Dynamics of Ocean Structures
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Module - 1
Lecture - 23
Natural Frequencies and Mode Shapes

In today's lecture, we will again take up one more examples, where the lumped mass points in different degrees of freedom will be different.

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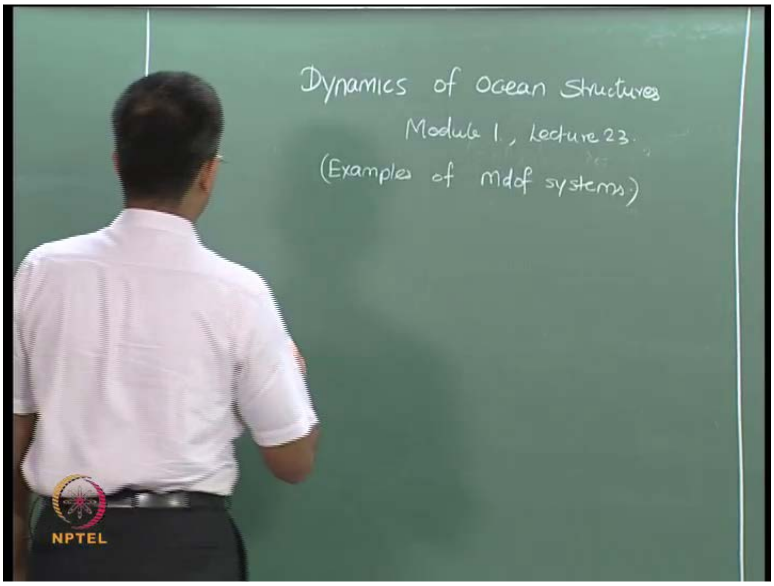

It may not be m; and as per as k will not be simply the value of identical in all the points; it will be different. So, let us say that, can I use Stodla method and influence coefficient and Dunkerley's method for comparing this quickly. Once we understand this, then in the next lecture, we will start moving on to Rayleigh-Ritz method and another technique, so that we can wind up the discussions on omega and phi; move on to the second module. So, this will be the pre-last lecture for the first module. So, one more lecture will there tomorrow on the first module; where, we will talk about Rayleigh's procedure for finding natural frequency.

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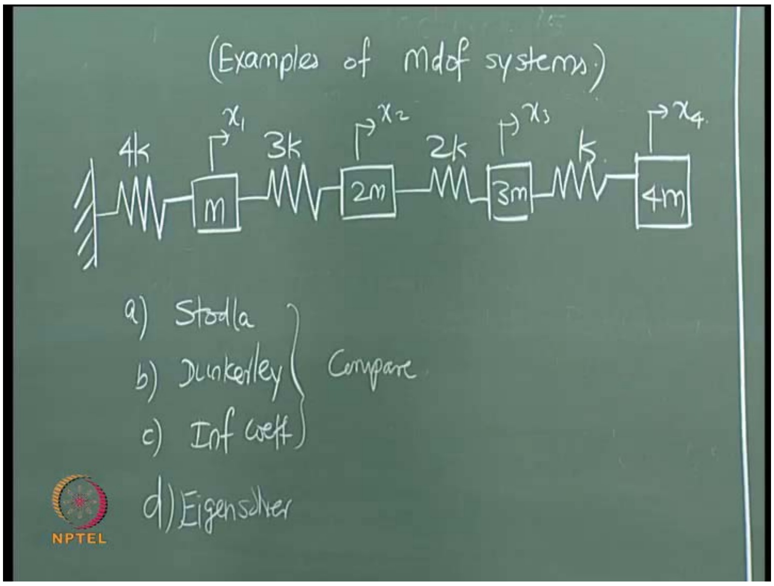

So, we will take up this example, which is slightly different from what we have seen so far. So, this will solve all your doubts if you have so in using it for multi-degree with different m and different k. I will take a four degree freedom system. So, this is 4k, 3k, 2k, k; m, 2m, 3m and 4m. Let us call this as x 1, x 2, x 3 and x 4. All these are free vibration problems; we are not look at the zeta value or the c value; looking for omega, which is function of k and m. So, let us say I will solve this problem by Stodla‘s method; solve this by Dunkerley; and also, solve this by influence coefficient method and compare them. Of course, I can solve it also using classical eigensolver and then we can compare. You will see this problem; or, this example is a very beautiful illustration, which will compare all the three very closely. Let us see how this is done. So, we will get back to the algorithm – how Stodla method is executed.

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This is description. So, we say assumed deflection. The deflections are the points where I am measuring the response coordinates. So, I must enter in the mass points here. So, this is…

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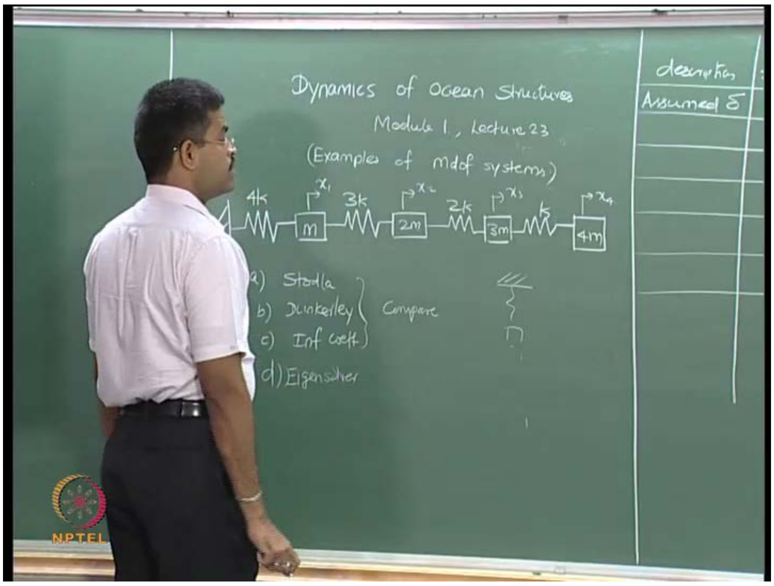

When I suspend this freely vertical like this and so on, the deflection given at this point will get added to the finial deflection here. Therefore, I say this is 1, 2, 3 and 4; which is proportionate to approximately either the stiffness or the mass.

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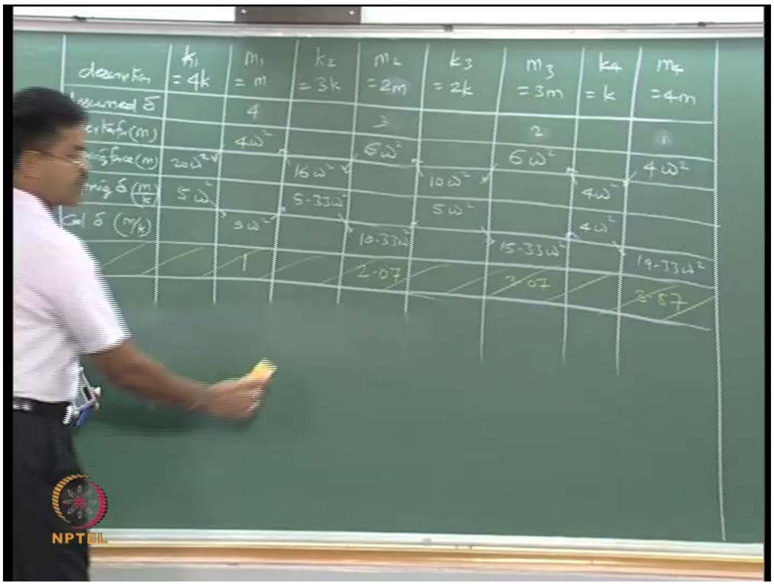

So, the inertia force; I take m outside. So, this is going to be m omega square; that is the inertia force – m omega square x double dot. That is the inertia force. I have taken m out; that is a multiplier here. So, I put 4 omega square here. Similarly, here I have taken m out; this is 3 and this is 2. So, I put 6 omega square. Similarly, I put 9 omega square here; and of course, I put 4 omega square here. This is 2m. So, this one is 6. So, let us say the spring force m constant out.

So, I must use this – 4 omega square, 10 omega square, 16 omega square, 20 omega square. Spring force; spring deflection, which is now m by k. So, 20 omega square by 4, 5 omega square; k is here. Similarly… So, calculate a deflection; again m by k… I can take a banded value say 1, 2.07, 3.07, 3.87. We started with 1, 2, 3, 4 and moving on to a different value. So, let me use these bands now. It is a banded value now. So, any doubt for anybody in the first state of iteration here? So, I am going to rub the values. Any doubt for anybody in the first stage of iteration? Any doubt? Shall I remove these values?

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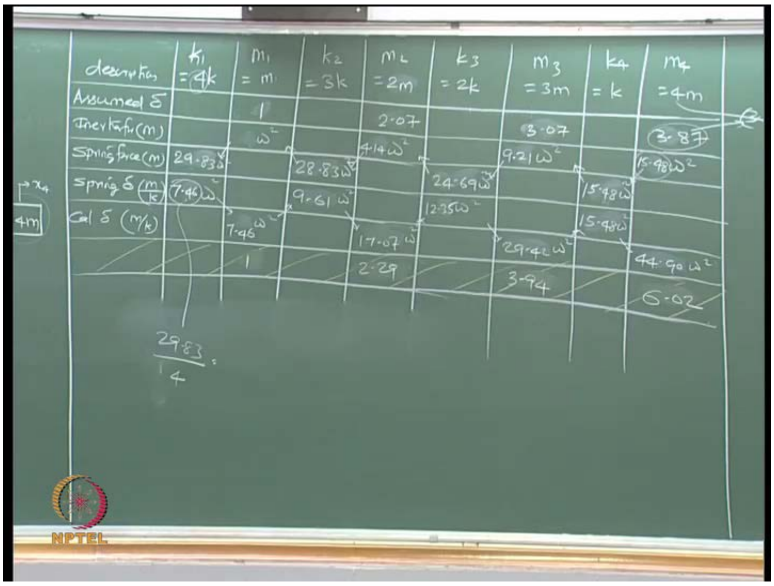

I will just rewrite them. You will extend the table and do it. I am putting it here as 1, 2.07, 3.07, 3.87… So, this becomes 15.48. How do I get this value, is nothing but 3.87 multiplied by 4. 3.87 is this value; 4 is multiplied. 9.21, 4.14 omega square. So, this becomes 15.48 omega square; 24.69, 28.83, 29.83, so 7.46. So, this value is nothing but 29.83 divided by the stiffness of 4, which is 7.46, so 9.61 similarly; 12.35 and 15.4. So, this becomes 7.46, 17.07, 29.42, 44.9. So, proportionately, 6.02, 3.94, 2.29, 1. So, that is the next iteration. So, let me take new set of values.

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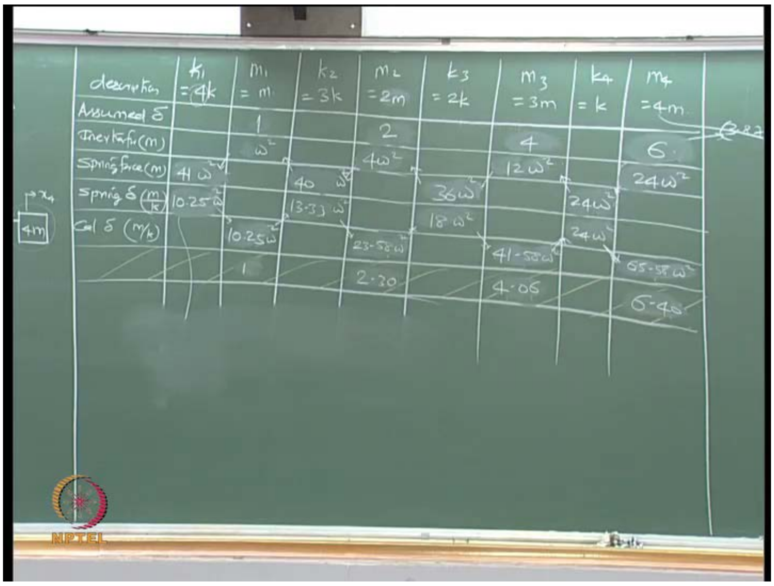

I will keep this as third iteration; I will replace all these values. It is my third iteration now. For the next set, I will use 1, 2, 4 and 6, because they were fractions here; making then as 1, 2, 4 and 6, because this was 2.29; this was 6.02 and so on. So, this becomes 24 omega square, 12 omega square, 4 omega square and omega square. So, 24 omega square… I can even say 6.4, 4.06, 2.3, 1. So, the next iteration, I will take this as 1, 2.3, 4 and 6.4.

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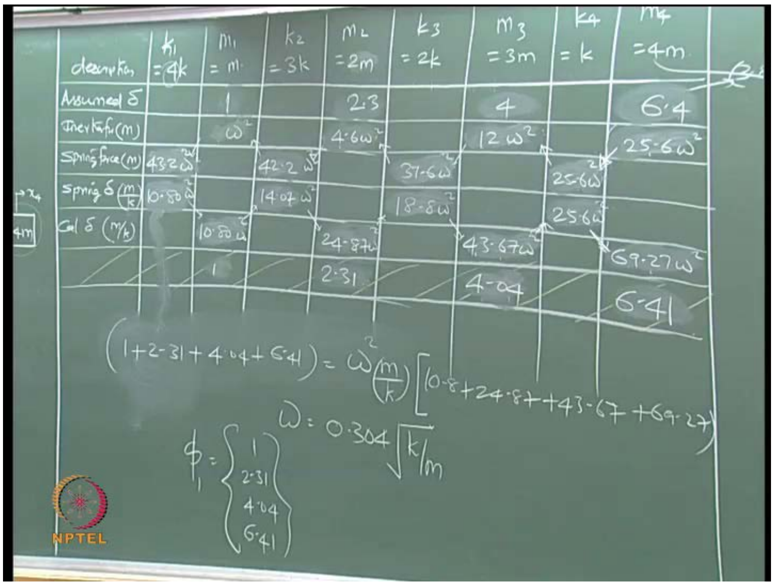

I will do that again. So, 25.6… 24.87… So, I take a proportional value now. So, it is almost converging. So, I can go for further steps. But, we will take these values as converged, because it is 2.31 and 2.3, 4.04 and 4; 6.4 and 6.41. So, I say it is converged. So, I must now say 1 plus 2.31 plus 4.04 plus 6.41 should be equal to… m by k multiplier is here; omega square is this side. So, omega square m by k of 10.8 plus 24.87 plus 43.67 plus 69.27; that will give me omega as… This is 6 7. Is it 0.304? Get this as 0.304 root k by m. And the first mode shape – 1, 2.31, 4.04, 6.1.

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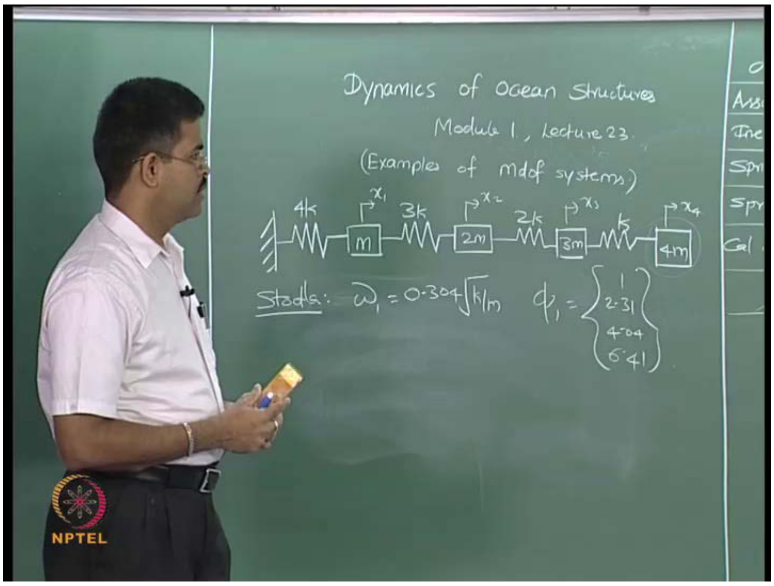

I am writing that value here. So, Stodla – omega 1 is 0.304 root k by m; and phi 1 is 1, 2.31, 4.04, 6.41. That is the first method we have. Now, let us do the influence coefficient matrix. I will draw this figure here – there in the right-hand side and try to draw a free body force diagram for this and try to find the influence coefficients for this.

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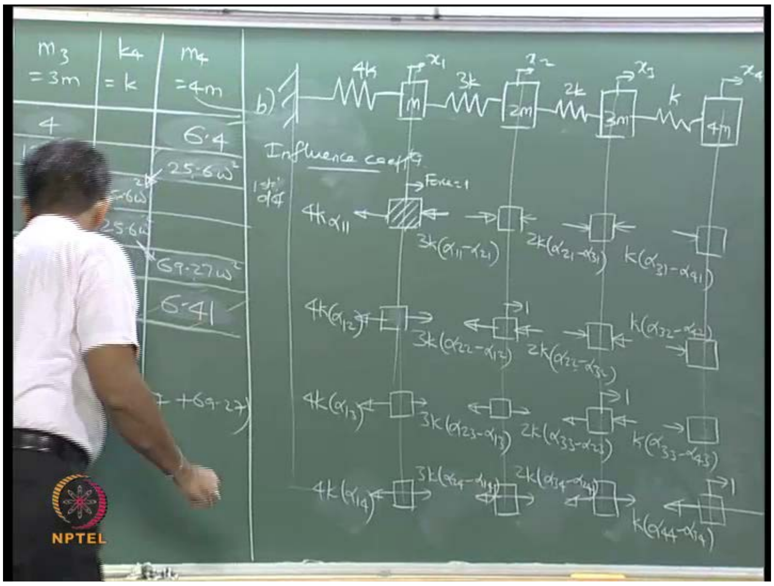

So, this is m, 2m, 3m and 4m. This is k, 2k, 3k and 4k. This is x 1, x 2, x 3 and x 4. I am trying to derive the influence coefficients; which are nothing but the flexibility coefficients. They are called influence coefficients, because they influence inertia force on the mass points. So, let me first… I want you to pay attention here; do not keep on writing; just see how we are doing it.

So, I want to first draw the mass position, which is m here; I give unit force here. That is how I derive the influence coefficient. So, when I try to give a unit force, push the mass towards right; this spring will oppose. So, you write this value as 3k of alpha 1 1 and 2 1. And the same will go here. Similarly, as this mass moves towards right, this will try to pull it back. So, that value is going to be 4k of alpha 1 1. Similarly, we will continue for all. This will move. So, this stiffness will try to push the mass towards this side and opposite. And that value is going to be 2k of alpha 2 1, 3 1. Similarly, this spring will try to push it opposite. So, I am looking here. So, k of alpha 3 1 minus 4 1. This is the first degree.

So, let us do it for the second degree. I want to give unit force in the second place here – unit force. All these are force values, because I am multiplying stiffness by displacement. These are all force values. So, I am actually drawing a free body force diagram. So, I am giving unit force. In the second degree, the mass will try to move towards right. This spring will try to oppose it. So, let me put the direction here. So, this is going to be 2k of alpha 2 2 minus 3 2. The second subscript will always stand for where are we applying the unit force and so on. So, you complete the force diagram; I will draw it. So, this mass moves towards right. So, this spring – 3k will try to bring it back. So, the arrow direction will be this way; which is going to be 3k of alpha 2 2 minus 1 2. The mass moves towards right. The spring will bring it back, so 4k of alpha 1 2. Similarly, this mass moves towards right. This will oppose it, so k of alpha 3 2 minus 4 2.

Let us do it for the third one. I am giving unit force in the third degree. The mass when move towards right, the mass 3m will move towards right. So, this spring will try to push it. So, that is going to be the stiffness of the spring. This is nothing but k of alpha 3 3 minus 4 3. And this spring will try to bring it back. So, that is going to be 2 k of alpha 3 3 minus 2 3. This spring will again try to bring it back. So, this is 3k of alpha 2 3 minus 1 3. This spring will again try to bring it back – 4k of alpha 1 3. So, do it for the fourth. Are you able to see from there if I write it here? So, I apply unit force here. So, this spring will try to bring it back; which is going to be k of alpha 4 4 minus 3 4 and so on. So, this is going to be 2k of 3 4 minus 2 4. This is going to be 3 k of 2 4 minus 1 4. This is going to be 4k of alpha 1 4. I want you to pay attention to this once again closely and try to understand how we have marked the arrow directions and how we have marked the values.

Afterwards, it is simply force balance equation, which is very easy to solve. Is there any doubt for anybody how we are writing this free body diagram for the forces? I am applying unit force and different displacements – 1, 2, 3 and 4. I am trying to multiply the equal length stiffness of the segment with the deflection. So, obviously, if we have two springs here; if we have one more spring here; find the equivalence stiffness of that and use it here as k equivalent and so on.

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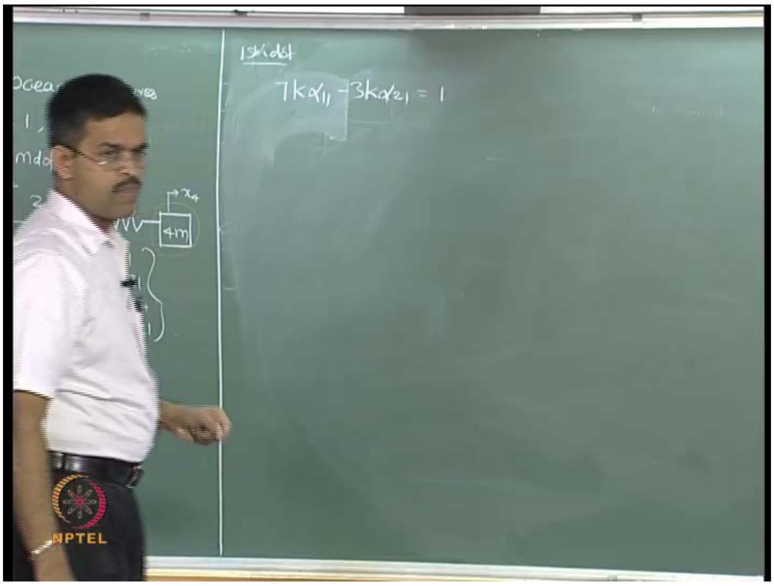

So, let us say I do for the first degree. So, look at this equation here. I am just writing simplified form. Look at this. So, 4k alpha 1 1 and 3 k of this is acting towards left; and one is acting towards right. So, I should say 4 plus 3 – 7k alpha 1 1 minus of 3k alpha 2 1 should be equal to 1.

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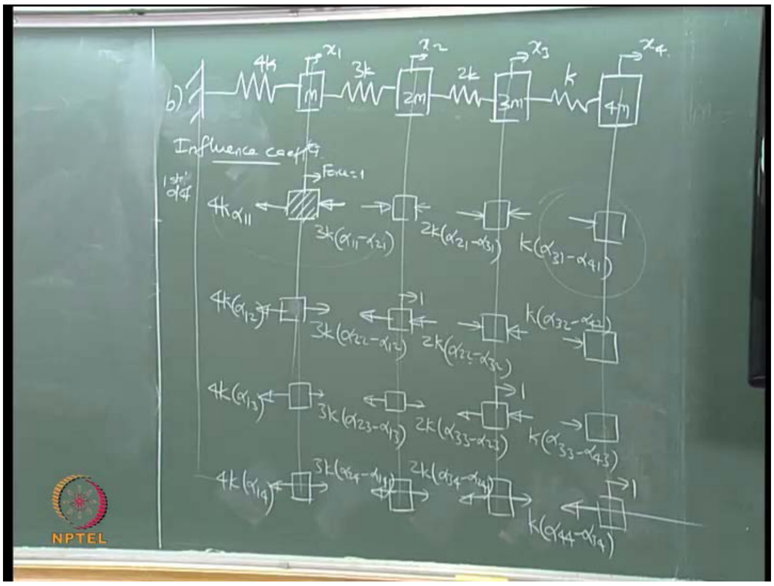

I come back to this equation.

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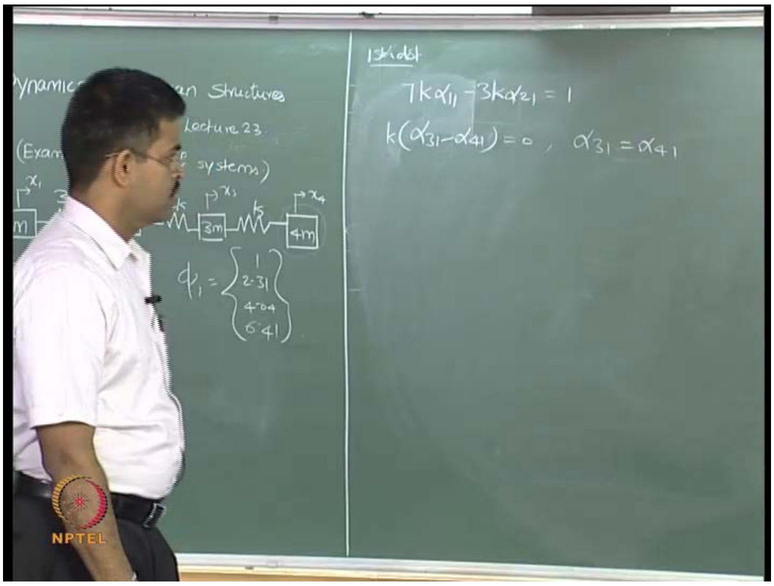

I say alpha 3 1 minus alpha 4 1 of k is 0. So, k cannot be 0. So, it implies that alpha 3 1 is equal to alpha 4 1.

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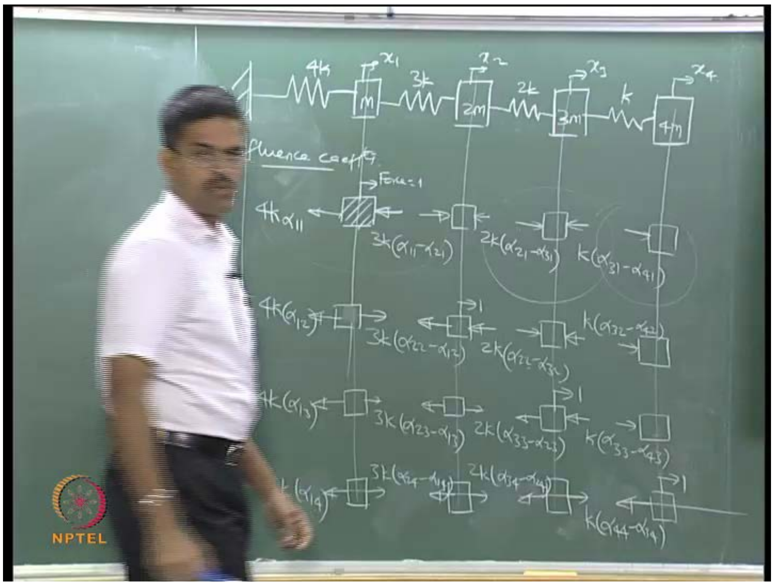

I come back to this level. These two are opposite; 2k of alpha 2 1 minus 3 1 should be equal to k of alpha 3 1 minus 4 1. We already know alpha 3 1 minus 4 1 is 0, because they are equal.

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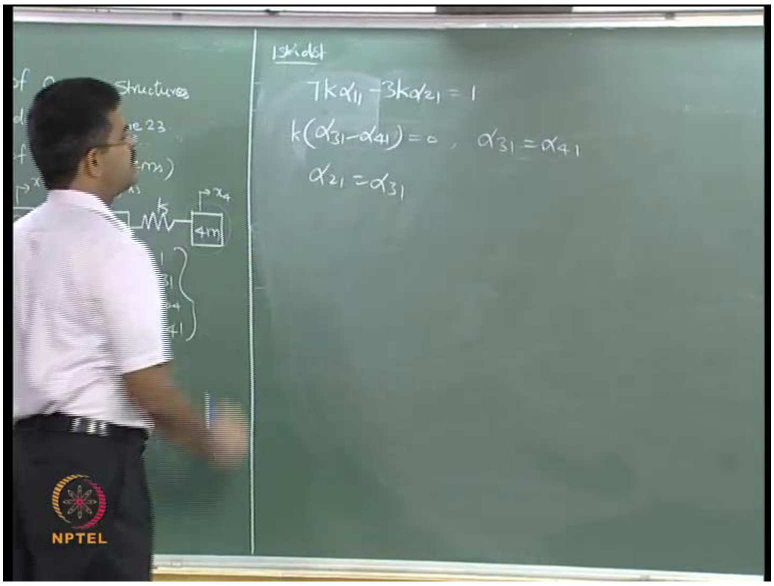

So, I can straight away say alpha 2 1 will be equal to alpha 3 1. Is that all right? I am writing the equation here at this level. I am saying 2k alpha 2 1 minus alpha 3 1 is 0, because this value is actually 0. So, k cannot be 0. Therefore, alpha 2 1 is equal to alpha 3 1.

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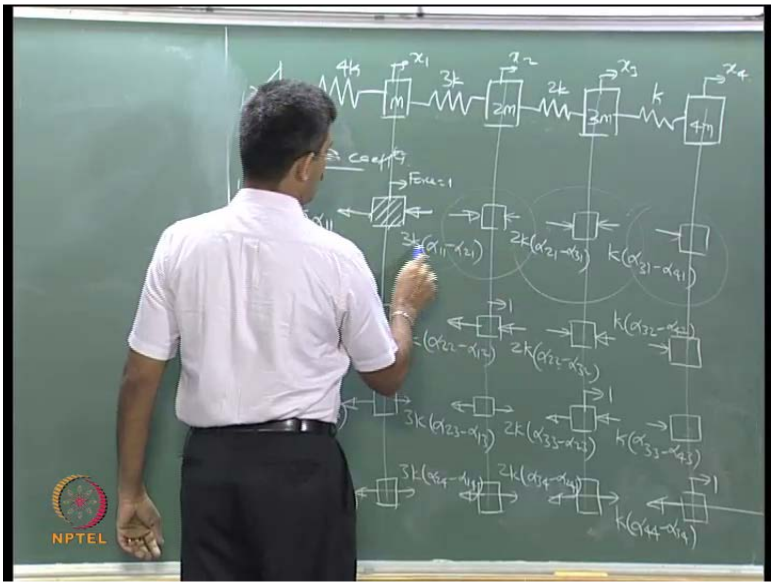

Apply back again in this level. These two are equal. So, 3k alpha 1 1 minus 2 1 is equal to 2 k alpha 2 1 minus 3 1.

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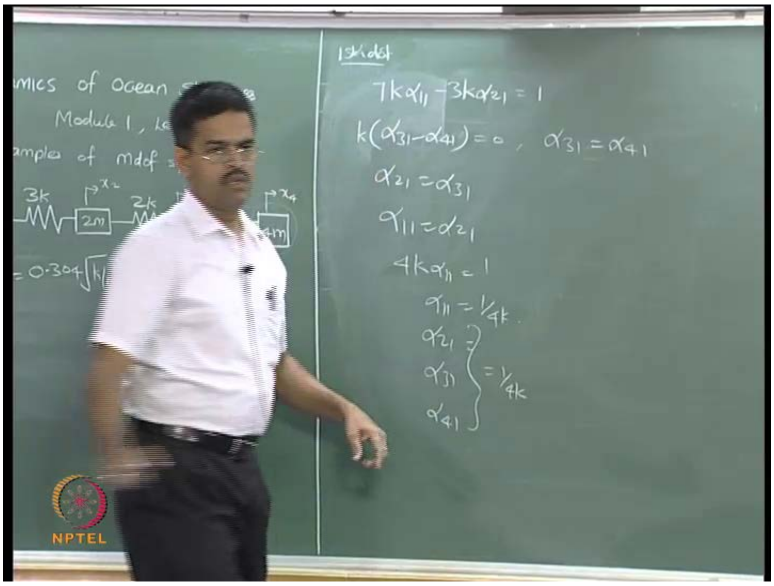

Already we know 2 1 and 3 1 are equal. Right-hand side becomes 0. Therefore, alpha 1 1 will become 2 1. Is that agreed? So, I put back here. I get 4k alpha 1 1 is 1. So, alpha 1 1 is 1 by 4 k. And alpha 2 1, alpha 3 1, alpha 4 1 – all are 1 by 4k. Are you following or not following? Please raise your hands who are not following. Please raise your hands who are following. Again there are people who are neither raising hands nor they are following; I do not know what happened. I am simply solving the simultaneous equation in a shortcut; that is all.

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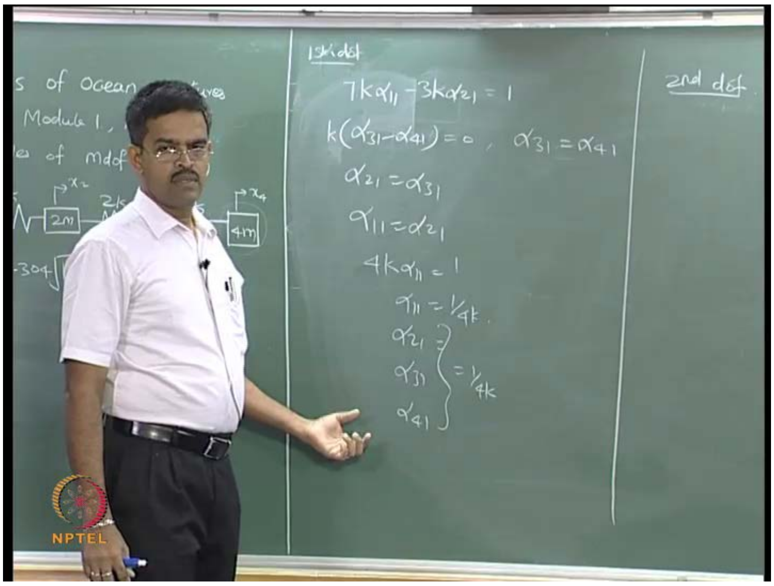

Let me do it for the second degree. Is there anybody still sincerely has not followed this? Please raise your hand. You have to follow it here; do not carry it back to your house and other things and say that you will check up later. You must follow it at this instantaneous moment. You must have followed this free bodied force diagram and you must have followed this. Is there anybody who is not following? I can again expand. There is another three more opportunities for you to learn here. There are three more equations I am going to write. Is there anybody who has not followed this – how did I get this?Nobody?So, we will then use the same shortcut back again here.

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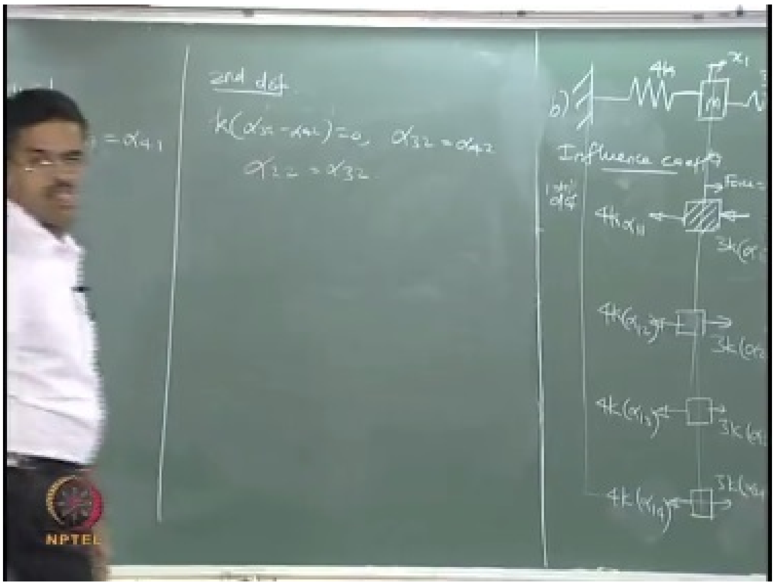

I will get back here. So, 3 2 minus 4 1 is 0. So, 3 2 is equal to 4 2. I will get back here. These two forces are opposite. So, 2k of 2 2 minus 3 2 should be equal to k of 3 2 minus 4 2; 3 2 and 4 2 are equal; right-hand side becomes 0. That implies alpha 2 2 is alpha 3 2.

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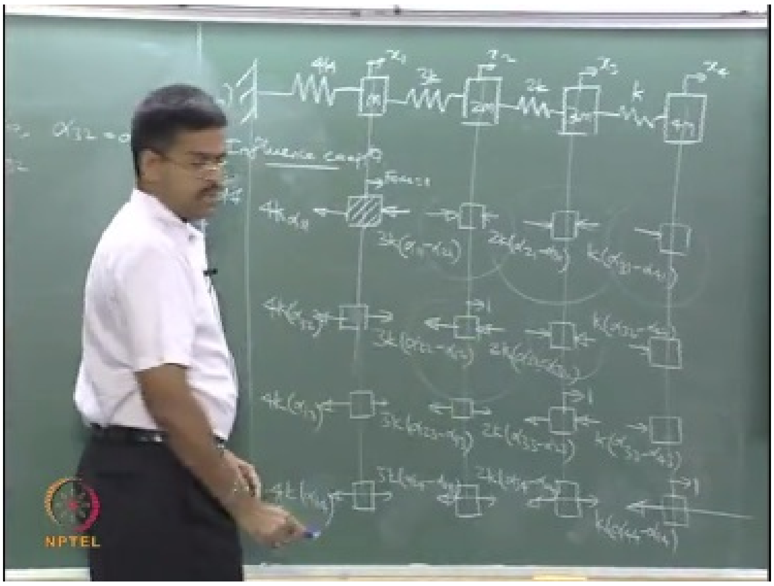

So, I put it here 3k alpha 2 2 minus 1 2 plus 2 k of this should be equal to 1. It is already 0, because 2 2 and 3 2 are equal.

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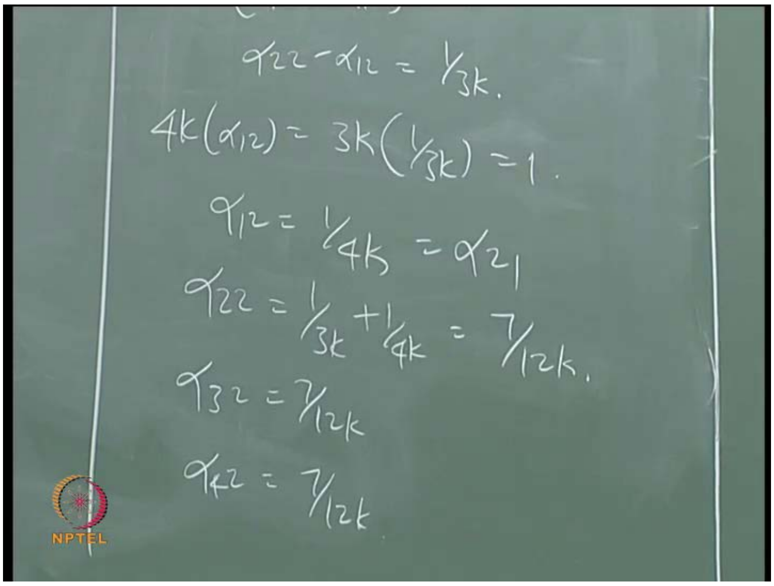

Therefore, I can write this equation as 3k of alpha 2 2 minus 1 2 is equal to 1. So, alpha 2 2 minus 1 2 is 1 by 3k. I will get back to this equation. 4k alpha 1 2 is equal to 3 k of 2 2 minus 1 2; which is 1 by 3k. This is again 1. So, alpha 1 2 becomes 1 by 4k; which is as same as alpha 2 1. So, this verifies reciprocal theorem automatically. So, once you know all the values, keep on substituting and find out. So, alpha 1 2 is 1 by 4k. Get back that 2 2. So, alpha 2 2 will be 1 by 3k plus 1 by 4k, which is 7 by 12k. And alpha 3 2 is same as 7 by 12k; and alpha 4 2 is same as 7 by 12k. I have got the second column of (( )) influence matrix now.

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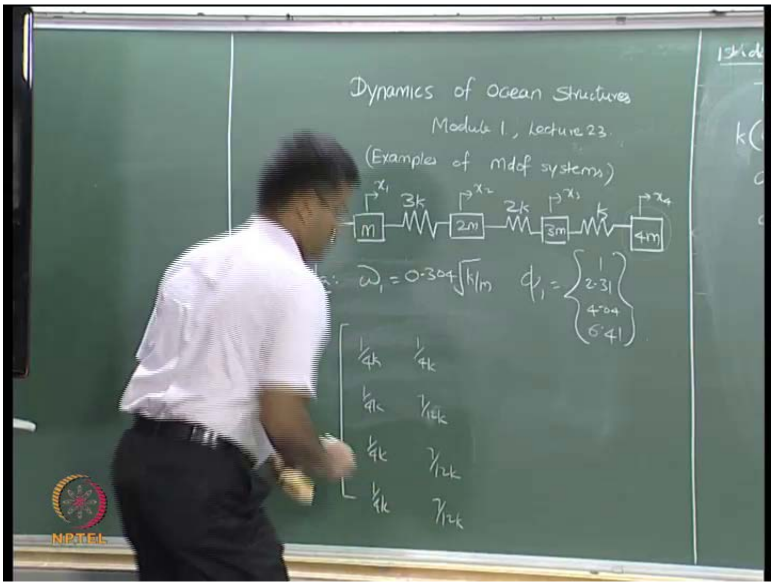

Let me write down the matrix here, because I will be rubbing this. 1 by 4k, 1 by 4k, 1 by 4k, 1 by 4k; 1 by 4k, 7 by 12k, 7 by 12k, 7 by 12k. Can I rub this?

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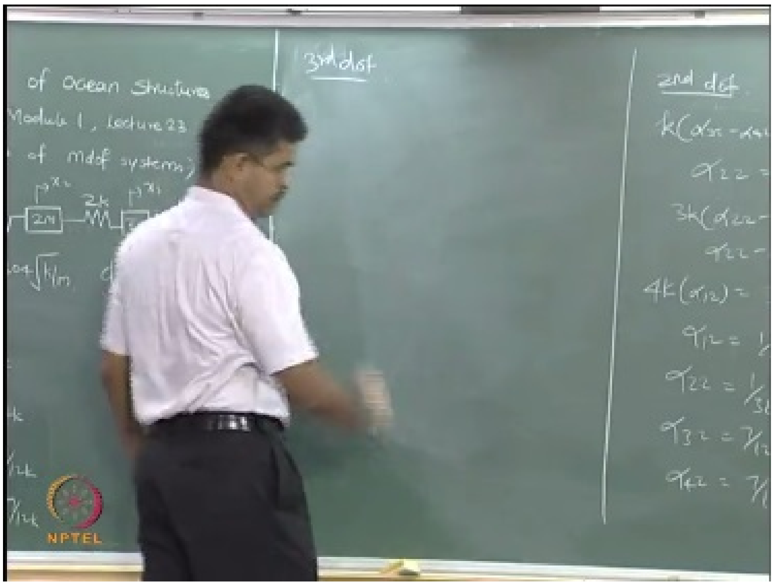

Let me do it for the third degree.

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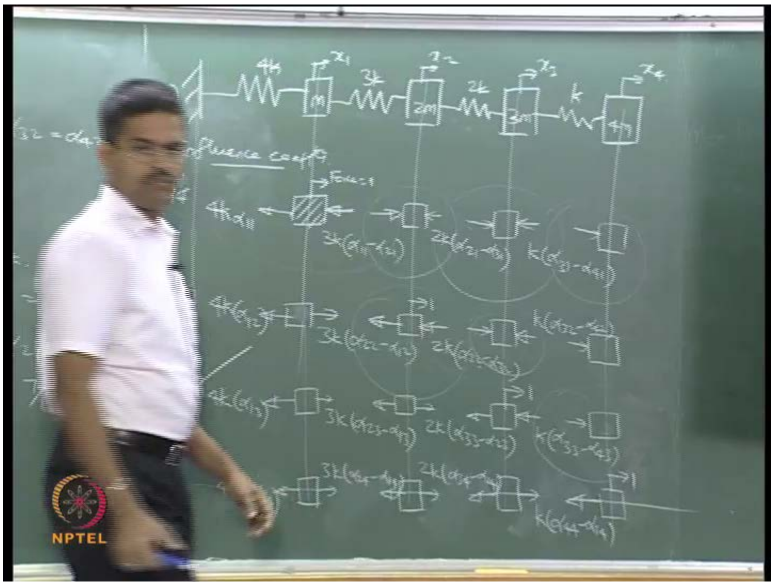

I am getting back here; starting from here, which implies alpha 3 3 and 4 3 are equal.

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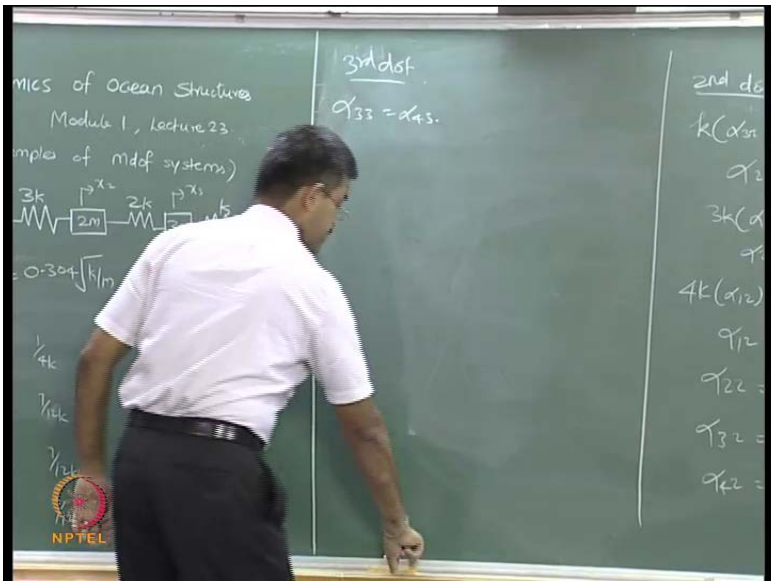

I think all of you agree that why I am writing this. Essentially, the equation is k times of alpha 3 3 minus alpha 4 3 should be 0; which implies alpha 3 3 is 0, because k cannot be 0. If k is equal to 0, then the problem does not exist. So, let me remove this statement, is not required here. It is understood. I believe you are understanding it.

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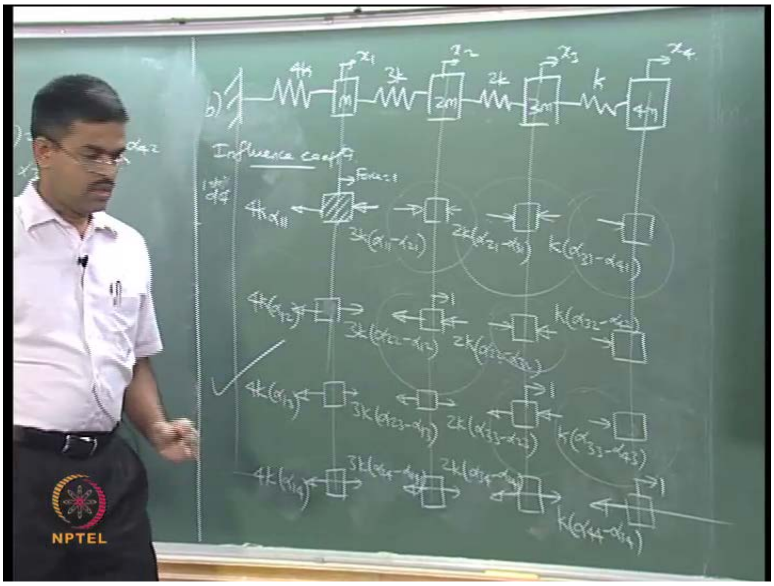

So, let me get back here. I should say 2k alpha 3 3 minus 2 3 plus k of alpha 3 3 minus 4 3 should be equal to 1. And this is already 0.

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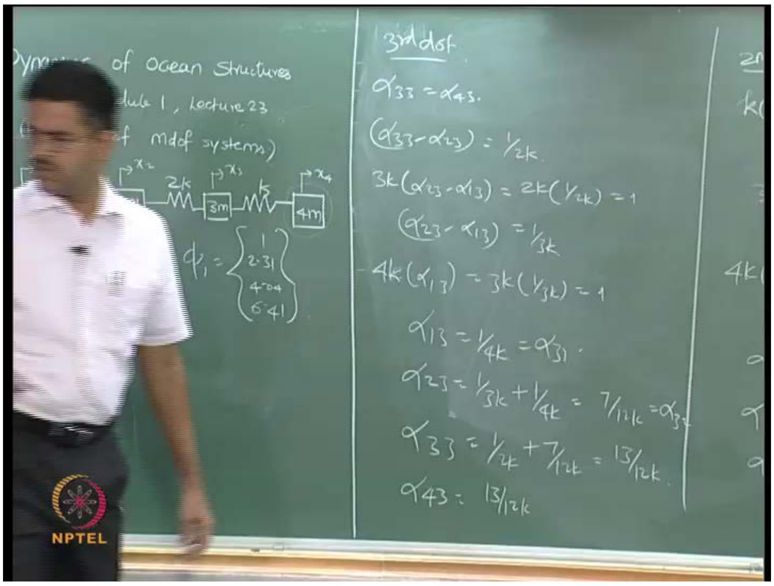

So, I can straight away say – alpha 3 3 minus alpha 2 3 is 1 by 2k. Coming back here; I can say 3k of alpha 2 3 minus 1 3 should be equal to 2k of 1 by 2k, which is 1; which implies 2 3 minus 1 3 is 1 by 3k. Coming back here; 4k of alpha 1 3 should be equal to 3k of 1 by 3k, which is 1. So, alpha 1 3 is 1 by 4k is as same as alpha 3 1; verifies the reciprocal theorem. And alpha 2 3 is 1 by 3k plus 1 by 4k, which is 7 by 12k; which is as same as 3 2; which again verifies the reciprocal theorem. 3 2 is here. And alpha 3 3 is equal to 1 by 2k plus 7 by 12k – 13 by 12 k. And alpha 4 3 is as same as this value.

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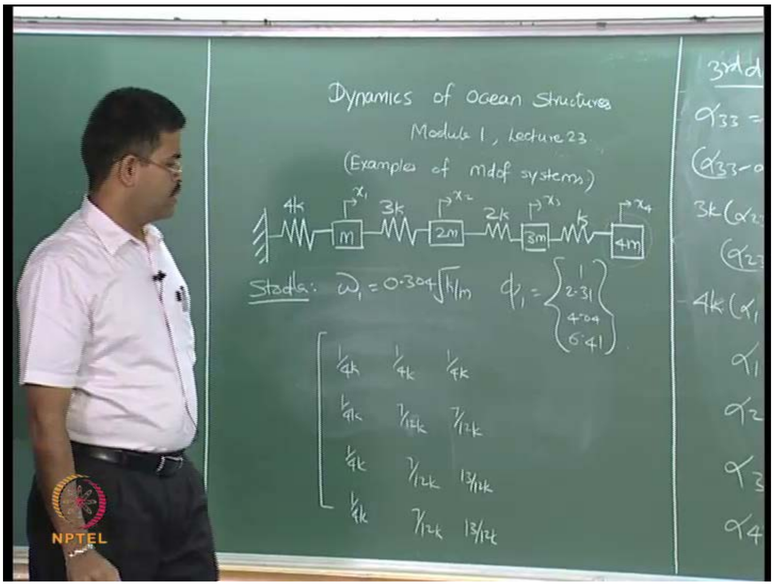

So, I write these values – 1 by 4k, 7 by 12k, 13 by 12k and 13 by 12k.

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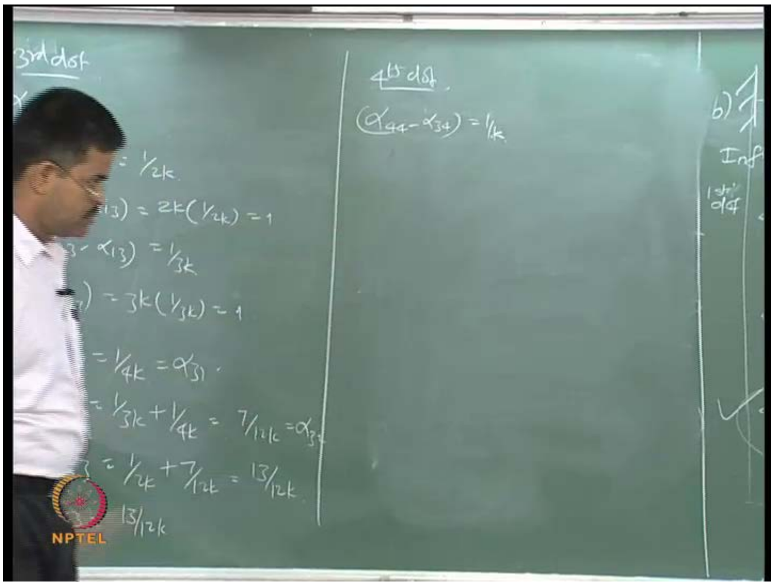

Let me write the fourth degree of freedom. Let me pick it up here – the last one; it says alpha 4 4 minus 3 4 is 1 by k; the last one. Please do not copy; try to understand how I am doing it; otherwise, it will be a trouble. I am picking up from here. I am picking up from here; I am writing this statement. I may also be wrong. If I am wrong; please correct me if I am making mistakes.

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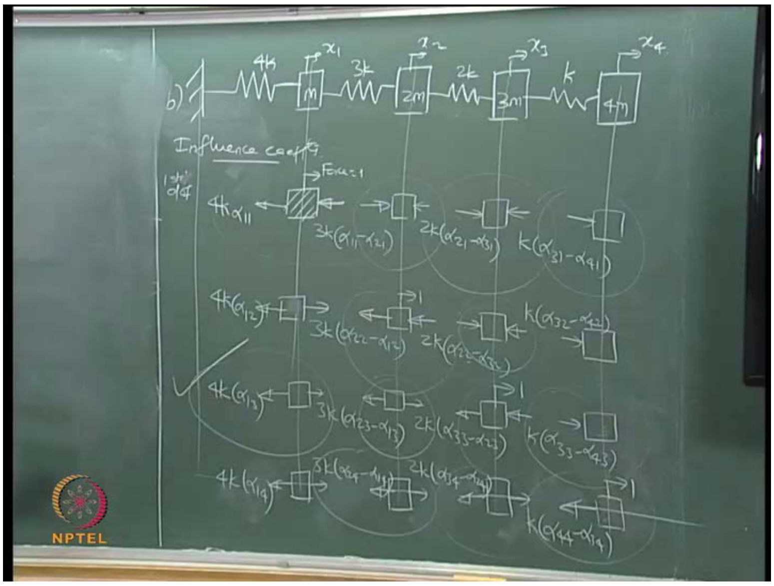

Get back here.

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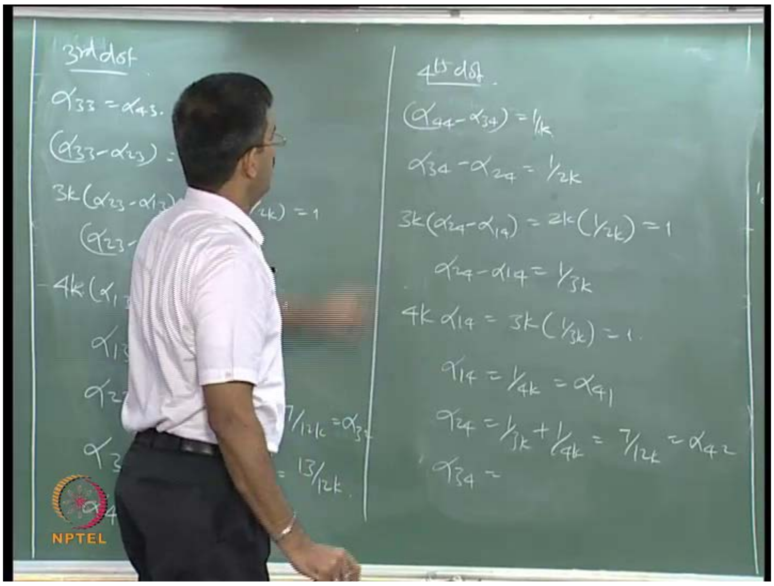

So, look at this equation. I can straight away say alpha 3 4 minus alpha 2 4 is 1 by 2k. Can I write? This is alpha 2 4; I think you are able to read it. This is alpha 2 4; this is alpha 3 4. So, alpha 3 4 minus alpha 2 4… because alpha 4 4 minus 3 4 is 1 by k. I multiply this with this; I get 1. 1 by 2k will give me this value. Come back to the second here. So, I should say 3k of alpha 2 4 minus alpha 1 4 should be equal to 2k of 1 by 2k; which gives me 1, so alpha 2 4 minus alpha 1 4; again 1 by 3k. Come back to the last; 4k of alpha 1 4 is equal to 3k of 1 by 3k, which is 1. So, alpha 1 4 is 1 by 4k; which is as same as alpha 4 1. So, it verifies the reciprocal theorem automatically. So, alpha 2 4 will be 1 by 3k plus 1 by 4k, which is 7 by 12k; which is same as alpha 4 2, because alpha 4 2 is 7 by 12k.

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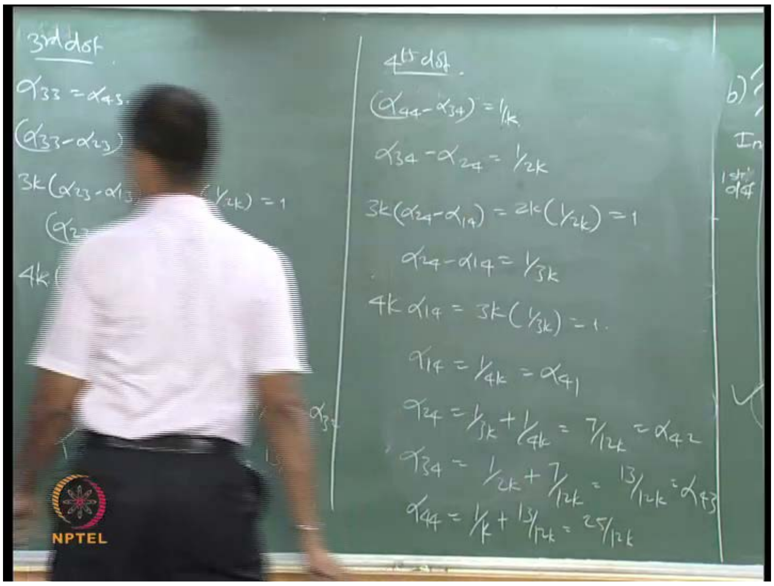

Alpha 3 4 from the equation here will be equal to 1 by 2k plus 7 by 12k, which is 13 by 12k; which is as same as alpha 4 3 from here – fourth row, third column. And alpha 4 4 is from here, which is 1 by k plus 13 by 12k; which is 25 by 12k.

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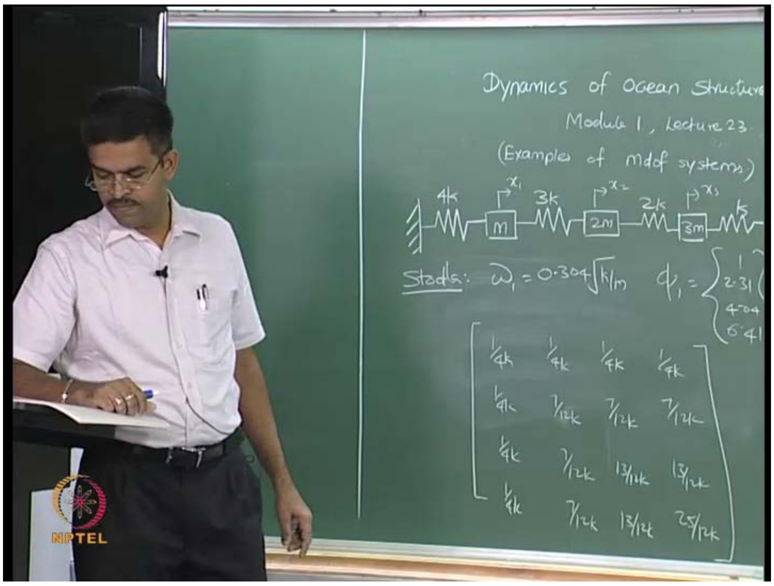

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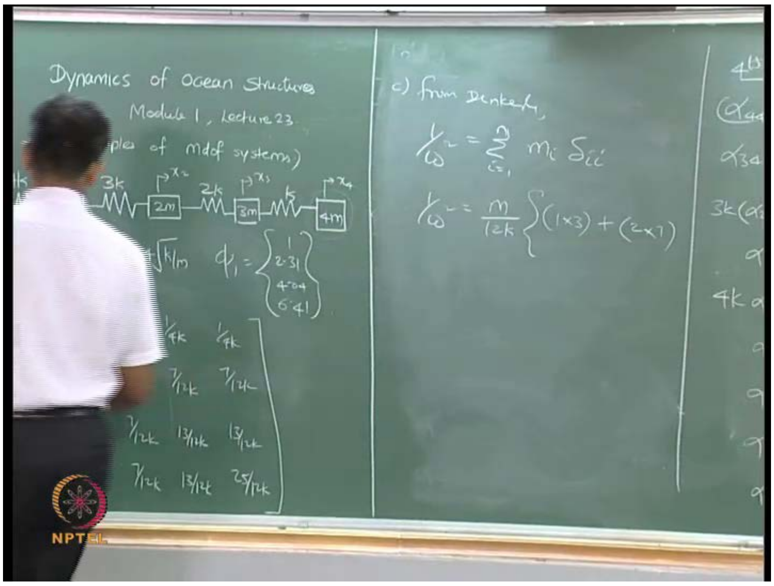

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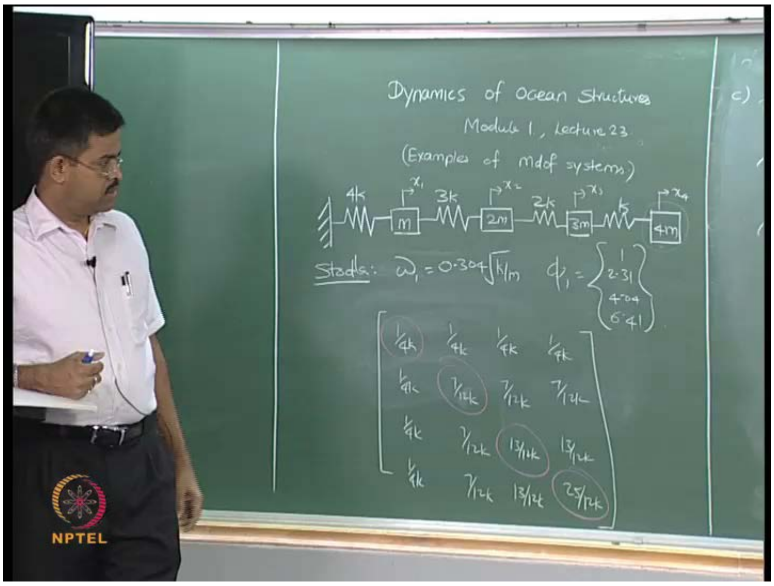

So, 1 by 4k, 7 by 12k, 13 by 12k, 25 by 12k. That is the influence coefficient matrix. Is it all right? I will remove this. Let me quickly verify the Dunkerley and see what happens. So, c – from Dunkerley, we already know 1 by omega square is sum of i to n m i delta i i – only the diagonal values of alpha matrix. So, 1 by omega square is equal to m by let us say 12k. The first m is corresponding to 1. So, I should say 1 into – I have taken 12 here; so it should be 3 – plus the second mass is 2 and the second value is 7. I am picking up only the diagonal values – plus the third mass is 3 and this is 13 plus fourth mass is 4 and this is 25; which gives me omega as… Quick. How much? It is 0.278 root k by m.

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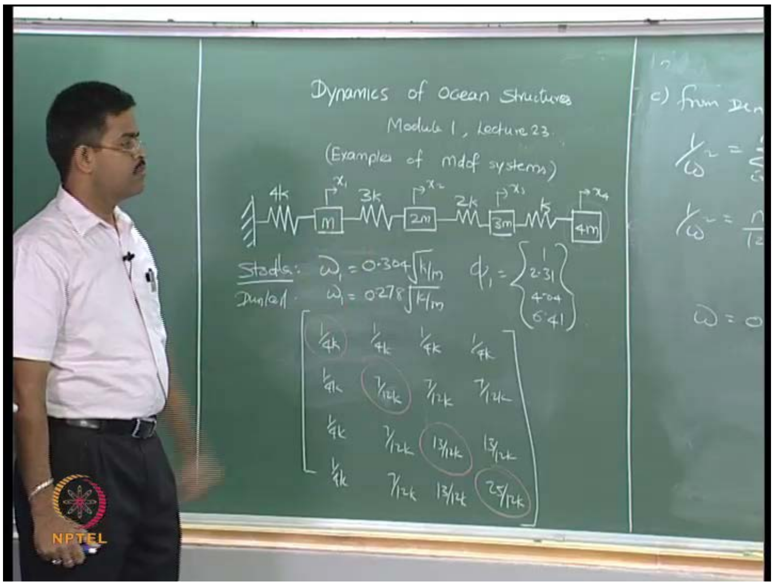

So, I should say Dunkerley gives me omega 1 as 0.278 root of k by m. Dunkerley does not give me the mode shape. So, it is closer. If you have another 10 minutes, I can solve this by influencing coefficient technique also to find out at least the first matrix; but we have only 3 minutes time. I think we will carry it over to the next class; there is no way. Anyway I will do it in the next class. I do not want to hurry up; I will do it in the next class. Maybe another five minutes.

So, what I want to show you in this example is that, a problem of four degree in order can be solved by hand by at least three methods and compare fundamental frequency mode shape within one hour. That is the speed of these methods. So, if I give you a problem for examination, which is going to be two degree or three degree; if I ask you to do three problems in one hour, do not get surprised. I am doing three methods in one hour. That is a four degree problem. So, you have got to speed up the method and try to understand. And each method should not take by hand. If it is 3 degree, maximum 10 minutes for finding out the solution. So, in 60 minutes, I must solve four problems by all the three methods. Dunkerley will take only 1 minute for me. So, be prepared for this speed along with the derivation for the theory; otherwise, you will end up in answering only one question, which will be having only just 5 marks or 3 marks.

And, I have already told you, you must write a program and demonstrate this for n degree of freedom and show me at an additional tutorial for this class. Do you have any questions here? Any doubt? So, I have demonstrated three methods. We will pick up another two more methods. Of course, we have demonstrated four methods; eigensolver is also one of the methods. So, four methods we have explained. We will give you another two more methods. There are six methods. We will also give you the Matlab coding for doing all these things at the end of the semester.

So, you write the coding first; let us see. So, you must be aware that, how to generate or how to find the fundamental dynamic characteristic of a multi-degree freedom system, that is, simply k, m and omega and phi and of course c if it is a critically damped system. Or, assume a percentage of zeta. You should be able to get all the matrices k, which I have derived in many classes earlier; m, which I have already shown when it will be diagonal, when it is going to be half diagonal terms, will be there. c of course is going to be a proportion of k and m; which we will discuss very briefly in the next class. Then of course, f of t is your job; you have got exterior force because of wave, wind, etcetera. We already discussed that. And you must solve this. Write the equations of motion; solve this for omega, phi as the first step. All these should happen in one hour of your research.

If you have the geometry ready; in another one hour of the geometry getting ready, all these answers and all the methods of six should be there within one hour; where people take years to generate this in the Ph.D. It is surprising; people do not know how to do it. And they will land up in using software. And I am here demonstrating by hand calculation, how I am comparing. I will show in the third method, the mode shape and frequency will exactly match at the second decimal here. I will show you here in the next class. The second decimal I will match and the fundamental frequency in mode shape in just 5 minutes.

So, one can do this. One has got to understand how this method has to be trained. There are many more methods available in dynamics. There is a popular course being taught in different universities. Dynamics for non-mathematicians – this is a non-mathematics course. I think any school child can follow this algorithm provided she knows how to operate the calculator. She may not understand what we mean by alpha 1 1, what we mean by deflection, stiffness, Dunkerley. She may not remember them, but she will be able to solve the problem faster than all of us. So, try to get hold of these methods and you must rehearsal yourself before you come for the test.

The test will be on this Friday at 3 O'clock. So, one of the class representative here should make arrangement to fix up any one of the classroom on Friday at 3 O'clock in the department. So, any one of you take the responsibility, fix up the class, keep it ready. 3 to 3:50 we will have the test. Whatever we will be discussing till Thursday's class will all appear in the exam; may be numerical, may be derivations, may be theory, may be objective, may be match the following, may be fill in the blanks, may be suggestions, may be inferences. Any kind of form can be there in a paper in 50 minutes. In fact, I will also open up the second module. I will also take up the derivation of k and m for TLP's. That may also appear in your paper. Whatever we discuss till Thursday, will appear on your exam on Friday. Friday there is a class in the morning also. So, one of you please take the responsibility to fix up the class on Friday 3 O'clock. So, tomorrow we will meet again; we will complete this problem.