# Dynamic of Ocean Structures <br> Prof. Dr. Srinivasan Chandrasekaran <br> Department of Ocean Engineering Indian Institute of Technology, Madras 

## Lecture - 8 <br> Iterative Frequency Domain II

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So, we will talk about the continuationof the last problem, which we discussed in the last lecture. So, in the last lecture we get an iteration scheme, which is evaluation of Jacobian at rth iteration which is given by the Jacobian at $r$ and the displacement vector at mth harmonic at r minus 1 minus the Fourier components of the loading at rth, which was listed as equation number 22. So, why we are defending or why we are using Jacobian for a Newton Raphson classical theory is that, the conversion becomes competitionally expensive, therefore I am evaluating Jacobian for a specific.

So, if you look at the classical Jacobian evaluation, the Jacobian evaluation for a mth harmonic looks simply as the partial derivatives of this components with respect to the displacement vector, because that is what I am trying to find out from this theory which is summarized as $\mathrm{K} m$ bar minus F m bar which read as equation 23 in your derivation where we also said F m bar can be expressed in a matrix form in a simple term which is dou F m c.

So, this vector is derivative of partial derivative of displacement with a sign component and this column is a derivative of partial derivative for 2 displacement of the cosine comp of both of course the harmonics of cosine and sine components of the force vector; that is how it is been derived. This reads as equation number 25 in your derivation. Is that ok?

So, we picked up one specific case and elaborated this how we can use the hydromantic loading chosen from the Morison theory or Morrison equation areas theory and we expanded it for non-linear drag terms.
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We explained this yesterday, this equation reads as duo Fem c by duo theta by m c was given as 2 by record length of integral of 0th record length $m$ sigma summation the drag component which is k j of course, h i v I, I multiply k j because I am taking the movements of this forces about the point at the base.

At 2 nodes i and i plus 1 minus there are 3 term percent in this as we disused in the last lecture writing term by term. So, this becomes 2 by $R L$ of 0 to $R L$, $k$ nu $t$ cos square $m$ sigma $t d t$ and then, 2 by $R L$ of 0 to $R L$ of I naught beta $t m$ sigma square cos square of the same term because there is a theta double dot here and they are connected by a negative sign of the same term we had it here. This reads as equation number 29 in your derivative.


Let us slightly rearrange this and see that how I will set my iterative scheme very comfortably for a specific term. So, I am rearranging equation 29. So, there are sin cosine components here, cos square component I have to integrate it, I have to bring it in a normal form. That is what I am trying to do here. So, it is nothing but, a simple trigonometric simplification, sin theta cos theta sin 2 theta $\cos 2$ theta 1 minus cos square theta $2 \cos$ square theta minus 1 and so on. I will simplify it bringing to the multiple values of thetas inside this integral. Ok. That is what I am trying to do here.

So, 2 by R L so, the first term will be let us say m sigma by R L, this 2 will go away because sin theta cos theta can be sin 2 theta by 2 the 2 gets canceled here.

So, m sigma by R L integral of course, kj so, I can say $\sin 2 \mathrm{~m}$ sigma that is a first term, minus. So, when a d compose this so, we know that $\cos 2$ theta to cos square theta minus 1 there will be 2 terms here. That is nothing but, 1 plus $\cos 2$ theta $2 \cos$ square theta there will be 2 terms here, there is a minus sign here so, both of them will remain negative I am writing them here. So, one term here, which is m sigma by R L so, I differentiate it $R L, 0$ to $R L, k$ nu of $t$. I will take the first term, $\cos 2 m$ sigma $t d t$ that is a first term.

Let me write the second term also, minus m sigma by $\mathrm{R} \mathrm{L}, 0$ to $\mathrm{R} \mathrm{L}, \mathrm{k}$ nu tdt that is, this term. Similarly, I do decomposition for this, both the terms will remain positive write it here plus. So, $m$ sigma square here so, $m$ sigma square by $R$ L because in both the cases
this will get divided by this they will go away, $m$ square by R L integral 0 to R L I naught beta $t$ which is cos twice $m$ sigma $t d t$ plus $m$ sigma square by $R L 0$ to $R L$ I naught $b t d$ t. Ok. I call this is equation number 30 . I just rearrange this term and got it here.

There are 3 terms here, I decompose it, I get 5 terms here. Now, equation 30.(()).Talking about this.[FL] This is partial derivative of the variable with respect to theta so, I will get m sigma here.
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Now, look at equation 30 and see what are the different condition I must assumed. So, equation 30, now can be solved, this is 1 example this is only duo F mc by duo theta m c only 1 value. This can be separately picked up. And if you look at the Fourier comp look at the Fourier comp of the loading vector which is k j .

So, I must look at the Fourier components, the drag comp of hiviplus hiplus 1 viplus 1 at the mth harmonic because all these values are working at mth harmonic. Pick up the Fourier components of that because I have to worry Fourier I am talking about the cosine components mth harmonic. So, picked up that components substitute here and evaluate these components of Jacobian. And already we know that $k$ nu of $t$, beta of $t$ are all theta dependent they are all theta dependent or I should say displacement dependent. So, look at the mth harmonic of that and partially differentiate this and get the first value of your Jacobian of the force vector.

Obviously, here you must know the value of theta at an i-th iteration whereas, you will know theta only at i minus 1 iteration. So, use that value a iteration and substitute and get and keep on iterating it until, the new value assumed by you, the old value assumed by you, the new value what to get the conversions.

So, the conversions set is rho mean square value of 2 successive iterations, do not look at the absolute, look at the root mean square value and compare it with tolerance. So, you set the tolerance limit may be 10 power minus 5 , 10 power minus 3 whatever the tolerance limit you want, set the tolerance limit check for the conversion of the RMS value and keep on iterating it until it is reached.

So, to start with, take theta as 0 . What do you mean by theta as 0 ? The platform or the tower is in un-displaced position that is, what theta 0 means. So, what will you get the outcome of this will be theta, theta dot and theta double dot. Is it not? because the equation of motion is written in the displacement vector like this.
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Now this is been apply and examine for this problem.


So, the data which has been done is for on A T whose, height of the deck is about 400 meters and operating at water depth of 350 meters. The deck wide is about 25000 kilo Newton to the structural weight is about 200 kilo Newton per meter.

The effective diameters are taken as the 34 meter for the central tower and for the buoyancy tank, it is 15 meters and for the height exposure in the wind drag is 25 meters. Structural damping in this example is considered as 3 percent which is amounting to 0.173 radiances per second.

So, the dimension buoyancy chamber and the effective diameters are given of course, the reference reads from the Jain's paper in 1989 where, the data has been borrowed.


So, that analysis has been imposed on this particular tower and the results are the following. So, there is figure which shows the power spectral density function of the heel angle theta which is plotted for different frequency band varying from 0 to 0.12 .

So, the figure shows 2 variations: 1 with the pressure drag and 1 without the pressure drag whereas the velocity is taken as 30 meter per second wind. The respond spectra what you see here for a heel angle is characteristic by single peak, there is single peak, there is only a single peak here there is only 1 peak. It means, his predominantly operating or showing the response only a specific band of frequency varying from 0.05 to 0.07 only in this band it is getting excited. So, the peak is occurring at the specific frequency where, the energy of the wind velocity was maximum. So, it is having a near resonance response.

What you see from this figure is that, there is no significant difference between whether you consider the pressure drag or not it means, the nonlinearity involve in the system is not significantly effecting the response.
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What it is mean is, the system is too flexible though there nonlinearities in different parameters of $k$, nu $t$, beta $t$ and $f$ of $t$ is not influencing the response ultimately because the system is too flexible it means, the net response which is between the velocity exerted by the wind or weather with respect to the velocity for the structures exciting by the force, the net value is very less. This very flexible it because of the reason the variation is not much, whether you consider the non-linear pressure drag or not.

So, if you looked again at the effect of the tower in term of the displaced position. Because just now we saw theta, theta dot and theta double dot are actually showing he base angel inclination at any instance of time.

So, if you include the instantaneous position effect and do not look at the instantaneous position effect, look at only there un-displaced position effect and solve the problem the variation are shown like this.
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Again, the expectation is seen only in the narrow band over let us say 0.045 the 0.075 wherein, the max velocity of the wind energy is being concentrated and the variation is not, the reasoning is gain same the non-linear effect are not significant because the system is too flexible.
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So therefore, the effect of nonlinearities does not influence the respond in this specific example. So, spectral analysis can be preferred for this case. So, non-linear
hydrodynamic forces preferred to be linearized otherwise, you cannot handle this problem.

So, you have to linearize. We already discussed how to linearize or drag term by given different approximation. So, system are shown 3 nonlinearities, already we have seen them. 1 is velocity sure term and wind drag force effect of large displacement because of the movement of the tower with also impose nonlinearity and of course, non-linear drag term available in the F of t .

So, it has been in this study that the nonlinearities do not influence the response for a fundamental reason that the structure remains too flexible. So, if you got the tower or any structural system which is excited under wind and waves the system remains too flexible, the nonlinearities accounted in the analysis which is correct form of looking analysis may not significantly differs from an un-displaced position of the analysis that is very important. In this example you have a time that that is what has been observed by the research as seen in the literature.

So, we have picked up an example of a single tower, excited to wave and wind action, divided tower into different parts, evaluated the mass stiffness and damping matrices for this, we justify why we are going for a spectral analysis.

We picked up the nonlinearities identified them, define the terms then, instead of going for a Newton Raphson technique which is generally applied for any non-linear system solutions.

We went for a Jacobian method of Newton Raphson evaluation scheme, we divided the matrices into 4 comp of 1-1-1, 2-2-1 and 2-2, picked up 1 specific comp elaborated it and show how one can estimate the force vector looking at the nonlinearities of different system.

We solved it as it is, from the research papers, we understood that even though a upon for the displaced position of the tower, the ultimate response of the tower works only in the band where the excitation force is getting concentrated, in this case wind force because the reason is the tower is too flexible.

So, the differential response between the drag of wind or the drag of waves with respect to the velocity of the structure is very less. It may not be true with all kinds of structures in ocean but, still in this examples we have understood that yes it is possible.

Now, because of this reason it has been seen that the structures getting excited in a specific band to a very high value near resonance value. So, people have down the line went and saw can be controlled this response because it do not want this kind of larger displacement rotation should the base because this will cost lot of uncomfortability for people working on board.

So, people came out again with the geometric innovation saying that instead having a single tower, can I go for a multi lake towers. Ok.

That is what we are now looking at. Then how the research went down the line and dynamics, saying that in case of multi legged towers, what were the differences and advantages obtain by the geometric difference but, still can be attempt control the response using fundamental principle of dynamics, will come to the down the line in this lecture.
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So, let us quickly see what are multi legged articulated towers which is abbreviated as MLAT is multi legged articulated towers. It is nothing but, the extension of the concept of single tower to the development of a new type of platform where there are several
columns which are parallel to each other. Now, one may wonder that where these examples are applied, where do they exist. I already shown an example in the last lecture like this has been used as a single leg anchor mooring system which we said as SALM which showed photograph and also the conceptual model for single leg tower.

Now, in MLAT is the columns are connected to the tower to the deck and the foundation with universal joints. So that is the very interesting part here. What people did is, in the geometric form innovation they said let me put the universal joints not only at the bed or at the bottom but, also at the deck level.
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So, I have a tower now, which is comprising a deck which is again having the columns now, I am drawing 2 in elevation. So, I am introducing a new element here of course, this element already is present so, both are articulation nothing but, hinged joints. Now, one can wonder why this fundamental deviation from a single tower. The fundamental deviation was made from a single tower because the single tower demonstrated a rigid body motion.

On the other hand, when I have a wave action here, when the tower is affected by wave and there is no wind, the wave action makes the rotation and the hill that is transferred to the deck. When we have a wind action here, when there is a moderate wave, when there is high wind velocity happening here, the deck motion is transferred back to the heat. So, there has been rigid body connectivity between the tower and the deck. It means, for any
wind and wave action the tower is constantly disturbed. That is why the effect of nonlinearity presents in the system. Otherwise, has not become advantages in controlling or reducing the response.

Now, I would not actually separate the deck movement with that of the legged. So, we introduce a hinged here. The moment I say the hinged here, mechanically or engineering wise we know the movement at this point obviously become 0 .

So, this is now going to be acting as a simple structural hinged which is only transfer as specific mechanism from the tower to the deck or from the deck to the tower. Remember, the buoyancy chambers are located anyway to restore the tower back to normal sea and there is any horizontal action.

Is that clear?

There are other way is there they are not modified at all. Now but, advantage of this is one can look for larger deck size, one can look for deeper platforms, one can look for comfortable working environment on the deck under the wave action. So, because of the functional reasons the form definition has been modified.
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So, now they said let use have multi-legged towers, multi legged towers are actually legs parallel to each other and connected to the deck using a hinged joint.

## Multi-legged articulated towers

- Extension of the concept of single-leg articulated tower led to the development of a new type of platform with several columns that are parallel to each other
- Columns are connected to the deck and to the foundation through universal joints. These are called Multi-Legged Articulated Towers (MLAT).
- Use of universal joints ensure that the columns remain parallel to one another and the deck remains horizontal even for large rotation of columns at the base
- there is no rotation (yaw motion) about the vertical axis of the columns.

So, of course, use of universal joints enables the column to remain parallel to each other and the deck remains almost horizontal not exactly horizontal even for large rotation column at the base. This was not happening in the earlier case. We just now saw the rigid body motion is there between the tower and the deck. So, now this isolates the tower from the deck.

So, this is one mechanism being thought by our ancestors a control the response on the deck under later loads is one mechanism. Of Course, there is no rotation being seen about the vertical access because of the symmetry of the geometry if you have a symmetric geometric, there will be no rotation about the vertical access no yaw motion will be seen.

So, maintain symmetry in the geometry and isolates the deck from the tower by placing a universal joint so that you can improve the workability of the platform in terms of its let us say displacement characteristics.

Is that clear?

- MLAT have pay loads and deck areas comparable with other conventional production platforms in moderate water depths.
- As connection to the seabed is through the articulation, tower does not transfer bending moment to the base and thus making the foundation simpler.
- Natural frequency of the tower is designed to be well below the wave frequency resulting in lower dynamic amplification factor.

So, what are the advantages of this, very quickly we will see because we are not talking about the evolution of the platform in this course but, still one must understand the genesis for the modification in the design. One must know that ok.

So, MLAT have increased pay loads, people have seen the payloads coming on the MLAT from the deck can be slightly higher than that of a single leg ok. And of course, the deck area become comparably larger, compare to the single tower. So, people started using it now for drilling also. Initially, you thought it is where use only for anchoring.

Now, people have extended this concept for drilling also because they can go to moderate water depths.

So, that is one functional advantage we gain by making the geometric form more innovative and of course, as we all know when the tower is connected to the sea bed using articulation, there is no movements transfer happening.

The advantage the biggest advantage of this is the foundation design becomes simpler, why? Foundation will not be now subjected to movements excel forces and the save bearing capacity reaction from the sea bed.

So, foundation design becomes much simpler compare to that of the earlier case. Above all interestingly, the natural freq of the tower is designed well below the wave freq which result in a very low dynamic application factor. That is one of the design parameter.

The mass and the size and the dimension the tower have been selected in such a way that the natural freq of this tower is falling much below the wave action. So, the dynamic application becomes much lesser. So, these are advantages which have been arrived from the geometric innovation cost on a design of a MLAT, compare to single leg towers.
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- Articulated towers are compliant type offshore structures that are proved to be economically attractive in comparison to fixed type platforms
- Used as loading \& mooring terminals, temporary naval base etc (Fulmer Offshore Field, Sanata Barbara Channel)
- best suitable for permanently mooring tankers in deep waters
- commonly used as riser support systems (FMC Corporation, Chicago, IL; Drillmar Inc, Houston, TX)
- Articulated towers are also used as production platforms in water depth between $300-400 \mathrm{~m}$ (Butt et al., 1984)
- But showed higher response due to increased topside weights as drilling, production and accommodation platform
- high degree of safety is felt necessary by the researchers (Butt et al., 1984)

So, where are they have been used. I will quickly read them because this is available in the literature anyway. Now, instead of using it for anchoring, this multi-legged towers I have been used for loading and mooring terminals, for tempering naval base as you see in fulmar offshore field Sanata Barbara channels. Further, they can also be used to permanently move tanker in deep waters. They have been used widely as riser support system as you see from FMC corporation, Chicago and Drillmar incorporation Houston Texas. So, people have started using this kind of towers for varied application starting from drilling, temporary naval base etcetera.

So, what does it mean is that this geometric innovation or the deviation from the standard has improved the operable condition of the deck one. Two: it has indices deeper waters it can now, the platform can be moved to the moderate depths also instead of shallow depths. So, people have started using this at the depth of even 400 meters earlier it was only 125 meters.

So, more than double the depth, people have attempted this kind of towers can see from butt et al 1984 they have been references available where 80s they have been used at
greater water depths. But, there has been some problems when start using it at higher water depths. What are these, where we are at the addressing this in this lecture? Unfortunately, this tower showed increase responses when you put them for drilling, production and accommodation. It means people felt that high degree of safety is not mandatory because you are using drilling it also; you are using it for drilling purpose as well.

So, must include the safety of the platform now, earlier it was only anchoring safety was not that important. Now, you are putting this platform for drilling there will be living quarter people will be serving or working in the deck. So, people said yes! I must improve my safety on the platform now in 1984.
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- As permissible rotation of columns about the base is higher, structural response of the tower increases with the increase in water depth under the combined actions of waves, wind and current.
- Tankers that are used for temporary storage or sometimes for production purposes are moored to the articulated towers;
- such combination of tanker with articulated towers showed complex and high responses under large wave heights
- hence motion control of these structures assumes significance.
- Articulated towers ensure better structural performance under the encountered environmental loads with few structural modifications (Sara and Marano, 2010)
- Modifications are made to enhance their capabilities to encounter the environmental loads (Sellers and Neidzwecki, 1992)

Subsequently, it has been also seen by the researchers that when the rotation of the column over the base is higher the structural response increases very high and this increase is phenomenally high when it goes to deeper waters. So, when you would not move this platform or this kind of structure for deeper waters the responses at the base of the tower under the combine action of wave wind and current showed very high values. So therefore, people then felt the necessity of control in the responses back again. Initially, people extended this geometry by introducing an articulation at the deck and tower connection they achieved a larger deck size, use it for drilling, production etcetera.

But, when they analyzed it in reality they found out the responses are relatively higher. So, people felt yes! I must now control the responses on MLAT. So, what are those philosophies available? So, examples of these are for example, tankers can be also used for temporary storage. So, people used tankers which are anchored to the MLAT which are use for temporary storage. So, such combination of tankers cost lot of impact load on the towers.

So, the response on the towers must have been reduced when use it for anchoring storage tankers. So therefore, structural modification was suggested in the literature in recent past, may be in 10 years down the line that yes! I must do further, modification to this tower to make it feasible under different environmental loads.

The researchers say Sara Marano says, yes! I must do further modifications to the design. So, modifications are made essentially by Sellers and Niedzwecki in 1992 which has been examined by Sara Marano in 2010 and said yes! These modifications are better and they have improved the workability of the platform. But, still the higher response with higher depth is still a challenge till 2010.
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Jain, 94 also said is flexible risers can be supported by these kind of towers, they show lot of application but, they have higher responses under impact of tankers. Oded et al also said that there will be considerable reduction will enable the larger deck size so, if
you reduce the response larger deck size can be plat for production and drilling and it can improve the operational features of the platform.
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So, based on these people suggested in 1994 an idea that why cannot we use tuned mass dampers, tuned mass dampers TMD for response control. So, this is the first time people thought yes! Let me control the response using the different external mechanism where, I can tune my secondary mass to the primary mass using principles of dynamics to control the response. So, what we call as tuned mass dampers.

So, relatively new attempt done in 1992 by Fujino and Abe and of course, Kirk and Jain also verified that this can cause lot of probability of failure then articulated joint therefore, it is essential that I must reduce the response by some mechanism.

Thompson in 83, 84 and Elven also study the complexities of these complaint offshore towers and they said yes! If you want to use, I can reduce the response using TMDs but, there is a chaotic motion is happening in the system the response reduction should address the chaotic motion of the tower. It means, the tower when you take it to deeper waters, when you attempt to control response, you must also address the response characteristics of the tower in addition to reducing or controlling the response.


With this background people thought let me introduce hinges in between let us say instead of having hinge only here and here let me use multi legged multi hinged tower. So this concept came into play which is called multi hinged ATs.
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Let us derive the equation of motion for this very quickly and see how equation of motion looks like for this tower. Any question here? So, I am drawing this figure back again anyway, I am drawing it here, may not have to see it, here I am going to draw this figure again.
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Let us say there are 3 hinges hinge: 1 hinge, 2 and hinge 3 . I call this stiffness of this rotational stiffness of this spring as k theta, the rotational stiffness of this spring as k theta 2 and this as $k$ theta 3 respectively. So, $k$ theta $i$ are the rotational stiffness of the hinges. Of course, m 1, m 2, m 3 are the respective mass. Delta 1, delta 2 and delta 3 are displacement of the length respect to vertical, undisplaced position. Un-displaced vertical, the bottom leg is vertical.
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So, let us picked up one specific leg, only 1 leg, I will write here the cross section is larger here because I have reason that I would not have a buoyancy chamber here. So, this is 1 hinge, this is another hinge. So, I call this as Li length of the ith member. So, there is centre buoyancy, the center of gravity, mass center and buoyancy center so, I call this as L 1, 1 stands for the first leg, B stands for the buoyancy center. And I call this L 1 g , g stands for the mass center. You would not add find out the force at any F 5, any point in this leg at F 5, I call that distance as L 1 F . Why this is required, because I am always taking movement about this point, my rotations are all my displacement degree of freedom is rotational in this case.

By the way what is the size of this degree of freedom? Is the single degree freedom system problem or multi degree? How many degrees? 3: theta 1 theta 2 and theta 3. So, it is a typical member. And remember the dimensions are all measured along the length of the member not respect any respects any access they are local, they are all local. Now, I can write.
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My equation of motion is given by, in my mass matrix which is going to be 3 by 3 is given by, I 1 mass movement of ratio with the first leg, I 2 mass movement of ration about the second about the third plus M 1 L 1 g square M L square plus M 2 L 1 plus M 3 L 1 squares. When I come to the second is going to be M 2 L 2 g square same minus plus M 3 L 2 square, when it come to the third place this going to be M 3 L 3 g square. Let us
talk about this term is going to symmetric talk about this term which is going to be M 2 L 1 L 2 g plus M 3 L 1 L 3.

I can write it here, M 2 L 1 L 2 g plus M 3 L 1 L 3 the symmetric. The third term will be M 3 L 1 L 2 g because sorry L 3 g . The second term will have 2 g , third term will have 3 g , first term will have 1 g .1 g is nothing but, the center of gravity even the desistance of c $g$ of the particular leg from the base. So, I can write here M 3 L 1 L 3 g . Remember, g is not a multiplier here, g is just a notation, it is a notation. I am not multiplying this with 9.81, it is a notation actually. So, this will be M 3 L 2 L $3 \mathrm{~g}, \mathrm{M} 3 \mathrm{~L} 2 \mathrm{~L} 3 \mathrm{~g}$. One can also understand this in a simpler manner. If you are looking for the first row, first row I will have L 1 everywhere. Second row, I will have L 2 everywhere third rows I will have L 3 everywhere.
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So, let us look at the stiffness matrix I will remove this, I will remove this again 3 by 3. Of course, which will be, I will explain it here. Shall I take away this?


So, k 11 of course, $k$ theta 1 plus $k$ theta 2 because 1 connecting 2 hinges 1 and 2 the member 1 is connecting 2 hinges 1 and 2 therefore, k theta 1 k theta 2 rotational stiffness of this spring of this joints plus d times of I am looking for the force stiffness is force require for unit rotational displacement or whatever it is, g times of displacement multiply by the length of someone values. I am looking for L 1 B now, because $k$ is nothing but, restoring force. So, buoyancy is the restoring force here whereas, when you look at mass I looked at g all kg s I have used. 1 b plus L 1 of the total displacement delta 2 plus delta 3 minus M 2 and M 3 minus of course, M 1 L 1 g .

There is no multiplier here, g is a suffix actually it is only a notation, there is no multiplier here not g its not square g is only here this is 9.81 . So, k 12 which is this value this of course, equal to k 21 symmetric matrix, which is nothing but minus of the rotation stiffness of the joint 2 . Obviously, it is expected that k 23 is k 32 which is this value, is minus of $k$ theta 3 . Again, remember this is not a multiplier, is not $k$ into theta it is not like this, it is k theta 2 it is a notation, nothing but, rotational stiffness of the spring.

Let us see k 22 so, 2 is connecting the legs 2 and 3 so, must have $k$ theta 2 and $k$ theta 3 as here 1 and 2 here plus $g$ times of delta 212 b same algorithm plus 12 of delta 3 minus M 3 minus M 212 g . Can you write k 33 ? I can write one easiest term here, I will write this. So, g of delta 3 L 3 b L 3 of M 3 g sorry, there is no multiplier its L 3 g , M 3 L 3 g which is negative. Is it not negative? So, I have all the components here.
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Now, the damping matrix C is assumed to be proportional to K and M . So, phi times cos c phi is 2 zeta omega $m$. So, from this you can find the C matrix that phi is the mode shape and zeta is assumed as 5 percent. So, what I have in this equation of motion m c and K, I have all the values. Now, I can solve this problem.
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So, interestingly people have solved this multi degree freedom system problem for a MLAT and they have understood that the response of the multi hinge of the multi tower is again higher because of improve flexibility they want a control. So, they have used a secondary mass systematic $C$ in the right side using a tuner mass damper, they have use a secondary mass system.

So, as Sara Marano Chandrasekhar examined it continuously they found out a vibration observation mechanisms vibration observations mechanisms using TMD which we will discuss in the next lecture for control mechanism.

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So, the references for this particular lecture are the following: Butt et al 1984, Carlson Spencer 1996, Chakrabarti, Chakrabarti and Cotter 111983 is all the references for this specific lecture.
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Fujino Abe who suggested the dynamic characterization multiple tuned mass dampers in structures, Jain of course, reviewed the flexible risers and articulated storage systems on ocean engineering. Jain and Datta where this paper has been basically based Kirk and Jain has suggested that the response can reduction in towers. Oded and Oded tell in 1992
said the analysis of non-linear characteristics can be study for towers. Sara Marano have optimize the design linear tuned mass dampers for non-linear behavior they are not applied for offshore structures but, they showed it is possible.
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Sellers Neidzwecki showed the response characteristics of multi hinged dot articulated as we just not saw here. Thompson of course, studied in 83 and 84 and showed the chaotic motion of 80 s onto the later loads and so on.
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So, the lectures stop's here. The next lecture we will talk about here secondary mass system attached to this as the tuned mass damper. How the response can be control and how do you handle this problem in a dynamic analysis format. So, we will talk about that in the next lecture. That will conclude the dynamic analysis on AT s. Then, we will pick up for TLPs then, we will discuss again the derivation of equation of motion for TLPs a response control on seismic expectation for TLPs. Then, we will talk about new generation structures like then, we will discuss about the experimental analytical studies which has been carried out to give a feeling that how they can converge for a very nice understating of dynamic analysis.

So, I urge you that you must go through the references of this try to understand how the tower has been analyzed for different kinds of load because we are not able to show the result here for MLAT but, I will show the MLAT with TMD attached within the next lecture.

