Dynamics of Ocean Structures
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Module - 3
Lecture - 1
Introduction to Stochastic Dynamics of Ocean Structures

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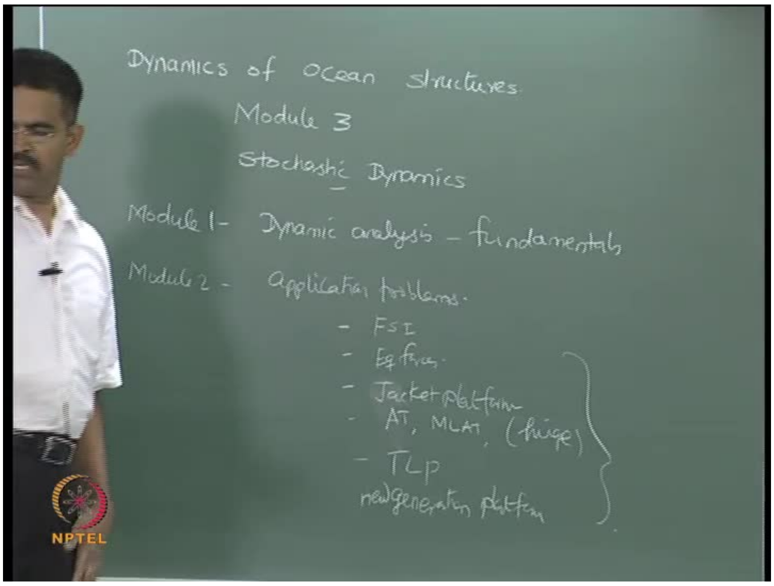

So, in the first module, we discussed about the fundamentals of dynamics analysis. We discussed some few fundamentals of this. We discussed about how to write equations of motions for different single, double and multi degrees freedom system and how to find out the Eigen values and Eigen vectors or on the other hand, frequencies and mode shapes of a given equation of motion. We have also classified and understood how free vibration and force vibration analysis can be carried out and what are the important control techniques, by which we can understand them.

In module two, we discussed about some application of dynamic analysis on different classified problems. There we saw how one can impose different kinds of loading using fluid structure interaction. You also studied how one can include the earthquake forces in the analysis and we picked up example problems from jacket structure, fixed jacket platform, articulated towers, multi lead articulated towers, and advantages of a new geometric form by providing hinge. We have also seen about the tension leg platforms.

We have also seen about the new generation platforms, the complete dynamic analysis with references to the literature. So now we look into some advance topics where stochastic dynamics also becomes interestingly important in research perspective.

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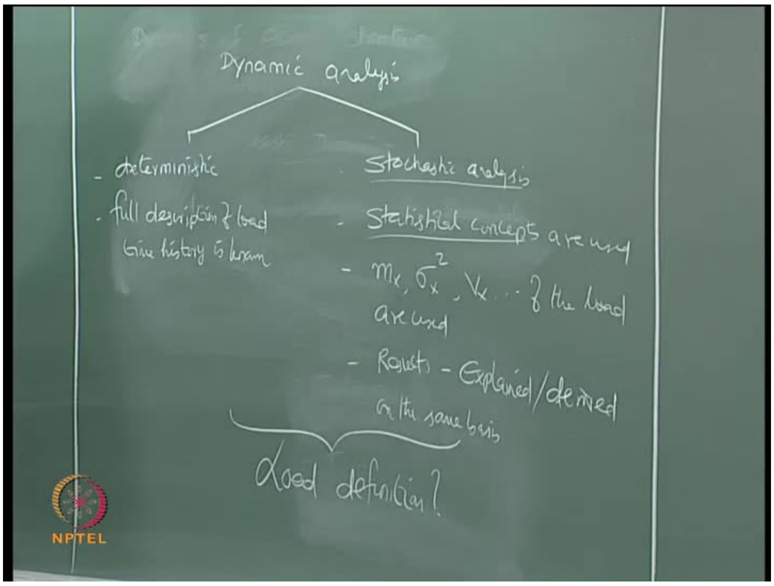

Now, let us quickly understand two different methods by which I can do dynamic analysis, which we already studied in the first module. We already said the dynamic analysis can be done in two formats. One is what we called deterministic analysis, where the full description of load history or load time history is known. Sometimes, the full description of the time history may not be known.

Then alternatively I can use the statistical parameters, which we now call as stochastic analysis. In stochastic analysis, the statistical concepts are used. Statistical concepts in sense, mean variance of the load are used and obviously, the results what we get from stochastic analysis will also be explained or derived on the same basis. So, fundamentally stochastic analysis deals with statistical characteristics of the loading. So, in general, both of the methods essentially divide the analysis pattern only based on how the load is defined. Depending up on how do you define the load, you can either do a deterministic analysis or you can do a stochastic analysis.

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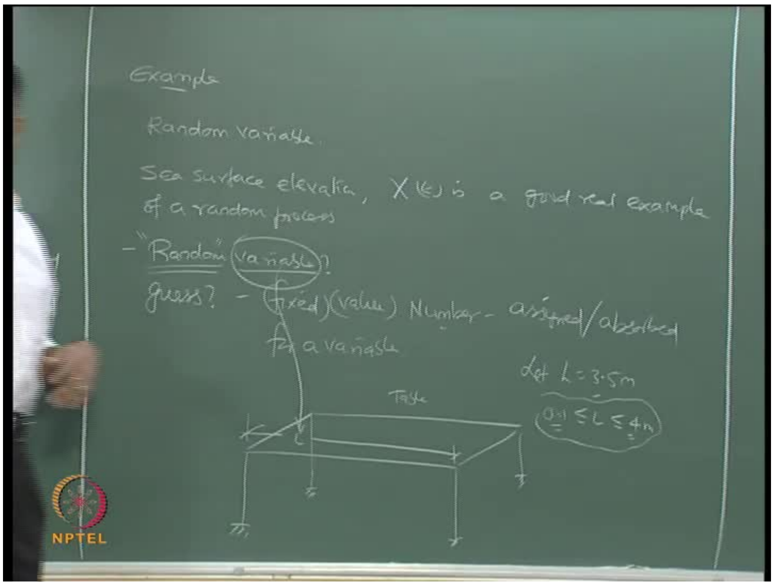

Now, let us have an example of a stochastic analysis, where will be leading with or we will be dealing with what we call random variables. So, sea surface elevation, capital x of t is a very good real example of random process. Actually, random variable has a very classical definition. Many literature, many authors, many books refer this slightly in a different manner. I will give you very simple definition for a random variable. Random variable is a number; I should say is a fixed value; I should say is a fixed number assigned or observed for a variable.

Now, the question comes why it is called random. The random term associated is, you do not know the number. For example, let us say I have a table in front of me. I have a table. So,, I want you to guess the length of the table. So, length of the table now is a variable because if the class sample is around 100, each one of you will have a value assigned to this length. Therefore, it is a variable. But why it is random because the value, the length of the table has a fixed number, may be 3.5 meters. Let l be 3.5 meters, but since we do not know this value, without measuring the length of the table, we keep on guessing this value anywhere varying from 4 meters.

So, the randomness is actually the value, which is getting skipped off within these numbers, between the lower and upper limits, which are reasonably proper for the dimensional what you are looking at. So, randomness is that guess what you make for the variable because the value of l keeps on taking any number of values in between this range as per your choice. So, stochastic process is exactly a combination or a development of a random variable. We will see, we will connect ultimately at the end of the today's lecture, as to how stochastic process can be connected to the random variable. So, how will we define a stochastic process? That is our aim.

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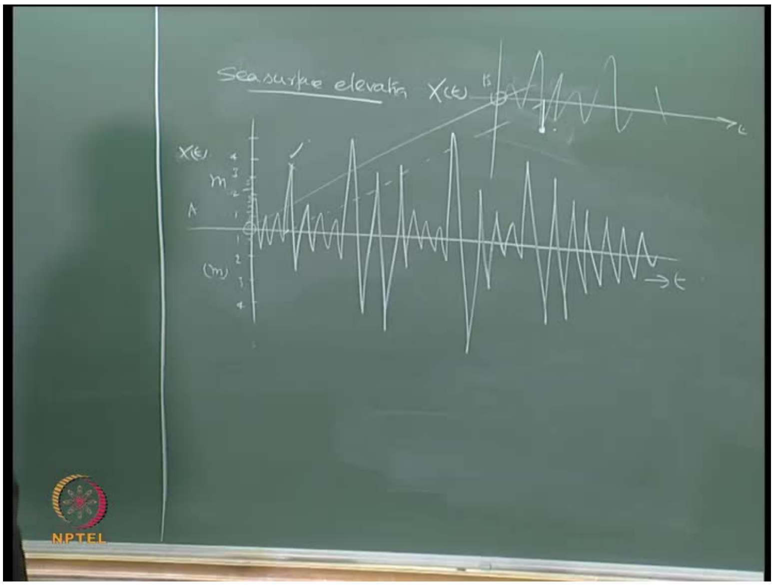

Now, let us plot quickly a time history of a sea surface elevation and see what are the randomness involved in this and how this can be generally handled in the analysis. That is, I plot a sea surface elevation and I call this as x of t. So, the x of t looks like this. It keeps on going and this is time and some value may be 0.1, sorry, 1 2 3 4 meters and this is x of t. This is also 1 2 3 4 meters. So, it is random in its time extent. This is one record what you are seeing. Similarly, one person standing at other end, somewhere at few kilometers away, will also have a similar record and will also have a plot. So, again the sea surface elevation and both of you, let us say, you are A and the other person is also started recording this and he is B. He is also having same.

So, at any given time t, the sea surface elevation what you see or what you record will be different from the sea surface elevation what this gentleman see and record. So, it is actually a random number because this varies from any value to any value. Physically, it cannot be 0, because you can see from the record, sea size elevation is never 0. Some number, we do not know the number. Therefore, it is random and the number I going to guess from any value from 1 to 5, which is an amplitude of the sea wave, which also a variable, so it is a random variable.

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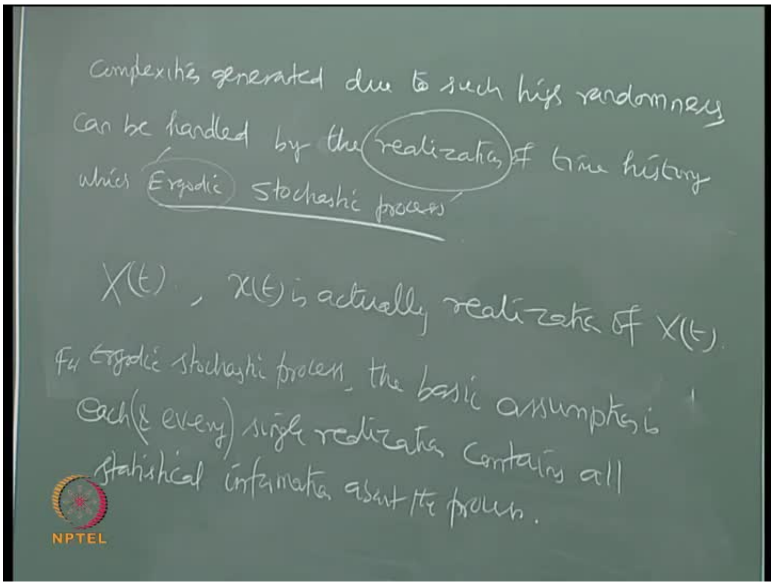

Now, there are lots of complexities involved in this. How will you handle this? Now, the complexities involved in such high randomness can be simplified using a simple technique in analysis. The complexities generated due to such high randomness can be handled by the realization of the time history, which is Ergodic stochastic process. Now, there are two keywords here. What do you mean by realization? Then of course, what do you mean by ergodic? Stochastic will anyway link to the randomness later.

Now, x of t is my load history or my amplitude of the time history of the wave, which can take any value and x of t is actually the realization of this value. What it is mean is, x of t is assigned a value, in the large bracket of capital x of t. We call that as realization. Now, the complexities involved in this can be handled by realizing the time history, if the process is said to be ergodic.

Now, for an ergodic time process or ergodic stochastic process, the basic assumption is, for ergodic stochastic process, the basic assumption is each and every single realization, let me put it here, each single realization contains all statistical information about the process. What do you mean by this? A is observing at one part on the coast and he is trying to measure the sea surface elevation by some technique, may be by visual observation; may be by using a wave pro; may be by using a satellite; he is trying to plot.

B is standing on another location on the coast somewhere else and he is trying to measure or trying to visualize the sea surface elevation, which is fluctuating as same as A is observed.

Now, both of this process has generated complexities. So, both of this process generated complexities because we really do not know at any given point of time t, in a record length of capital t, what is actually the elevation. High randomness is there. To simplify this, out of all the infinite number of records available at A B C etcetera, observing the sea surface elevation, I pickup any one x of t, which is a segmental representation of capital x of t, which I call as realization, because x of t is a random process and that is ergodic, if this data, which I am picking for my analysis or the load history, which I am using from a dynamic analysis contains all statistical information that any record contain in my example.

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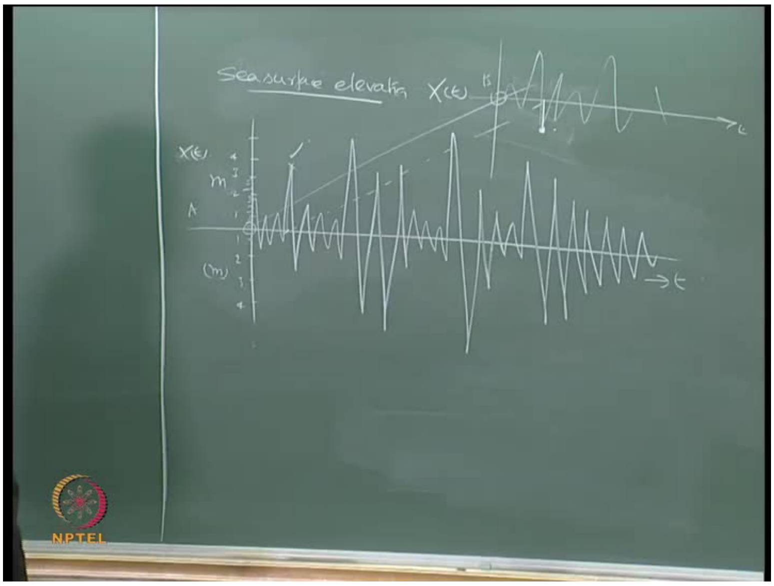

So, if you can pick up any one such process which can have realization of all the statistical parameters of the entire record, then that process can be said as ergodic process.

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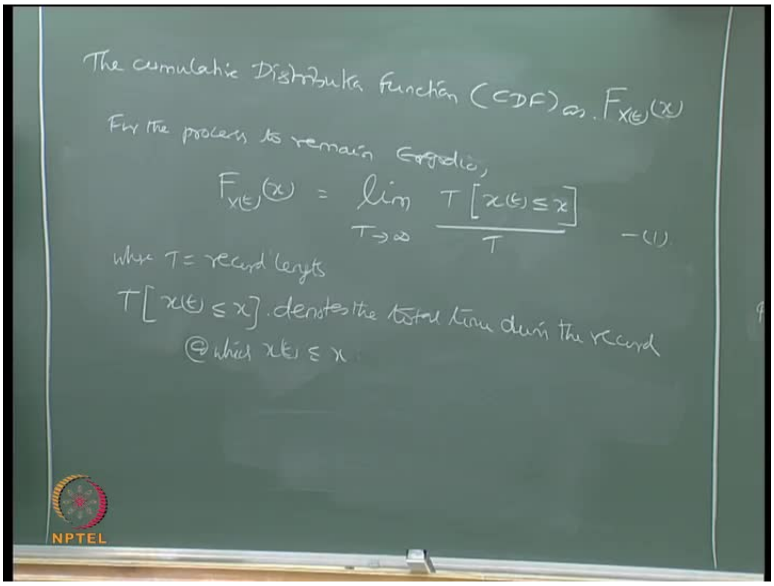

Once we said the process is ergodic, and then we define the cumulative distribute function as f of x t of x. Write it carefully. I am rewriting it again. This is f of x t of x. This is cumulative distribution function called as c d f in the random process. Now, for the process to remain ergodic, f of x of t of x is given by limit t tends to infinity t of x of t less than equal to x by t, where equation number one, t is called record length and t x of t less than of x denotes the total time during the record at which x of t is less than or equal to x. What does it mean? How do we understand this graphically? So, I am picking up a sea surface elevation observed by observer A in a record like this.

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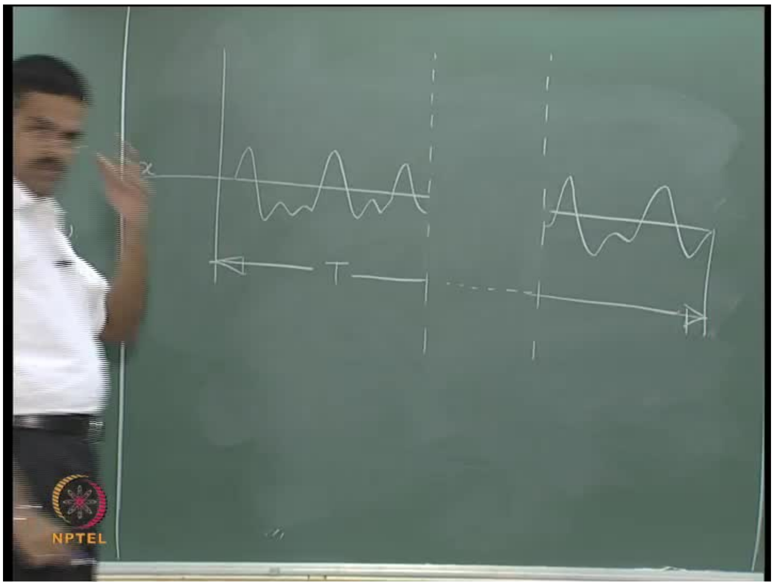

t is infinitely large. The record length is very very high. This is my x value, some pre-assigned value. Not 0 because we know sea surface elevation cannot be 0. This is my record, where this is my t. I said the cumulative distribution function will be given by the value of realization of x, which is less than or equal to x. I have a pre assigned value of x. So, I pick up those segments in the record, delta t 1.

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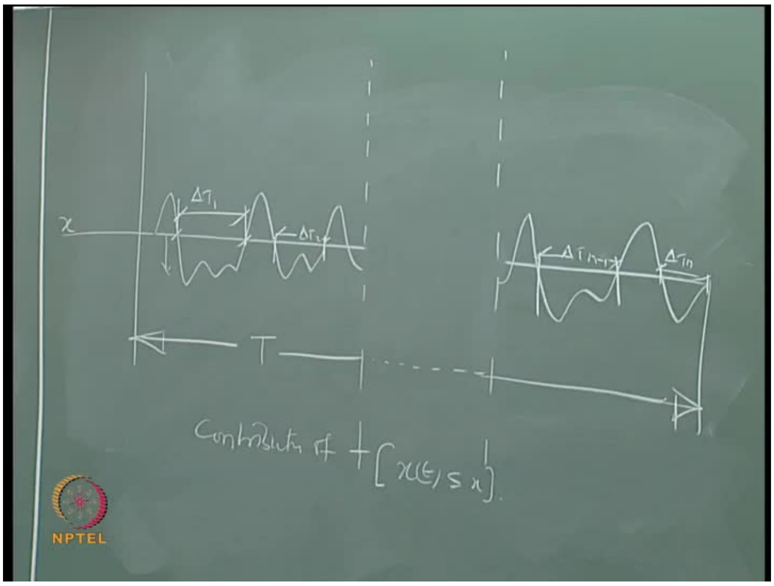

These are all nothing but the time. So, time average, actually I am picking up the time, at which the time, at which my value of x of t is a random variable is less than or equal to a pre assigned value x in a given example or a sample. Similarly, delta t 2 and similarly, delta t n. Similarly, delta t n, n minus 1. You can see here very much that the sea surface elevation or x of t is a value lower than x. This is what we call contribution of t x of t less than of x. So, if the stochastic process is considered to be ergodic, then the cumulative distribution function can be easily given in a closed form as we see in expression number 1 or equation 1. There are further terminologies involved in stochastic process which you must understand. Let us quickly see them one by one. Any questions here?

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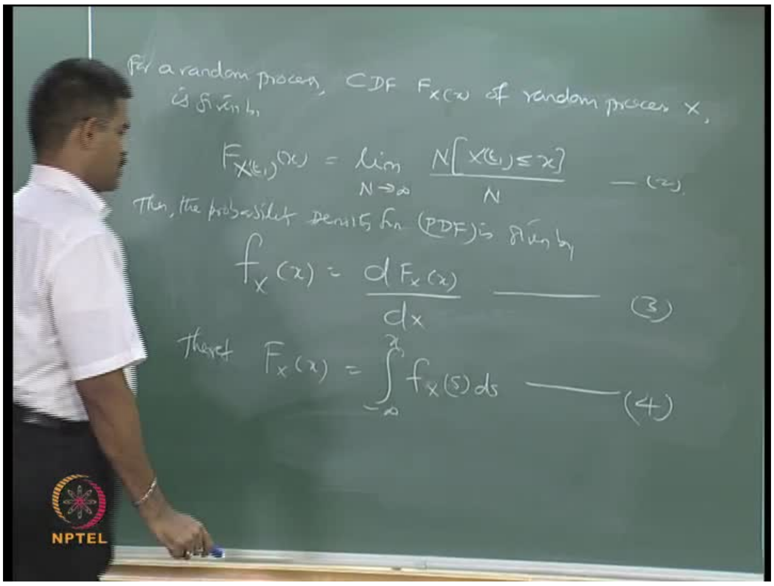

So for a random process, the cumulative distribution function, not t, any process, not time. It may be random with a value itself. Not necessarily with respect to time of the random process x is given by f of x of x. Let us say the variable is again at t 1 interval of x. This is capital x is given by limit n tends to infinity. Looking for the number of records n of x of t 1 less than or equal to x by n, equation number 2. Then the probability density function p d f is given by f of x of x, the derivative of, provided that there is integral exists. So, equation number 3. Therefore, f of x of x can be said as x f of x s d. This is converse of this.

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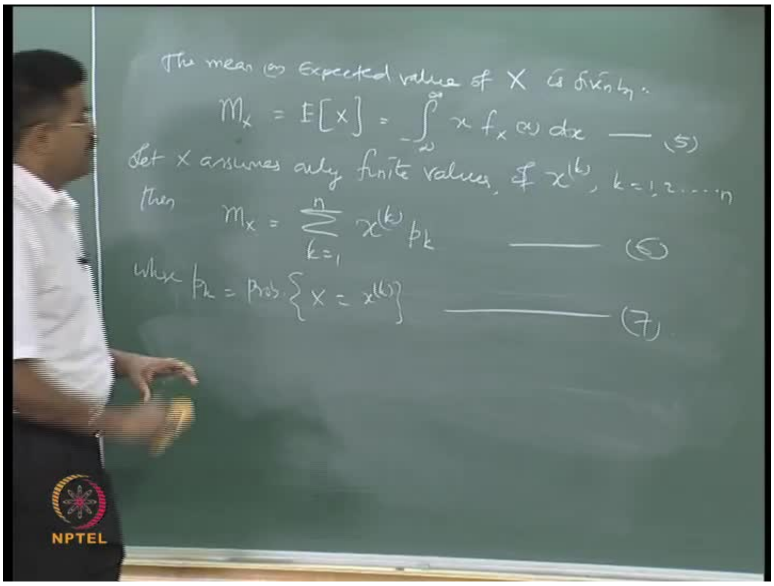

Once I said this and then the mean value or the expected value of x. The mean or expected value of x is given by, let us say m of x, which is equal to the expected value of x, which is the product of d x; variable is small x. Now, let us say x assumes only finite values, like length of the table of x of k, where k is 1 2 n. Then the mean will be now given by varying from; it is not going to be infinity, the realization is only till the value of n as a whole number or finite values, which is p k, where p k is given by it is probability of x equals x t. Now, interestingly we can also find the expected mean or the first order derivative of this process by a different technique and then we will compare both of this.

So, what we have so far said is, for a given ergodic stochastic process or a random process, we are going to connect with both of these two at the end of this, I can find the cumulative distribution function as an average of the time, where x value or the variable of x is less than or equal to a specific index value x, which is pre agreed upon. Or alternatively I can also find c d f as an integral of this value from infinity to x and based on which, I can always find the probability density function f of x. If you know the probability density function f of x, I can always find the mean value or expected value as a product of these two in general. but if x is going to take only realized value of finite number from 1 to n, not infinity, then. I can still find m of x or mean value from varying from k to 1. k is a value, which is now taking the, assign the realization of x for a fixed value of 1 to n, which is given by this equation, where p k is nothing but the probability of x being equal to x 1 x 2 x 3 x 4 x 5 etcetera, till n. Any questions here?

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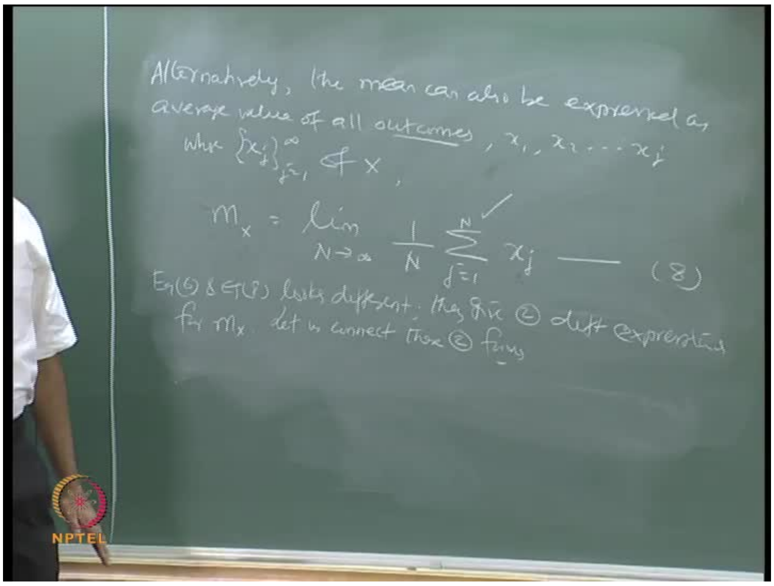

Alternatively, the mean can also be expressed as an average value of all outcomes. Not the assigned value but the realized value x 1 x 2 x j, these are all realized values, where x j varies from j equal 1 to infinity of x or realized values of the random variable x. In that case, the expected value or the mean can be given by limit n tends to infinity 1 by n is a classical equation, which we all use, j equals 1 to n of x of t; equation number 8. This is classical equation what we all use for finding the mean. Now, the equation what we had, which is equation number 6 is different from equation number 8.Now, equation 6 and equation 8 looks different. I do not have 6 here. You can see in your note book. They are different and they give two different expressions for mean. We can connect these two. Now, let us try to connect these two forms. First, let us understand what are these two forms. One form which gives me the equation 6 is generic and the other form of m of x is actually the realization of x value from 1 to j, where j varies from 1 to capital n. In the earlier case, there is a small n. Now, let us connect these two because both these expressions are giving the expected value or the mean value of the process. So, they should give me the same meaning.

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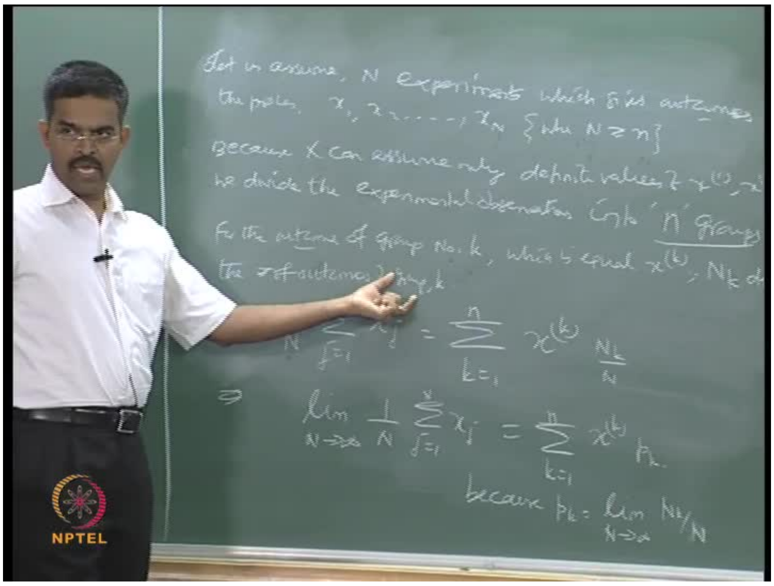

Now, let us assume that n experiments are conducted. What are experiments? Experiments are nothing but observation of sea state at a b c d n and n experiments are conducted, which gives outcomes. What are the outcomes? They are nothing but the x values, small x value from the capital x value and outcomes of the process, which is x 1 x 2 and so on, which takes you till x n, where n is much greater than n.

Now, there is a problem here because x can assume only definite values of x 1 x 2 x n, because we have a problem that x value cannot assume values till n. Capital n can assume values only till realization of small n, where capital n is much larger than small n. So, we have discretized x value to assume only between x 1 to x n, which is small n. We divide the experimental observations. What are experimental observations? The sea state elevation into n groups. Is that clear? So, I have got a, let us say for example, 400 values. But I have a sample space which can accommodate only till 100. So, I will give 4 groups of 100 values.

So, what exactly you are trying to say is, equation number 6 is generic for capital n, whereas equation number 8 is generic only for small group of small n. So, we want to combine these two. So, for the outcome, what is outcome? Outcome is a realization of small x on the sample set of capital x. For an outcome of group number k, some group number, which is equal to x of t. It may be 50, k may be 50 and k may be 100. So, I got a small group of k whose realizations can be or whose outcomes can be referred as x of k, which varies from x 1 x 2 x 3 till x k and n k denotes the number of outcomes of group k.

In that case, 1 by n of summation of j equals 1 to n, so this is capital n, because my summation is on capital n, x j is given by summation of k equals 1 to small n; that is the group, x of k n k by n. These are the number of groups we have, which will give me limit n tends to infinity 1 by n of summation of j equals 1 to n of x j is 1, for n being very large. Very large is higher than small n, which is now given by summation of k equals 1 to n x of k p k. This is the equation what we already had because p k is nothing but the probability of x less than or equal to x of n, which can be simply limit n tends to infinity n k by n and that is p k. Is it not? That is the probability.

So, what we have done here is, we had two expressions. One is on generic of capital n, which are the complete set of observations and one is the discrete values of small n, which are groups, which can take any number less than capital n. So, I divide small n by capital n, let say and I get number of groups of k groups, and I call n k as one of the outcome of the group k. I am now trying to connect equation 6 in equation 8 by generic form like this.

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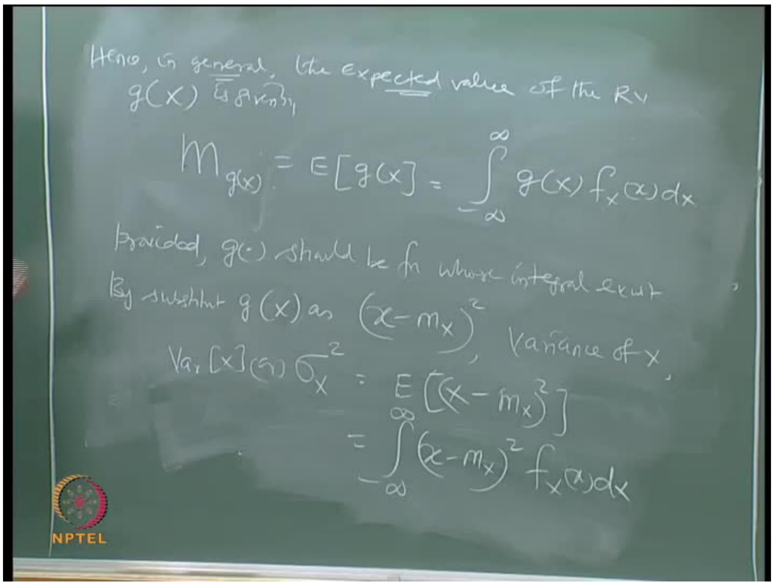

Now, this can be further simplified very interestingly to get a closed form expression, which is commonly used in stochastic process. What is that expression is we are going to see now. Any question here? You have to go through this repeatedly to understand this. I am sure you will understand this. Hence, in general, because I have connected now a mean value or, sorry mean or an expected value of a variable x in a generic form and a specified form of group of small n. The expected value is nothing but the mean of the random variable; I am using r v, random variable g of x. I am having a general form g of x is now given by, because I am looking for a general expression for the mean value, which I can say m of g of x is not, this is not capital m, this is small m, capital m is moment, that is second order derivative statistics. Small m is the mean. This is small m. I will rewrite it again. This is small m g of x is given by expected value of g of x, that is the mean, which is given by a classical expression minus infinity to x infinity g of x f of x of x d x, which is the probability density function for a given distribution.

Now, the question is, what to be this function g of x first, provided, interestingly and mathematically g of x or g of any variable should be a function whose integral exists. That is very interesting. Otherwise, you cannot find out the mean value. Now, for substituting g of x as x minus m x the whole square, can be any function, which you must integrate. Now, I can find the variance of x, which is given by the variance of x or mathematically, sigma x square. It is nothing but the expected value of x minus m of x the whole square, which is given by integral of minus infinity to plus infinity x minus m x the whole square of x of x d x. Sigma x is called the standard deviation and square of standard deviation is what we call as the variance. Now, any questions here?

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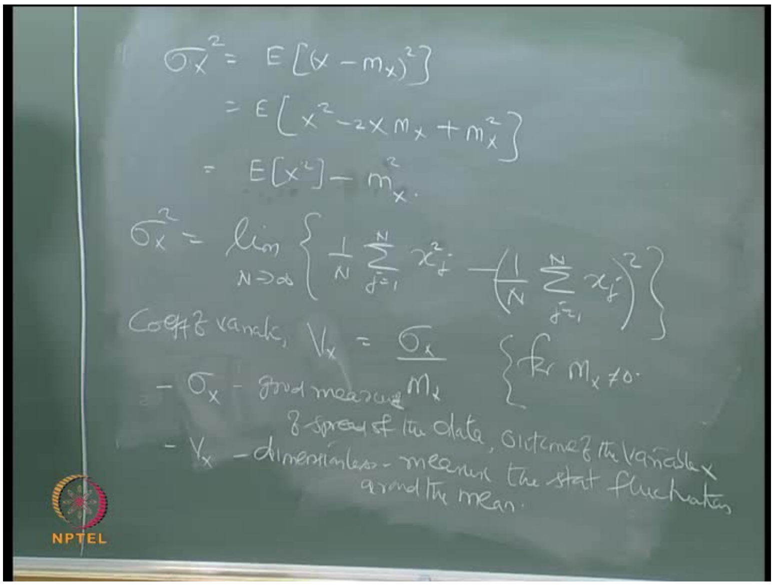

So, I am picking up g of x as any function which should be in an integrable form or the integration should exist. I pick up that function as a function given here. Once I do that here, this expression will turn out to be the variance. So, having said this, sigma square from this expression can be again its expected value of x minus m x the whole square, expected value of, which can be separated as expected value of x square minus m x square. Therefore, I can write sigma x square as limit n tends to infinity e of x square, which can be 1 by n of j equals 1 to n of x square j, because this is expected value of the square of the value, minus mean of the whole square, which is 1 by n of j equals 1 to n. Here we are talking about capital n. We are doing for a generic process, which is going to be x of j. Now, I want to have whole square of this.

So, it is going to be whole square. So, this is my variance. I can also find the coefficient of variance. We use v x, which is given by sigma x by m x, of course, for a non zero mean process. Mean cannot be 0. Now, what are the physical meanings of the standard deviation and the coefficient of variation? The standard deviation can give a good measure of spread of the data. In our case, spread of the outcome of the variables, whereas coefficient of variation v x is a dimensionless quantity, which can be used to measure the statistical fluctuations around the mean.

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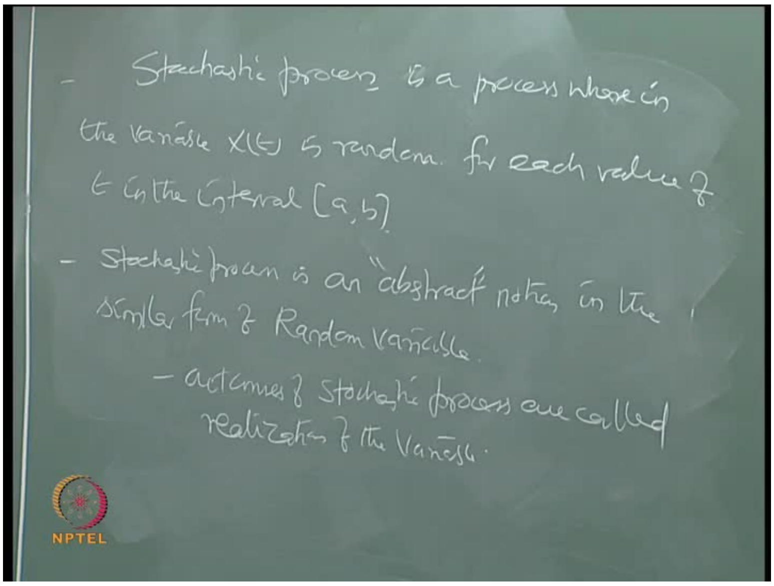

Therefore, based on these expressions, how do I connect stochastic process to random process? Alright, random variable. Therefore, stochastic process is a process wherein the variable x of t is random for each value of t in interval of a n. So, stochastic process is an abstract motion in the similar form of random variable. Here, the outcomes of stochastic process are called realizations of the variable. So, stochastic process is nothing but the abstract notion of random variable.

So, it gives you meaning for a random variable. It is an abstract notion. Random variable can be a number which can observe any value in a given frame of relativity. The moment I say the process is stochastic, it is having an absolute meaning of the variable. So, it is giving an abstract notion of the random, and that is variable. So, interestingly in the next lecture, we will talk about one important aspect. For a linear system, how can we connect the response to the load directly in a stochastic process? What I am going to call as response spectrum. In a given stochastic process, if I got a load spectrum and the realization of load also, where capital f of t is a load and small f of t the realization of the load and capital x of t is the response and small x of t is the realization of the response, how can I connect these two using what we call as a response spectrum for a linear spectrum.

So, in the next class, we will derive the expression for a stochastic process and what will be the expression for a response spectrum and then we will talk about what do we mean by a return period in a given stochastic process. Having said this, we will try to use the modal analysis and the modal participation factors and truncation of higher modes in dynamic analysis using classical theory, what we call modal truncation theory. Subsequently, we will use stochastic process back again to find out the fatigue of structures. How can I do that? Because, to find out the fatigue, we need to know the 0 crossings or any value about the threshold number, like x of t as we seen in the previous example.

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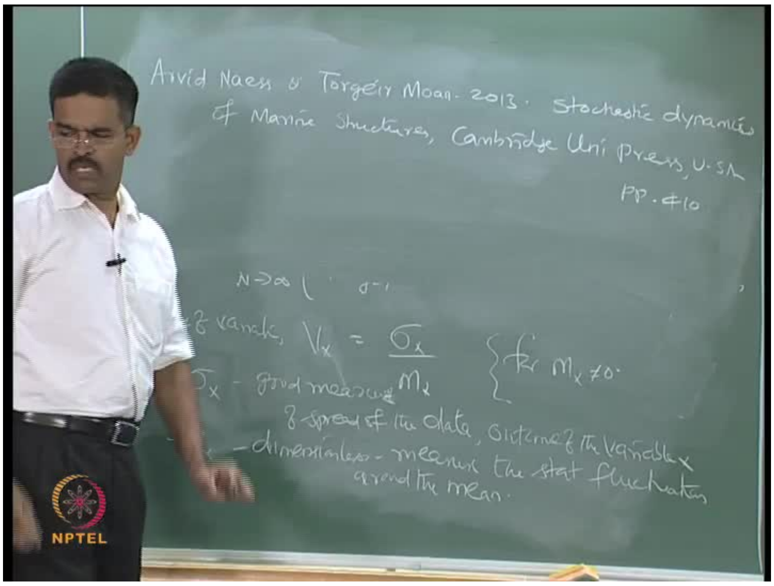

So, that will be the path for another 5 to 6 lectures, in module 3, which we will talk about the alternate method of dynamic analysis. What we have seen so far in module 1 and 2 is the deterministic method, by which we know the time history completely in terms of its history. If you do not know the history, but you the statistical content of that, then I can still do a dynamic analysis and that is what is called as stochastic dynamics, which we are going to do.

In this module, there are some important terminologies we are going to understand. We will talk about that slowly in other lectures and then we will lead to the end of this module, how can I also do a stochastic dynamics using this kind of variables and these terminology is being commonly used. So, we have got three terminologies that are very popularly being recommended in stochastic dynamics. One is the mean, standard deviation, coefficient of variation and of course, the cumulative density function. So, we will talk about this subsequently and will pick up examples and show how this can be applied very quickly. So, people otherwise call this analysis as frequency domain analysis. When I used it for dynamics, I will talk about that. Any doubt here? So, go through this. There is an interesting reference on this. You can read that because you may require very badly, this one reference. There are many other. So, we will discuss the further part in the next lecture.