, c is given as 0.12, cc we know, so our zeta is expressed in percentage of… Are you getting this? Calculate zeta and c. So, what is the logarithmic decrement? What is the expression for logarithmic decrement? Delta is 2 pi zeta by 1 minus zeta square. You can compute this. So, 2 pi 0.0052 by root of 1 minus, how much is this, how much, 0.0 double three. So, the first thing asked is logarithmic decrement.

The second thing asked is, find the ratio of any two successful amplitudes. So, I want amplitude ratio, which is x 1 by x 2 because it is decreasing, it is not x 2 by x 1. So, it is nothing but e delta, is it not? It is exponential decay, so e power delta 1 point… Is that ok? So, the successive amplitudes keep on decreasing by so many times of the earlier one; that is what asked in the first question. Second question, is this clear to all of us? Shall I remove this?

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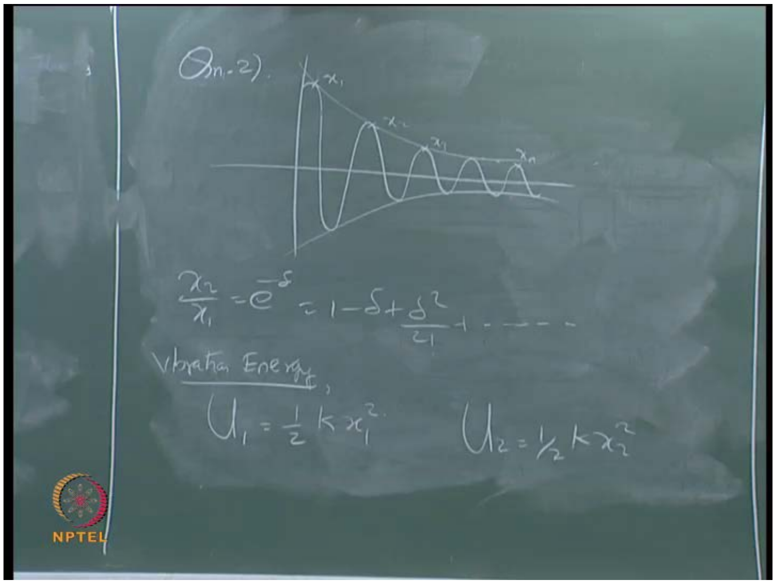

Let us see question number two. For small damping, zeta very insignificant, show that the logarithmic decrement can be expressed in terms of vibration energy and the energy dissipated per cycle. So, how do we approach this problem? Let us say, this is my plot, I understand. Let us say look at the positive peaks, x 1, x 2, x 3, let us say x n. So, I already know x 2 by x 1 is e power minus delta, which is given as 1 minus delta plus delta square by 2 factorial and so on. Is it not? We can expand this e power exponential series.

What is the vibration energy? Let us say, I am looking for the energy at the first level U 1, which is half k x 1 square. Look at the second amplitude, U 2, half k x 2 square. System is same, so stiffness will not vary. What is the loss of energy? U 1 is higher or U 2 is higher? Loss of energy, U 1 minus U 2 with respect to U 1, let us try to find out that, I will write it here.

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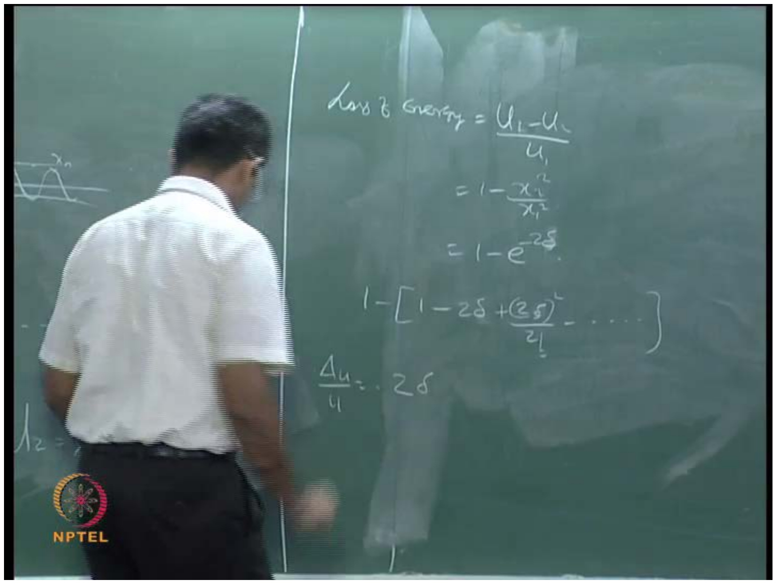

So, loss of energy, U 1 minus U 2 by U 1, substitute for U 1 and U 2, apply this equation, what do we get? You will get this as 1 minus x 2 square by x 1 square, is it ok. Are we getting it? So, x 2 by x 1 already I know as e power minus delta, so 1 minus x 2 by x 1 whole square can also be written as 1 minus e to the power of minus 2 delta, I can expand it.

So, how do I expand it? 1 minus 1 minus 2 delta plus 2 delta the whole square by 2 factorial minus and so on, is it not. For smaller values of delta, why the problem says, for small damping, for smaller values of damping what is the ratio? Delta U by U will be equal to what? 2 delta; is it not? I think it should be delta, I think I am writing wrong, please correct this. So, I can connect the logarithmic decrement to my vibration energy. I know the logarithmic decrement, I can find out the vibration energy difference from this expression. Any doubt? Can I take away this?

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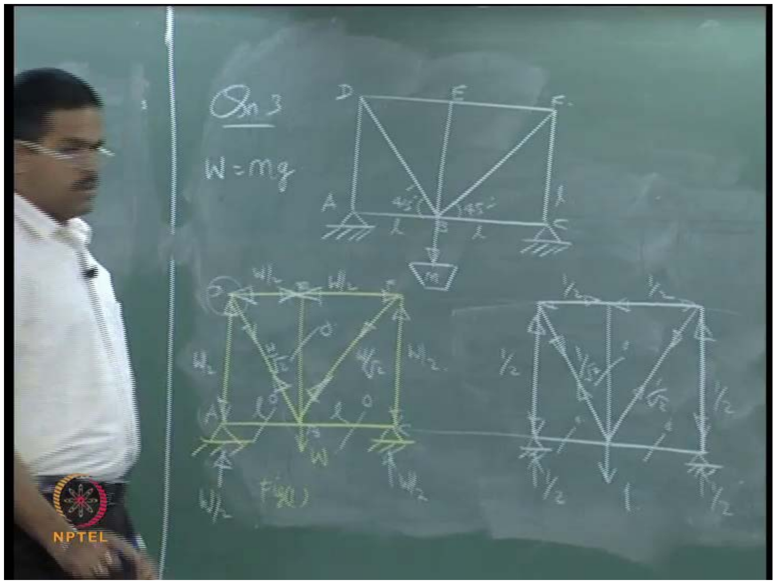

Question number three, direct application problem, let us say, I have an offshore platform whose deck is support by a truss system; whose deck is supported by a truss system. A typical truss system is shown, I am drawing it here though in reality, the truss system for which an offshore platform deck is supported will be complex, but I am taking a very simple system for demonstration.

So, it has got a specific support condition, both ends are hinged, all the cross-section areas of the members are equal length of the members are equal. So, l, l, l to impose this condition, this degree will be 45. I name this joints as A, B, C, D, E and F. I want to find the frequency of vibration of this truss system for a load suspending here, that is the question asked. A deck of top side is truss supported system as shown in the figure below by neglecting the self heat of the truss. So, mass of the truss by itself is neglected in this problem. Estimate the frequency of vibration of the truss by idealizing a simple spring mass system. Assume that the area of cross-section and length of all the members are same. How do we approach this problem?

So, I draw 2 truss here, I put w. We already know w is mg and ideally, replace this m by this equation. So, let me put w here because I am going to apply a force, which is going to cause free vibration to the truss at this point the moment I apply a force here. Let us look at this joint. I must select a joint where there are not more than two unknowns. First, let me find the reactions. This is symmetric, therefore obviously, this will be w by 2, this will also be w by 2. Is that agreed?

So, select any joint and do, by method of joints select any joint where there are not more than two unknowns. What are the unknowns? All the internal forces in these members are unknowns. What are these members? A, B, C, D, E, F. The force in the member: AD, BC, FC, AD, BD, BF, EB are not known to me, they are unknowns. So, select any joint where there are not more than two unknowns. Select this joint, for example, there are three unknowns. Select this joint, for example, there are 1, 2, 3, 4, 5 unknowns, so I cannot handle this. Select this joint, there are two unknowns. One is the vertical force on the member, AD; other is the member AB. Is it not? So, since there is no horizontal force acting here in the whole system, I can say this is 0 and this will be w by 2 because this has to oppose this force here. I use a simple equation of equilibrium here and try to solve this similarly by symmetry and of course, 0.

So, can you solve the remaining? Quick, this angle is 45 degrees; that is given here. So, what would be the next joint you select? You cannot do the joint B, you select either D or F. So, select the joint D. I hope we all know how to solve this, right. So, I get this as a tensile member, which will be w by root 2 and this becomes a compression member as w by 2; same way here, w by 2, w by root 2. So, at this joint B there are no horizontal forces from these two members. These two, they are opposite, they will resolve. These two put together will be equal to w. So, this member will become a null member again. Is it not? I will draw one more truss with the same conditions. I apply unit force, so I can just divide all of them with w. I will get the same value here, which will be half, which will be half. I quickly make a summary table, I will remove this figure, I will make a table here.

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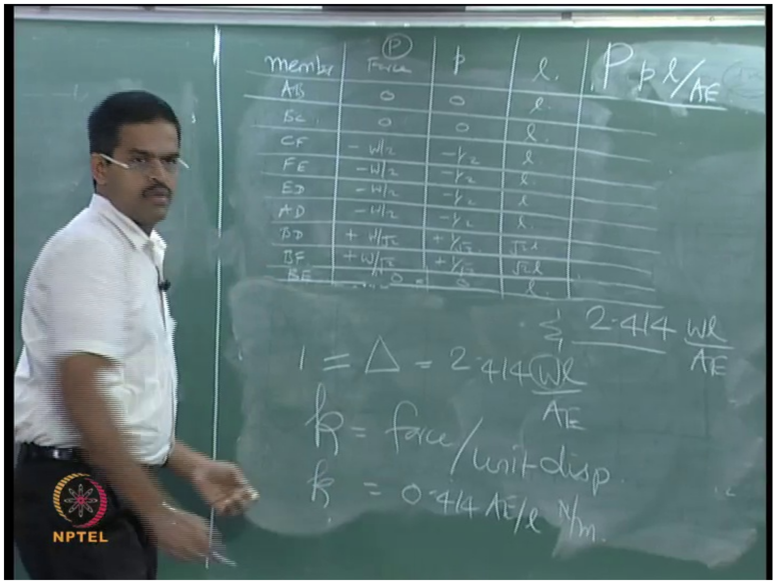

A member force, I call this as P diagram, I call this as small p diagram, this is unit load. So, force due to the P diagram and force due to the small p diagram. So, let us enter the members AB, BC, CF, FE, ED, AD, BD, BF and BE, we will have one more, write it here, it is a null member, anyways, so no problem. It is w by 2 as a length of the member; here I take tension as positive. Now, which are tension members? Which are compression members? I already told you, if I have a member, these are my joints or support condition. If the member is pushing the joint, this is compression; if the member is pulling the joint, it is tension. Look at the force application at the joint, is it pushing or pulling? So, put positive for tension, fill up this table quickly.

So, AB is 0, 0, length of the member is l; BC is also null member; FC is compression, minus w by 2, minus 1 by 2, l, this is l, this is e; EF is this member, so again minus w by 2, minus 1 by 2, l; ED, minus w by 2, minus 1 by 2, l; AD, minus w by 2, minus 1 by 2, l; BD are diagonal members, so root 2, l, and they are tensai, so plus w by root 2 plus 1 by root 2; BF is also a diagonal member, so plus w by root 2, plus 1 by root 2 and root 2; and of course, BF is a vertical member; BE is the vertical member, is a null member, so 0, 0.

So, I have summarized the forces from the P diagram and from the small p diagram, unit load diagram in a tabular form like this. So, I will remove this. Let me try to find out Ppl by AE, P is from here, p is here, l you know, AE is the constant. Why it is constant? The problem says, all cross-section area of the members are constant, l is also equal, even l is not equal. I am having l value here, I can use it appropriately and I want you to give me the sum of this with plus minus appropriate.

Sorry, last column, yeah, you have to fill up this value, multiply these two with l by AE and put it here, Ppl by AE. You are not able to see this, is it? So, I will write it bigger. This is Ppl by AE. The sum will be equal to 2.414 wl by AE. Is that ok? So, what is this value now? This value now is nothing but the vertical displacement of the truss at the point we are suspending the mass, which is 2.414 wl by AE. What is stiffness? Stiffness is force by unit displacement, is it not, set it to 1 and find out the force. Set it to 1 find out the force, how much it will be, which will be 0.414 AE by l, is it ok, Newton per meter, mass in kg. But I am holding the problem in w; it is Newton because it is mg. Now, I have k, the problem says ignore the (( )) of the members. If you know m, I can find omega. I know k if you know m in the problem, whatever may be the value, I can find omega of this truss.

Now, what is the interesting part in this problem is, that if you have got only a single member subjected to axial pull or axial push, stiffness would have been only AE by l. You have got parallel members or in series members, you would have got equivalent stiffness. But in this problem, there are members parallel, there are members in series, there are members diagonal also. So, what is the net effect of stiffness in the whole truss, that is, what I have got?

So, this is stiffness of the whole truss system, which is having parallel members with axial tension or compression series members with axial tension or compression. In this case, there are no series members and diagonal members. So, this is a very easy mechanism, which we have already studied in our undergraduate level, same principle I am applying to solve the dynamics problem. So, I am getting now k, if I know m given in the problem, I think I can easily find out omega n. So, omega n is root of k by m and m is given, is it not. It is given in the problem. So, if you have m, you can easily compute this. Any doubt here?

Similar problems of this type, I have one more problem, I do not want to rush through, that is a beam problem. This will take about 15 minutes time, I think we will do it in the next class. This requires more discussion slightly; we will do it in the next class. I do not want to rush up, I have only 3 minutes left over we will do it next class. So, similar problems of this type is expected in your quiz as well as theoretical derivations as well as inferences from the figures and response behavior of single degree freedom systems and dynamic response behavior in physical understanding of different offshore structural systems and coastal protection systems, which will be covered in the quiz 1. In addition, I will also talk about two degree and multi degree in the coming lectures.