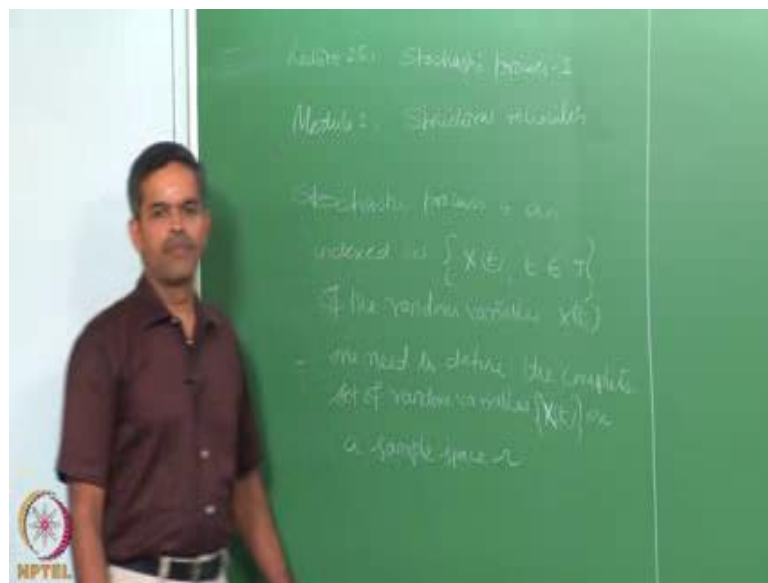


Risk and Reliability of Offshore structures
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Indian Institute of Technology, Madras

Module – 02
Reliability theory and Structural Reliability
Lecture – 25
Stochastic process I

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Welcome friends to the 25 lecture on module 2 where will talk about stochastic processes. This is a lecture on module 2 which is focusing on structural reliability on the online course title Risk and Reliability of offshore structures at NPTEL, IIT, Madras. Now, we already said that to explain the probability of failure or the reliability index which is converse of the failure. One can always look in to the Boolean variables and one can compare the systems with the elements in parallel or the system with the elements in series. And you can assign the Boolean variables to the failure or non failure state of these elements and ultimately one can result in probability of failure which can indirectly give me the reliability index of the given system.

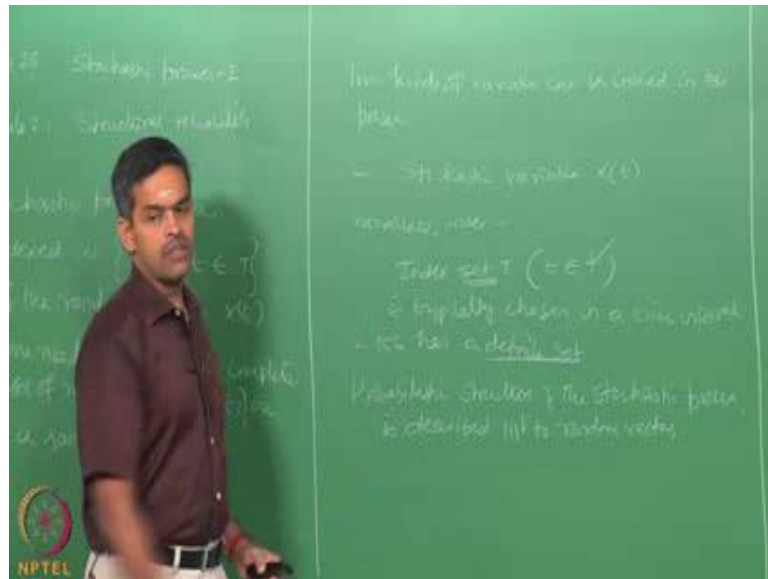
Due to several nonlinearities in another inconsistent and variables present in the given system which leads to lot of uncertainties. It is very essential that one should find out the

process by which these uncertainties in terms of qualifying random variables can be addressed. Alternatively, various researches have identify different processes by which this can be handled two of them were using synthetic data simulation using Monte Carlo simulation, where the random variable data set are generated using a simulation procedure. To qualify the data which can be used as an input variable for the reliability analysis? Alternatively, one can also use perturbation methods to do the same. But in the last lectures we understood that the consequences which arise from non mapping of proper variables which can be input variable to reliability analysis can lead to a wrong answer for the reliability analysis. Therefore, we understood that certain issues which are random variables in reliability analysis need to be investigated you through mechanical modeling.

So, we studied the coupling between the mechanical modeling and reliability analysis, what are the different players in the whole game. And what are the factors that influence the mechanical coupling. And we also stated in the example in the last lecture showing that how the moment, rotation characteristics of a hinged joint can be estimated and how the estimate can go wrong if you are not able to do this through experimental investigations. So, in the whole scenario one can easily understand that the variables which are used in reliability analysis unfortunately vary both in space as well as in time. Whereas when you convert them in the stochastical models one can get rid of the space variation by substituting a vector which we discuss in the last lectures and we make the variable only vary in respective time.

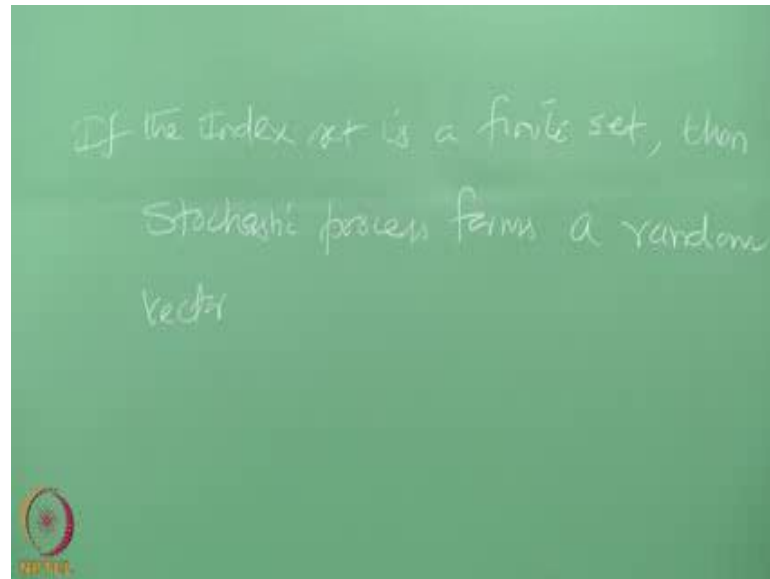
Therefore, now look into detail more about the stochastic process. So, stochastic process is a indexed is actually an indexed set. Let us called the set as x of t which is a time variable let say elements of small x of t and the whole the set is x of t . And where t is a element of the period of time within which you are estimating the reliability analysis. So, it is a set of x of t of the random variable which we say as x of E , now one need to design or explain or define, one need to define the complete set of random variables x of t on a sample space.

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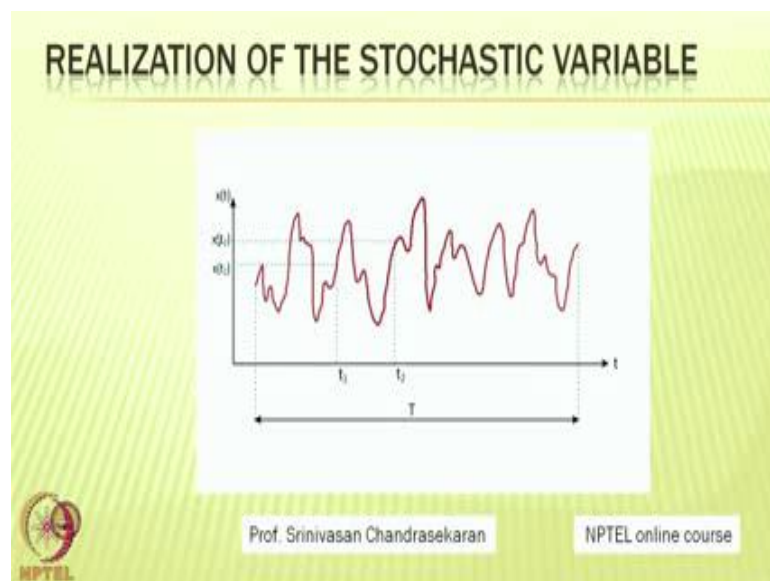
So, there are two different kinds of variables which are involve in this process; one is what we called a stochastic variable which is indicated as x of E in our discussion. It is also called as index the variability; it is also called as index. The index set T , because you know index set T , why because t is a element of T . The index set T is typically chosen in a time interval, but interestingly this has a definite set because the upper bound of the choice is fixed. So, it is got a definite set. Therefore, if we look at the probabilistic structure of the stochastic process, you describe in a similar way as set of random vectors. This is due to the reason that they assume any value within the given set index set T and of course, the assumed value on the set is a definite set, therefore, the stochastic process description is similar to as set of random variable in probabilistic point of view.

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Now, if the index set is a definite set or if the index set is a finite set, then the stochastic process forms random vector. Interestingly, friends the fact that the stochastic process is a set of random variables makes it natural to describe its probability structure in a way similar to that of random vector. But in this case the only difference is the index set what we explain in stochastic process is a finite set.

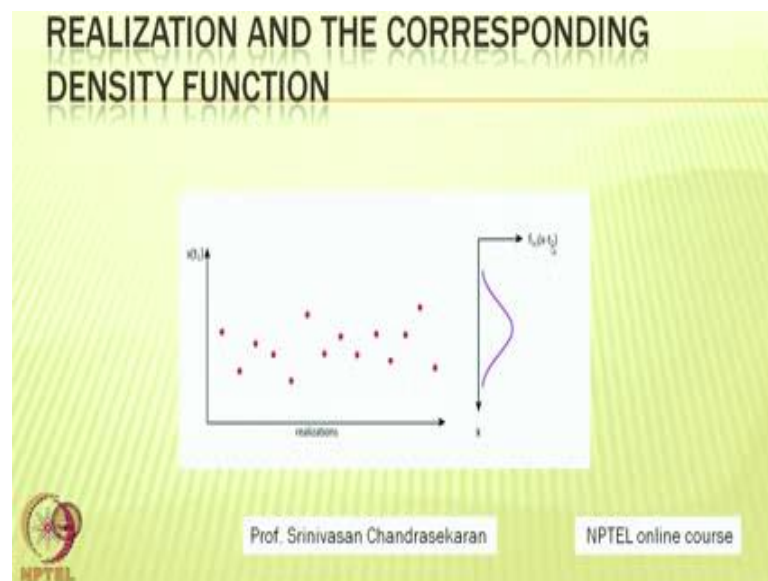
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Please pay attention the figure shown on the screen now. The figure shows realization of a stochastic variable where the variables are x_{t_1} , x_{t_2} and so on at t_1 and t_2 . For example for a given set of t index value of t let us select two values arbitrary t_1 and t_2 . I can call that value as x_{t_1} , and I can call these respective value as x_{t_2} which are assumed by the samples in the interval of 0 to t , there I am talking about capital T as a index set.

So, interestingly, these variables are similar to random vector because I can assume any value between the index set T . But in the probabilistic angle the only advantage of a stochastic process or a stochastic variable is the other than the random variable in comparison is that they have a finite set of values that is very important and that is where the problems becomes much more simpler when you handle the whole exercise using a stochastic variable instead of a random vector.

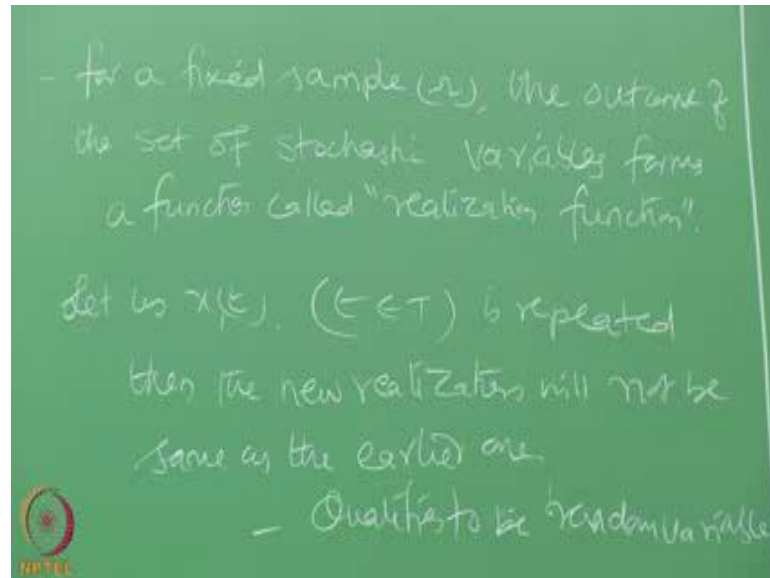
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Please pay attention to the figure shown in the screen again the figure shows realization and the corresponding probability density function of the specific variable. Now from the picture what we had in the last screen, which is shown now. Let us pick up one set of variable which is x_{t_1} . So, for x_{t_1} , one can easily find out the approximate or

appropriate probability density function $f(x, t)$ where we are talking about a specific variable whose values have finite set within the given index time capital T .

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So, therefore, for a fixed sample space, which is usually the case in reliability analysis of offshore structures the outcome of stochastic variables the outcome of the set of stochastic variables form an ordinary function which is called realization function. Now, choosing this variable let us say x_1 of t choosing this variable within interval of t as an index set is repeated then the new realization will not be same as that of the one what is on the figure. So, then the new realizations will not be same as the earlier one. So, in that way the stochastic process or the stochastic variable also qualifies to be random variable.

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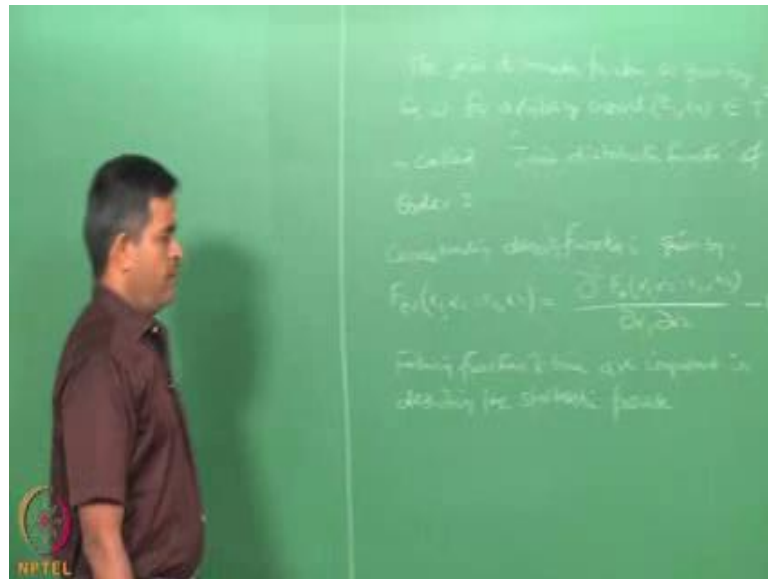
Interestingly, the probabilistic content of this variable will remain same. The realization will be different when we repeat the experiment, but the probabilistic contents of the stochastic process or the stochastic variable will be the same. So, considering this statement or considering this property, one can say the joint distribution function which is $F_{X_1, X_2}(x_1, x_2, t_1, t_2)$ which is going to be in the interval of t_1, t_2 can be defined as $F_{X_1, X_2}(x_1, x_2, t_1, t_2)$. Can be simply said as probability of $X_1(t_1) \leq x_1$ intersection $X_1(t_2) \leq x_2$. So, let us see equation number 1.

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So, the joint distribution function as given by equation 1. For arbitrary interval t_1, t_2 which is element of t square is called the joint distribution function of order 2. The corresponding joint density function is given by let say $f_{22}(x_1, x_2, t_1, t_2)$ is going to be dou of x_1, x_2, t_1, t_2 by dou x_1 dou x_2 . Let me call this question number 2. Therefore, friends in describing the stochastic process following functions of time are important. So, following functions of time are important in describing the stochastic process.

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The mean value of a function let us say mean value function which we call μ_x of t is defined as an expected value of x of t is given by is the expected value of x of t which is minus to plus infinity x f of x of x and t of $d x$ equation number 3. Now, the auto correlation function between the arbitrary interval chosen which is $R \times x \times t_1 \times t_2$ it will be equal to the joint moment of the random variables x of t_1 and x of t_2 .

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Which can be given by equation 4?

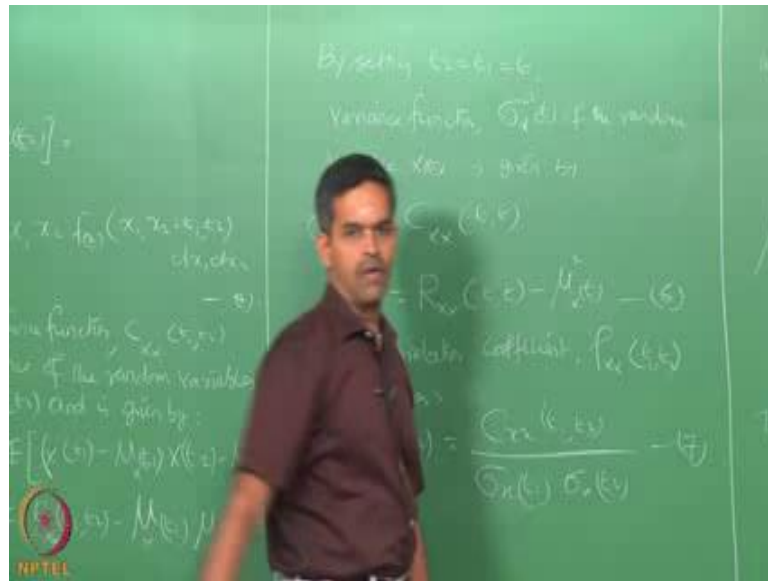
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The auto-covariance function, $C_{xx}(t_1, t_2)$ is the covariance of the random variables $X(t_1)$ & $X(t_2)$ and is given by:

$$C_{xx}(t_1, t_2) = E[(X(t_1) - \mu_x(t_1))(X(t_2) - \mu_x(t_2))]$$
$$= R_{xx}(t_1, t_2) - \mu_x(t_1)\mu_x(t_2)$$

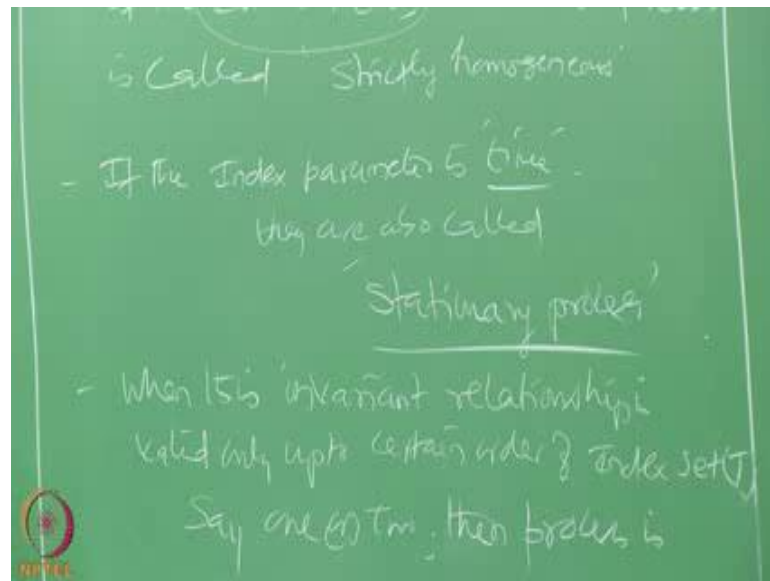
Now, auto variance function, which is $C_{xx}(t_1, t_2)$ is a covariance of the random variables $x(t_1)$ and $x(t_2)$. And is given by that is the $C_{xx}(t_1, t_2)$ is expected value of $x(t_1)$ minus $\mu_x(t_1)$ times $x(t_2)$ minus $\mu_x(t_2)$ which can be $R_{xx}(t_1, t_2)$ minus $\mu_x(t_1)\mu_x(t_2)$ let us say equation 5.

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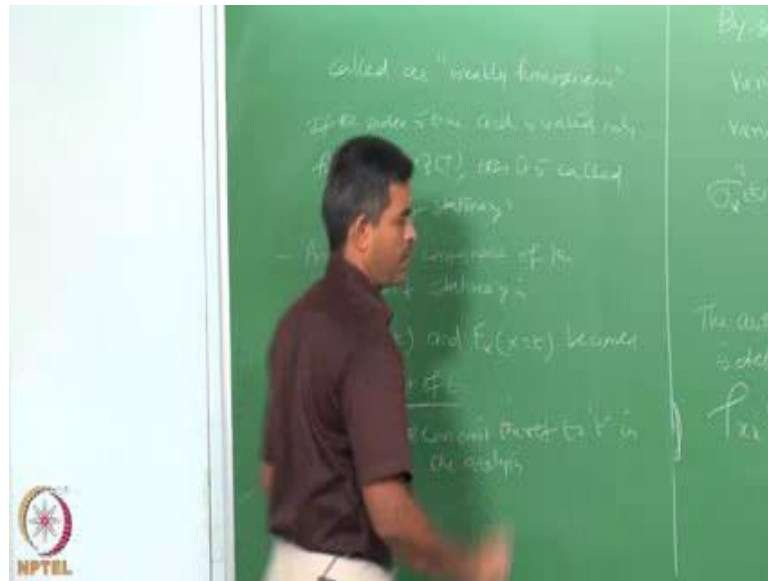
Now, by setting $t_2 = t_1 = t$ the variance function which is $\sigma_x^2(t)$ of the random variable $X(t)$ is given by the variance of the variable is the auto covariance function which is now going to be $C_{xx}(t, t)$ because I am now setting $t_1 = t_2 = t$. And we already have here expression for auto covariance t_1 and t_2 given by equation 5. Now, we reproducing them slightly in the different manner because this now equal to $R_{xx}(t_1, t_2)$, therefore I simply say $R_{xx}(t, t) - \mu_x(t)^2$. So, equation number 6 the auto correlation coefficient in this case which is indicated as $\rho_{xx}(t_1, t_2)$ let us say $\rho_{xx}(t_1, t_2)$. Let us write a general expression is defined as $\rho_{xx}(t_1, t_2)$ is given by $C_{xx}(t_1, t_2)$ divided by $\sigma_x(t_1)\sigma_x(t_2)$ equation number seven. Now, friends for an important group of stochastic processes all finite dimensional distributions are invariant to the linear translation on the index set.

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So, if you have a property of this order that is if the finite dimensional distributions are invariant to a linear translation of the index set T then this process is called strictly homogeneous such process as great advantage in the reliability analysis. Now, further if the index parameter is time as in this case then they are also called stationary. We all know in probabilistic and statistic the stationary processes has lot of advantage in handling them with probabilistic tools. When this invariant assumption holds good only for distribution of order one or two when this invariant assumption, because now we said it is a general one it is available as invariant linear translation for the entire subset of t . But unfortunately or incidentally if this invariant relationship is valid only up to let say certain order of T say one or two, let us say then the process is called

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Weekly homogeneous, if the index is time and is valid only for lower orders of index t then it is called weekly stationary. Now, what would be the consequence if I have a stochastic process which is either weekly homogeneous or strictly homogeneous or on the other hand if my process is indexing time then if it becomes weekly homogeneous or weekly stationary what would be the consequence. Now, one can look at an important consequence which arise from this an important consequence of the assumption of the stationary is f of $x \times t \times t$ and f of $x \times t$ becomes independent of t , so that they can omit in the whole analysis reference to t . So, this is the case one can omit the reference to t in the analysis.


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If I talk about second order distributions let us say second order distributions will only depend on the difference of index parameter τ , which can be the difference between the intervals t_1 and t_2 chosen within the index time t . Now, this statement is true for all other statistics therefore, friends in practical applications modelling of physical quantity by a stochastic process is based on a very important statement. So, in physical quantity, if you want to model the quantity by a stochastic process then it is based on an important statement that it depends on single realization of a stationary process. If only one realization is at hand then the mean value is very easy.

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If there is only one realization, then the mean value is given by.


$$\mu = \frac{1}{T} \int_0^T x(\tau) d\tau \quad - (8)$$


So, if there is only one realization then the mean value is given by the subset or the space set T 0 to T x of τ d τ let say equation 8.

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$$\mu = \frac{1}{T} \int_0^T x(\tau) d\tau \quad - (8)$$

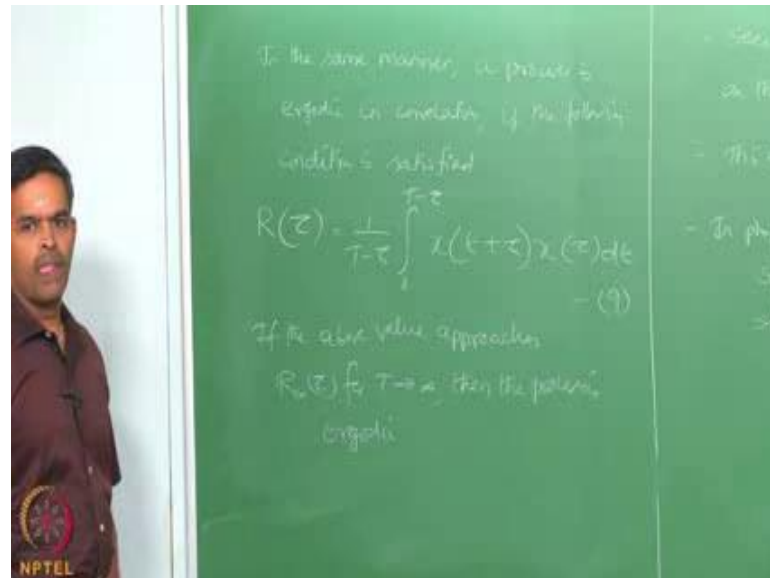
- If this time average approaches μ_x for $T \rightarrow \infty$, then the process becomes Ergodic in the mean value.



Therefore, equation 8 simply shows as it is the time average process. So, if this time average approaches a condition μ_x as T for T greater than infinity then the process let

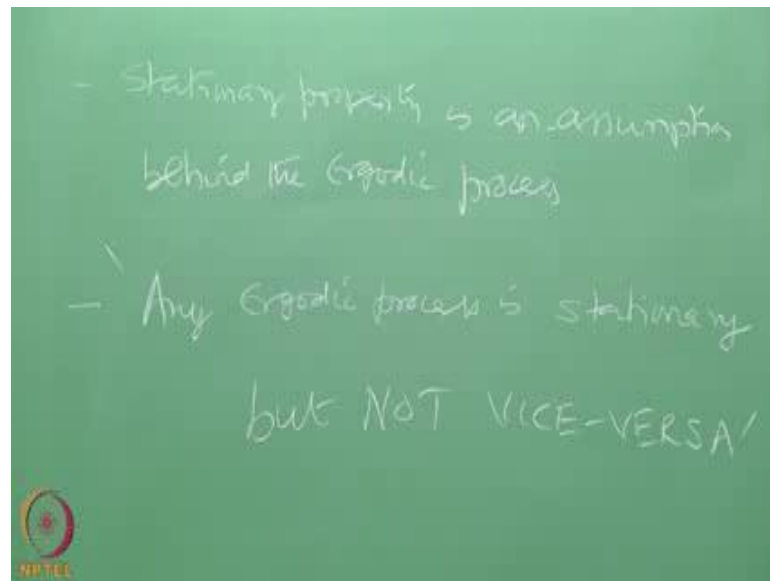
say tends to infinity not greater tends infinity then the process becomes ergodic in the mean value.

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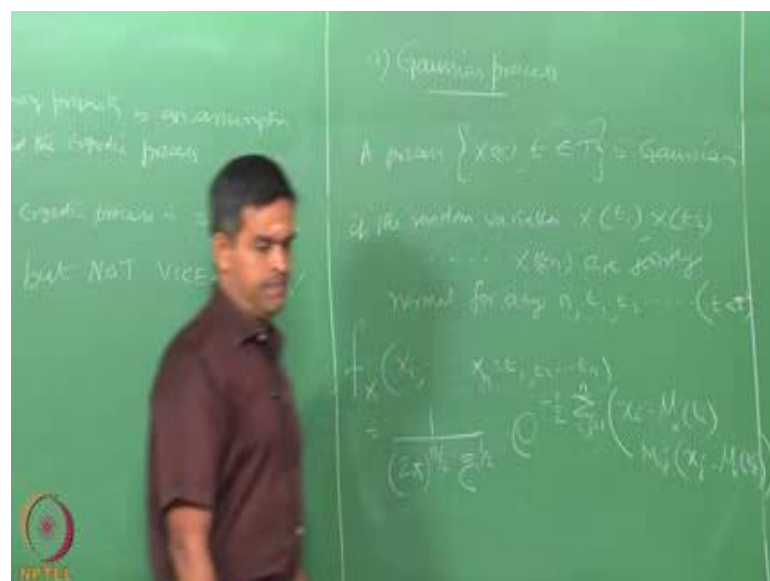
In the same manner, a process is ergodic in correlation if the following condition is satisfied $R(\tau) = \frac{1}{T-\tau} \int_0^{T-\tau} x(t+\tau)x(t)dt$ sorry $d t$ call equation number 9. Now, if the above value approaches $R_{xx}(\tau)$ for t tending towards infinity then the process can be called as ergodic.

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Interestingly, in the whole explanation the stationary property is an assumption behind the definition of an ergodic process that is important the stationary property is an assumption behind the ergodic process. So, that any ergodic process is stationary, but not vice versa. So, very important statement any ergodic process is stationary, but not vice versa that is important.

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Now, let us apply this concept of understanding to a Gaussian process. A process which is $x(t)$ and t is an index time capital T is said to be Gaussian if the random variables $x(t_1), x(t_2)$ and so on $x(t_n)$ are jointly normal for any n ; any n, t_1, t_2 chosen with in index capital T . Therefore, I can write $f(x_1, x_2, \dots, x_n)$ is given by $\frac{1}{(2\pi)^{n/2}} |C|^{-1/2} \exp\left[-\frac{1}{2} \sum_{i,j=1}^n (x_i - \mu_{x_i}) M_{ij} (x_j - \mu_{x_j})\right]$ equation 10.

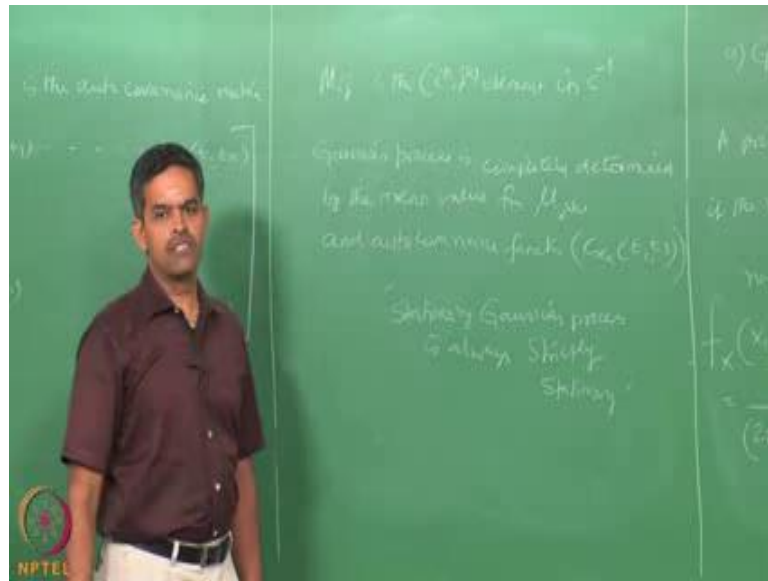
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Where \bar{C} is the auto covariance matrix

$$\bar{C} = \begin{bmatrix} C_{xx}(t_1, t_1) & \dots & C_{xx}(t_1, t_n) \\ \vdots & & \vdots \\ C_{xx}(t_n, t_1) & \dots & C_{xx}(t_n, t_n) \end{bmatrix}$$

Where \bar{C} is auto covariance matrix, the \bar{C} is actually given by a matrix, which can be $C \times x \times t_1 \times t_2$ varying from and extending till $C \times x \times t_n \times t_1$. Similarly, $C \times x \times$ varying $t_1 \times t_n$. So, it is going to be t_1, t_1 and this varies and extends as $C \times x \times t_n \times t_x$, so that is going to be the auto covariance matrix which is used in equation 10. I will call this as equation 11.

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And in the equation C_{ij} is the i th j th element in C minus 1. So, it is clear from the definition that Gaussian processes completely determined by the mean value. So, one can say the Gaussian process is completely determined by the mean value function which can be said as $\mu_x(t)$ and autocovariance function which is given by $C_x(t_1, t_2)$. Therefore, interestingly a stationary Gaussian process is always strictly stationary; it cannot be weakly stationary at all.

So, friends, in this lecture, we introduce you about the stochastic process, what is the stochastic variable, how a stochastic variable compares by the random variable, and what are those statistical properties in terms of auto covariance function cross correlation coefficients, how they can be derived for a given process of stochastic variables within interval t_1, t_2 in a subset of index T . In the process, we have also explain what do you mean by ergodic, stationary, and ultimately we fixed up a specific case of a Gaussian process ensure that Gaussian process if it is stationary will remain as strictly stationary and not at all weakly stationary.

Thank you very much.