Dynamics of Ocean Structures
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Module - 01
Lecture - 09
Characteristics of Single Degree - of - Freedom Model

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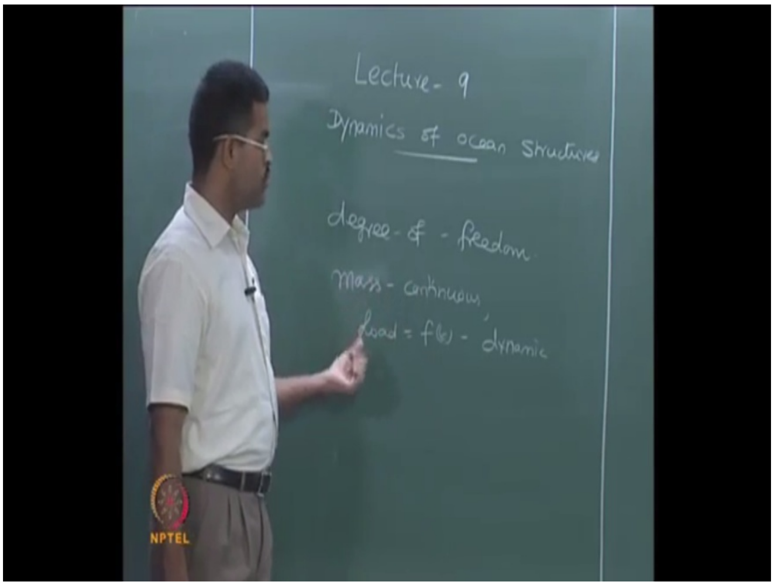

So, in the last lecture we discussed about the degree of freedom, we have said that it is the number of independent displacement convicts, which can express the significant contribution of inertia forces in the given system. We have also very clearly said it is not the points where the mass is lumped because we have shown couple of examples where the mass can be lumped still at one point, but the structure can have more than one degree of freedom.

For example, a t l p For example a tri sera top extra, so it is number of independent displacement components that will explain the significant contribution of inertia force. Since, we said degree of freedom is associated with the inertia and inertia is coupled with mass. Therefore, we said how to discretize mass, so we said that mass can be either continuous or it can be lumped. When the moment I say mass can be continuous the problem becomes slightly complicated because than we all know that load is a function of time because it is dynamic, as well as the inertia forces which is cast from the continuous mass will also vary with the span of the number.

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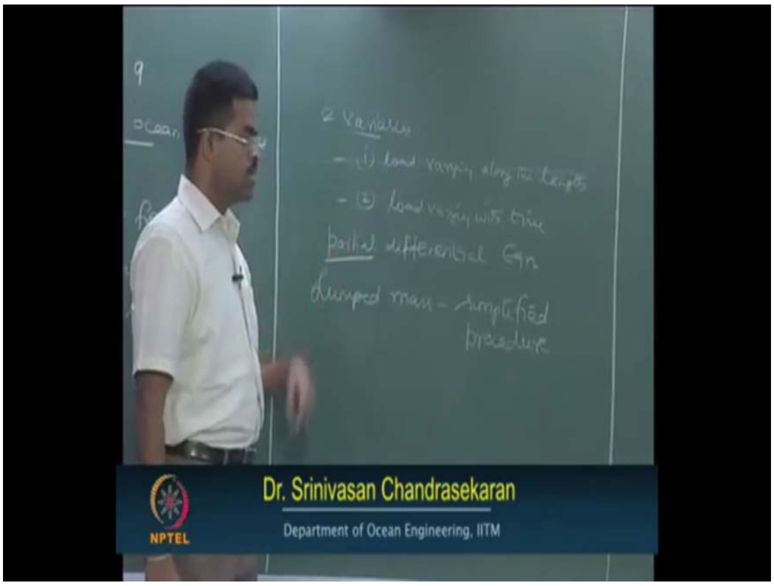

So, there will be two variables now if you take a continuous mass system one will be the load varying along the span, or along the length of the member because mass is not lumped at one point, but it is continuously spread for the entire length of the member. The second variable of course, the load will vary with respect to time. So, once I have two variables in the given system, then I should go for partial differential equation to explain my equation of motion. So, this is complicated that is why we generally say the lumped mass discretization is a simplified procedure, that is the reason because here one of the component which is varying along its length.

For example, this is eliminated; we have said that the mass is lumped only at specific discrete points, therefore the variation of load along the length is eliminated from the discussion. We will only look for the time variant of the load which is purely of course, dynamic in nature therefore, the analysis can become simple. So, when you have a lumped mass system then the analysis is simpler compared to the continuous mass system.

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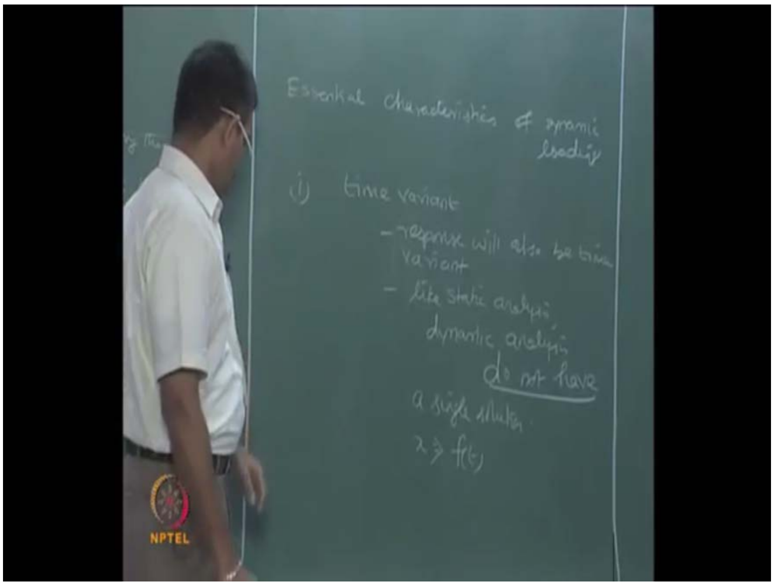

Now, let us quickly look at what are the essential characteristics of dynamic loading_ we already said that in the last lecture just to recap the essential characteristics of a dynamic loading, foremost the load will vary with respect to time. So, it is going to be time variant, as you understand the load is time variant the response will also be time variant. Hence, like static analysis, like static analysis dynamic analysis do not have a single solution, it is time variant because it will be a function of time x the response will be time variant. So, there is one important characteristic of a dynamic loading. The second important characteristic of a dynamic loading will be coming from the system.

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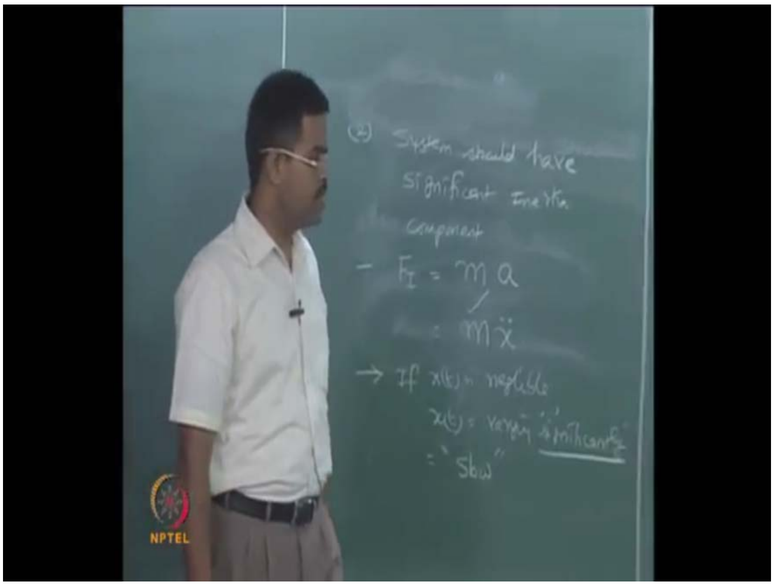

The system should have significant inertia component, what do you mean by this? It explains like this, we already know the inertia force is simply given by mass into acceleration. If you take x as function of time as a response of the system I can say the inertia force can be mass into acceleration of the system, there are two components here. So, if the mass is significant, which will make the inertia force or the inertia component significant, I must do dynamic analysis.

Alternatively if the response is negligible that is x of t is not varying significantly variation is there, but it is not significantly varying. So, what we say is crudely the system has slow response, if the system has slow response even though mass of the system may be predominantly high, we need not have to do dynamic analysis. There is no need to do dynamic analysis. So, we are not explicitly saying mass of the system should be significant, we are saying the inertia component of the structural system should be significant, which is having two components within itself. One is the mass of course, other is acceleration that means the mass is moving.

So, if we have the time variant of the response or the acceleration is not varying much significantly with respect to time, we may not do a dynamic analysis. A classical example maybe let us say dikes or coastal protection structures, which are massive, but they are fixed to the sea floor or they are fixed to the bed. Therefore, there is no need that we have to do dynamic analysis for them because the variation or the response of the system in respect to time domain may not be significant, it will be there, but not significant. So, in that way people resolve to what we call Queasy static analysis more simple static analysis it is an approximate method. So, these two are considered to be essential characteristics of a dynamic loading any questions here. We move on further now I want to mathematically model a given system to do dynamic analysis.

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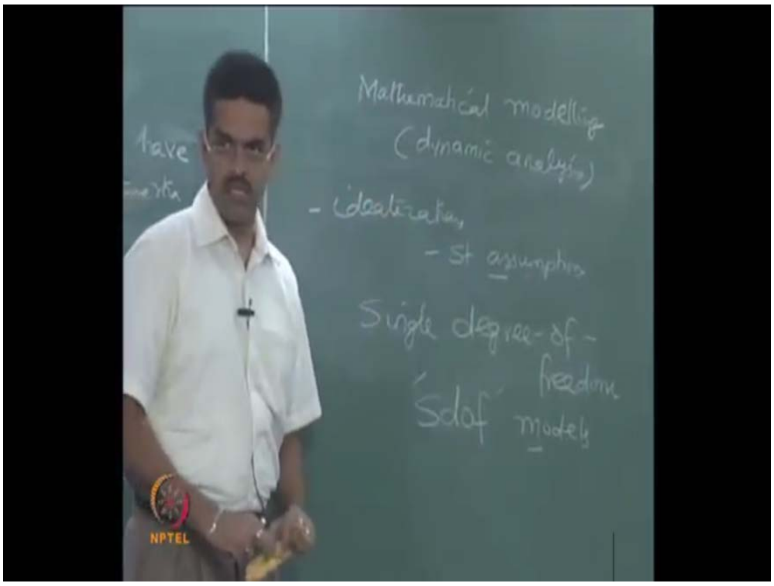

So, the movement I say, I want to do mathematical modeling of given system to do dynamic analysis. So, the moment I say I want to do a mathematical modeling of a given system for doing dynamic analysis then I have to make certain idealizations, what we call as structural assumptions. The first idealization what I will start with the dynamic analysis is I will pick up a system, which is having only one degree of displacement what we call single degree freedom system, or in brief SDOF models.

So, we will again recollect the degree of freedom is independent co-ordinate explaining the inertia component in a given system and it is not the point where the mass is lumped. So, it is talking about the displacement, independent displacement components. So, in a single degree of freedom system, I must restrain the motion of the mass in only one displacement co-ordinate. So, how I do that and what are the basic features of a single degree mathematical model.

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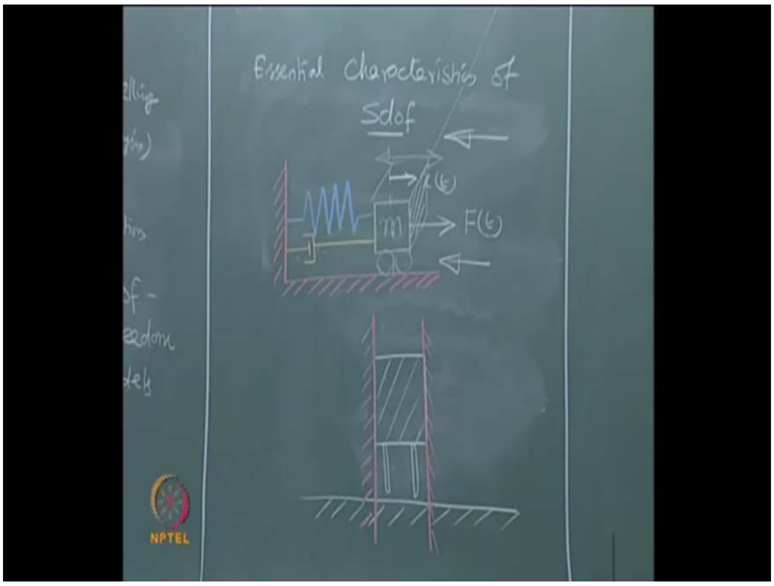

A single degree freedom system has essential features or essential characteristics of a single degree freedom system and referring this as SDOF this is single degree of freedom. So, classically the single degree freedom system will look like this, I have a mass component, I do not want this mass component to move in the vertical direction, I am putting a constraint. So, you may wonder if I am putting the mass component exactly resting on the floor, then there will be some resistance created by this mass component by the friction of the floor. So, I want to make it easy, so let us say I put a wheel here and enable the mass to move only in this direction.

Of course, to and fro now as long as we do not connect this mass to any of my fixed support system, the mass will be infinitely keep on moving and I will not get what we call as a vibration. So, I want to connect this mass with a support using some component and I also want to represent the effect of the friction, generated by this mass while moving by some ideology I put another component. Now, above all if the whole system is in position, the mass will not activate or initiate to move at all. So, I need some external force at least to start the movement of the mass.

So, I apply that and call that of course, as function of time which is the dynamic problem. And once the mass starts moving I should be able to measure the response of the mass I am interested in measuring the response of the mass. Since, the mass is continuously moving I should measure the response with the reference of certain data, I take the datum as c g of the mass itself and say with respect to this point. I will measure the response of the mass called the function of time because we just now saw in a given system which is essentially dynamic in nature, when the force is time varying the response accounted by the inertia component or the mass component of this will also be time varying, I am measuring x of t.

Now, in the whole exercise this is an idealized model of a single degree freedom. Now, you may wonder that how this is a single degree freedom system? Look this problem from the front face of the board you will see the mass is moving in this direction, so it is one degree, but where is the guarantee that the mass is not moving normal to the board. You are looking from the board side here, you are able to see the movement of the mass here, but where is the guarantee the mass is not moving the normal side. So, let us look at look at the view of the whole problem in this angle.

So, I looked at this in this angle I will again plot the mass like this, it is a three dimensional model let us say, the surface what I am hatching here is the surface what I am marking here. And the wheels what i see here i am just trying to mark slightly in a different manner just to make you to understand that these two views are not same. And of course, this is the floor this as same as this floor and I will not see anything else because these are all hidden behind, I do not see any anything of them right when I see from here.

I am imposing an additional constraint to this mass saying that the mass is not moving anywhere in this direction, is it clear? So, if you put a mass locked between these two surfaces and of course, resting on this surface by a wheel and try to pull the mass this way now, the mass will move only in this direction. On the other hand if you see this from this view parallel to the board the mass will now enable movement only in this direction, which I have shown here. And the mass will not have any opportunity or any liberty or any freedom to move in the direction normal to x of t, is there a possibility the mass can jump? Is there a possibility the mass can jump?

We are assuming that the mass internally connected through a wheel and resting here and we gently apply this force only purely in the direction of motion of what we are measuring. There is a possibility that if you apply this force angular and try to lift the mass and release the mass may jump. So, the direction of motion of the force application of force is exactly in the direction of motion of the body. So, the mass is now made to move only in one direction ideally in the other direction it is restrained. Now single degree of freedom therefore, it is an idealized model of…

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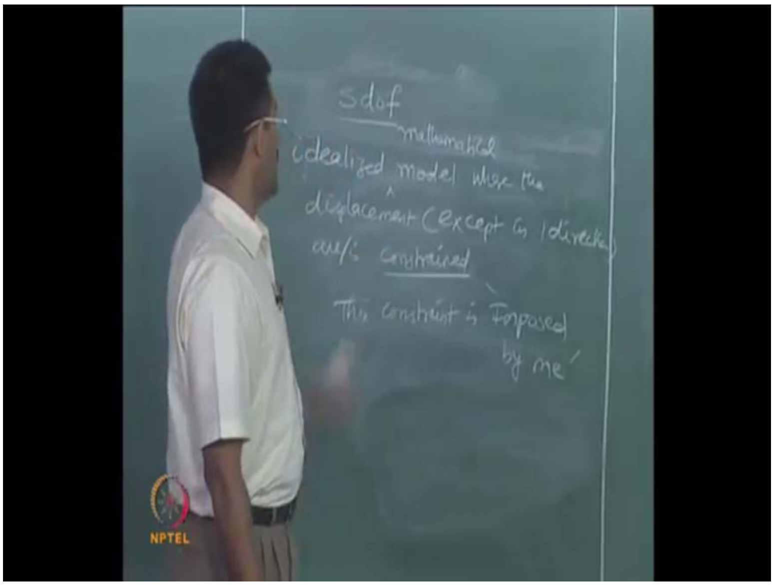

Single degree of freedom is therefore, an idealized model the moment I say a model, it is not a physical model, it is a mathematical model. In an idealized model, where the displacement except in one direction is whatever may be constrained, this constraint is imposed by me, I am imposing this constraint it is not existing I am imposing this constraint. Why I am imposing this constraint? Because I want to be assure that the mass will move or get displaced only in one direction, but in general if we have any system like this mass will have freedom to move in any direction the mass wants.

So, single degree of freedom or those mathematical models which impose constraint to displacement directions, in all other degree except one of your choice that one can be translational one can be rotational also. But only one not necessarily translational, it can be rotational also it means how many search idealized mathematical models in single degree freedom exist in the literature is this the only model exist or any other models exist?

Before we see those respective models which exist in the literature, which impose constraint on any other degree or direction of motion except one, we will first understand what are the essential characteristics of this idealized model. And what actually it will represent in physical terms, because they are important for me to solve the problem. Any questions here? Any doubt? any questions here will remove this. If you look at this figure which I have drawn I will call this as figure one, and let us say figure two if you looked at figure one.

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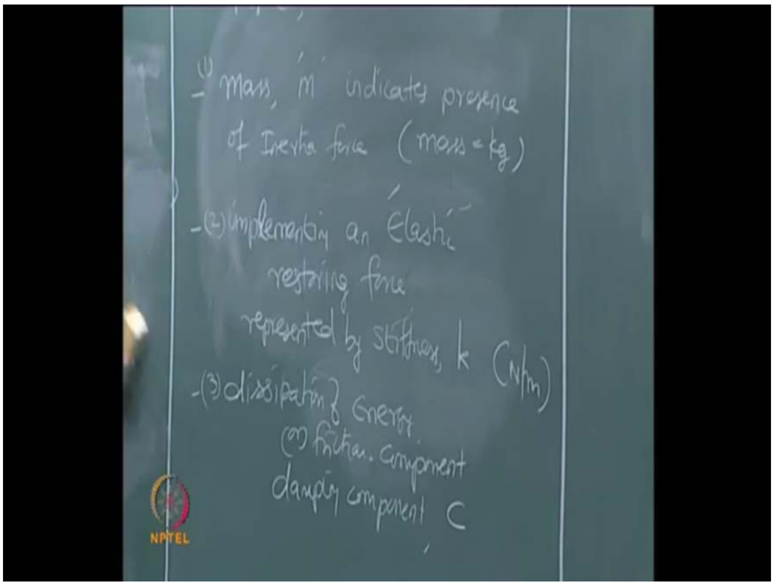

Mass represented by m indicates presence of inertia force in the system, the spring attached to the mass here having some stiffness, let me put that stiffness as a. So, actually it is not stiffening the moment of the mass it is implementing an elastic restoring force, it is restoring the force, what does it mean is when the mass is made to move towards right the spring attempts to bring the mass back to its position. When the mass is made to move to the left, the spring pushes the mass to the right or to its initial position. It means it is restoring the original position of the mass, and it remains elastic.

It means this element will not impose a permanent deformation or displacement to the mass. This is what we call as represented by stiffness indicated by a letter k unit be Newton per meter. And unit will be in this case mass units will be in k g. When you try to move the mass towards right, the spring tries to bring the mass to its left, it restores the position of the mass. Now, when this mass is being moved to and fro, there is a probability that in addition to the force exerted by the restoring component, the movement of the mass is also attempted to be depleted or stopped by a friction generated between the wheel and the surface.

So, there is some dissipation of energy or a friction force, which is applied to this mass that is indicated by a damping component, which is indicated by c there are many models about damping, we will talk about that later. It can be velocity proportional and so on we will talk about that later. So, the first component of an essential characteristic of single degree will be mass, second will be stiffness, third will be damping and of course, fourth will be…

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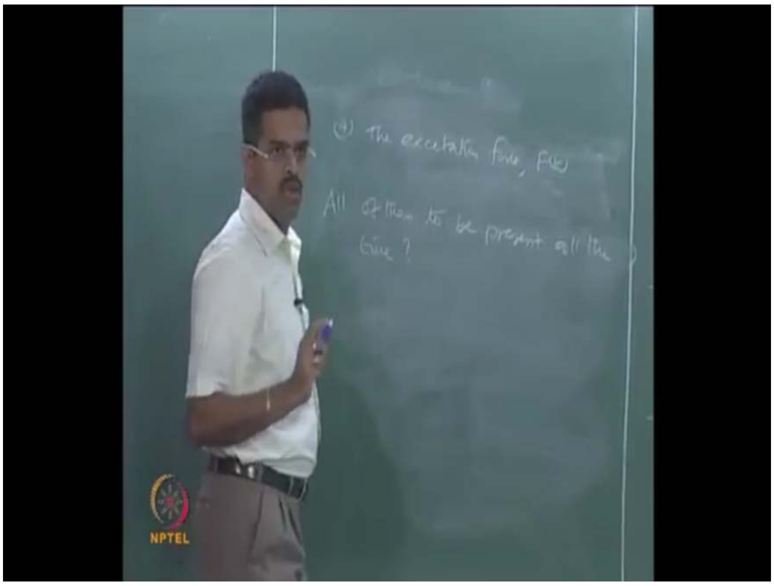

The excitation force indicated by f of t which is a function of time. Now, the question comes if these are all essential characteristics of single degree of freedom system, do we need all of them to be present all the time I mean all the time you want m to be present, k to be present c to be present, and f of t to be present. So, literature in dynamics classifies them slightly in a different manner, they say the method by which you want to do the analysis accordingly you can say that which should be present which should be absent.

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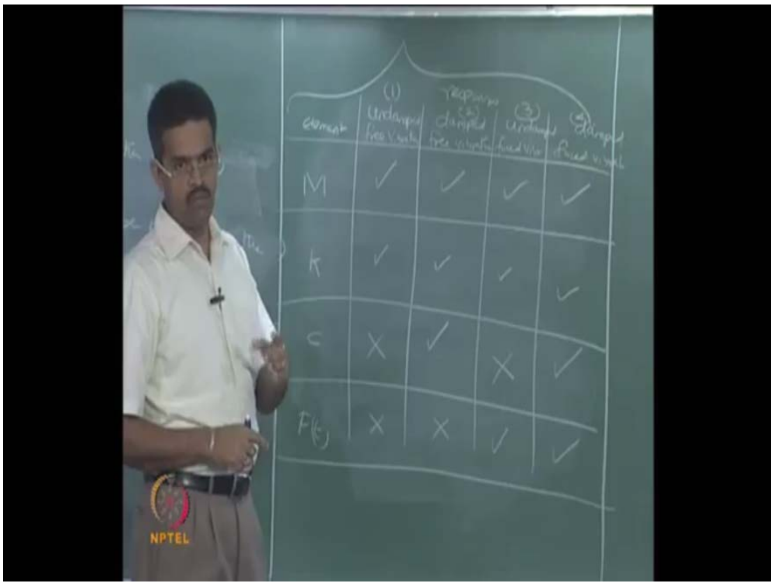

So, let us say this is element or now we talk about the responses because I am interested about the responses of the mass for a given loading; the responses can be categorically divided- the response can be un damped free vibration, that is one category of the response. The second can be damped free vibration, the third category can be un damped forced vibration, the fourth can be damped forced vibration. Now, how do we understand them symbolically, let us say my contents which are present to be will be mass, stiffness, damping and f of t this is what I have. So, if I try to see which will be present and which will be absent under this category we can quickly make a matrix to understand this.

For doing a dynamic analysis the essential characteristic of dynamic analysis is it should show significant presence of inertia component, inertia component generates from mass in all category whether you do analysis in any format you must have mass present in the following. The mass should be connected to my support system to restore its position to its normal seek, the restoration can be done by two ways. One can have an elastic restoring component present in the system can also model the friction present in the system as a damper.

Now, there is an option- if we have got two options, can one have at least one of them in the system. So, the basic model says always have the elastic restoring component in your analysis, it should be always present. Now, comes un damped so damping is not there un damped, damping is not there damped damping is present, damped damping is present.

You may wonder that if you do not have f of t which is a function of time which is acting externally on the body, how will I apply dynamic analysis to this body? Because essential characteristic of dynamic loading is force should be time variant and inertia component should be significantly present in the system.

So, we have proved the second one inertia has significantly present in the system, but do we have to say that the f of t will be always present in all the cases. Now, there is a classical difference here in dynamic analysis, people say do what we call free vibration. What does that mean is, you apply the force initially and then release the force instantaneously. So, you have to activate the moment of the mass, the force may not be present always.

Now, the question here what I am asking is, do all of them are present at all the time? that is the question what I am asking; I am trying to answer this matrix. The moment I see free vibration I need not have f of t present all the time the movement, I say a forced vibration I want f of t to be present all the time. So, you may please do not get confused that if f of t is not there how the body or the mass will actually get excited, the question here asked is do we need f of t to be present all the time during the analysis, there is a question asked. So, initially if you want f of t to start the motion of the mass then you stop applying f of t to the system, the mass will keep on vibrating at its own characteristics.

So, the response analysis for a single degree of freedom system has got four classifications, un damped free which we will see first then damped free then un damped force and damp force same all. So, this will give me a clue which components will be present and which components will be absent in my analysis. So, the name itself can give me a clue or when you look at any literature or any paper, which talks about free vibration etcetera it is understood that f of t is not there.

Remember all of them are essential characteristics of a idealized single degree freedom system model as we see here, but dynamic analysis classify them into a matrix like this. Now, why this has been done what is the necessity for doing this classification, why we should do this? Why it is to be necessary that this classification or the study on dynamic analysis response of a single degree should be done in four different analogies? What is the necessity? Any idea? Well that is a very important question if you do not understand the reason why to calculate.

Student: The natural frequency.

Natural frequency can be calculated in, do you mean this can be calculated in all the the analogies? It is necessary because we have not spoke about the natural frequency yet, because when the body vibrates it is vibrating at specific frequency. We are going to speak about that in the next component of the lecture. So, a natural frequency, what he is trying to say is, the analogy is there because I want to estimate the natural frequency or vibration of the given system therefore, I need it; that may not be the right answer. There is a very important categorical understanding and compulsion why I have to divide this into different components, very important. Let us try to understand this then we will move forward, let us talk about the f of t.

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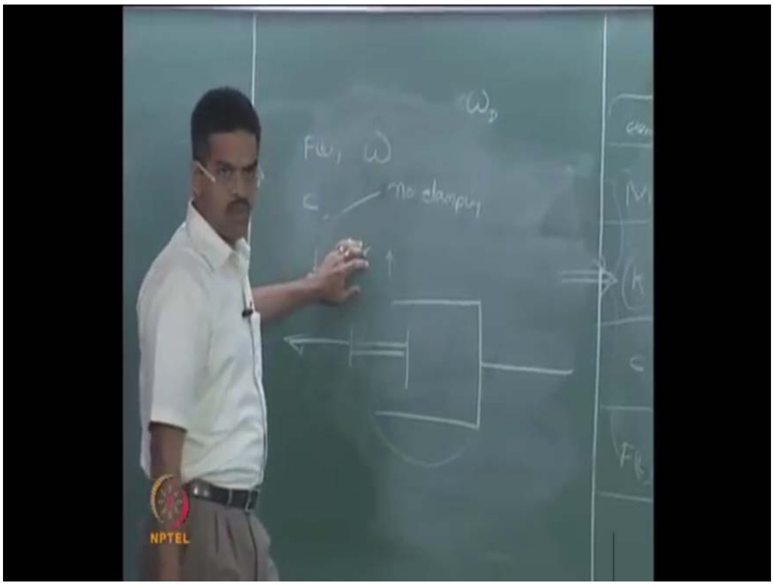

The moment I say a forcing function is varying with time, it will be the oscillatory motion or a vibratory motion the force will also be oscillatory. The moment I say oscillatory this will have a specific frequency at which it is vibrating, suppose I eliminate this in my study I will not see the influence of the forcing frequency on the natural frequency of the model. Suppose, when I include this in my study I will know the influence of the forcing frequency on the natural frequency of the model.

So, distinctly f of t present f of t absent has got a very distinct meaning in the dynamic analysis, talk about damping because we talk about un damped and damped systems. If we eliminate damping, you are not including any loss in the material or in the displacement caused to the mass either because of friction, or because of content of environment present in the system. If you include damping you will know the influence of this on the response, how do you know that exclude damping get the response, include damping get the response; compare them, you will know what is the influence of damping on the response. So, damped un- damped will have a significant understanding in the classification of dynamic analysis like this. Now, I have a question why stiffness should be always present in my model? Sorry it will restore.

Student: It is property of the material.

No, no ,let us answer one by one quickly is this rigid body cannot vibrate, but I am asking about, why stiffness should be there rigid body cannot vibrate then the rigid body is remaining static no dynamic analysis to be dynamic because I already said when x of t is not significantly varying that there was no need to do dynamic analysis. So, rigid body means it has to vibrate, then only dynamic analysis is important for me. So, the question is that answer may be or maybe it is ruled out, any other answer.

Student: It is a property of a material which cannot be concluded.

You mean to say the stiffness is the characteristic of the material, it is an elastic restoring force present in the system, it may not come from the mass component it can come from some other supporting component of the system also. We do not know that because of the body is vibrating. Suppose I eliminate k and give c only still also the body will vibrate,

Student: ((Refer Time: 34:44))

If there restoring force is removed I have a damper, you understand how does it work this symbol how does it work, it is very simple to understand I have a cylinder. Let us say a piston there is a fluid inside this when I am connecting this to a separate piece and this to a separate piece, the relative motion between these two will be controlled by this fluid inside. No, not necessary, not necessary it will not be equal to f(t) not necessarily it can be equal to f(t) this will also restore this will also bring back because the fluid will have a capacity to bring back its motion or damp the motion. For example, do not try to imagine you are pulling the body; you are pushing the body, how can you cast vibration.

Student: Harmonic motion.

No, it is not depending up on the characteristics of the motion at all.

Student: Then we cannot analyze the natural frequency of the system if.

Why?

Student: Sir if only damper will be there then it cannot means vibrating the natural system natural frequency.

You mean to say it will vibrate in a different frequency or it will not vibrate at all.

Student: No sir it can vibrate, but slowly it is frequency can change.

That is fine because it will vibrate at specific frequency called omega d, which we call damped frequency we are not talking about that here fine frequency will change, but when frequency is there it is vibrating. But there is a basic problem with this, talk about this elasticity I am talking this as elastic restoring force will this guarantee an elastic restoration? It will not, restoration may be there what does it mean at any given point of time damper cannot ensure return of mass to the original position at all. That is what elastic is or non elastic is, it will create a permanent displacement to the mass I will never know, what was the initial position of the mass after lapse of time capability. Whereas elastic means it will restore the mass even after infinite time expected to restore the mass of the original level of reference or frame of reference.

Therefore, k or the component of stiffness which is indicating elastic restoring force in a given system, should always be present for your analysis. Adding to this when you have only the damper and not k then obviously the damped frequency will be different from the natural frequency. Therefore, we cannot capture the natural frequency or vibration of the given mass agreed. So, each one of these distinct divisions in dynamic analysis have very interesting consequential meaning, why they have been divided. Now, the question comes, what are those idealized single degree of freedom systems, where only one direction of displacement is imposed on the system remaining all are constraint. One simple example is spring mass system.

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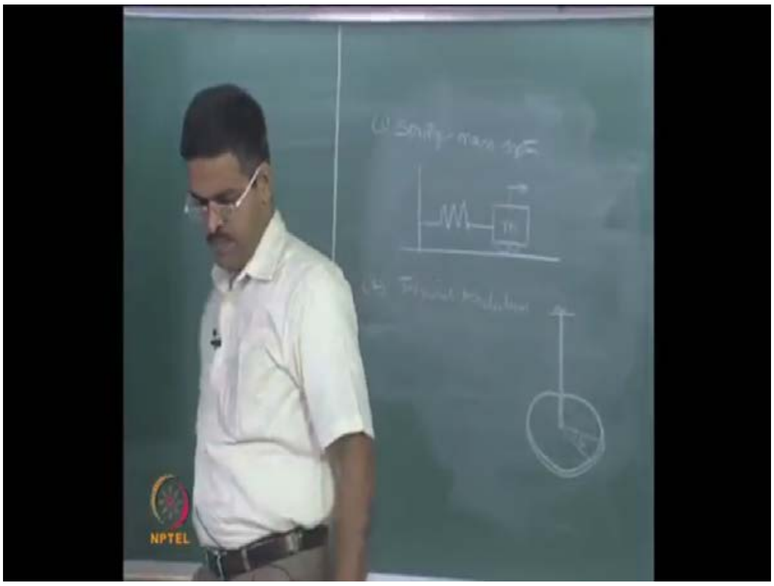

This is a single simple example of spring mass system I am not putting a spring mass dash box system. Now, this can move only in one direction as we saw there, the second can be a torsional pendulum. So, at any given point of time the pendulum will have rotation theta, which can be moved with respect to the vertical axis. I think you people have something very different or you both, please leave the class; before attending next class you come and meet me. Both of you, please leave the class. If you do not leave I will stop the lecture. Before attending come and meet me personally.

Because I think we are teaching to people who wanted to learn, we do not want to teach people who are Einstein’s born right, if any Einstein’s are born here please leave the hall you can disconnect and discontinue with the course, I am not interested. We will stop the lecture today we will see tomorrow next class.