Risk and Reliability of offshore structures

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Module – 02
Reliability theory and Structural Reliability
Lecture – 11
System Reliability – III

Dear friends, welcome to the 11th lecture on the course Risk and Reliability.

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This lecture is the eleventh lecture in module 2, where we are discussing about the reliability theories and structural reliability. So, this is going to be lecture 11, where we will continue with the system reliability as a continuation of the last lecture. I call this system reliability 3.

In the last lecture, if you remember or recollect we discussed about general system and we say it is actually a combination of system in series and system in parallel series, systems are those systems, where if any one component fails, certain parallel system are those systems where if all components fail, then the system is fail. So, for a given general system we identify the cut sets and the path sets from the cuts sets and the path sets we did not derive minimum cut sets and minimum path sets.

Now, based on that system you are going to discuss the failure theory, but let us recollect a new general system now, which is going to either comprise of minimum cut sets or minimum path sets. So, let us reproduce the same example again for our understanding and let us draw a general system with minimum cut set and minimum path set.

So, if you look back this was the general system we had we said, this is 1 and this is 2, this was 3 and this was 4, this was a general system which is of course, the combination of series and combination of parallel system. As you see from here, for this of course, is expandable in neither way for this, we identified the minimum cut sets which are C 4, which comprises 2, 3 and 4, which comprises 1 and 4. Similarly, we also had minimum path sets which where p 1 p 2 and p 3 p 1 is only 4 and p 2 is 1 and 2 and p 3 is 1 and 3. Now, we are going to form a system which comprises of either minimum cut sets or minimum path sets. So, now, we are going to form a general system.

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Assume to have a combination of either a series system with its minimum cut sets or a parallel system with its minimum path sets. Let us represent this graphically, the general system with cut sets which is going to look like this, the cut sets are 2, 3 and 4, the other cut sets is 1 and 4 of course, they need to be connected because I want them to remain in series that is what I am looking at because a general system with minimum cut sets will behave as a series system. So, I have put them in series, this system is now subjected to a load. So, this was my C 4, this was my C 1. There is now general system which comprises the minimum cut sets and this system is now going to behave as a series system. Please understand system which is in series is set to be failed, if any 1 component of the system fails.

Similarly, if we say a general system with minimum path sets the path sets are 4. So, let us connect them is 4, then it has got 1 and 2, it has got 1 and 3. So, this was my p 1, this was my p 2, this was my p 3, when I reproduce a general system with minimum path sets the system will behave as if it is a parallel system. Now, they are behaving as a parallel system when I reproduce this general system with minimum cuts sets. The system will behave as a series system now, they are in series, and these are 2 cases.

Now, I have modified the general system, which is the combination series and parallel where identifying the minimum cuts sets and minimum path sets. I have reconstituted the general system either with minimum cut sets or with minimum path sets. So, I will behave as if it is a series system 1 will behave as it is a parallel system. So, now, the complexity of identifying the failure of a system either with all components failing which is a parallel system or any 1 of the component failing which is series system is now simply to either series or a parallel for which independently 1 can easily identify the failure systems.

The complexity in a general system is now simplified to a series or a parallel system. Of course, understand this system now comprises of components of minimum cut sets or minimum path sets therefore, a very important for a given system to identify the minimum cut sets and minimum path sets. Now, interestingly I call this as figure number let us say, 1 for our understanding and 2 this is series system this is parallel system. So, as shown in the figures here, each minimum cut set is actually a sub system of parallel components.

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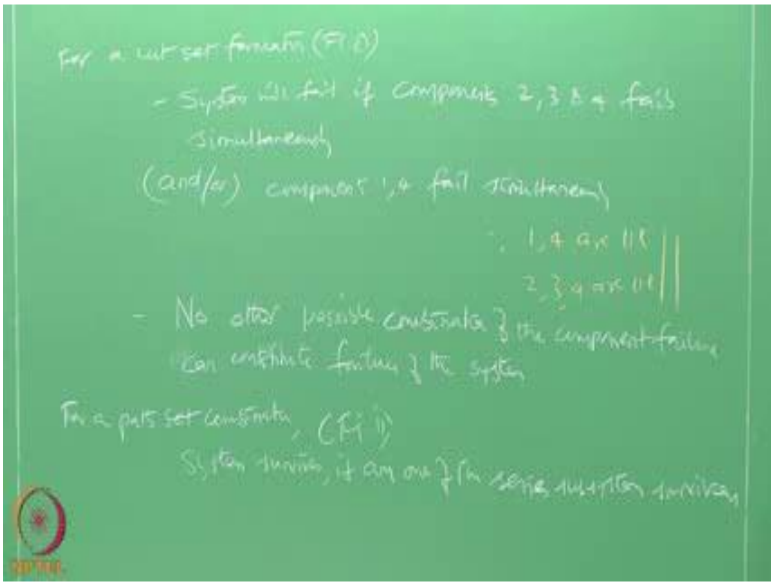

Correct is actually a sub system of parallel components is not and each minimum path sets is actually a sub system of components and series. We can see here components and series, but it is a parallel system. Now, for a cut set formation which can be seen in figure 1, for a cut set formation, system will fail if components 2, 3 and 4 fails because it is a parallel system, is it clear?

Simultaneously and or components 1 and 4 fail simultaneously because 1 and 4 are parallel. 2, 3 and 4 are again parallel if you carefully look at this figure and this combination you can always say, no other possible combination of component failure exit is it not. So, I can say no other possible combination of the components failure is it not can constitute failure of the system, correct. Now, for a path set combination which you can see from figure 2 as you see here, it is very easy to verify that the system survives if any 1 of the series sub system survives is it not because they are in parallel. So, the system survives, if any 1 of the series sub systems survives. Having said this, let us say having said this.

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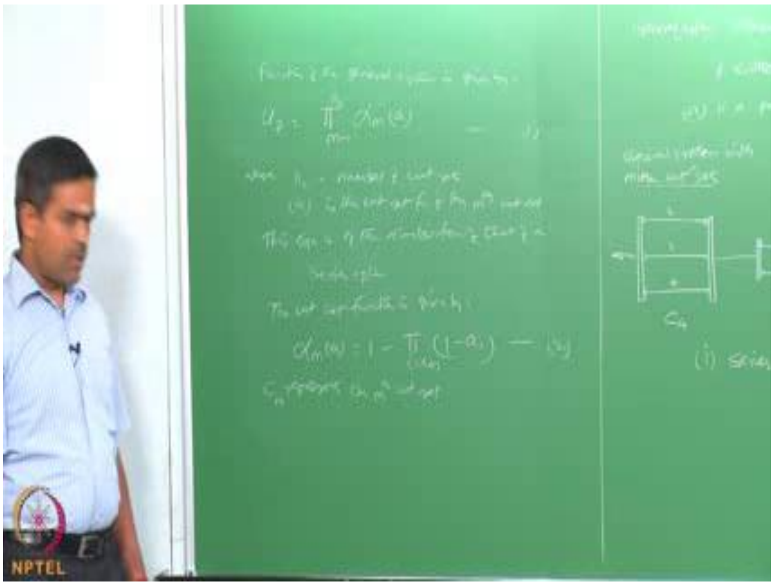

Function for the general system that is the failure function performance, function of the general system is given by a z, which is pi function of m minus 1 to n C of a where I call equation number 1, where n C is the number of cut sets, a is the cut set function of the mth cut set.

Now, you can see here. This system actually this equation is of the similar form of that of a series system is it not. So, pi function for which the cut set function is given by alpha m a see the cut set function is again in parallel, is it not therefore, that is going to be 1 minus pi of that is a parallel system, function i is equal to C m 1 minus a equation 2. So, alpha m which is used here is given by because now, in this system each 1 of them remains as parallel is it not therefore, the function failure is going to be happen from this equation. In this case, C represents the mth cut sets, C m actually represents the mth cut set.

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Therefore, for the general system function now, given by a z which is 1 minus, 1 minus a 2, 1 minus a 3, 1 minus a 4. These are the combinations; we have of that of 1 minus 1 minus a 1, 1 minus a 4. I will call this equation number 3. Now, for the Boolean variables, we have a special property. These are Boolean variables why because they take only values 0 and 1, failure or success for the Boolean variables, we know that a i k is equal to simple a i of k or a i, if k is greater than 0.

Applying this the above condition, we get a z as a 4 plus a 1 a 2 plus a 1 a 3 minus a 1 a 2 a 3 minus a 1 a 2 a 4 minus a 1 a 2 a 3 plus a 1, 2, 3, 4, equation number 5 same argument. Now, we can apply for the general system with minimum path sets.

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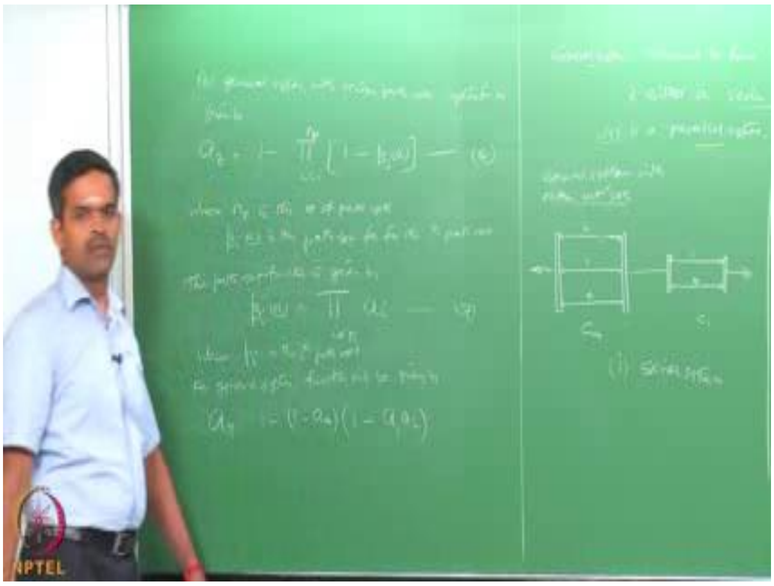

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So, for general system with minimum path sets system function can be given by a z. So, 1 minus i is equal to 1 2 n p 1 minus beta i r equation number six this is n p where n p is the number of path sets and beta i of a is the path set function for the ith path set the path set function is given by beta i of a is pi i element of p i of a i because in parallel system in path sets all will remain in series. So, we use that equation the equation number seven.

Where in this case p i is ith path set therefore, for general system function will be given by a z 1 minus 1 minus a 4 1 minus that is what you are looking at you look at this figure a 4 1 2 1 3 1 minus a 1 a 2 1 minus a 1 a two.

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So, expanding equation eight we get a z is equal to a 4 plus a 1 a 2 a 1 a 3 minus a 1 a 2 a 3 minus again a 1 a 2 a 3 minus a 1 a 2 a 4 minus a 1 a 3 a 4 plus a 1 a 2 a 3 a 4 you will; obviously, see this is as same as you got it for the function expanding in the series system. So, 1 can follow either way of finding out the failure or the system reliability for the general systems either comprising of minimum path sets or comprising of minimum cut sets you land up in the same function of failure.

Now, actually I am interested in computing the system reliability now. Now for a, i, j, where j equals 1 to n for the components and a z for the system which contains Bernoulli variables, let p i be equal to p of a j set to 0, which denotes the probability failure of the ith component and let p f equals p of a z to 0 denotes probability of failure of system. So, z denotes system and j denotes the component, I mean, in fact, it is a j having said this for the ith component.

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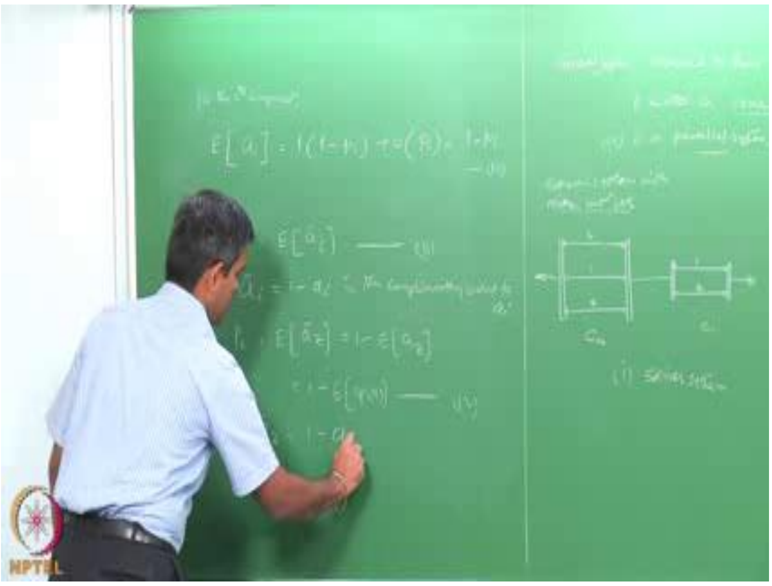

We can write accepted value of a i 1, 1 minus p i 0, p i which amounts to 1 minus p i equation 10.

Hence, p i is accepted value in this case of a bar j, where equation number 11, a bar i is 1 minus a i, this is going to be i right because we are looking at p i 1 minus a i is actually the complimentary event a i is it not its 1 minus a. Similarly, p f probability of failure of the system is accepted value of a bar z, which can be 1 minus e of a z, which can be 1 minus e of function of a, where a bar z is 1 minus a z in terms of component level reliability analysis.

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One can say that event a i set to 0 is equivalent to that is what you see here. is it not a i set to 0, a i set to 0 is equivalent to some function, which is g i of x is less than or equal to 0, which is the failure function. In other words, the ith component that is ith component fails then the outcome of the random variable in a given problem defines a point of failure domain that is what it means. So, the ith component fails then the outcome of the whole random variable is lying in the failure domain.

Now, for a system in series which is considering of n components system fails, if any 1 of this component fails, we know that therefore, the failure domain of a series system is nothing, but the union of component failure domains which can be given by let us say the expression which is union of i equals 1 to n that is a number of components g of x failure domain. So, less than or equal to 0 equation number 14 for a parallel system, which is considered n components system fails at all components fail is it not.

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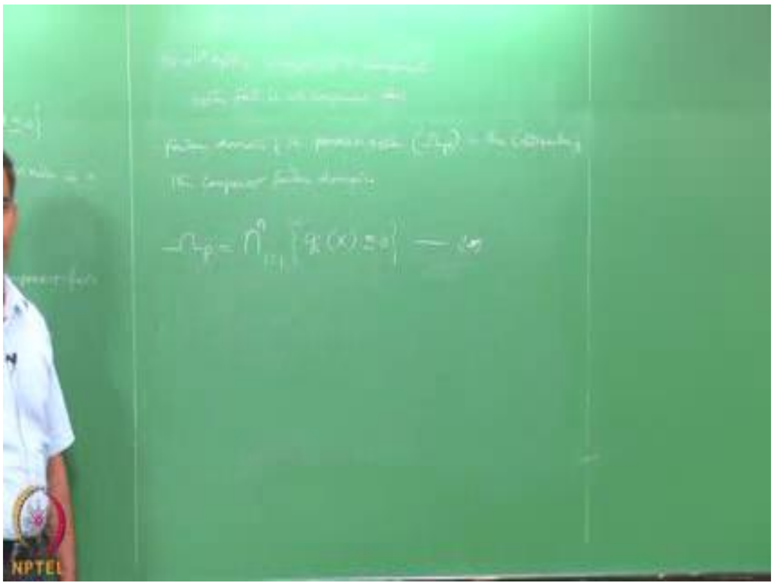

Therefore, failure domain of the system are let us say, to be very particular of the parallel system, which I am going to say as sigma p is the intersection of the component failure domain is it not, which is given by n number of components equation number 15. So, we will try to plot this failure domain and try to understand the limit state function surfaces, which indicate the region which is safe and which is corresponding to the failure.

So, friends in this lecture we extended the discussion of general system as a combination of series and parallel system. We replace the general system with minimum cut sets or with minimum path sets, one system becomes either cut sets in series are path sets in parallel and the sub sets in each of them is vice versa. The sub sets in a series system are parallel the sub sets in a parallel system are in series.

So, 1 now understands a failure of either a series system or a parallel system, which is going to amount to the failure of the general system. So, we did derivation for both independently and showed that both of them exactly gives you the same limit state function a z, where z is the system variable, and i is a component variable, then further extended the discussion to understand the failure function or the failure domain of the series system and the failure domain of the parallel systems. Once, we understand this, try to plot them graphically and show the failure surfaces for both of them independently and see how we understand really, the system failure in detail which we will see in the successive lectures.

Thank you very much and bye.