Dynamics of Ocean Structures
Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Lecture - 11
Numerical Examples (SDOF)

Now we will talk about the eleventh lecture on module 1, where we will solve some numerical examples on single degree freedom system, also help you how to write the equation of motion for Multi Degree Freedom System using different techniques. Just for the benefit of all the readers let us quickly summarize what we have done so far, in about few minutes. We started with the matrix of presence of mass presence of restoring components k, presence of damping force which is dissipating component and the external agency on the external force.

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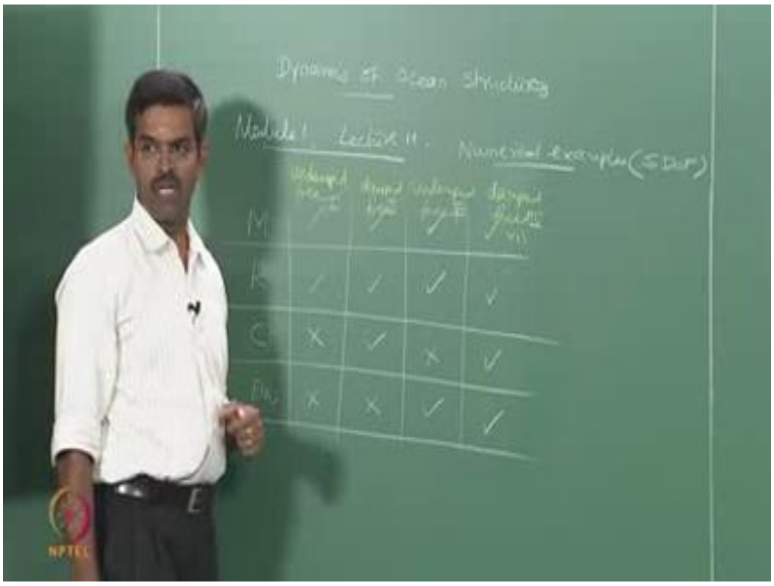

We said that in all cases, mass must be present and restoring component should be present. There are 2 reasons for this; 1 mass is always present in an offshore platform because, essentially in the initial conceptual design it was actually gravity based flat forms subsequently the top side details are so massive. Therefore, mass presence will be always; obviously, there in all type of structural systems. And generally offshore platforms are design in such a manner that, 1 must get a very commendable restoration capacity or recentering possibility should be there, bead gravity base platform, bead jacket platform, bead complaint system, even bead semi submersible. 1 must ensure that there is a proper recentering. So, that the displacements or the vibrations given to the connecting elements like, umbilical cables, pipe lines are minimized. So, that exploration and production can happen conveniently.

So, K will be always present in the system. Now, 1 can always ignore the dessipation of energy or the response because of damping? The damping is always present by 2 external sources, 1 can be aerodynamic or air presence of air will also cause damping. The second could be aerodynamic. Which are; obviously, an inadvertently present in offshore system, but for hypothetical consideration we can neglect them. To find out the essential characteristics of a single degree freedom system model, we may sometimes ignore the external force also, we call this as undamped because damping is not have present free vibration. Because, f of t is 0, similarly we moved on to the presence of mass and presence of K; we never wanted f of t whether damping may be present, we call this as damped free. Subsequently M and K are present, damping is not present, but f of t is considered we can call this as undamped forced vibration and of course in the last case all are present. So, we can call this as damped forced vibration.

Now, we have seen the response behavior of all these categories separately, in a closed form solution which is derived analytically from the basic spring mass model which we have established has 1 of the standard models for representing a single degree freedom system. So, for understanding let us quickly write those equations back again for recollection. Let us see that can I write all this equations of all these systems back, quickly recollect that how the response where different and different segments where the transient response was focused? Where this study state response was focused? What is the upper bound of the response at resistance? What happens to system we have discussed this for the past 10 lectures. Let us quickly summarize this in few minutes write down the equations. Because 1 can understand this lecture is a summary lecture of all the 10. Even if we miss some of the earlier lectures, even with this lecture it will be able to understand yes these are my equations, these are my close form solution, can I be derive it back from the fundamental.

So, with that principle let us write down this equation, I will call this as let us say case 1, case 2, case 3 and case 4. Of course, in case 3, when you have force vibration there will be 1 more sub division in this, The sub division is if the forcing frequency is matching the natural frequency then the the behavior is different. Similarly in the case 4 I will again have a sub division if the forcing frequency is matching with the excitation frequency or the natural frequency, then I have different case. Similarly when c is present I can have under damped critical and over damped all this sub division will any way play a role we have done all of them, let us see quickly how they can be written.

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So, I will say case 1, just for understanding purposes m, k present c, f of t absent. Just for our understanding that is case one. So, we know that the response is given by x naught cos omega nt plus x naught 0 by omega n sin omega n t.

Let us say, case 2 were m k are anyway present c is present, but f of t is absent the moment we say damped there are 2 damping we study, 1 is the coulomb damping which is frictional model. The otherwise is a viscous damping which is proportional to the velocity. In our case we already defended that why viscous damping is applicable to offshore structures; because of very important reason that offshore structure generally in the presence of the water. Therefore, viscous damping or the damping proportion to velocity of the water particle will be a better assumption considering the coulomb damping. We have also technically and analytically proved that viscous damping has got more deceleration or more recentering capability compare to coulomb damping because, coulomb damping is the number of cycles taken in case of critical damping. Where c is c r is equal to c r which is purely hypothetical, we said x of t is given by a plus b t of e to the power of minus c c r by 2 m of t, where a and b are constants which can be evaluated. And c c r is a critical damping which is 2 m omega n, and for under damp system v x plus damping has a ratio which is c by c c which is x plus zeta always in percentage.

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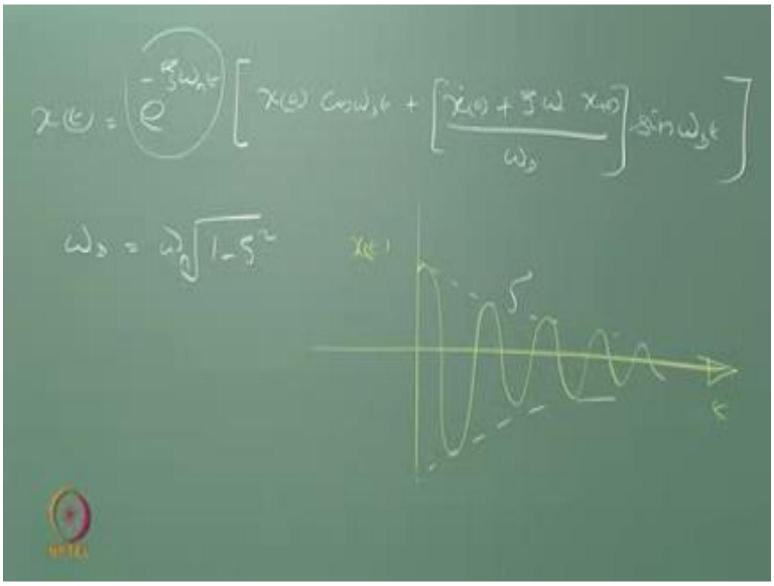

So, x of t was given by e to the power of minus zeta omega n t, x 0 cos omega D t plus here omega D is called damped vibration frequency, which is function of zeta omega n 1 minus zeta square where zeta is the damping ratio. We have already seen that the response typically will decay, x of t omega x of t Let us say time typically in D t, this is decaying because there is an exponential negative sign here. So, decays, once we say it is decaying then the ratio between the successive amplitudes. Let us say I want to know the ratio between positive amplitudes, or the successive negative amplitude I will call this as logarithmic decrement we derived this equation.

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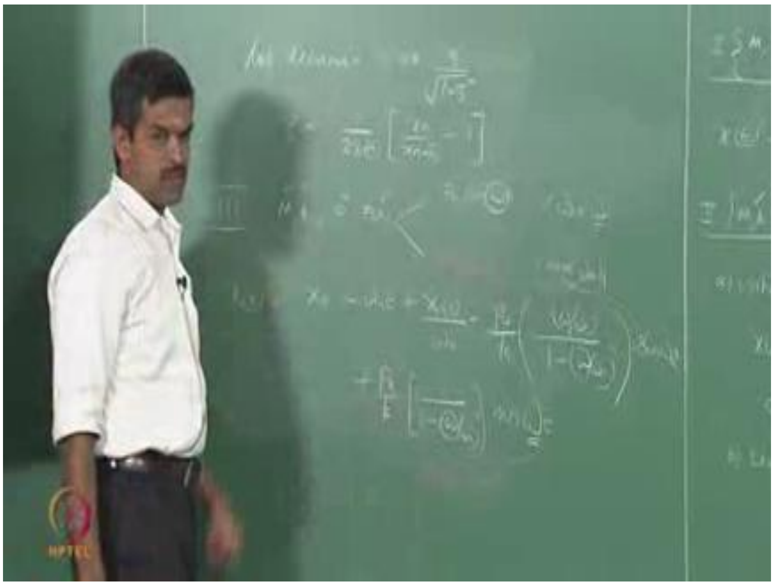

We can write the equation here, the logarithmic decrement. This logarithm is an a p s logarithm base e it is not natural logarithm I mean this is not log to the base 10. So, logarithm decrement is given by 2 pi zeta by 1 minus zeta square.

If you want to make it more general then I should say, zeta can be given by 2 pi m xn by xn plus m minus 1. Where m is a number of successive frequencies peaks either in positive or negative and n is the first point what you are starting. We can started x naught you can start at x 1 etcetera. So, 1 can find zeta and zeta will be expressed in percentage, because zeta is generally given in percentage. Then we moved on to force vibration that is 3. So, undamped Let us say case 3, I say m and k are present c absent, but f of t present. Now there are 2 cases here; f of t is considered to be a general function of P0 sin omega t, where omega is called the forcing frequency. Now the case can be either omega equals omega n the natural frequency system or omega less than or greater than omega n is not equal to omega n.

Of course here also equation of motion, we know that omega n is root k by m. So, we know this. So, there are 2 cases where this can be, 1 this can be 2. So, we will write the equation of both of them, because both equations are different. So, let us write down for this, where it is not equal. So, x of t given by x 0 cos omega nt plus x naught 0 by omega n minus P0 by k omega by omega n by 1 minus square of sin omega minus n t, plus P0 by k 1 by 1 minus omega by omega n to the whole square of sin omega t, now this omega is the forcing function frequency and this omega n are the actual frequency system given by this equation for a single degree, and we already said that the response contains 2 parts; one set of the response depends on the initial condition given 2 the problem we call this transient response. The other does not depend on initial condition we call this as steady state response, which will always be present. So, for our interest we generally consider steady state, but already told you in ocean structures sometimes transient response are also important. The specific application of this will be discussed in the next module where we call springing and ringing responses.

So, it is also important we cannot simply, because we very well understand in case of the natural frequency matching with excitation frequency, the development of response though it is gradual it becomes bounded or it gets unbounded. So, the growth is there. So, when this growth can damage the system we need not have to ignore that part. So, we are interested in focusing on transient response also for the time being you focus only on the steady state response in general. This is what we have discussed as far as x of t in this case concern. Now when omega equals omega n you will; obviously, see the denominator become a; you cannot use the same solution back again. So, we said that p a is different the particular integral is different. Therefore, we gave an equation which ultimately results in what is called deformation response factor Rd.

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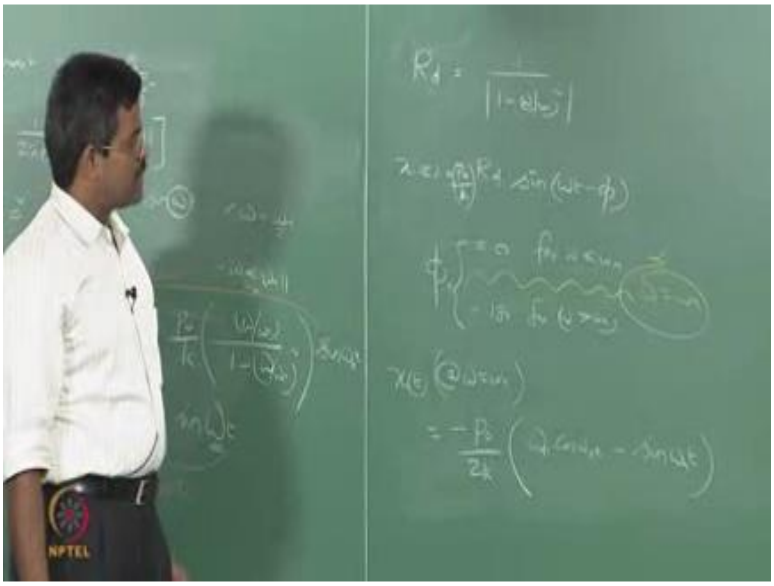

So, we made it simplify to understand Rd has deformation response factor which is 1 by mode of 1 by 1 minus omega m whole square, and we generalized x of t in this case as Rd, P0 by k by m, P0 by k sin omega t minus theta r let us say pi and we said this phi value has 2 values; one 1 will be 0 for omega less than omega n will be 180 for omega greater than omega n.

Now, it is silent when omega equals omega n it is silent. So, at omega equals omega n at this specific case x of t at omega equals omega n, is given by a different expression now, Which is minus P0 by 2k omega nt cos omega nt minus sin omega. So, whether I write omega here or omega nt here does not matter because this a validity I wanted to kindly check all these equations back from the notes and see that I am writing the same equations back again, and we have seen that when the damping is not present because we are looking at the undamped case here.

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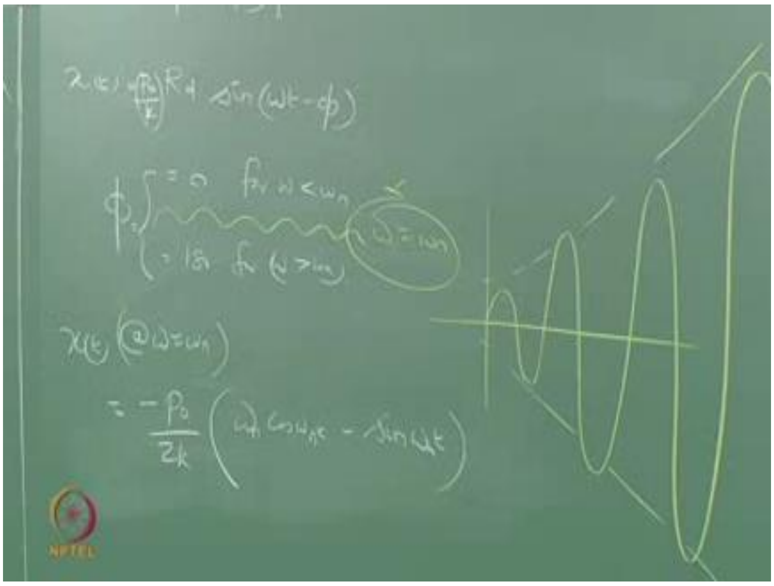

The damping is not present the response it builds up, expected to reach infinity after large number of cycles and we have shown mathematically for every cycle the increment is pi times like a bell. It is grows like a bell these undesired behavior we do not want because I cannot allow the response remain infinity because we have no control of this condition in the reality because the forcing function frequency can match the natural frequency of the system, at that situation the system should not entering to a undefined unbounded response. We do not want a system design like that; therefore we now considered damping which is present inherently in the system which is case 4.

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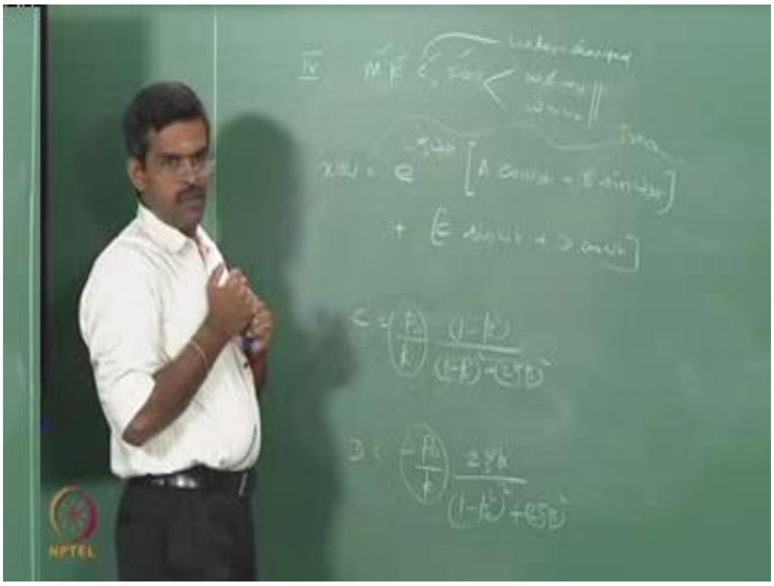

So, case 4 is more or less a realistic system where M, K, C and F of t all are present in c usually we talk about under damped case, in f of t we will talk about both cases omega equals omega n sorry omega equals omega n both we will talk about. So, we derived this equation x of t is given by a to the power of minus zeta omega nt a cos omega D t plus b sin omega D t plus c sin omega t plus D cos omega t now one may ask a question here that the response is again in 2 part; one on omega D which is the function of omega n and other is omega, but again there are constant c and D constant a and b.

It may give confusion at this stage to people that does response analysis of damped forced vibration depends on initial condition or not, please understand the constant c and D or not dependent on initial conditions, they are simply for our understanding I have written c and D is a long equation. So, it is nothing, but P0 by k, 1 minus beta square by root of 1 minus beta square square plus 2 zeta beta square where as D is minus p naught by k remember the negative term in our derivation is associated to cosine. That is why I do the organ diagram 2 zeta beta by 1 minus beta square square plus 2 zeta beta square.

So, they are not depend on initial condition of x naught and x. Whereas, a and b if you open up the transient response you will be see that they will be function of x 0 and x dot 0. Therefore, 1 can say that this will die down because it is an multiplier of exponential here it will not die down because it does not have an x naught here at all most importantly, please understand that the steady state response is nothing, but a magnification of a static response. So, there is a factor which magnifies P0 by k, but P0 by k is generally called as static response. So, let us see what is the defector? What we call dynamic amplification factor. We made an organ diagram for this and understood the as a resultant of the components of cosine and sin and we know this expression which we already derived in the last lectures.

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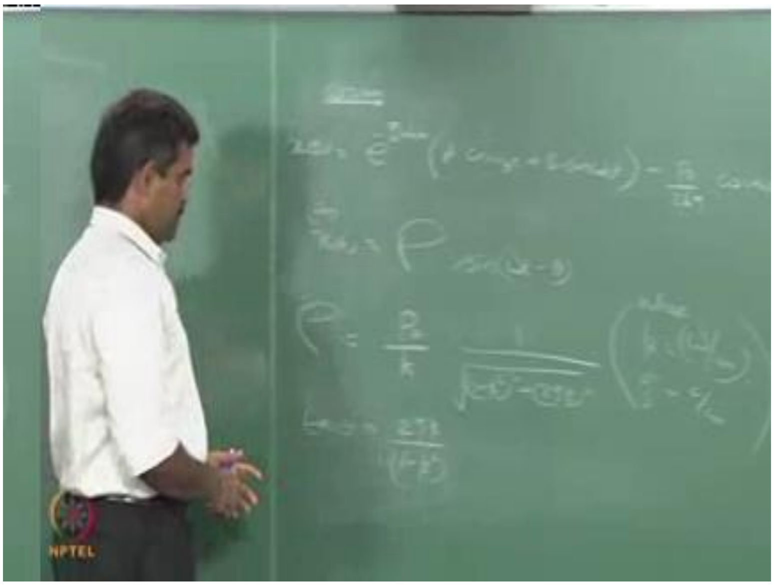

So, the dynamic D equation factor or dynamic magnification factor please understand the term is, amplification is only applied to the static response. So, you are magnifying the static response.

So, is again given by D please understand that this D is different from this D. Is actually some mathematical constant I have indicate you can if you want avoid confusion; you can even make it as D 1 or e etcetera. So, that this D and what I am writing it as different the nomenclature unfortunately remain same. So, before writing D let us say, we write omega equals omega n I want to capture the response. So, we already said that x of t looking only the steady state part because transient part we are not interested for the time being, the whole response is nothing, but e minus zeta omega nt; whether I write omega n or omega does not matter because this is the validity. The transient response part does not change it does not change the sin minus P naught by 2 zeta k cos omega nt. That is a generic expression, I can always generalize this as x of t or x of t as some amplitude sin omega t minus theta it is always preferable to express the response in terms of some factor of that of forcing function the moment, we say then this value rho is given by p naught by k 1 by root of and we said tan theta is given by it is other way and we know the angle of theta is resultant respect to the x of sin omega t part.

So, from this I can say x of t by x static which is my ratio of response, which will now give me the dynamic amplification factor why because you see that the amplitude of the response finally, is getting magnified by static response for some times, we all know that beta is ratio of the frequencies and zeta already we know c by c c. Where there is no multiplier here separate.

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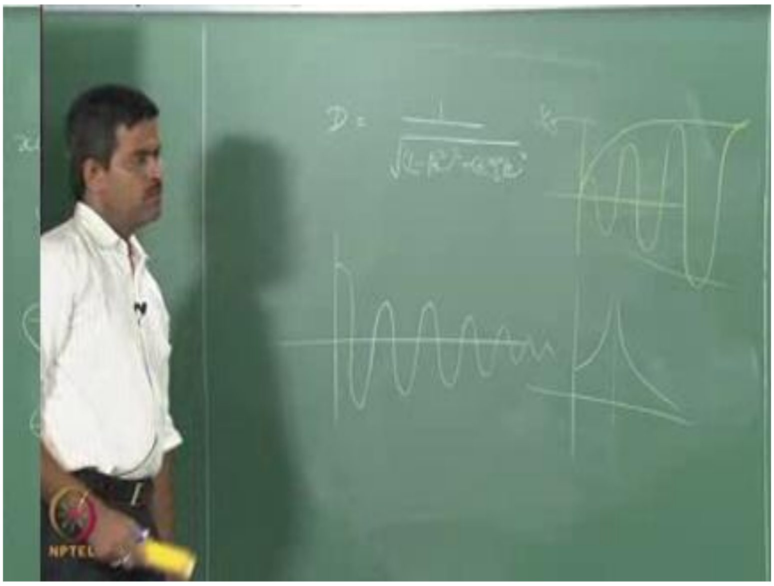

Therefore the dynamic magnification factor D is given by 1 by root of 1 minus bets square square plus 2 zeta beta square. And we also understood that even if omega equals omega n the response builds up no doubt, and it is bound at 1 by 2 zeta. Then we compare both the responses of the (Refer Time: 25:39) increasing response of undamped system at resistance which was not able to capture because the Rd plot. If you look at it was not able to capture at resistance is a unbounded and this captures at resistance because, this function is derived at omega equals omega n and we have discussed both of these as compared them and said how we have understood the response build up in both the cases.

Having understood this, the one more small focus which we have to understand here is how to estimate damping experimentally? We have already seen that the example if I conduct an experiment and try to measure the displacement of the platform using an lvdt linear variable differential transformer or using an explorometer, then I will get a plot of this order because I given initial displacement to the system. I can find out the peaks and try to get the value using the equation what we already discussed I can get zeta from this there is one more way of getting the damping ratio zeta which is called half power bandwidth method to obtain damping ratio data.

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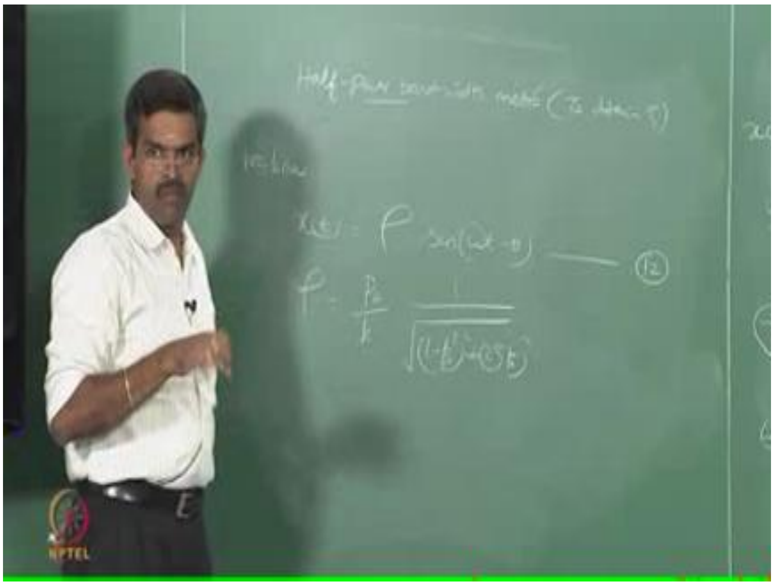

Now, we can proudly write we know, if you ask me how you knew just now for 20 minutes I said we know that. Therefore, please look at that we know x of t is given by some ratio some amplitude sin omega t minus theta and rho is given by P0 by k1 by - we have it here.

So, what it does is a pickup this equation may be equation twelfth. If I am just slipping by one number you can correct me if I am wrong may be 11 I cut that to the last lecture. Let us say 12, this we know this equation I want to plot this equation for different forcing frequencies that is the only variable here because P0 is the known amplitude of the forcing function k is the known value. Because m k are all known to me beta is the ratio of omega versus omega n omega n is known to me, for every value of omega I know beta, I select zeta which I want, but in this case I do not known. Therefore, what I do is I pot this for different ratios and get a function or a plot like this. If you look at this plot a typical plot of this will look like a bell.

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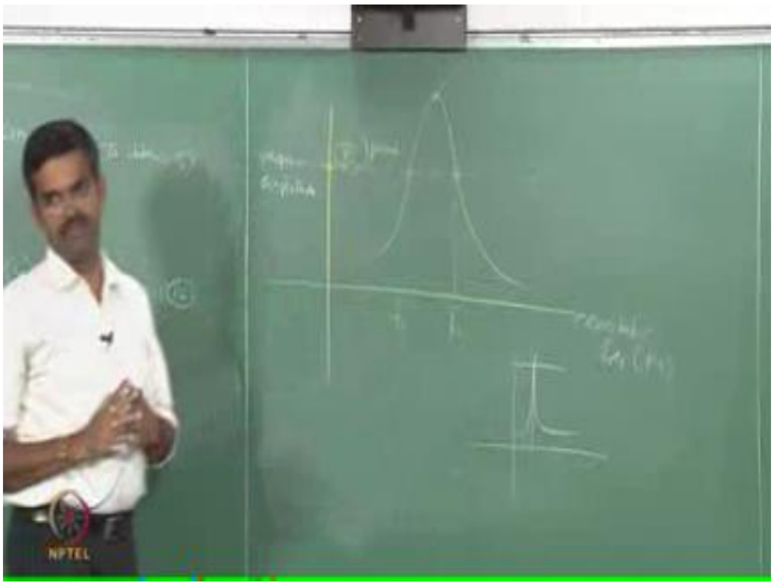

Now, the sharpness the crust the width depends on what zeta you are using for plotting? Now you will certainly give a peak value, Let us say this is my peak value now the plot will have a x axis as excitation frequency in hertz and this will be response amplitude. Which is this part only the amplitude?

You will have a peak I know this value from the plot pick up this value and try to mark an horizontal line I do not know whether you have to see it or not I will mark it in yellow an horizontal line whose value will be equal to 1 by root 2 half the peak. Pick up this value fortunately we will cut this bell at 2 points, projects those frequencies down because I will call this as f 1 this is f 2 f for the frequencies forcing function.

So, f 1 f 2 can be now obtained for the horizontal line cutting this curve which has been plotted for equation 11, for an assumed zeta I want to know what is the zeta that is the argument here you may ask me sir you assume a zeta and you are checking the zeta it is true in this case also see here, you have got a response the structure is decaying the response is decaying is decaying at a given zeta you do not know the zeta the whole exercise which captures the zeta. Here also same argument, right I want to capture the zeta I want to know what the zeta estimate from this is. So, this point are called these points are called half power band points that is why it called half power bandwidth method. Now interestingly if the curve becomes so narrow and your 1 by root 2 comes here; obviously, you will get practically only 1 one f, that is why it is called bandwidth method. So, you have to design in such a select such a manner that you get a proper bandwidth getting f 1 and f 2 properly since this dependent on various factors this method is not accurate.

So, I know f 1 and f 2 now, I want to estimate zeta let us see how we do that.

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We already know the dynamic amplification factor is given by 1 by root of 1 minus beta square square plus 2 zeta beta square we also know for beta equals 1 what does it mean omega equals omega n because omega by omega n is beta substitute you will see the maximum amplification what you will get for this case at resistance is 1 by 2 zeta. Now one may ask me why it is called maximum amplification it is called maximum amplification. Because this amplification is ascertained at resistance system is expected to vibrate to it is maximum response only at resistance. Therefore, we call this a maximum excitation that is why we say this is the upper bound of your response. The one by 2 zeta is what we are having, now I have 2 equations 1 x of t is rho something which we wrote the rho has P0 by k, I call that as x static. With some amplification factor which is root of 1 minus beta square square plus 2 zeta beta square. You can easily understand this equation physically. Physically in sense x of t of a given system is nothing, but magnification of static response and that factor is this.

Now, this can be equal to the maximum value is known to me, I know 1 by root 2 of this what I am going do is, I am going to substitute these 2 values considering at these frequencies they are the peak values. So, you had this frequency they are the peak value the excitation will be almost equal to this. So, what I would do here is, this value will be also equal to x static of 1 by root 2 half 2 zeta, because that is the maximum response I get for some zeta, and why I am saying x static we know already very well that x of t is nothing, but a magnification of static response. I write in the same here this goes away you may say why I am equating this analytical constraint. If you really wanted to find out at what zeta this is happening you have to equate this, we equate this cross multiply square them it will become a quadratic in beta. So, I will get beta 1, beta 2, two values now is it clear.

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So, with famous mathematical sentence neglecting higher powers of zeta and solving this is very important, you must know how to solve this right we write directly beta 1, beta 2; 1 minus zeta minus zeta square 1 minus zeta plus zeta square, sorry you please change this is plus and this is minus, please make a change here. So, I should say beta 2 minus beta 1 will be 2 zeta. So, I can easily get zeta now has beta 2 minus beta 1 by 2, I expand beta which is half of omega 2 minus omega 1 by of course, omega n I can also write instead of omega also half of f 1 sorry f 2 minus f 1 by f n. Because I have used f in the previous figure you can write like this 1 and the same and we all know f n is omega 2 plus omega 1 by 2 where there is a mid point.

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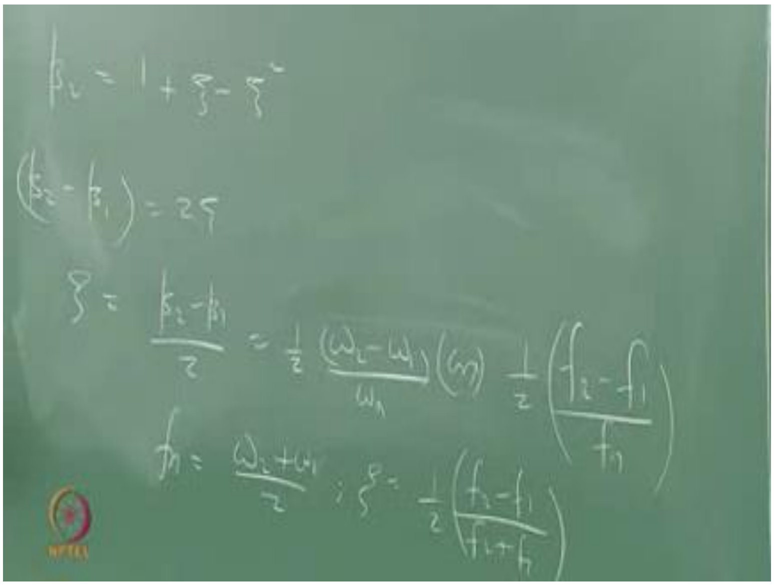

Therefore; zeta is half of f 2 minus f 1 by f 2 plus f one. So, if you know these frequency points from the half power bandwidth method, we can easily find zeta by this equation this is 1 method to find zeta other method is of course, experimental to conduct experiment and get zeta. Do you have any doubt till here? Now it has summarized completely about the single degree freedom system model starting from equation of motion explaining all analytical procedures deriving all equations step by step, summarizing and understanding physically and relating the understanding to the design of offshore structures. Do not isolate dynamic analysis and design, they have to be integrated all the time we have been talking about the physical characteristics of a plat form in terms of it design.

Design is nothing but finding out the geometric form of the plat form and the member dimensions a n of course, the material. So, if you know them for a given form how the dynamic analysis can be integrated with the design that is what we are looking at now never isolate dynamic analysis and design separately they are integrated you may study them in 2 different courses. But you should be able to integrate them at any given thought point of time you should do it.

Now, let us some couple of problems understand how. So, for the equations developed and generated and understood and by hearted and re written can be applied in numerical problems this is very important, because that is how you will realize how mathematics can be understood. Because if you see if you do not see the numbers, if you see only zeta beta x naught x dot you will not appreciate once you see the numbers you will know how they can be, because numbers will have unit conversion. Let us see that to start with, will take up the simple problem writing equation of motion for multiple degree difference system models because, we have written equation of motion using only 1 mass, and 1 spring system let us deal with couple of them and see how we can write equation of motion. So, I take up an example one.

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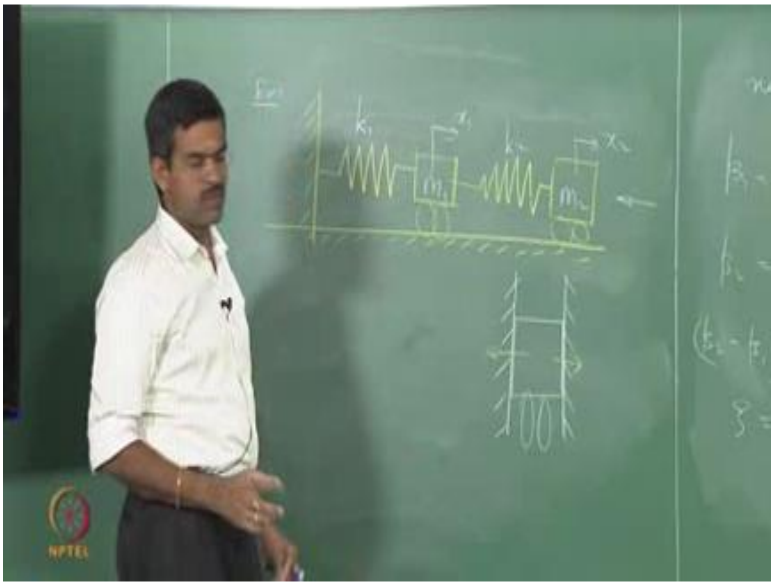

Let us say the system is like, this let us for 2 mass points and to prevent the mass from oscillating in the vertical axis or the vertical plane of the board. I have to put a constraint to this mass, let us the mass rest on roller, why rollers? Rollers will offer no friction to the mass on this plane and roller also prevents mass from oscillating the vertical plane the mass if at all can move only move in this direction.

One can ask a critical question, the critical question is like this. View the system from here now you are viewing the system from here the mass cannot move vertical fine the mass cannot move can move horizontal fine, but view from here I will draw the figure like this, this is my box these are my wheels just to differentiate the wheel in allegation and plan I am doing like this it is not that is a oblique wheels now you may have a question when I pull the mass will the mass move in the direction. Then it is not a single degree I put an artificial constraint here which is assumed that the mass does not move normal to the plane also now the mass as only 1 freedom each mass as different freedom now, why? Because the stiffness of the springs are different if the stiffness are same what will happen even then the mass can be different even if the mass is same and frequencies are same will it have a single displacement no because the half force, half force from 1 point transfer of the force this mass will have different mechanism. Therefore, x 1 and x 2 will be different now let us come back to degrees of freedom.

Degrees of freedom are not those points where the mass is lumped degrees of freedom are those points or independent coordinates which will needed to require the position of mass are configure the portion of mass for a given instant of time. So, at any given instant of time mass m 1 and m 2 will be located independently though they are connected to spring case 2 as well as their totally connected by spring k 1 to the support system. So, you need 2 coordinates independently to locate m 1 and m 2 they cannot be connected. Therefore, it is a 2 degree freedom system. So, I am not putting a dash plot here we will put a dash plot in the next problem. So, I want to write equation of motion. So, it is very easy, let us follow this principle easily and you can write equation of motion comfortably. I will use Newton's law there are many methods I will also solve the same problem using energy principle. I will write equation of motion in the same manner you will see both methods will give the same equations of motion.

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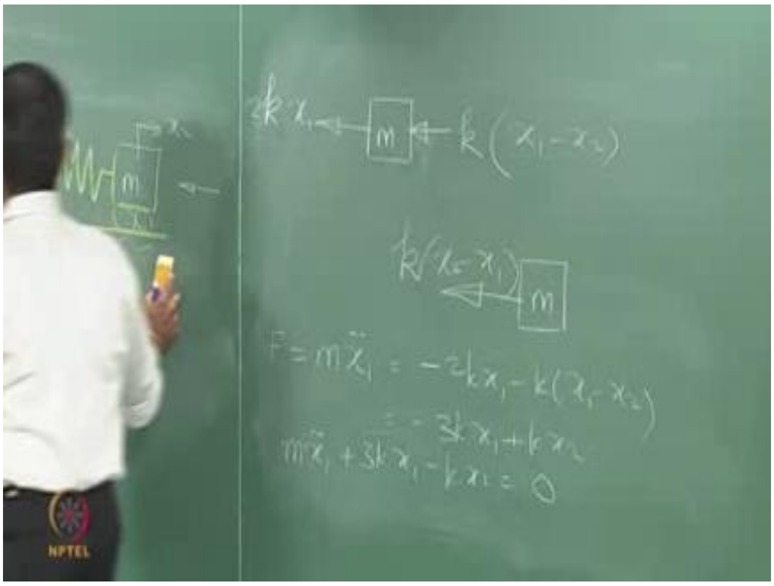

So, this is my first mass point, I call m 1 we know that if at all I want to set a motion I must give a either a displacement or a force. The moment I pull this mass please listen carefully this is very important this is the point where generally people get completely confused. So, they are not 2 techniques; one without understanding whether this s x 1 and this is x 2, without understanding this m 1 and m 2 they have a blind Rd they will write mass matrix and stiffness matrix immediately without knowing the problem. It means they are by hearting. So, which is very wrong and very bad if both case are equal and both ms are equal they are really gone, because by heart will not help the second issue is which value will be negative and positive.

So, therefore you will not be able to remember at the age of 30. Till 30 you can remember, when you cross 50 you will not remember if a fundamental is not clear you cannot remember at all the every time you turn the book and unfortunately and which is the truth, every author explains writing equation of motion in it is own style ultimately the equation motion will be same for all authors, but the explanation given by authors are actually confusing.

So, the best mechanism to avoid this confusion of follows one author. So, that you get confused the same and I see is confused. So, that you follow the same path. So, the 1 author which you will get confused will be me. So, follow this. So, I tell you a very short technique how to remember this that is very important if you remember this I think you will not make mistake.

When I pull this mass to the right that is what my displacement is this spring will try to pull it back. So, let us mark it here, now I am interested in writing out the force we know spring has got a stiffness value which is force per displacement. If you really wanted to find the force I must multiply the displacement by this spring constant I get the force I am writing force equation that is what Newton's law is. So, stiffness multiplied by the displacement will give me the force.

To make the problems more interesting, what I will do is I not say k 1 and k 2; let us change these values slightly is not because, I have an answer here it is 2 k and k let us make it like this let the mass be both m. Now this spring when I try to move to the right it will get compressed we will offer a force in opposite direction. So, let us do that force in the opposite direction, as stiffness of the spring, so I write a 2 write a 2 I have replaced I have replaced it. So, I multiply the stiffness of the spring with that of identify the coordinate where you are starting the equation, write that coordinate first and then the next coordinate second x1 minus x2 similarly for the second mass m I try to move the mass towards right you may ask me why sir always right? Because the boss is always right. In this case I am having the displacement to the right.

So, I move it to the right the mass will move to the right this spring will not allow the mass to move because recentering concept. The mass the spring will try to pull it back and I want to find the force multiply with the constant of the spring, and now same argument first write the equation where you are writing and then coordinate of the next

So, I want to write Newton's law force this mass into acceleration, because the displacement is x 1. Therefore, x 1 dot will be my acceleration second derivative of displacement which will be given by minus 2 k x 1, why minus because I have given the force to the right, but the spring is bringing to the left I have to algebraic express this. This D'Alembert's Principle, 2 a k x 1 minus of k of x 1 minus x 2 which is minus 2 minus 3 k of x 1 plus k of x 2 I have bring it back here. So, m x 1 double dot plus 3 k x 1 minus k x 2 is 0. I will write down the second equation here itself. So, it is easy for us to capture.

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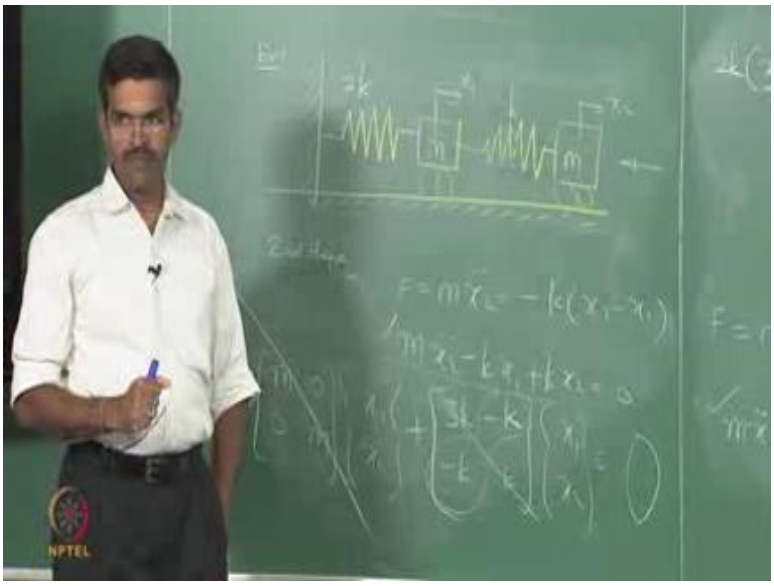

So, for the second degree f m x 2 double dot, will be minus of k of x 2 minus x 1. So, m x 2 double dot minus k x 1 because here it is positive brings it here negative, plus k x 2 is 0. Now I got 2 equations this is the first equation, the second equation. I can now write in the matrix form both the equation now I want to ask you question is anybody will not able to understand how I am marking this arrow directions, how I am writing these values. We can repeat it very quickly again, I will not rewrite it again. I will write only the figure, I have a mass which is m indicated here the mass is going move to the right, but the spring will not allow the mass to move. Therefore, it is the force which has spring constant multiplied by only x 1 that is the displacement, but at this spring is connecting 2 displacements, and this spring when you try to pull the mass towards right the spring will compress the mass or push the mass towards left.

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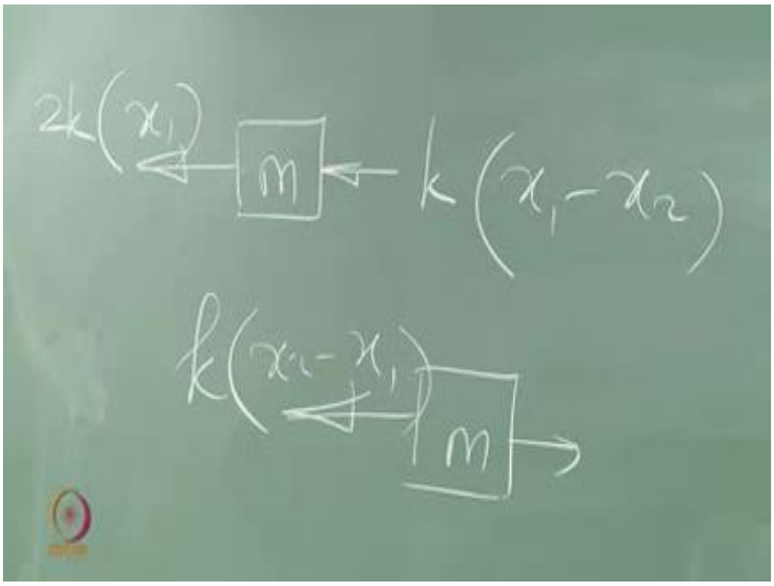

Therefore the direction is this way k is the constant, multiplied by the point where you are discussing is the first coordinate and then the second coordinate. Similarly for the second case I say m pull the mass this way, but the spring try to move now the coordinate of the first and then the second. That is what we wrote here we reassembled them and we got the equation now this is the first equation the second equation I write them in matrix form. We already know how to write them in matrix form we already know how to write them in matrix form.

So, m 0, 0 m of x 1 double dot x 2 double dot you can read it m x 1 double dot that is the first term here plus k is a 3 k minus k minus k k of x 1 x 2 is said to 0 let us read this matrix m x 1 double dot plus 3 k x 1 minus k x 2 is 0. Similarly m x double dot x 2 double dot minus k x 1 plus k x 2 0 that is, what I am having here more interestingly when your displacement coordinates or selected at the point, where the mass is lumped mass matrix will be always diagonal the half diagonal elements will be 0. And usually the stiffness matrix will be symmetric usually and the stiffness matrix will be diagonally dominated. That means, the value is here on any row will be higher. So, these are some checks if they are not diagonal dominant you cannot take in inverse of this nothing, but flexibility matrix f of flexibility matrix exist. That is why we get stiffness matrix here. So, I can easily derive the equation of motion in a minute for a given problem like this.

So, I can keep on adding degree of freedom keep on writing equation of the motion keep on developing matrix method and what people generally think is, they actually follow a principle 2 k k 3 k minus k minus k k. So, if this is let us say 4 k and 3 k they are simply as a seven k minus 3 k minus 3 k 3 k this is actually a blind method of following do not do this may be right may be wrong do not do this always derive from the first principles because arriving or deriving stiffness matrix itself is a menace in ocean structures. I will show you in second module how the stiffness matrix cannot be derive as like this. So, very tedious process, always use first principle to derive the equation of motion and from that plug out stiffness matrix and mass matrix separately, now since damping is not there the damping vector on the damping matrix is absent. If it is there it will also be present. So, I have derive the equation of motion using Newton's method I will also derive the equation of motion using energy method next class and compare this that I will get the same equation of motion whatever method I follow.

So, in this class we summarize very quickly, what are the different elemental stages of understanding the dynamic response of the single degree freedom system or spring mass system starting from undamp free, damped free, undamped force, damped force, resistance vibration, response buildup, response deformation factor and dynamic amplification factor, estimating zeta from half power bandwidth method and also deriving equation of motion for multi degree equation system explaining back, what are degrees of freedom? What are restoration forces? How are the arrows marked? How are the coordinates fixed and how do we write? How do we assemble? How do we check, whether the matrix is OK, we will discuss all these things in next class. Any doubt you have please post it to NPTEL, any appreciation you have please post it to NPTEL back.

Thanks.