**Dynamics of Ocean Structures**
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**Module - 3**
**Lecture - 4**
**Return Period Fatigue Prediction**

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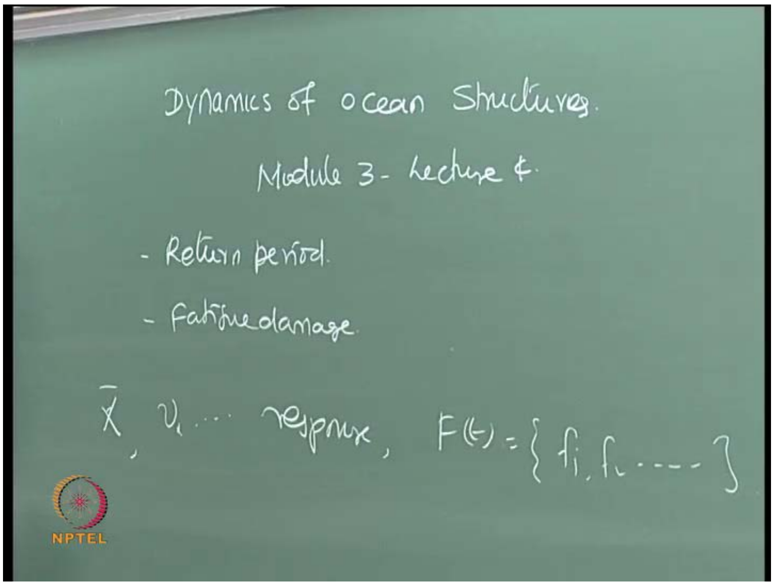


So, looking into the stochastic content of dynamic analysis, we have already seen that, how does the stationary process can be used and advantages to derive the first order second model values as x bar and auto covariance function etcetera, for the response, which are nothing but the realizations of a stationary stochastic process of f of t, which has ensembles as f 1, f 2, and so on.

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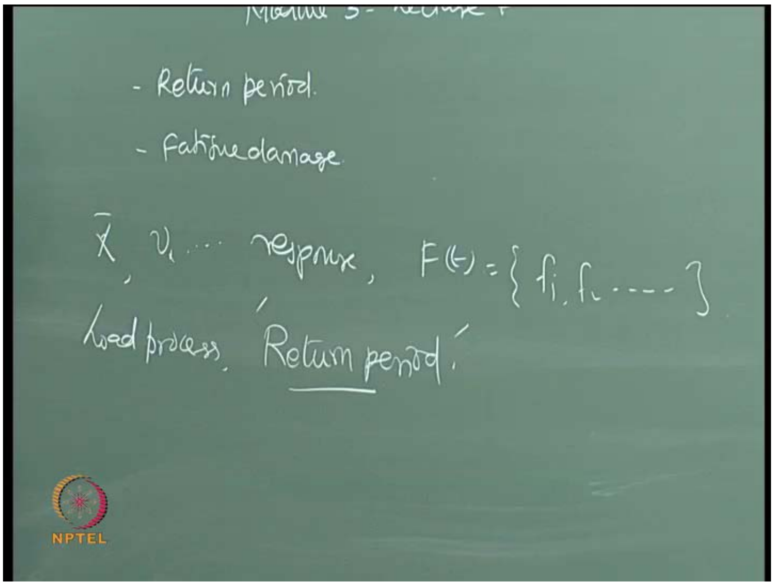


Now interestingly, when you talk about the load process, there is a very important terminology associated with this load process, what we called as return period. Let us quickly see how do you define a return period and how to compute it for a stationary narrow band process.

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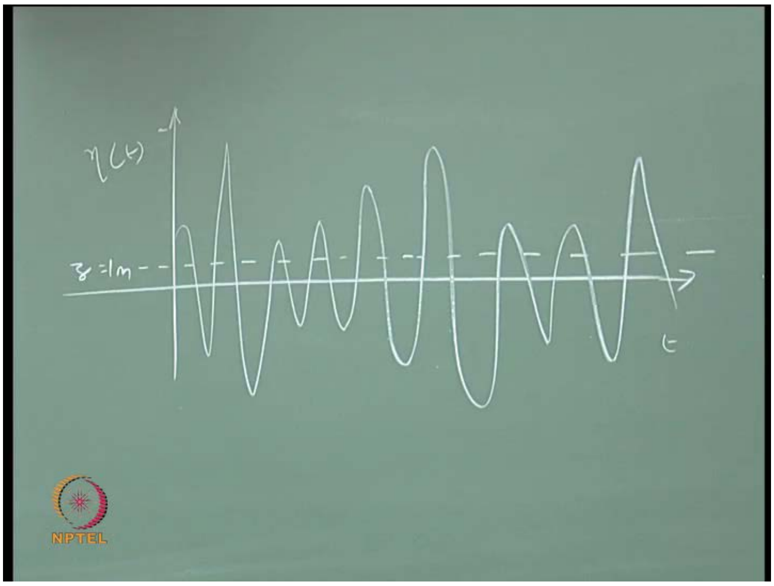


Now let us say for example, I am standing on the sea shore and trying to measure by eye judgment the sea surface elevation or using some mechanical methods or some equipments or instruments, I am trying to measure the sea surface elevation x of t on a time frame t. I can always fix up plus minus values; talk about the plus value, I can always fix up a plus minus values of the specific number, maybe 1, 1 meter is above, let us say 3 feet. So, I always look into or I am interested in looking into only those wave heights, which cross at least, let us say my belly level, maybe 3 feet. It is a physical observation.

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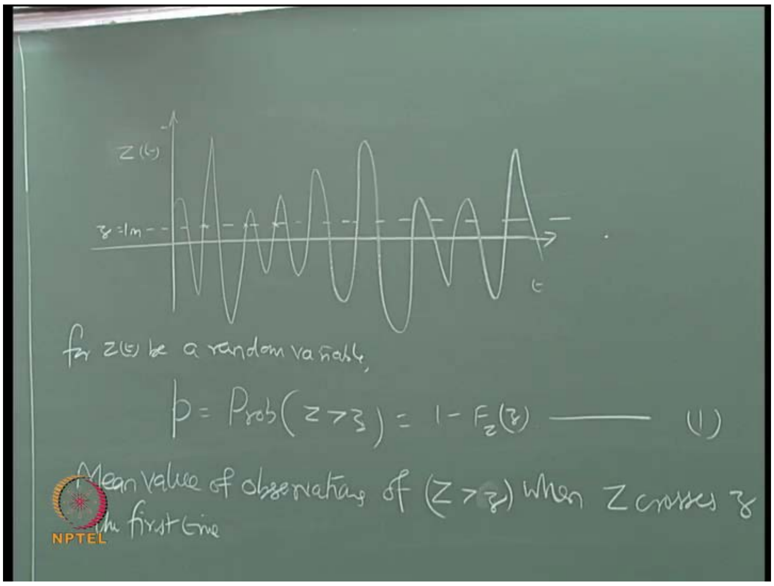


So, now if we replace the sea surface elevation zeta of t by a variable, so for z of t be a random variable. If you want to express the value of z of t exceeding any pre agreed number zeta, then I will always look at the probability associated with this. Because I really do not know how many times in a given observation of time record capital T, the value z of t, exceeds a pre agreed value zeta. I do not know this. That is a probability. So, I can write the probability p maybe as the probability of z exceeds zeta, which can be given as 1 minus f of z of...

Now, the mean of observations, you have made some observations and you are trying to find out the mean value of this observation. What observations have we made? The mean observations of z exceeds z, that is the observation you have made. That is the observation you are making, z exceeds z. When were we making it? When, z crosses z for the first time. See, it is interesting that z will cross z many number of times. For example, see here, 1 2 3, many number of times it will cross in a given time record.

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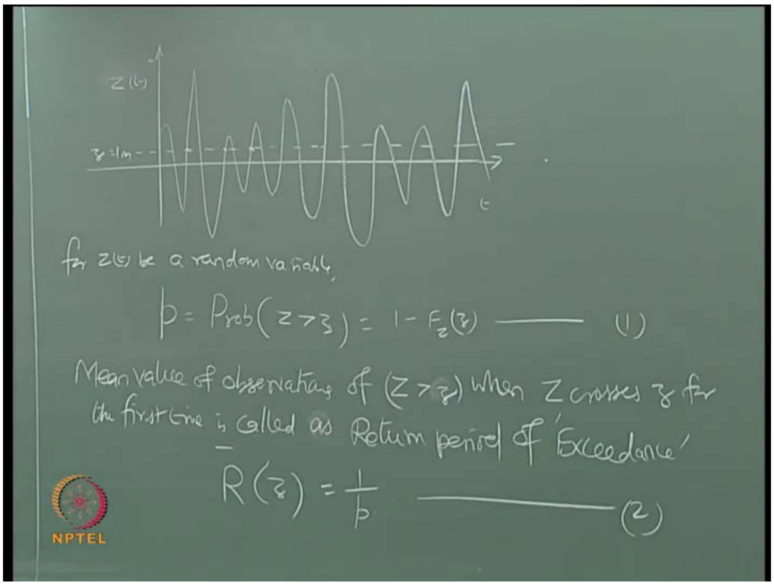


So, you try to find out the mean value of observation of z crossing z, when z crosses zeta or z at the first time and I call that return period of exceedance. Why I am calling exceedance because I am looking for a condition, where my sea surface elevation random variables z of t exceeds a pre agreed value z. So, it is period of return period of exceedance. What does it mean is, what is the period at which first time you have observed that, second time you observed that, what is that period at which this occurrence of z exceeding z is happening in a given time space or time record capital t or record link capital t. Now, I can say r of z, which is the return period of exceedance of z. This argument is always the value which you want to exceed. It is not the data, so always the value which you want to exceed. You want to exceed zeta or z. Small z is simply given by the inverse of p's.

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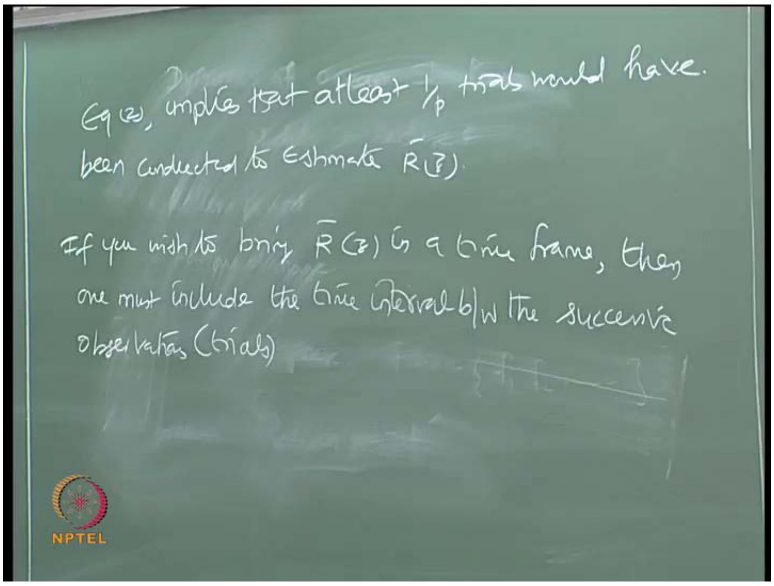


So, interestingly equation 2 implies that, at least 1 by p trials would have been conducted to estimate r z. You need at least 1 by p trials, otherwise you cannot take the mean. So, here in the return period of exceedance, we are talking only about the probability of the variable, which is exceeding a pre agreed value in a given record length capital t. Can I convert this argument on a time frame? Can I look into this on a time frame, because one is interested in always explaining the return period on a time frame. But, here in this case, return period is not on the time frame. It is exceedance of a specific value, which is pre agreed by you. May be 1 meter, may be 1.5, 0.5, whatever may be the value. It is a pre agreed value. Now, if you wish to bring r dash of z in a time frame, then you must include the interval between the period of observation. Ok? Then one must include the time interval between the successive observations or I can put it in brackets, successive trials, because these are all trials.

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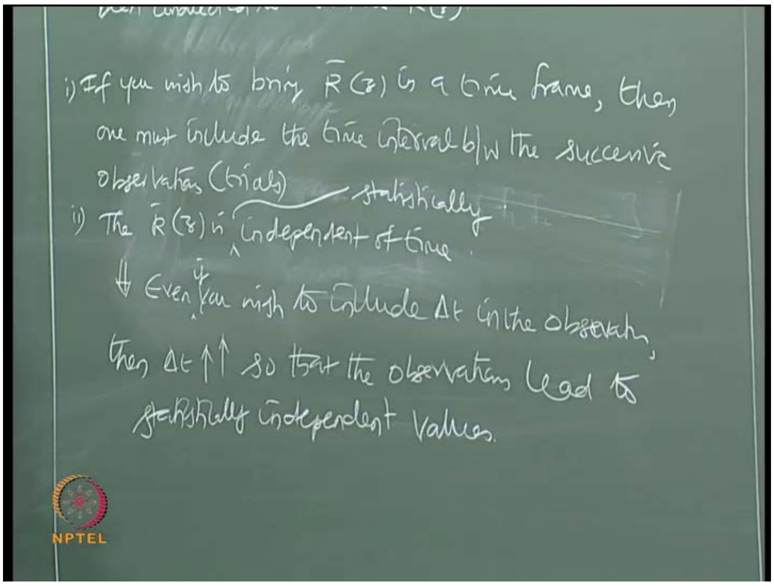


That is one part. The second part is in this argument, we are inherently assuming that the value of return period is independent of time. There is not time dependency. Here, it is independent of time. So, what does it mean? This implies that, even if you wish to, this I should say it is statistically independent. Let us be very clear. Even if you wish to include delta t in the observations, then delta t value should be sufficiently large enough, so that the observations lead to statistically independent values. You cannot take this delta t so close. It should be sufficiently large, so that the successive observations do not get influenced by the previous observations. They should remain as statistically independent.

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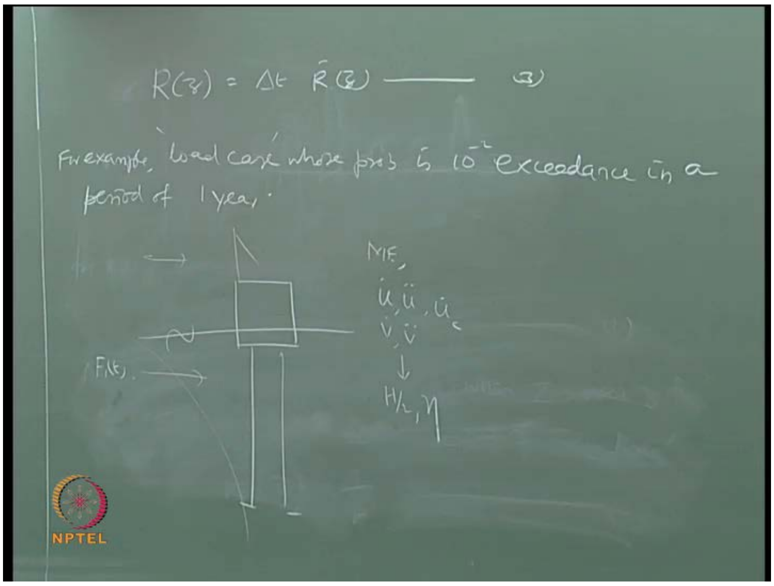


Having said this, I am curious to convert the equation 2 into a time frame, where I want to include the discussion of delta t. That is, the period between successive observations or the time interval between the successive observations into my calculation. So, I can even write the following equation, which says r of z now. The r bar has gone, which is delta t of r bar. Let us quickly look at an example. Let us say for example, I have a load case whose probability is 10 power minus 2 exceedance in a period of 1 year. Let us say, I have a case. I have some load of case. What do you understand by a load case physically? For example, let us say I have an offshore platform. I have to subject this platform to wave action and wind action. Let us take wave action as f 1 of t. f 1 of t is a random process. It is stationary, but I want to fix up the amplitude because for finding out f 1 of t, working back, let us say I have a compliant system d by l less than 0.2. I am looking for Morrison equation forces. When I look for Morrison equation forces, I need water particle velocity and acceleration in horizontal and vertical direction and of course, the current also.

To do that, I must know the amplitude as well as sea surface elevation. So, sea surface elevation is a value, which has an exceedance of the mean sea level or a mean value, which you are interested in working out now for a return period. I am looking for a value of h by 2 or eta, which will be imposing the load on a system, which has got a specific return period, which I am working at. Let us say, I have to look at that consideration and say, I have got an amplitude, which cause the force, which has an exceedance of probability of word, let us say 10 power minus 2 in a period of 1 year. What does it mean? I fix up a value and I say this value, which I am pre fixing, gets exceeded in a year only by a probability of about 10 power minus 2. So, I pick up that value as my load amplitude and imposing it get velocities and acceleration, water particle kinematics and apply it to Morrison equation, get my forces, do the analysis and do my design. Ok?

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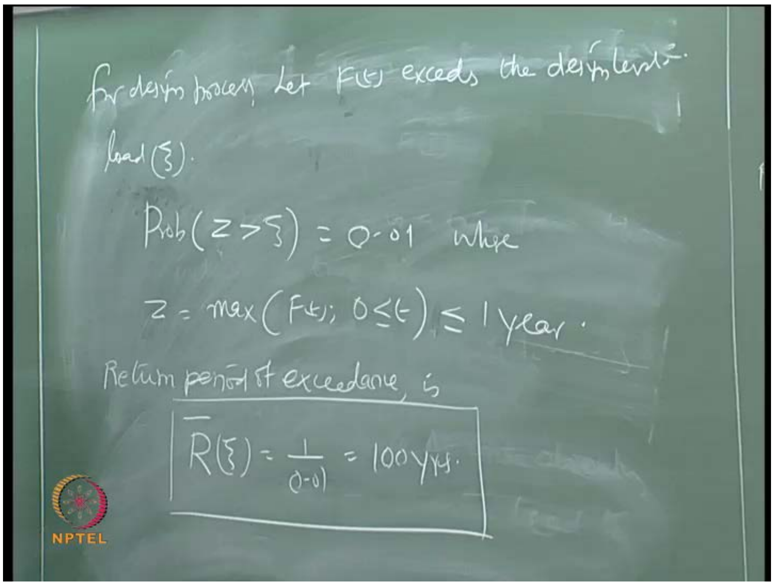


Now, for design process, let f of t, the load exceeds the design level load, let us say z. It is not damping. Some design level load. I have to fix up the design level load zeta. I am taking about the load and not the amplitude. From the amplitude, I get the load. Now, I fix up the load directly and my f of t may be f 1 f 2, whatever maybe, exceeds this amplitude, the desired load level or the design load level zeta. Then I can write probability of z exceeding zeta in my problem is about 10 power minus 2. Is it not? where the value z here is nothing but the maximum value of f of t. For t exceeding 0, I will write it like this, 0 less than or equal to t, less than or equal to 1 year.

Now, my return period of exceedance of f of t exceeding zeta value is r bar zeta, which is 1 by 0. That is, the 1 by probability. That is the equation we had, which is 100. Is it not? This is a typical return period for design waves in offshore structures. So, physically what does it mean is, the load which causes the design value on a system in an offshore structure has a probability of exceedance by its value itself by 10 power minus 2, in a given period of 1 year. Then what is this 1 year called? Because return period is 100 years, this 1 year period, which you are using in this calculation, is called reference period. Now, sometimes in literature, people also express return period in terms of risk associated with the loading. So, this is one case, where I am only checking the exceedance of the load with a specific amplitude value. Indirectly one can also look atthe return period of exceedance with respect to the risk associated with this. Let us see what is that.

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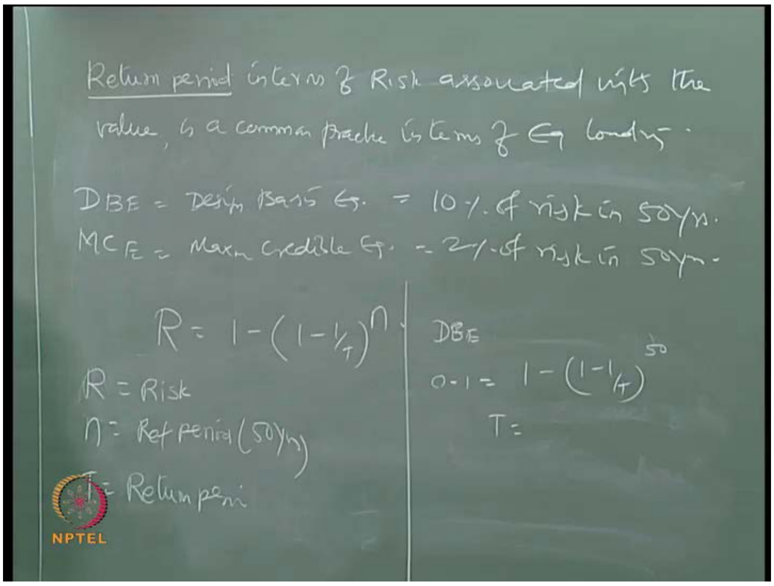


Generally, when we talk about return period computations in terms of risk associated with the value, so you must always understand return period is nothing but physically the probability of exceedance of a specific value or your variable exceeding a threshold value, which is pre fixed by you. There is always a risk. If you explain return period in terms of risk associated, this is the common practice in terms of earthquake loading. For earthquake loading, people talk only about the risk level.

Let us say there are two kinds of earthquake loading, which is generally applied in offshore structures. One is what we call d b e. This is called design basis earthquake. Another is m c e, which is called maximum credible earthquake. Design basis earthquake is that earthquake signal, which has a risk of 10 percent of risk in 50 years. Whereas, maximum credible earthquake, some literature will consider this as maximum considered earthquake. That is wrong. This is credible. Ok? Maximum credible earthquake. This is 2 percent risk in 50 years.

Now, I have the risk associated with each one of the level of earthquakes, which I am going to consider as one of my input loading in my structural system. What I am interested is not the risk. I am interested in finding out the return period of this earthquake in the life span of the structure or the link record, link well structure. So, this is simply given by then of n, where r in this case is the risk and n is the reference period in my example, in 50 years and of course, t is my return period. If I substitute for d b e, for design basis earthquake, I should say 0.1 1 minus 1 minus 1 over t of 50, which gives me t as, how much? What is the value of t? 474.06

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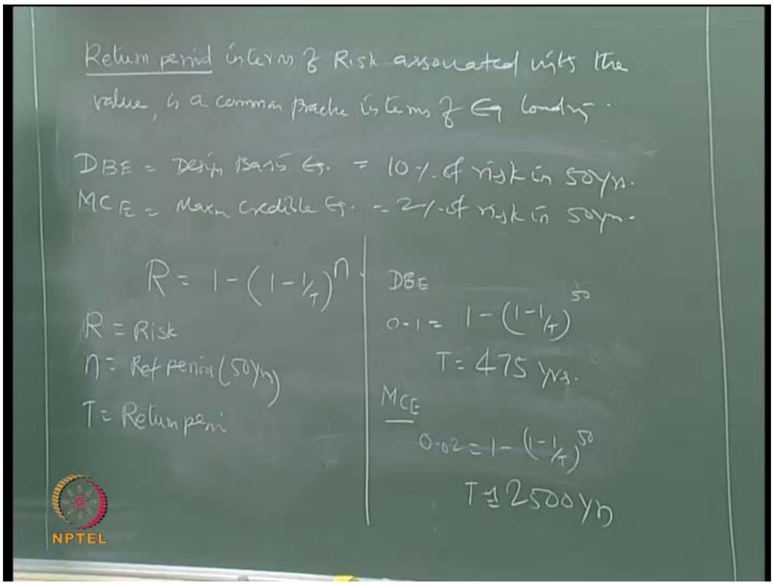


We will take it is as whole number and say, 475 years. Is that ok? Can I do it for maximum credible earthquake also, which is 0.02? 2475.4, 2500 years. So, one can always find out what is the risk associated with an event whose value is 100 years. For a reference period of 1 year, what would be the risk? You can easily calculate from this equation, right. So, one can always estimate return period based on the risk associated or return period based on the level of exceedance of probability of an pre accepted value zeta. Both ways are ok. Now, one may wonder why in earthquake engineering or earthquake loading people talk about risk. Why not in wave loading or wind loading people do not talk about risk. What is the difference? Why there is a difference between these two? In fact, if you look at the earthquake loading, earthquake loading is generally described in two scales.

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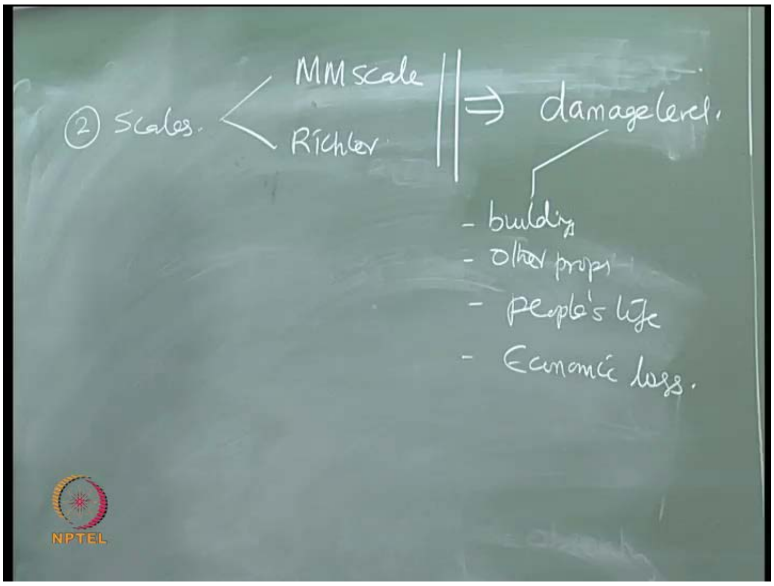


There are two scales at which earthquake loadings are generally described in literature. One is called modified Mercalli scale and other is called Richter scale. Both of these scales in turn will try to explain the damage level. The moment we say damage, it can be to buildings, it can be to other properties or it can be to people's life. It can be of course, economic loss. But, you would always observe in the paper release or the media release saying, whenever an earthquake occur, people always focus directly to this any of them. May be Richter scale of 8, maybe Mercalli scale, modified Mercalli scale of roman. These are all roman numbers. These are all in roman numbers. These are all numbers 5 7 like this. These are all numbers. So, Richter scale of magnitude 7 or modified Mercalli scale of magnitude 8 will always talk about, in general, damage associated with the people and loss.

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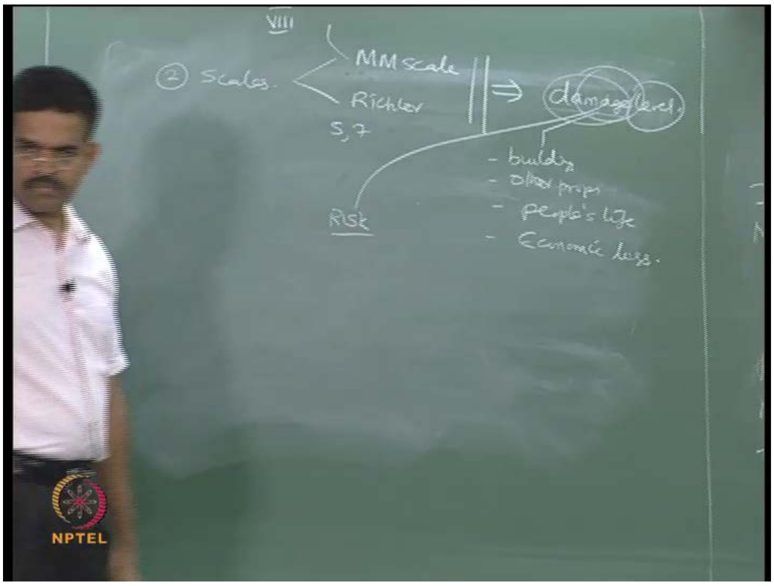


The moment I talk about loss or damage level, I am always interested in a term risk. Ok? That is why generally, in case of earthquakes, since the focus is on damage levels caused on any one of the parameters you see here, people talk about risk. Therefore, return period is expressed in risk in earthquake force. That is the reason.

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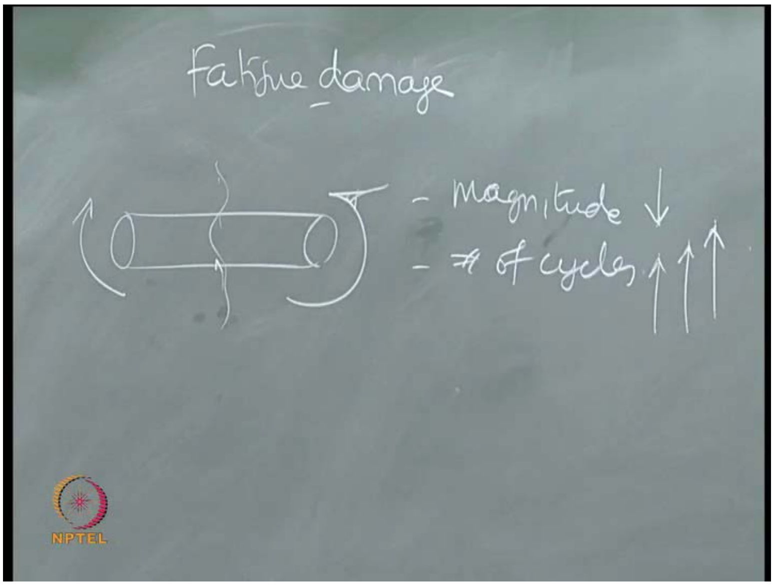


The next argument what we will see today is a brief introduction to fatigue damage. We all know fatigue is nothing but a cyclic stress. So, when I have a bar or any element, may be subjected to a couple, for example. Applying this at a specific cross section of our choice, not necessarily at the centre, anywhere of our choice, a specific critical cross section of our choice and keep on applying this at that specific section. There are two important things which you can observe from this. One is the magnitude of this load. Second is the number of cycles you apply this load. You will notice that, even for a lower magnitude, large number of cycles will make this material to fail.

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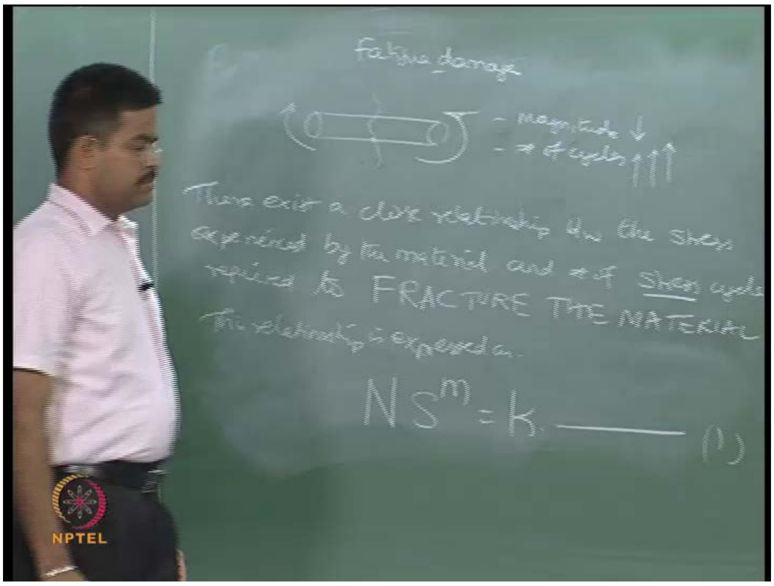


So, the magnitude may not be important. May be very low magnitude, but large number of cycles will make the material to fail. On the contrary, with a very large magnitude, very very low number of cycles, the material may not even fail at all. So, from an understanding of this nature, we know that there is a relationship between the stress experienced by the material. You may wonder, sir, we talked about load magnitude and why we  are talking about stress? I said, load applying on a specific area of cross section, so it becomes stress automatically and the number of, I should say, stress cycles required, now I got slightly a greater extent, required to fracture the material. My focus is not to simply keep on applying. I apply till the material is fractured.

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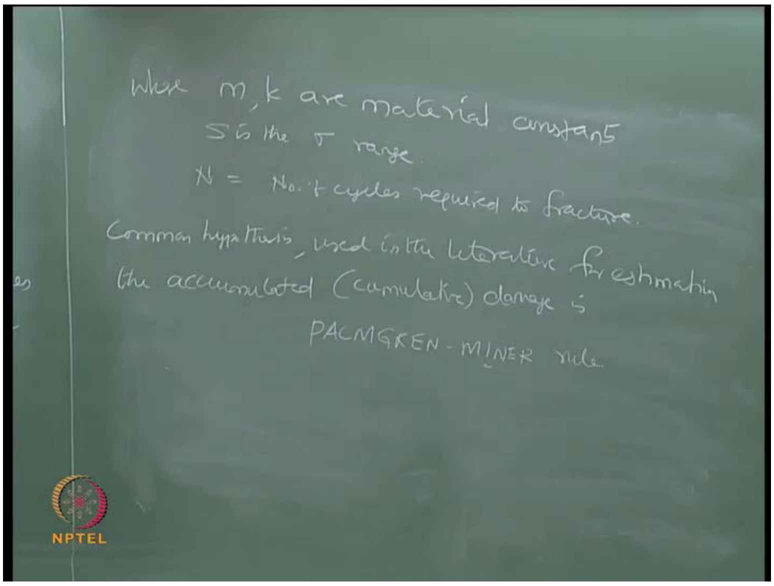


This relationship is expressed as n s to the power m as k, where m and k are material constants and s is the stress range, which we are bothered about. Of course, n is the number of cycles required to fracture. There is a very interesting and common hypothesis used in the literature. The more common hypothesis used in the literature for estimating the accumulated, I should say, cumulative damage. One may wonder, why physically this material will fail at this section. Whenever you apply any one cycle to this material of a specific low magnitude, when one cycle is applied, the material starts accumulating some loss because of the stress and it fractures because this damage is cumulatively occurring and that sum of the total damages makes the material to fracture. So, there is a common hypothesis available in the literature, which can always be used to estimate the accumulated damage, which is called the Miners rule, but we say Palmgren Miners rule.

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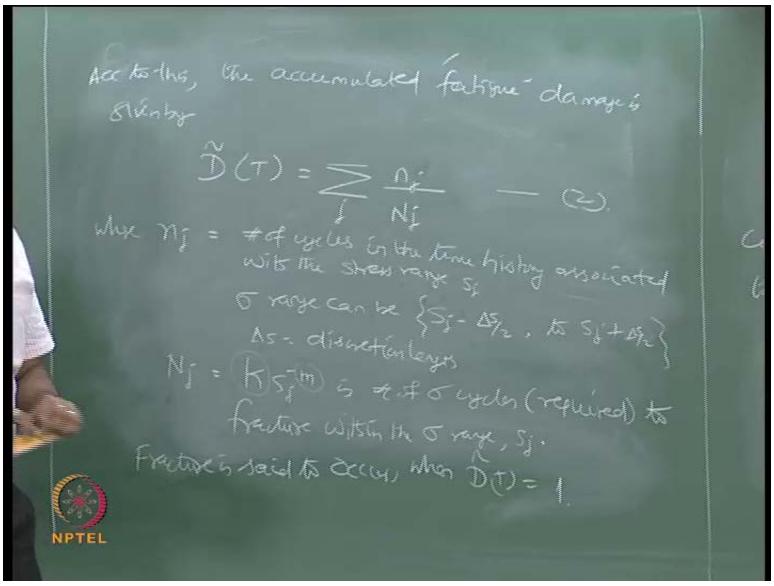


According to this, this hypothesis, the accumulated fatigue damage, in the first time the word fatigue is brought here because it is due to reversal of stresses. The accumulated fatigue damage is given by, you may wonder, how we are connecting stochastic dynamics to this. I will just connect it at the end of this lecture. It is given by d tilde of t as sum of j, which is n j, equation number 2, where small n j is the number of stress cycles. So, number of stress cycles in the time history associated with the stress range s j. Now, one has to fix up the stress range s j, which can be said as, the stress range can be, we can say some level minus delta by 2 to some level plus delta by 2, delta s of course. This is my range. So, I call delta s as my discretion length, s by 2 to plus s by 2, I get s. It is called discretion length, whereas, of course, capital n j is given by the same equation of 1.

I am just re writing the equation here, which can be said as, k s j minus m. Is that ok? Equation number 1. I am finding about n j phenomena. This is the number of stress cycles required to fracture, I am putting required here, required to fracture within the stress range s j. Within the stress range s j, I want the number of cycles. So, in this case, k and m are actually material constants. s j is that stress range, which I am talking about here.

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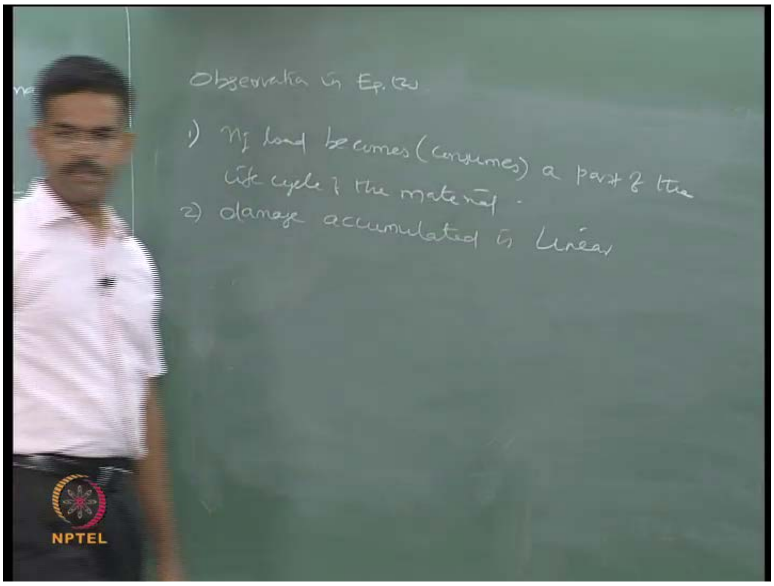


Now, when can you say the fracture is done or achieved or occurred? Fracture is said to occur when d tilde t is unity. Is it not? When they are equal, fracture has occurred. There are two critical observations in equation 2. Let us see what are they. The first observation is, small n j is the number of cycles in a given time is 3, which is associated within the stress range s j, which can be having a discretion length of delta s. It means, the n j load, what do you mean by n j load? The load responsible for causing the number of cycles within the stress range s j. That is called s j load. It becomes or consumes a part of the life cycle of the material. It consumes a part of the life cycle or life time of the material, Number 2, the damage accumulated in this model is linear. Now, one is interested to know from the accumulated or cumulative damage occurred to the material within a specific stress range s j. One is not interested at to know how many cycles will be responsible to cause this damage. That is one part of the argument, but what I am interested to know is, what will be the life time of this material or the structure or the member.

So, I must convert the whole argument of this failure into a life time estimate of the material or the member. Now, we understand here that the load cycle, which is responsible for causing this kind of damage is a random process. Now, connect this argument to stochastic dynamics and estimate the life cycle of this material or the member. Let us see how this can be done. Any questions here?

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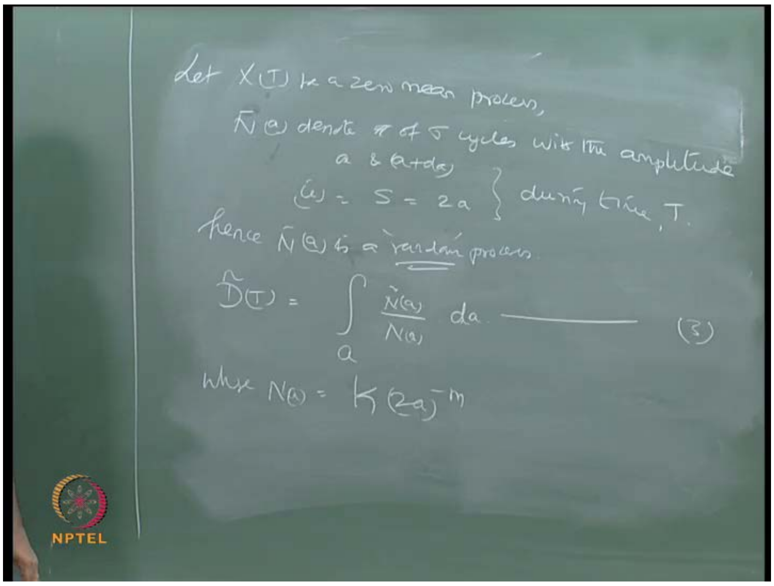


So, let x of t be a 0 mean process and let n dash a denote the number of stress cycles with the amplitude a and a plus d a. That is why we call this n bar a. That is, my s value is 2 a, during the time t. During the time capital t, this is happening. That is why it is x bar x of t and hence, n bar a is a random process. I already told you what is random. Random has got a fixed number. The question is, we do not know that number. That is why it is random. Now, the accumulated damage d tilde of t can be expressed as the domain of integral a of n tilde a by n of d. Equation number 3, where n a is as same as the first equation with the small modification because the stress range now is 2 a. Now, the expected value, because I am interested always for a random process, I am interested always the mean value of this.

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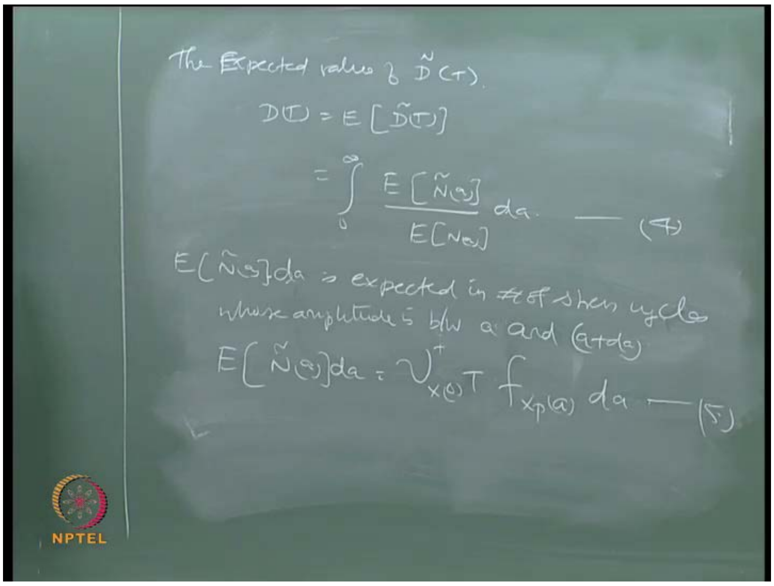


So, the expected value of d tilde of t can be given as d of t expected value d tilde of t, which from a conventional expression, integral 0 to infinity e tilde of, sorry, e of n a by n a d a. Is that ok? Now, the argument here is, call this equation number 4. Now, the expected value of n tilde of a d a is expected value in n number of stress cycles, whose amplitude is between a and 2 a. Is that ok? Or, not a and 2 a, a and a plus d a. That is my amplitude. That is why d a is here. So, I can write this value as v of x plus 0 of t f x p of a d a, where F of x p a d a is the relative number of peaks.

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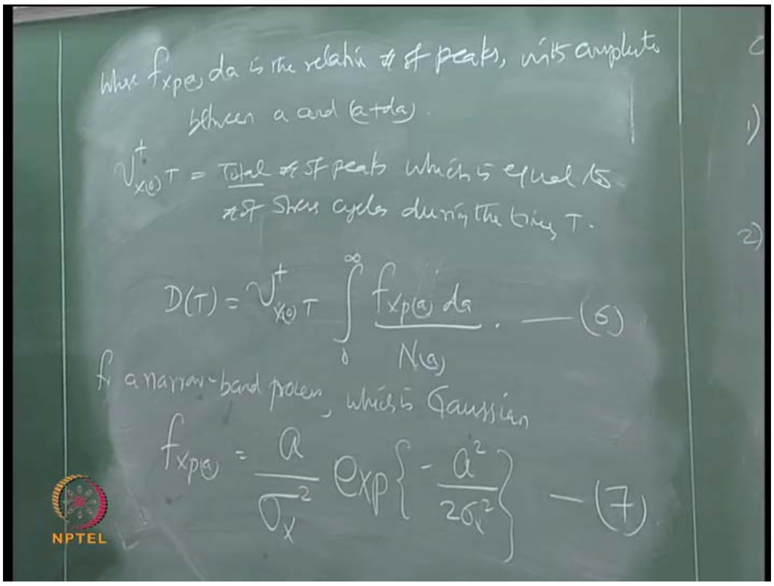

It is a relative number of peaks with amplitude between a and a plus d a and v x naught t plus is the total number of peaks. So, total number of peaks, that is why we are saying plus, you are adding it up, which is equal to the total number of cycles, I should say. Should I say stress cycles, during the time t. Now, I can write d of t as v x naught t plus integral 0 to infinity f x p a d a by n of a. Equation number 6.

Now, the period for a narrow band process, that is f x p of a. For narrow band process, we already spoke about what is a narrow band process and white noise in the last lecture. For a narrow band process, which is Gaussian, because Gaussian distribution has a specific characteristic of mean and standard deviation. So, f x p of a, you may wonder why I am taking an example of a narrow band. I already told you why it is narrow band in the last lecture. Why Gaussian because generally the expression of this distributive has a Gaussian relationship. So, f x of p can be given simply by a sigma x square exponential of minus a square by 2 sigma x square. I can call this equation number 7. By substituting 7 in 6, because I have x p of a, substituting 7 in 6 and I already have equation for n a. I can rearrange them and I get a new equation. Please do that. I will write the equation here. I will remove this.

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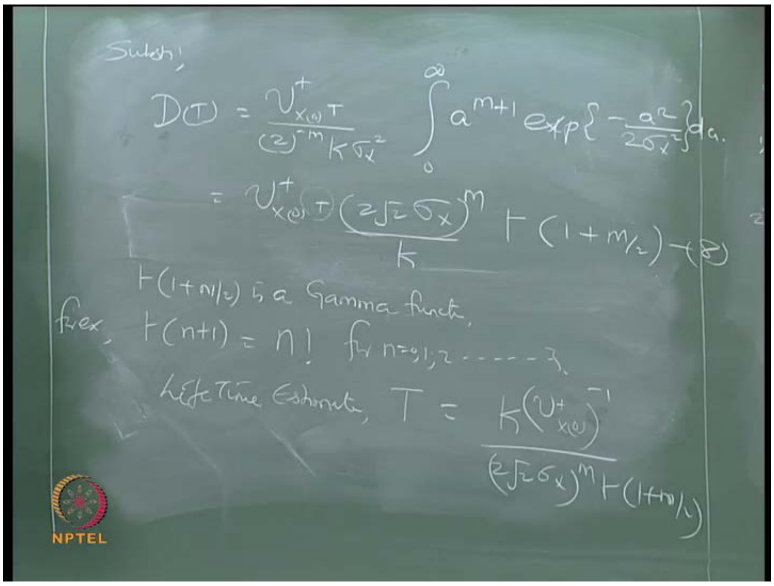


So substituting, I can now write d of t is simply given by v x 0 t plus by, I am writing first step, 2 to the power of minus m, because there is a 2 a there, k sigma x square sigma x square is here and of course, there is a here. I will take it inside, which will be integral

0 to a m plus 1. A is here. Already I have a m inside in my n of a, exponential of minus a square by 2 a sigma square.

Simplifying it further, which can be v x naught of t plus 2 root 2. You can evaluate this integral for a variable 0 to domain 0 to infinity, sigma x m by k. These are all material constants k and m and I have got a gamma function, which is 1 plus, m by 2. I call this equation number 8. This is called as a gamma function. For your understanding, if you really wanted to know gamma of n plus 1, it is nothing but this actually equal to n factorial for n 1 comma 2 and so on. It starts even from 0, for example. Now, I am interested in working out the life time estimate. So, the life time estimate can be computed. Because, in this case, the only estimate which I am interested is, the life time estimate t. This is the estimate I want.

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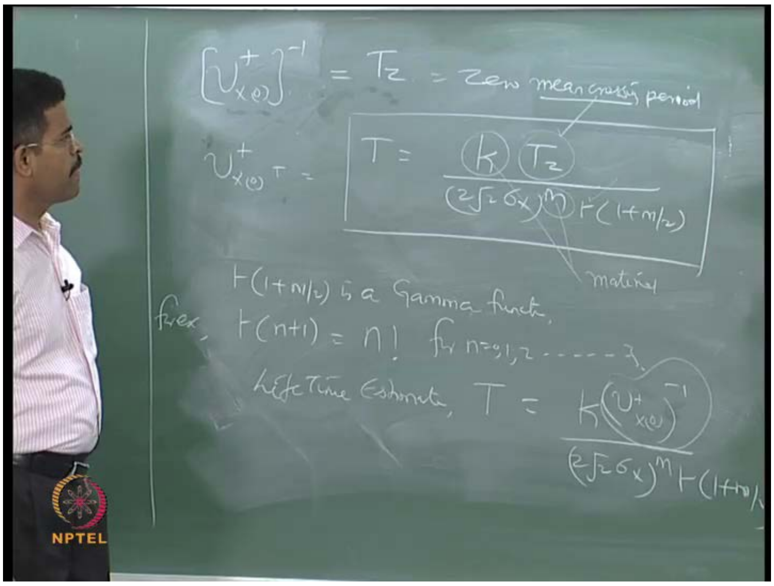


Cumulative time is already known to me from the minus rule. So, I can say, this can be given as, I can write down, t, I am working out this t, which is k v x naught t. Sorry, minus 1 by 2 root 2 sigma x to the power m gamma function. Now interestingly, if you look at this function, I will remove this. If you look at v x naught of plus is nothing but 0 mean crossing period actually. Because, this is, if you look at the definition of v x plus, turn back a little pages, v x 0 of t of plus, is nothing but the total number of peaks, which is equal to the number of cycles or stress cycles during the time period t. So, this will give me the 0 up crossing period.

So, I can now say my lifetime estimate will be k t z by 2 sigma 2 root 2 sigma x of m gamma function of 1 plus m by 2, where m is a material constant. Instead of up crossing, let us say this is 0 mean crossing period. So, one can estimate the lifetime (()). k and m are material constants and t z is observed from the data. Since, you know m, you can find the gamma function of that and the variance of the standard deviation of your load is known to be, so it is a stationary process. We have assumed as Gaussian in this problem. It is known to me. So, I can easily find the lifetime estimate of this. That is what we wanted to discuss here.

So, in the next lecture, we will talk about the modal response. May be one or two lectures we will do some example problems also in modal response and then we will wind up. By this Friday, I think we will wind up the lectures. That is the last lecture. This Friday we will have a last lecture. I will try to give a quick summary of this on Friday, if it is possible because large volume has been covered in different segments. We will try to give it on Friday, if I am able to complete my lecture on Thursday. So, I have one class tomorrow and one class on Thursday, which I will complete the modal response derivation and then the problem applied to modal response. Then on Friday's class, we will summarize module 1 2 and 3 quickly, that what we have learnt and what has been applied so for. That will be the end of the third module lectures. Any questions?

So, we have learnt two things today. One is the return period of exceedance of any specific value and return period can be computed based on the amplitude or based on the risk associated with the force level. Different segments, both we have seen. The other is from the fatigue damage or conventional minus rule, congruent minus rule, how can we estimate the life estimate of any variance specific material or a member, whatever maybe in the case. So, this will help us to find out the fatigue damage estimate and it is also based on my stochastic data. Because, now fatigue based or let say, stress based design levels are also coming up, which we called as performance based designs, where stress level can also be one of the important performance index in a given design parameter. So, this is an important item, which we must understand. Of course, I have only given just an introduction to it. It is a long term process. It is not going to happen just in half an hour lecture. It itself, fatigue damage estimates itself can be a full course in fracture mechanics. So, we are not touching on that detail. Just to give an idea how this can be computed. Any questions?