Dynamics of Ocean Structures

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Lecture – 25
Damping Matrix By Super Positioning Method

Today we will discuss about another new method of damping estimate. The last lecture we discussed about Caughey's Damping matrix method by which you can easily estimate the damping matrix the co-efficient Cl is given by this equation as we derived in the last lecture, where the coefficient l for the variable l, varying from 0 to n minus 1. Where n is the number of degrees of freedom or j which can be much lower than n depending upon how much you want to truncate the modes. You can always find the coefficients C1, C2, C3. So, for assemble them and expounding damping ratio in the nth mode will be give by this equation. Again it is varying from l to n minus 1 and r j whatever we discussed yesterday. So, if you know the frequencies you can easily find out the values of zeta in every frequency or every mode.

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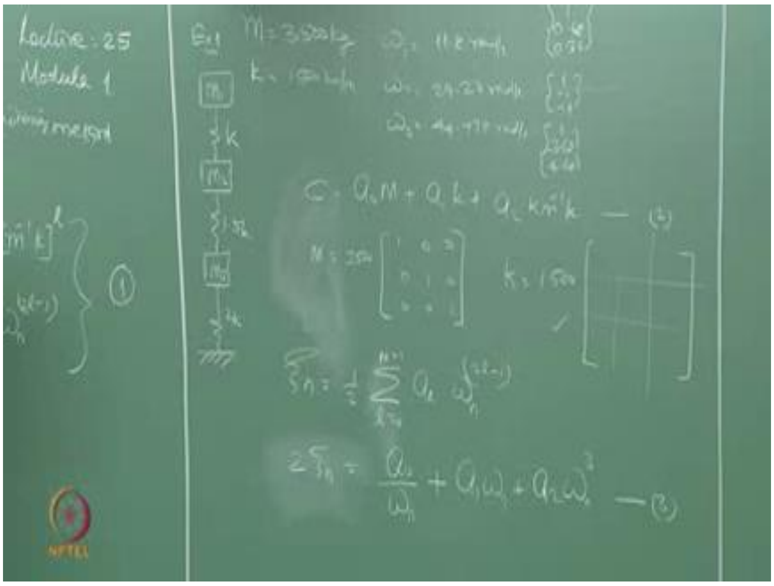

Let us take an example quickly and demonstrate this, before you move on to the new method by which I can estimate damping matrix by super positioning technique is another way. So, let us write example will solve we already has relay damping which me. So, let us try to solve this for a Caughey Damping Method. So, this was the problem given earlier. So, this was taken as k this was 1.5 k this was 2 k and in this problem m was given as 35000 k g and k was given as 1500 kilometer per meter we already have omega 1 values with us which we computed or you must have computed from different methods on the corresponding mode shapes also we have. So, all positive no 0 crossing h ere 1 0 crossing and here 1 here and 1 here back again. So, first mode second mode and third mode. So, when you substitute l is equal to 0 to n minus 1. Let us say we will take 3; zeta 1 zeta 2 zeta 3. So, let us try to find out what is C naught and C1 and C2. So, we already evaluated them in the last class separately. So, let us say I will write C matrix simply a 0 m plus a 1 k plus a 2 k minus k.

So, mass matrix for this problem we already have stiffness matrix I think we should write do you have the stiffness matrix with you anyway; we I think we have it written it already should be available with you. So, substituting back in zeta n, let us say half l equals 0 to n minus 1 a l omega n 2 l minus 1. So, 2 zeta because you know all these values have a naught a 1 and a 2 will have half as a multiplier. So, 2 zeta n will become a naught by omega n plus which already derived. In the last class we have this expression with us a 1 omega n plus a 2 omega n cube.

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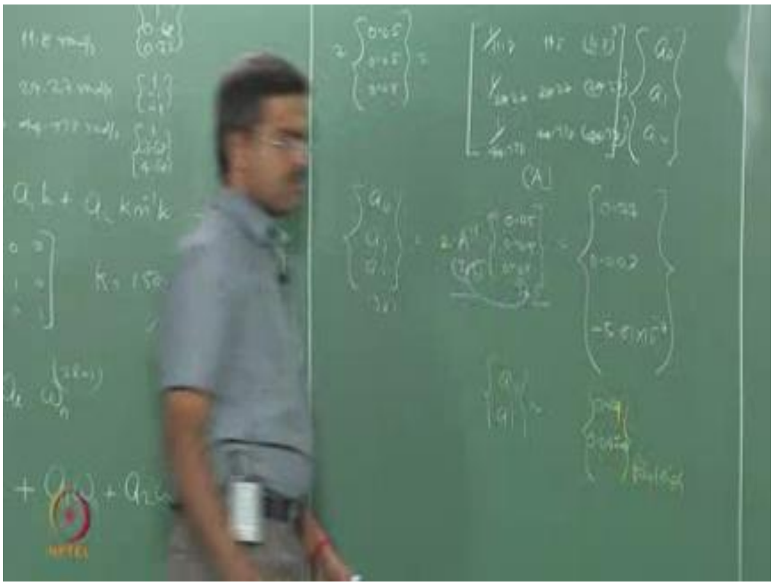

Now, let us assume that I have 5 percent damping in all the considered modes for the study. So, the omegas are 11.8, 1 by 11.8 1 by 29.27 and 1 by 44.778. So, a 0 and then a1 ka 1 omega n; 11.8, 29.274, 4.778 then third 1 could be a 2 omega n cube. So, 11.8 cube 29.27 cube 44.78 cubes. So, let us read 1 column let us say 1 value 2 zeta will be a0, a0 by omega 1 plus a 1 omega 1 plus a 2 omega n cube. So, similarly matrix now, I want to find a0 a1 and a 2. So, it simply the inverse of this matrix multiplied by this, I call this matrix a let us say. So, find a inverse and multiply with this vector. So, 3 by 1 3 by 3 and 3 and 3 by 1 comparability answer. So, can we get me the value of a0, a1, a2 quick ith a calculator easily you can invert 3 by 3 matrix comfortably. So, get me the value of this vector the coefficient vector a 0 a 1 and a 2.7, 0.00.

Student: point (Refer Time: 07:23).

Can I quickly compare these 3 values with that of relay what we had in the last example we also had this answers from relay of course, a 0 a 1 only not a 2.84 naught naught. So, it is obvious if I multiply this because you know a 0 which is equal to 0.77 in this case you know a 1 which is 0.003 and an of course, you know a 2 which will multiply by this and this term generally for this problem goes in significant. So, it is very clear 2 inferences from this example 1 Caughey's damping though it is extending for higher modes effectively relay would represent more or less the same contribution from that of Caughey.

Secondly, only the first few modes are important in a given m d a system remaining modes do not actually contribute, is very important method by which we can ignore the model participation of higher modes we need not have to go for n minus 1 modes at all is not required. So, Caughey was interestingly working at the higher end contribution of different modes, but where as in relay truncated to in the initial modes it was fine. So, now, once you know these values you know m you know k you can perform this as well and easily find the damping matrix which is required for this problem which I am not working out you can easily find out.

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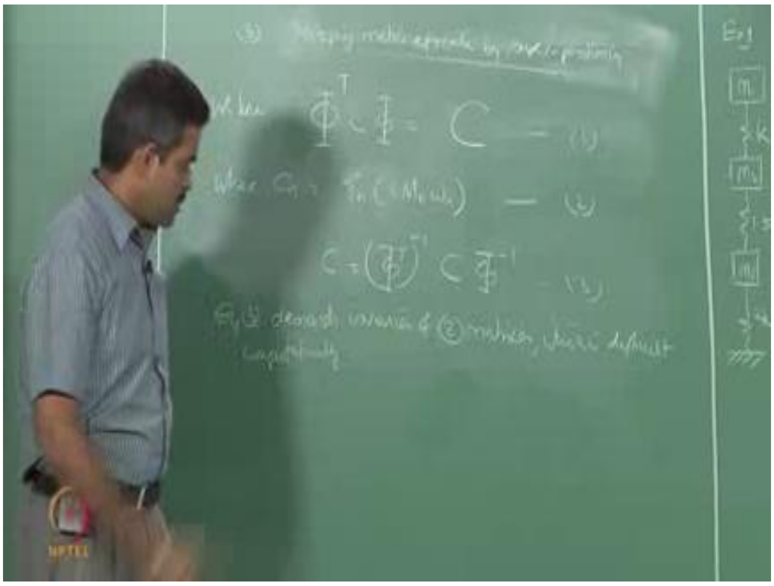

Now, let us look at another method of the same problem, which I am going to do by module super position. So, I will retain the problem as it is, I will just only rub this up this part because the example is going to be the same. So, by the third method which is damping matrix estimate by super positioning will try to see this method, let us say we know phi transpose where phi is a matrix each column of phi consist of each eigenvector or a mode shape.

For example, it can be a 5 matrix where, each 1 of them is called as small phi small phi indicates mode shape where as capital phi indicates a matrix. So, phi transpose C phi is generally a C matrix please note this is coefficient this is a full matrix. So, standard logarithm, where C n at any nth mode is given by we know this value 2 zeta n omega n let us call equation number 1 this as 2. So, from equation number 1 if you want to really find the coefficient I must say phi transpose inverse C phi inverse this equation 3 because, I am free multiplying post multiplying the inverse of this. So, I get identity I do the same thing on the right hand side I get this.

Now, interestingly if you look at the equation 3 it is very difficult to work out the matrix because, you have got invert 2 matrices equation 3 demands inversion of 2 matrices which is difficult computationally can we have an alternative method for this I do not want to invert both the matrixes, but still I want to find C I am only doing super portioning that is all this an alternative technique available for this. Let us see how I will rub this part from orthogonality principle we know.

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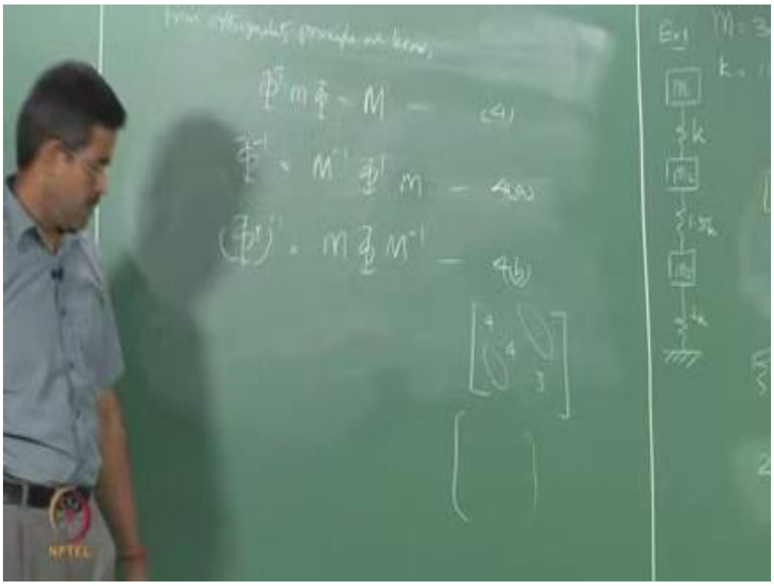

Let us say what do we know phi transpose m phi transpose m phi is m equation four make phi transpose m phi is capital m fine. Let us first find out phi inverse phi inverse of course, m inverse phi transpose m call this as 4A. Let us now get phi transpose inverse which will be m phi of m inverse call four b now this multiplication is easy for a and four b for simple reasons m is actually diagonal matrix all the elements are 0. When say performance this operation I am not inverting a actually five which is difficult I only inverting m inverting m is very easy because only diagonal elements are there. For example, if your n matrix could be 4 4 and 3, let us say inverse straight away could be 1 by 4 by 4, 3 you can easily do that.

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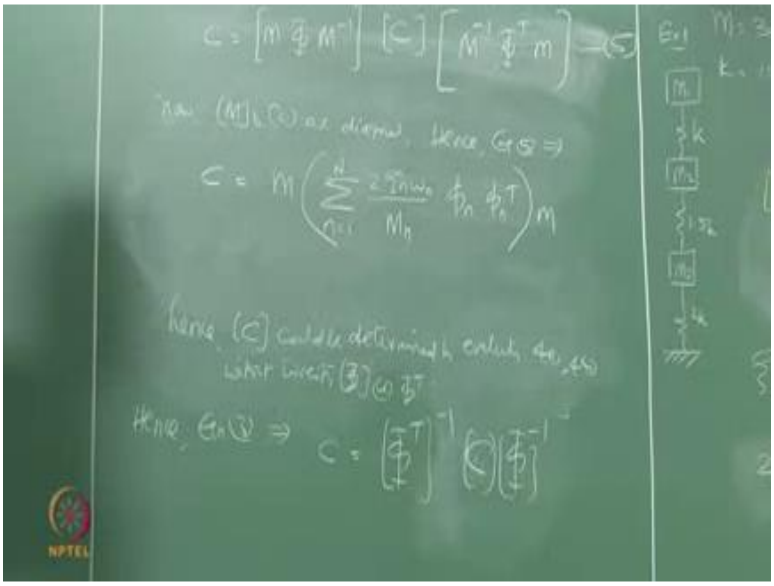

Now, by performing the operations of four a and four b I will get actually phi inverse and phi transpose inverse which was required to compute C at equation 3. So, look at equation 3, if I know these 2 in terms of this I can easily find C, C could be determined by evaluating 4 a and 4 b without inverting phi or phi transpose hence equation 3, now becomes which was actually C phi transpose inverse C phi inverse now C is therefore, phi transpose inverse was m phi inverse which was this value from 4 b C phi inverse phi transpose i.

Now call this as equation number phi now m and C are diagonal hence equation 5, now becomes I can take this m out which is common in both the cases summation of MN. Let us use the count n because you n everywhere 2 zeta n omega n by normal M N phi N phi N transpose of M this is true, because this is diagonal this diagonal this also diagonal this gets set up to this particular value and you get phi n and phi transpose equation 6.

Now, let us look at this equation I know the mass value for a given matrix I know zeta n and omega n. Of course, omega n I know zeta n I will have to substitute 5 percent, 2 percent whatever may be of course, I know the mode shapes I can easily find the transpose of these vectors easily pre and post multiply with mass matrix I get the stiff damping matrix easily without any inversion. I am not inverting any matrix here let us apply this to this problem and compare this matrix with what we got from relay and Caughey and how this is compare. So, let us do this it is got a very important advantage.

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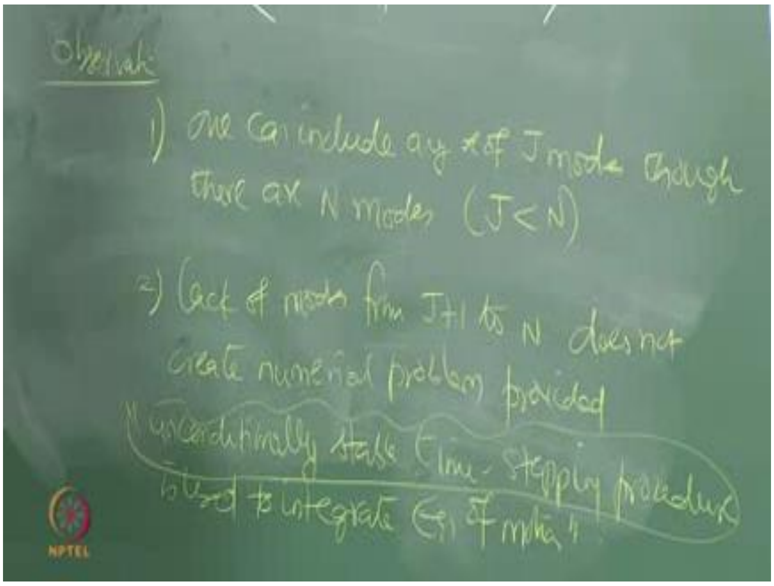

Let us say observations 1 1 can include any number of j modes though there are n modes j is less than what is it mean for your choice you can select this upper limit of this same summation to any j n number. One can ask me a question, if n is 3 and j is 2. So, therefore, you not have the contribution of the jth is equal to 3 in C it means the damping matrix will have no contribution of damping ratio from the third mode.

Suppose in a given matrix you have only C1 and C2 do not have C3, but you have third mode also is implied that the damping ratio from the third mode does not contribute significantly to the overall damping matrix damping matrix is dissipation of energy it is response control response decay it has been found to be propositional to mass and stiffness separately independently, but practical examples in experiment showed that they are neither mass proportional nor stiffness proportional. So, relay suggested we take component of mass and component of k and say a 0 m and a 1 k and found out a matrix and that gave me a liberty of truncating higher modes, but Caughey extended that saying the let us go for higher modes also, but it did not show me significantly for this example it is going to be as similar to that of relay damping.

So, this becomes more simple because it does not have any inversion at all and you can its gives a liberty of selecting any number of modes, where as in this case we cannot do that you have to keep on evaluating them see you came to understand that the contribution of the third mode is not efficient only. When you know a 2 until then the calculation was wrong, but here you can easily do the truncation as per your choice any where you want that advantage is there the second advantage is lack of modes lack means absence lack of modes from j plus 1 to n because, these modes are not present when I say it is we are truncating till j from j plus 1 till n these modes are not present in a calculation of damping matrix c.

So, lack of modes from j plus 1 to n does not create numerical problem provided unconditionally stable provided unconditionally stable time stepping procedure is used to integrate equation of motion will talk about this unconditional time stepping integration procedure later there are many methods. If we choose this will not create a numerical problem of deleting higher modes from j plus till n there will no problem numerically in this method. So, these 2 are considered to be great contributory advantages for using model super portion method for estimating damping matrix will apply this concept of equation six to this problem and try to evaluate C matrix and see how does it compare with my relay and Caughey BTU.

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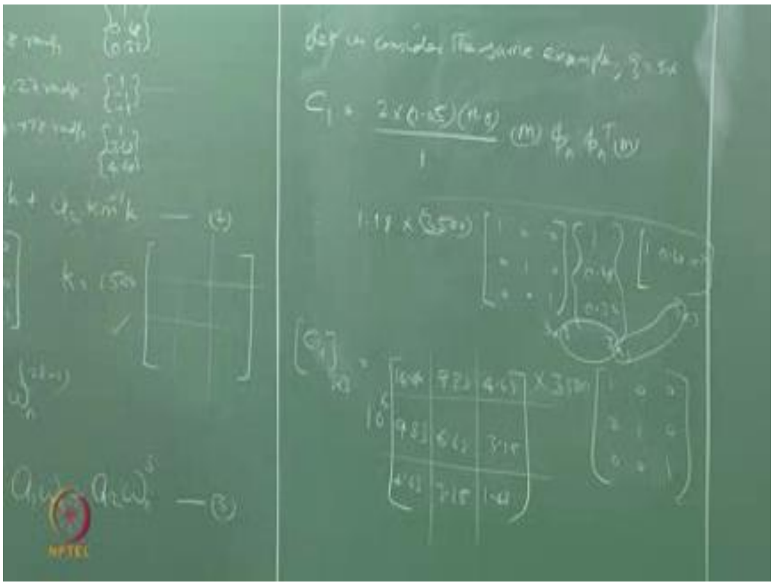

So, for simplicity we will take only 2 modes because I want to show what is the effect of that I will keep this as it is. So, let us consider the same example the example is here this example 3 degree freedom system M1 M2 M3 k 1.5 k 2 k we already have omegas and phi s for this system. We of course, do I am not going to use this in this Caugheys relation if it eliminate this relays relation, but I am not use I am going to use this relation equation number 6 to estimate damping matrix here are you getting the point. So, I have m matrix and of course, I will have k matrix, but I do not require actually k. So, there is no botheration about this I want only m matrix available with me here let us do this.

So, let us first find C1. Let us substitute C1 and see what happens the internal term, let us evaluate this first 2 times of 0.05 assuming zeta is 5 percent 0.05 omega 1 is 11.8 divided by normally mass matrix 1. We have multiplied 35 later m phi n phi n transpose yeah it means can you give me what is this value 1.18 know? 1.18 multiplied by 35 100 that is the mass value phi N phi 1 is 1.6 8.32, you can see here comparability 3 by 3 3 by 1 1 by 3 can you find out this matrix can you find out this again there is a mass multiplier here, please understand after this again there is a mass multiplier 35 100 of, so do all the multiplication get me the C1 matrix C1 matrix. I shall write it down for you please check 14.46 9.83 9.83.

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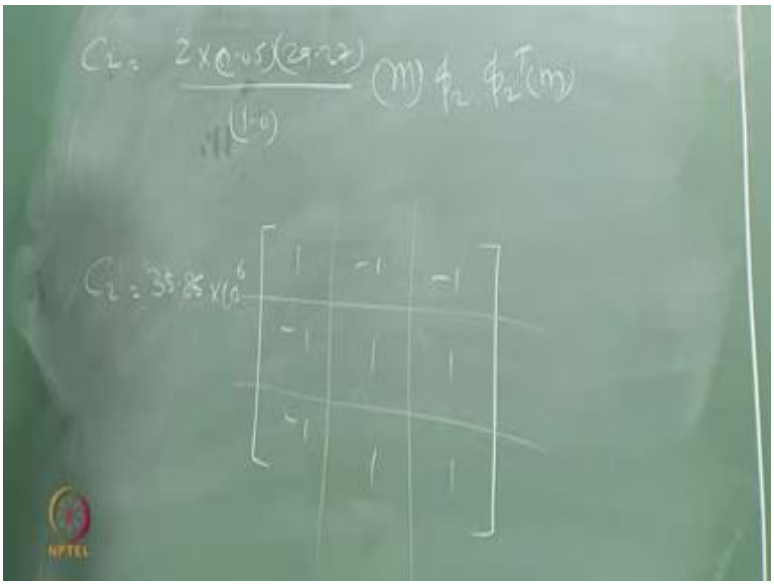

Similarly, let us try to get C2. So, I will substitute the values only 2 of 0.05 because I said zeta will be same in all the modes. One can ask me a question why zeta should be same in all the modes? What is the requirement you know the answer for this why it is. So, so 29.27 that is my second frequency divided by again the mass matrix of that value I am talking about this 1 1 multiplied by m phi 2, phi 2 transpose yeah. So, I have all the values because phi 2 I know here it is phi 2 transpose I can find mass matrix available here and pre multiply 35 10 in both these cases, as I did here, get the value and I now writing C2 in a C2. Now, one is not interest in getting pertain partial damping matrix he wants the whole matrix.

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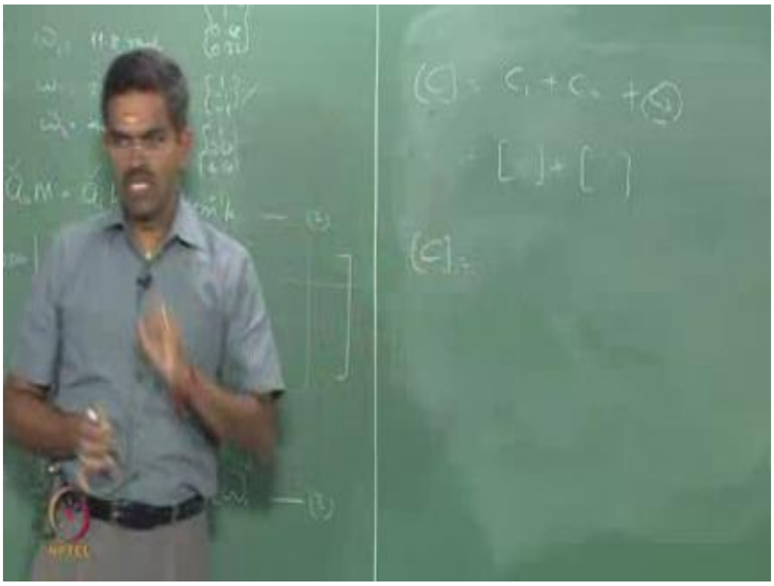

So, the C matrix will be given by C matrix is now given by C1 plus C2. Now can you please answer me why it is C1 plus C2? Yeah why it is C1 plus C2? Why? The whole method is on super portion? Here we are talking only about the addition, which is why we started it is only a summation C summation of this algebraic sum of n varying from 0 to j. So, I keep 1 to j keep on adding them. So, it is simple super position.

So, add C1, add C2 get C now I will give you a tougher task I want to compare the C with super position with relay and Caughey and tell me which is higher and which is lower and now we have 3 damping matrixes. So, 1 should able to tell me. So, I have another question to you. Now C3 is missing here what do you mean by C3 missing here? Initially the system has 3 these are freedom. I think we go back to the first lecture do you all believe the system has 3 degrees of freedom? Yes; how can you say that? I do not I am not getting I am not getting you I am not hard any 1 of you why are saying it has 3 degrees of freedom.

Student: (Refer Time: 30:26).

Sorry.

Student: 3 d (Refer Time: 30:28).

Yeah is anybody trying to answer that there are only 3 mass points is anybody trying to answer there are only mass, find is it so, but the lumped mass position does not indicate anything about a degrees of freedom at all. But still in this case there are 3 damped mass and therefore, is a 3 degrees of freedom is correct can you tell me why.

Student: We are considering only 3 independent.

Agreed, but in these example 3 mass points is a 3 degrees of freedom is correct why the answer is on the board sorry.

Student: (Refer Time: 31:14).

There is no stiffness matrix the answer is not in the board because, the mass matrix diagonal. The mass matrix is diagonal it means displacements are measured the lumped mass points. If you do not get the basics will be rounding only will not be able to get through the dynamics at all. I am sorry, but you are not able get through what actually want this clearly illustrate the displacements 3 degrees of freedom are marked only because the mass matrix is diagonal this will happen only when x1 x2 x3 are located at the point where, the mass lumped this was explained n number of times in the class now answer me it is got 3 degrees of freedom there are 3 frequency and 3 mode shapes. But see as only contribution of first and second mode shapes only what is it mean why C3 is not present here.

What does it conclude this C matrix will conclude that the contribution of damping from the third mode is insignificant it is not the third mode damping ratio is lesser please this, where people misunderstand the damping ratio is constant in all the modes why it has got to be constant first question is asked. But you have not answered them it is required because that is what is called classical damping; it should be uniformly throughout spread for the entire structure. So, I have zeta a 5 percent in all the modes no doubt on that there is no decrease on that. But still the contribution of that zeta on the third mode is lower, why the mode shape is compromising on each other when you got a positive displacement; you got a negative displacement some where they are compromising. Therefore, there is no dissipation of energy happening because of this vibration in the entire system. So, if you keep on adding higher modes you will see that the contribution generally of the higher modes on the damping is decreasing it is because of the reason people said.

Let us truncate the modes and bring down to 3 or 4, but the number is not a magic number you have got a justify where should be stop the mode counting again it will related to mass participation factor because, every where you see truncation of mode related to mass truncation of C related to mass you see here mass and mode shapes are very important frequencies are also important in this case, but mode shapes also equally important, but they will tell you what is the effective contribution of that particular vibration mode in the overall damping because, you are compromising 1 is positive 1 becomes negative. So, they may compromise on each other in terms of the vibration mode after all modes are nothing, but relative displacements are mass points at a given frequency of vibration.

So, that is how I do not have an influence on C3 on C. So, I have a very small homework for you. You compare C of super portion method compare this with relay and Caughey and try to compare the percentage error between this considering may be relay as a base considering relay as a base you can try to find out how much the error given by this methods. So, this ends the discussion on damping evaluation now, as we all understand from this lectures now that higher modes need truncation you no need to consider all the modes on the other hand, if there are n degrees of freedom you are actually not required to workout omega ns and phi ns it is foolish you do not require them.

So, not required therefore, classical Eigen solver which will give me all the mode shapes and all the frequencies is unpopular is not required and it is because of this is more popular because at least for sure gives me the fundamental frequency popular because, it gives me fundamental and the mode shape also inference coefficient method is popular. Because it gives me the frequencies and mode shapes in order and they are orthogonal automatically.

So, if you do not understand these clusters in a chain of lectures dynamics will become very dry applying will look like matrix algebra; you have to relate it to design it actually. So, you see here this mode of vibration is not going to contribute to the vibration or mass participation of third mode damping ratio is not reduced in third mode. It is same as the first mode, but still it is not contributing it is because it is only because phi 3 is compromising higher modes need to be truncated. How do we understand truncation of modes? That is the next lecture which we will address.