Dynamics of Ocean Structures Prof. Srinivasan Chandrasekaran Department of Ocean Engineering Indian Institute of Technology, Madras

Lecture – 05 Single degree of freedom

In the last lecture we discussed about the, 2 types of approaches, how do we do that dynamic analysis? Now we will say that, there are 2 basic types of loading, where you can do the dynamic analysis. One is what we call as a periodic loading, the other one is non periodic loading.

(Refer Slide Time: 00:33)



In the earlier class, we said that there are 2 approaches, deterministic and non deterministic approach. The deterministic approach will give you the time history of the responses. We can also find out the stresses and displacements etcetera, as a derived quantity, whereas in non deterministic you will get the statistical responses, you will not be able to derive them, you have got to do the independently.

Periodic loading is that, which has a same variation, for large number of cycles, whereas non periodic loading is that, which is for a short duration, and creates, what we call impulse loads.

(Refer Slide Time: 02:13)

Now, let us see what are those essential characteristics, or essential features, of dynamic analysis. One, the loading should be time variant, 2 it should produce inertia forces, which is significant. And we all know inertia force is mass into acceleration, if we take access the displacement, the double derivative of that will be the acceleration.

Now, for example, if the displacement of the system is very low, if the displacement of the system is very low, and is unable to produce, inertia force significantly, even though the loading is time variant, we do not do dynamic analysis. We can perform dynamic analysis only, when both these conditions are satisfied. Load should be time variant, and the load should produce significant inertia forces. Inertia force of course, depends on 2 parameters, one is the mass, and other is the acceleration. We all know for offshore structure system, mass are generally very large.

So, the question does not arise, that it will not cause any inertia force at all, because of the lesser mass. But there is a possibility that, the displacement can be very small. I can give one example. If the structure is bottom supported, it can have a very less displacement, therefore, the inertia force can be lower, but the statement does not tell you, that for bottom supported structures, like jacket structures, do not perform dynamic analysis, because this is qualitative, this subjective. We simply say, if the inertia force is lesser, insignificant, do not do dynamic analysis, but what is your quantification? That is not specified.

However, if the structure imposes a very large displacement, you have to do a dynamic analysis. Now that is the contraceptive of the problem. So, there is no question of not doing a dynamic analysis, for a compliant structure at all. But he can do a (Refer Time: 05:35) analysis for a fixed structure. That is the contraceptive of the issue. Therefore, when bottom founded structures or supported structures, were installed initially in 1930s, people did not bother about dynamic analysis for a large extend, as it is been talk of the day. Because now those structures are totally obsolete. We do not have any bottom supported structures, meant for oil exploration production, what is generally called as E and P. E and P are exploration and production. We do not have any structures, which is bottom supported, as on today

They do not; they are not actually used at all. It is totally obsolete, about forty years back. So, there is no research innovation happening in that side. So, if these 2 are the characteristics, which govern, the necessity of performing a dynamic analysis. Let us talk about the form, where, 1 of these 2 parameters govern, for example, water may be the form of the structure, may be fixed type, may be compliant type, may be floating type. Water may be a structural form, water may be the assembly of members, water is may be the diameter of the members, cross section, thickness etcetera; load is independent of the form. Because you have no control on what kind of load should be applied to the structure, we saw yesterday list of forces.

These forces are not dependent on, what kind of structure or the form it is. Of course, certain form is generate forces, because of complains action; however, force is not dependent on the form, but the condition of inertia force being larger or insignificant, depends on the mass therefore, mass should be quantified for a given form. Now it is very interesting for all of to understand, mass in general, is present continuously for the entire length of the member.

(Refer Slide Time: 07:53)



Now, you look at the complexities and dynamic analysis. Mass is a function of, mass becomes a function of, the length of the member, is continuously present by entire length of the member. And force is a function of space and time. Time, of course, dynamic, space of course, in arrow dynamic, or wind forces, depending upon the height, the z value which we saw in the last lecture, the z naught value from the mean terrain quantifies, the wind force. So, space variation is also there. Now you cannot do a problem, where, both are connected, if it is so, you can only do, the equation of motion in a partial differential form, because of them are inter-connected.

As I said, force depending upon the type of the system, can also generate. For example, vortex induced vibration. So, they can be connected. Then they are connected, when the mass is continuously distributed, the problem of definition for solving dynamic analysis, becomes in a partial differential equation mode. We all know there whether there are 2 variables. if you want to hold down 1 variable with respect to the other, you have to do a partial differential equation. And we all know that considering the differential equation, PDs have more complexities and solution procedures. Because we know, for a given an ordinary differential equation, we have got 2 sets of variables and answers, one is what we call as the complementary function, other is called as a particular integral part. Whereas in partial differential equation, the complexity is further large.

So, people thought to make it simple, one of them should be, made simpler than what we see here. So, they do not want to test the force. Because force, is considered to be variation of time. Variation in terms of space is not accounted. Force variation in terms of space, for example, I have a cylindrical member, this is my mean water level, you may ask me a questions, sir, when forced variation is put to space, that is along the depth, is not counted, how we do we encounter for this? We already know that the force variation becomes non-linear, and the variation of the force, in terms of water particle kinematics, that is velocity and acceleration of the water particle, is given by the respective theories.

For example, Aries theory, strokes, will give you the variation, of water particle acceleration and velocity, in horizontal and vertical direction, varying along the depth. So, the space variation is taken care of, by some other form. The variation of the force in terms of time is taken care of by me in dynamic analysis. So, we do not touch the force, now the second point of discussion could be, if the mass is considered to be varying along the length, I have got to do something for this, this is what we called as discretization of mass.

So, I discretize mass, at specific points, we call this as lumped mass. So, I lumped the mass that specific points. Now the question comes, there are various engineers doing dynamic analysis, one can lump the mass at five points, one can lump the mass at 10 points, one can lump the mass thinking that, the results will be more accurate, let me lump at hundred points. And one can say that there no lumping, I want to do a continuous variation, I will get more accurate results.

So, there is a diversification in the interest of dynamic analysis. So, to answer this, the question asked indirectly is, where should I lump the mass? The first question answered was, why should I lump the mass? If we keep the mass continuous, the problem will become partial differential equation, it is complex to handle therefore, we do not want to handle that. So, therefore, I am lumping the mass. This procedure, of continuum mass system to lump mass system, is called mass discretization.

We discretize the mass, at different points. Now where are these points? How many points can be taken, and why? To answer this, let us come back to the fundamental back again. We are not interested in the mass; we are interested in the product of, mass and the inertia, that is, mass in the acceleration of the mass. So, wherever I want to measure the

inertia forces, I will lump the mass there. Wherever I want to know, the displacements, I will lump the mass there.

So, now the question comes, in a given tower, or in a given column like this, which is fixed at the bottom, which is free at the top. We all know from mechanics that, the displacement at the bottom which is fixed here is practically zero. So, one would not like to lump the mass at this point, where there is no displacement, because, this term will become insignificant. If we really wanted to know the maximum effect of inertia in the system, you would love to lump the mass at the point where, the displacement is maximum. So, given form, given boundary condition, you automatically emerge in situations where, mask can be lumped if I have a system like this.

(Refer Slide Time: 14:19)



So, simple template structure, these are called nodes; they can be also called as joints. So, one can interestingly think of, an idealized model of this, as a simple stick model, where correspondingly, of course, this is the point where, the center of gravity is focused, on the top side let us say.

So, pick up these points, and lump the mass at these points. So, we are lumping the points, at discrete points, along the member, or along the structure, where, we are interested to measure the displacement, or the responses. So, this tells me, how many degrees of freedom, the system actually has. Now there is a small confusion here, is degree of freedom defined as, the number of points of mass lumping, or something else.

(Refer Slide Time: 16:51)

The answer is, it does not depend on, how many points the mass is lumped. It depends on something different. Then what is degree of freedom? Degree of freedom is explained as, is explained as, or defined as, the number of independent.

Displacement components, of a structural system, that is necessary, to quantify the inertia forces present in the system. So, these are the catch words. The mass should be lumped at the point where, inertia force can be measured. And they should be independent. So, here the focus is, not the number of points where mass is lumped, here the focus is, the number of points, or the coordinates, where displacement is measured.

So, this is nothing to do, with a number of points, where mass is concentrated, but unfortunately, or hypothetically, these 2 statements are actually linked. Because, it is only those number of points where, inertia force will be significant. Therefore, one can blindly say, generally, degree of freedom will be the number of points, where the mass is lumped. For example, this structure has about five degrees of freedom. But we can also have a structure with 1 mass, I lump the mass at this point, lumped the mass at one point, but allow the mass to have acceleration, in all 6 degrees of freedom.

(Refer Slide Time: 19:18)



So, 1 lumped point, 6 degrees of freedom, 6 lumped points, each one has 1 degree of freedom, again five, and so, on and so, forth. So, please do not be confused that, the number of points where the mass is lumped, will tell me, what is the degree of freedom. Because here the definition which prove this to be wrong. We can have 1 point where the mass is lumped, but still you can have 6 degrees of freedom for this. So, it is therefore, not valid the correct statement could be, independent, displacement coordinates, or components, which will quantify, or which will help me to quantify, the inertia of forces, present in a system. I should be able to capture, the inertia force, a system, with help of these independent coordinates, whatever may be the number that would be the degree of freedom.

So, expressed as D-O-F, degree of freedom, with hyphens in between, that is the classical way of writing degree of freedom. Now, the question comes, generally how many degrees of freedom, a structural member, or a system in offshore as generally. Because if we talk of beam element, there can be a four degrees of freedom, there can be 6 degrees of freedom, there can be 12 degrees of freedom, depending upon you do 2D, you do 3D analysis you ignore actual deformation, you add actual deformation. There is much confusion in mechanics whereas, in offshore structural system, these are all converged. So, single easy definition, how many degrees of freedom, a system can easily have. Let us see how many degrees, and how they define.

(Refer Slide Time: 21:31)



So, let say we have 3 axis. This is x axis, this is y axis, this is z axis. Let says, this is an origin, this origin can be anywhere in the platform. Let us say I have a TLP. This is super structure; these are called as column members this is called as a pontoon member.

Of course, you have super structure, where you have got crane derricks. You can have living quarters; you can have a helideck etcetera. This is got to be anchored with set of tethers to the sea flow, and that is my mean sea level, this is my wave direction. These are tethers. Of course, this is a sea bed and therefore, this is what we call as small d which indicates water depth, and, this is what we call as capital D, which indicates diameter of the member, essentially the outer diameter.

Now I can place this origin any where I want. It can be the center of gravity of the top side. Let us say it is somewhere here. One can calculate the center of gravity of the mass center, depending upon how the mass is distributed. One can easily calculate this, there is a easy mathematics available, we can make a spread sheet, in excel, you can easily find out, with reference to MSL, or with reference to sea bed, where is my CG located for the given mass distribution. It will be easy to find out.

So, let say this is the point where the mass is concentrated, for the entire super structure. I can locate this origin here, I can locate this origin anywhere in the center of this column, and anywhere I want, on the top, on the bottom, anywhere I want. So, the botheration of locating the origin in the analyzer is not important, the importance is at this point the entire mass, is set to be concentrated therefore, it is unique, and is simple and easy to locate this origin, in the same point, where the center of gravity is situated.

So, the entire mass is said to be concentrated. So, let say this is the point, now this has got 3 axis of motion, X axis, Y and Z axis. Therefore, they can be displacement along X along Y and along Z. There can be rotation about X, about Y, and about Z. So, the displacement along X is, is called the surge degree. The displacement along Y is called the sway degree. Displacement along Z is called the heap degree heap displacement. You can easily remember this, because, Y will coincide with Y, it is called sway, surge, and heap. These are technical names given for displacement along X Y and Z, respectively

Now, I want to mark the direction of rotational displacements. Use right hand rule, I put my thumb, towards the arrow direction, remaining four fingers, will show me a direction. So, I am marking it. Similarly this way, similarly this way, I am marking it. So, this becomes rotation about x axis, I called this is as roll rotation about y axis I called this is as pitch, rotation about z axis I call this as yaw. So, there are 6 degrees of freedom for the system. Why they are called degrees of freedom? Because they are independent displacement coordinates.

In a given system, which can help me to identify, the inertia force, in the respective displacement coordinates. And they are independent. Though the mass is at only 1 point, but there are 6 degrees of freedom. Therefore, degree of freedom is nothing to do with a number of points, where the mass is concentrated. Degree of freedom is classically defined as independent displacement coordinates,, which will help me to access to capture, to understand, to quantify, the inertia force present in the system.

(Refer Slide Time: 26:42)



Obviously, the inertia force will be generated, by the external force acting on the system, because the inertia is a product of mass into acceleration. The system does not move, does not accelerate, there is no inertia force. So, degree of freedom is classically defined and explained in offshore structures. So, maximum, we have 6 degrees of freedom. What does it mean is, your matrix size, in your dynamic analysis, will never ever exceed 6 by 6, whereas in other buildings, or any other structure like bridges etcetera, it can go very high. So, people use condensation procedure, people use banded with matrix procedure etcetera, to make it simple. But in this case, it will not more go more in 6 by 6, worse by worse 9 by 9.

We will talk about slightly later, in the next lecture, but let us say, my analysis is in a closed form, and can be easily handled, with the known mathematics. So, having said this, now I want to know, that we have agreed upon 2 cases, one, the loading will be periodic, or it is time varying. The system will have a significant mass, because system will produce significant inertia force, therefore, I will do dynamic analysis.

(Refer Slide Time: 27:53)



Then the question comes, what are the essential characteristics, of a mathematical model, through which I can do dynamic analysis. Ideally speaking, a single degree freedom system model, D-O-F stands for degree of freedom, single stands for 1 degree of freedom. 1 degree of freedom in the sense, there is only 1 displacement coordinate, and; obviously, indirectly, we also understand within our internal mind, there is only 1 mass. To start with, the most simple, ideal, mathematical model to do dynamic analysis, this is single degree of freedom system.

Now, one can have a doubt in mind, when offshore platforms, do have 6 degree of freedom, why are we starting your discussion with single degree? If you know the procedure and basics and fundamentals and dynamics based on single degree, you can always up rises to infinite degree of freedom because, the procedure is same. But unfortunately, which is the fact, I am openly in the class, 99 percent, of the class room teaching, unfortunately, stops at single degree. They think that single degree is all enough. We will take the lectures to, 9 by 9 degree of freedom. I will tell you how to derive a stiffness matrix mass matrix for a given offshore platform. We will solve it here in the black board, using calculator. So, I will, I will, I will demonstrate to you, how dynamic analysis is done using a simple calculator in the class.

So, essentially, we will understand first single degree, and then we will move on further. The moment it is single degree, we only have one thing in mind, there is only 1 displacement coordinate. And we also indirectly have in mind, that there is only 1 mass point. But what are the essential characteristics that must be present, in a single degree? Ideally speaking, a single degree freedom system model, mathematically, looks like this. There is 1 mass, there is 1 spring, there is 1 dash pot, and there is 1 coordinate system, where the displacement is measured, and the force is applied (Refer Time: 30:31).

So, it is got 1 mass capital M, it has got 1 spring, whose stiffness is K, it has got 1 dash pot, whose damping is C. We will talk about all these things now quickly. This is an idealized single degree mathematical model. So, the mathematical model should have 4 components. Obviously, the 4 components are F of T, M, K and C. Of course, this is not a component, this is a displacement. I am marking the displacement, from the CG of the mass. You can also mass the displacement from the front end, of the mass I can mark it from back end of the mass, anywhere I want, but I am taking the CG of the mass because, it is ideally to understand, at the CG the mass is set to be concentrated.

The foremost content of this will be the presence of mass, indicated by M, which is an indication of inertia force. Now imagine there is a mass, which is allowed to move only in 1 direction, may be to or fro, or forth and back. Because it cannot move vertical, there is a wheel here, it cannot move lateral, because it is going to move within a channel, let us say. So, it is going to move only in 1 direction. But as we keep on moving, the body will be keeping on moving, right? The body has to restore to it is original position. It has to vibrate. If it does not vibrate, there is no displacement generated, right?

Now, I want to bring the body back, or the mass back, to it is original position. So, I should have a restoration force, which is given by the stiffness of the spring. So, this will give me restoration force which is offered by the spring. Imagine where spring does not exist, the mass will not restore it is position back, it will be keep on moving in the direction, as the force is pulling the mass. Therefore, we do not have any such structural system, in offshore, where the mass can move, wherever it has to move, the mass has to stay at a place, platform has to stay, because we need to do exploration or production. Mass has to remain, at a place. So, we need some mechanism, by which though the force will pull or push the mass, the mass should restore it is position to some extent.

So, the restoration is essential, which comes from the stiffness of the member, what we call as K. I indicate this as stiffness of the spring. Mass is indicated in KG, that is in

system international, stiffness indicated a newton per meter, force, per unit displacement in stiffness, Newton per meter. The third content is, mass resting on the surface, this is a wheel, let us say, but still the mass will move, because the force is very large, the mass will move. But, there will be an opposing force to the mass, which will restrain the mass to move. That can be a frictional force also. So, there can be some, dissipating force, which will prevent the mass to move. It can be also offered by the air. Air can resist, the mass should not move. There can be friction, between the surface of the mass or the surface of the platform here, which will not allow the mass to move.

So, there is some mechanism by which the mass will be stopped. So, that is the dissipating force, which is offered by C, which is called as the damping force, damping coefficient, which is given in newton per meter per second. So, newton second meter. That is the unit. Let us say, I have a system where, mass is present, spring constant is available, dash pot is also available, which offers need some resistance, but there is no force. So, the mass is static, it is not moving at all.

So, when the mass does not move, X of T is not activated, X of T is not activated, X double dot T does not come, therefore, no inertia force no dynamic analysis. So, the forth component is external force, which essentially should remain as function of time, because time variant is an important character, in the dynamic loading. It can be periodic, it can be prescribed, it can be non-prescribed, the procedure can be deterministic, it can be non deterministic, but, it has to be remaining as a function of time.

Anyone which is function of time will become continuous derivative, if you differentiate the response, with respect to time. On the other hand, if X of T is a displacement, then DX by DT should give me X dot, and DX dot by DT, should give me X double dot. All are continuous. So, in this case the, the function is differentiated with respect to time. So, the whole story is happening in the time series.

Now, it is become very simple. When, you idealize this, as a lumped mass system, which is descritized at the point, where the mass is concentrated. Imagine, the mass is present all over the media, along the length, along the height, and along the volume, continuously. So, mass itself is depend on XYZ parameters, and every XYZ you have got 6 degrees of freedom. So, you can an infinite number of complications in the

analysis. So, to control this, people said, let us lump the mass only 1 point, let us assume the mass, moves only at 1 degree of freedom.

Then let us see, what would be the dynamic response analysis, of such systems. Suppose if we have a question in mind that is single degree freedom system is highly idealized? It is not in critical. I mean, can we name a single degree freedom system? Can be show it? Can we see? Can be draw, a single degree? Which is existing, because if does not exist, it becomes totally hypothetical. We should not study that. For understanding sake, we can study, but as I you told, 99 percent dynamics is concentrated on single degree first, and then extrapolate the multi degrees. It is not vice versa, people explain with multi degree, and then try to make you to understand single degree. No, single degree is the alphabetic level A, multi degree can be Z. So, that is the order.

But people do not have energy to carry from A to Z, therefore; they stop basically B or C, and say that is all dynamics about. So, how does it practically relate to the understanding? So, characteristics of mathematical model, of a single degree, should essentially contain the 4 components, without which, the model is not complete. There are 4 essential types of single degree, which is idealized in the literature. There are 4 types. Let us see what they are.

(Refer Slide Time: 39:03)



Let say, for the completion sake I will draw the same figure, slightly in a different model. Because I have drawn that on horizontal plane, I can draw it in a vertical plane also. Let us say, in this case the support is horizontal, which is normal to the board, you can always make the supporter, at the board itself. So, I am drawing it like this. Similarly the same figure I am re drawing, with this. So, this is my mass, this is my stiffness, this is my dash pot, this is my X of, sorry let us make this, Y of T, and this is my F of T. This is what we called as Spring Mass System, spring Mass dash pot system. This is called as a dash pot.

There is a very interesting meaning of this symbol. Let us enlarge this. Imagine, there is a fluid present inside. This is connected to a body; this is further connected to another body. So, let us say, this is moving and this is static, it does not move. So, out of the 2 components, you connect 1 to static, or 1 to fix. Let 1 move. So, as I move this, this portion will start moving, and there is a limit beyond which this cannot move, because there is a control mechanism inside.

Once I move and release the force, this body will move back, or move up, will go on hit. The body should not, go here and touch. Therefore, this will prevent the motional. So, this keeps on moving. That is why, it is called dash pot. It keeps on dashing, and there is a pot which does this. Ideal example of this is nothing but a simple, 2 strokes or 4 stroke engines, in automobile. That is how it works. This is connected to the crank, this is connected to the wheel, as the wheel moves a crank operates, then therefore, this controls a wall, fuel is pumped in, vehicle moves forward. It is a same idea here, but here we use fluid as a damping media.

Suppose I do not have a fluid here, there will be keeping on ringing; this will get damaged, ok? The energy should be absorbed, right? So, this body, or this space, will absorb energy, and they will dissipate the energy. Why it is dissipating? Friction will stop the motion. So, it is arresting the motion. Therefore, it has to dissipate the energy and the force. Therefore, it is dissipating force, ok? That is a dash pot. So, it is a Spring Mass system. The second could be a torsional pendulum. This is called a torsional pendulum, where the degree of freedom, is a rotation.

(Refer Slide Time: 42:54)



The third can be, where I subtend the mass at one end, I subtend the spring at the other end, and pull the mass. So, that is the stiffness, and it is a simple cable which runs through this and of course, this is it can be YFD, not instead of let us say YFD. So, it moves only in vertical direction. You may ask me a question, sir, there are 2 movements here, 1 is this mass moving, and of course, if this, pulley also has a mass, this will also rotate.

Let us say this is theta. So, there are 2 degrees here. 1 is the mass of the pulley, which is focused here, which is having theta of T. The other is M, which is moving vertical, which is having Y of T. How we can say, it is single degree? Interestingly, look back the definition. Degree of freedom is number of independent displacement coordinates. Now, in this equation, I can connect theta and Y easily. Sine theta T will be equal to Y of T.

So, they are not independent. Either you measure Y of T, or you measure theta of T. If we know one, you can find the other. If you know the radius of this pulley, is it not? So, I want independent displacement coordinates. So, in this case, independent is only 1, either theta or Y that is the third example. The fourth could be the primary mass, and the secondary mass, sitting on set of springs or series of springs. Now again, this can have a motion of X1 of T, this can have a motion of X2 of T. One can say there are 2 mass sirs, there are 2 displacements. Is it double degree of freedom or 2 degree of freedom? No, the answer is only single degree of freedom, provided, we are not considering the mass M1

to move, we only consider mass M 2 to move, therefore, there is only 1 degree. That is one example, or one definition. If both of them are moving, we looking for a relative displacement between these 2. So, X1 minus X2 of T, we are looking for, because if we know one, I can find the other, provided I know the stiffness of the springs.

Therefore, independently only 1 will be there. That is X1 minus X2 of T, whichever is higher. So, these are the 4 ideal single degree freedom system models, mathematically available in the literature, which will help us to understand, the dynamic analysis of an idealized, single degree of freedom system model. So, you can always, make or assume, a mathematical model of any platform, any structure, as 1 of these 4. So, if your platform is idealized, as any 1 of these 4 we call them as assumption, or limitation in the study.

I am assuming the model, to remain as, single degree freedom system, as a Spring Mass system. Then do analysis for this, and apply this to the structure. It is valid, it is mathematically acceptable, and researchers keep on doing like this. As you understand this, then you want to move on higher stages. Let us say, I have do not want only 1 mass, there are n number of places where I want to measure the inertia of forces. There are n degrees of freedom. Therefore, I wanted to do a dynamic analysis with n by n matrix, yes we can do that. Let us first understand this.

So, in this lecture we summarize that, what are the essential characteristics of a single degree of freedom system, what are the important components, what are the ideal examples of single degree of freedom system, why they are single degrees still there are 2 movements? Because we are talking about independent coordinates. We also discuss what is periodic and non periodic loading, and why at all, we do not want a mass to remain continuously present in the structure, because then it becomes partially differentiable, the complexity is higher, we want to lump them.

Once we agree to lump them, where to lump them. We want to lump them at the points where I want to measure the displacement, because I am interested in inertia force. An inertia force is a product of mass and acceleration. So, acceleration is a derivative of displacement, in a time domain, as we saw there. That is how it is explained. Therefore, in the next lecture we will talk about, different conditions of, single degree of freedom system model, and how equation of motion it can be written for this, by different methods.

We will see that in the next lecture. Then we will start solving the equation of motion, for different applications. Then we will take up, 2 degree, multi degree, etcetera, and see how I can solve the problem.