Risk and Reliability of Offshore Structures
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Module - 01
Lecture – 04
Probability and Plausibility

Friends, we now move onto the next lecture which is the fourth lecture title Probability and Plausible reasoning under the online course, title Risk and Reliability of Offshore Structures in module 1, organized by NPTEL, IIT Madras. In this lecture, we will discuss about some important rules of probability and introduce you to what we call as plausible reasoning. We already said in the last lecture that reliability is governed by uncertainty and reliability is one of the effective tools to handle uncertainty, and therefore reliability is always advantageous to be expressed in probabilistic terms.

Now, the question comes if the uncertainty is not properly handled in the analysis stage, how does it propagate, what could be the influence of unattended uncertainties in forms of data in the final results; on the other hand, how uncertainty propagates. Uncertainties are primarily responsible for making the reliability studies probabilistic in nature. However, it is very interesting and hypothetical of course to inform that if every event present in the analysis, design, construction, planning, layout of offshore structures are completely certain with high degree of confidence and occurrence of the event and there are no uncertainties involved with these events, then one can say that reliability study can be always become deterministic. You need not have to actually look at the probabilistic tools at all. When you are having a guess or a chance of not covering certain events or influence of certain events on the other what we call cross correlation factors.

If you have, a fear that such factors might have been over looked upon in your analysis then reliability studies should be governed under the braces of probability theory. So, probability tag that is associated with reliability makes it more mathematical; otherwise, reliabilities generally visualized as a study based on engineering judgment. Reliability is nothing but a physical or engineering judgment based on packed experience. This appears to be an abstract statement; therefore, reliability is comfortably converted into a mathematical modeling using probabilistic tools. Uncertainty propagates in many ways;

it can be probabilistic, it can be non-probabilistic.

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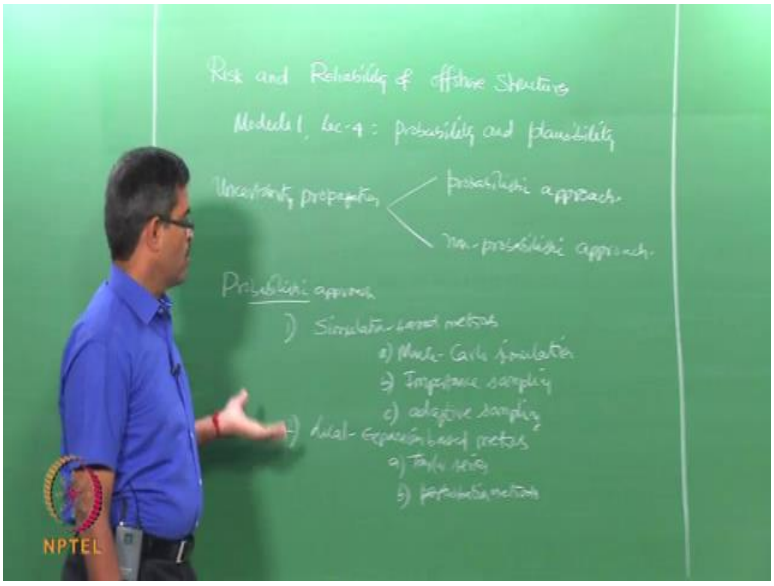

If you look at probabilistic approaches, how uncertainty propagates. It can happen by two ways. You can look this problem by two ways; one is you can look using probabilistic approach; can also solve this using non-probabilistic approach. You look at probabilistic approach; there are various methods which can be generally adopted for understanding the propagation of uncertainty.

Under this, the first class of methods which come in probabilistic approach is simulation based methods; we can name a few of them Monte Carlo simulation, importance sampling, adoptive sampling. We can also have local expansion based methods; under this, we have got Taylor series. One can also include perturbation methods as non expansion based methods. These methods are good to deal with small variability of input and output without high non-linearity. If the input and output variables have high non-linearity in nature, obviously, these methods both simulation based and local expansion methods are not very effective to handle them.

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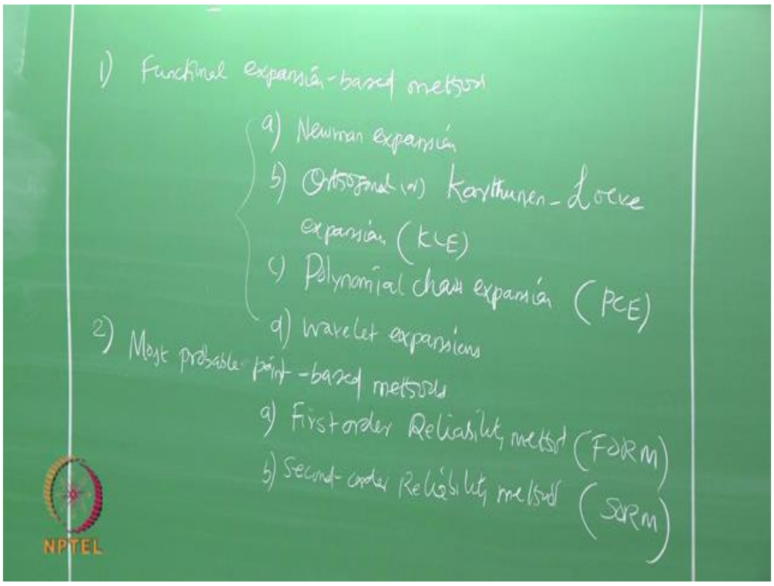

There are alternate methods which can also be used which are also probabilistic approach to handle or to model the propagation of uncertainty. Under this, the first class of methods will be functional expansion based methods we can cite a few example on this; Neumann expansion, orthogonal or Karthunen-Loeve expansion which is abbreviated as KLE approach. One can also have polynomial chaos expansion, which is abbreviated as PCE in the literature; of course, one can have wavelet expansion also. So, all these will be included in the subset of functional expansion based methods which are alternative methods compared to these two, and of course, they are also included under the braces of probabilistic approach to model the propagation of uncertainty in the variables.

You can also have the next set of methods, which is most probable point based methods; under this, we have first order reliable method, which we can abbreviated as FORM; second order reliability method which is SORM. The third set of methods as alternate methods to the classical methods as we see here based on simulation will be numerical integration based methods.

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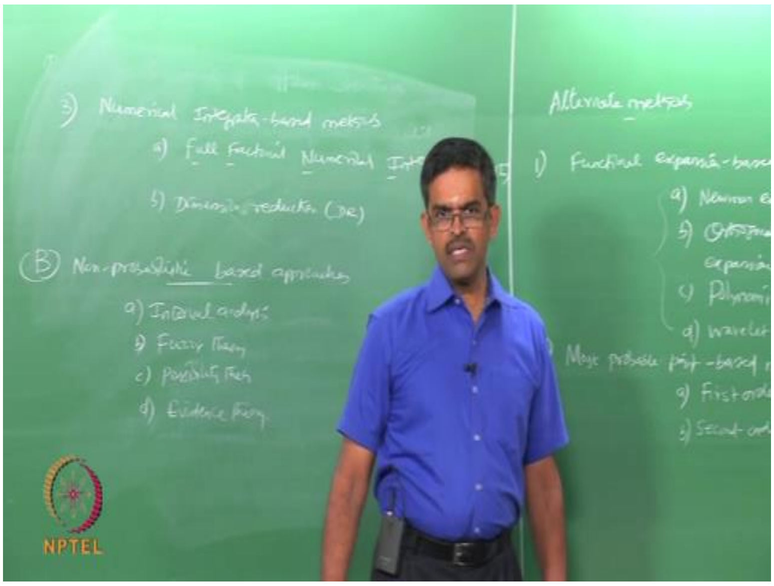

We can also name a few of them under this heading, which is full factorial numerical integration - FFNI. The method related to dimension reduction - DR method is also useful which is one of the numerical integration based methods; which can be used for modeling the propagation of uncertainty. Alternatively, you have non-probabilistic based methods like interval analysis, fuzzy theory, possibility theory, evidence theory. Probabilistic approach is of course, a rigorous approach to do uncertainty analysis. It is because of a simple reason because it is consistent with the theory of decision analysis. For example, in regression analysis and least square problems, standard error of the parameter is readily available So, this can be further expanded into the confidence interval to verify or ascertain the solution obtain from the analysis. Having said this let us try to list out what are the major challenges in reliability analysis as a whole.

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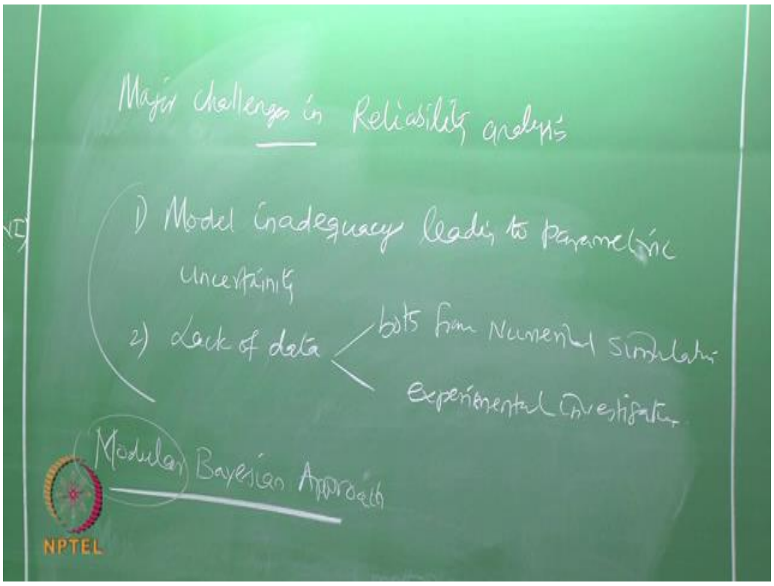

Essentially, there are two major challenges, which are focused in the literature. One is arising from model inadequacy, which of course leads to parameter uncertainty. Second could be the lack of data, which is required to do the analysis, which can arise both from numerical, simulations and experimental investigations.

Now, to solve these issues which are dominating the challenges in reliability analysis, people used Modular Bayesian approach. Modular Bayesian approach can be used to address such complexities. This approach used to inverse uncertainty quantification, and it is a better way to express the uncertainties in terms of circumscribing which is shown here which are posed as major challenges in reliability analysis. Now the Modular Bayesian approach has actually as name suggests it has got four modules, we will quickly see each module one by one and talk about what is the use or role of each module in the whole analysis.

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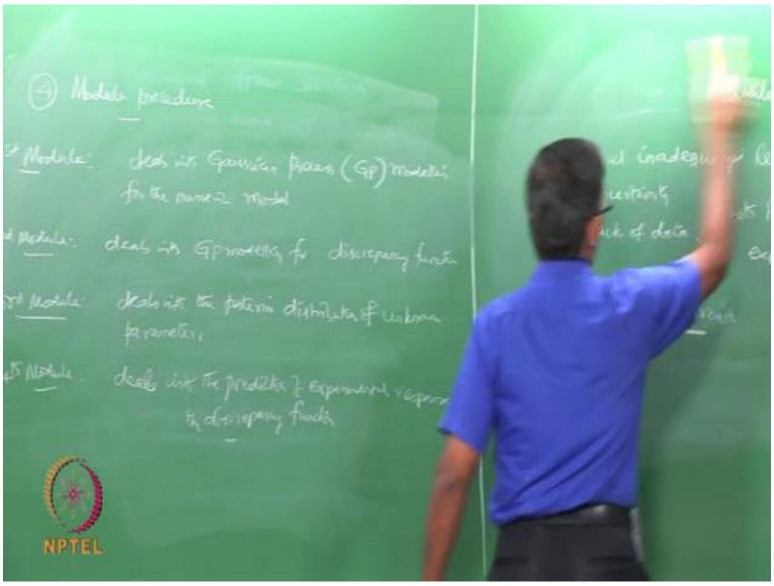

So, Modular Bayesian approach does the analysis in four module procedure. In the first module, it deals with Gaussian process modeling; I will use GP for Gaussian process modeling for the numeric model. The second module deals with Gaussian process modeling for the discrepancy function. The third module deals with the posterior distribution of unknown parameters. And the forth module which is the last module deals with the prediction of experimental response and the discrepancy function. Let us quickly elaborate each module and see very briefly how are they contributing to the whole analysis.

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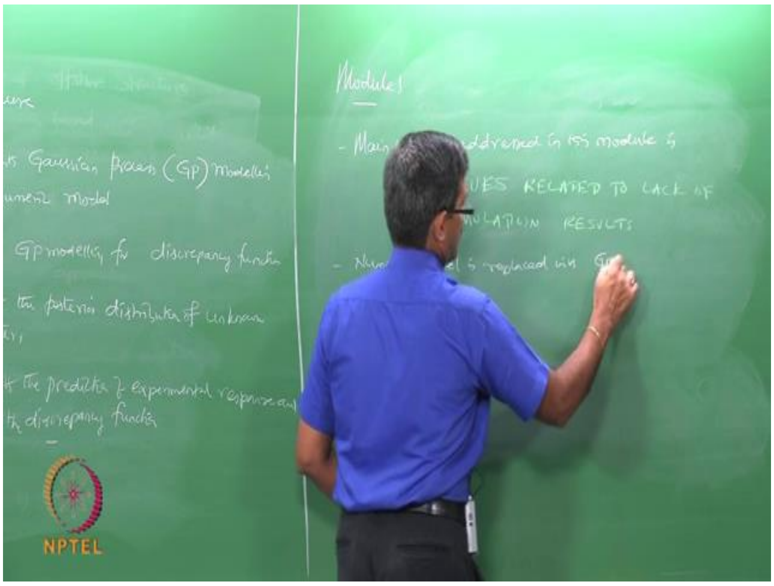

We talk about module one in Modular Bayesian approach, the main problem addressed in this module is issues related to lack of simulation results. In this module, numerical model is replaced with the Gaussian process model. So, the numeric model is replaced with the Gaussian process model. The replace Gaussian process model, which is used instead of numeric model, is given by the following equation.

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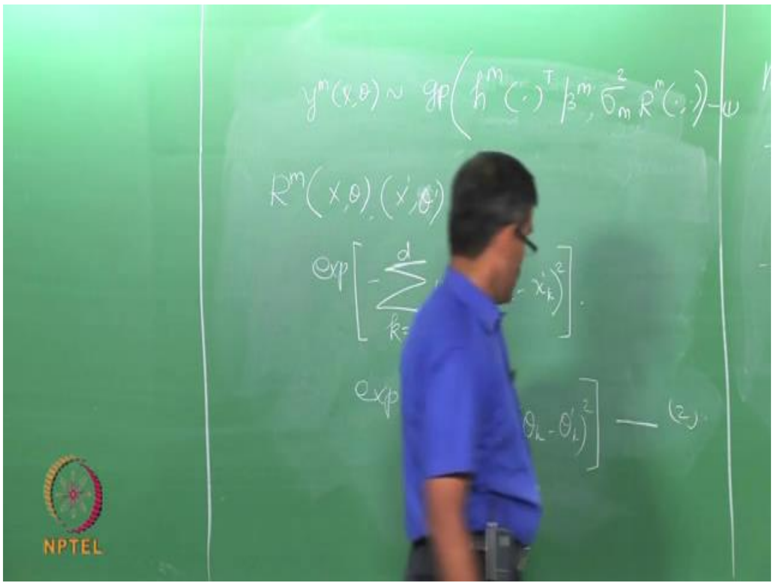

Y of m x, theta is approximately equal to the Gaussian process of h m of the function transpose beta m and sigma m square R m of the parameters. I will call this equation number one. Where R m of the parameters x theta and x dash phi or theta dash is actually given by exponential of minus summation of k equals 1 to d, and the summation is applied through the function k m x k minus x dash k square multiplied by exponential of summation of k equals 1 to r, which now goes with w d plus 1 m or d plus 1 d plus k m of theta k minus theta k dash square called equation number 2.

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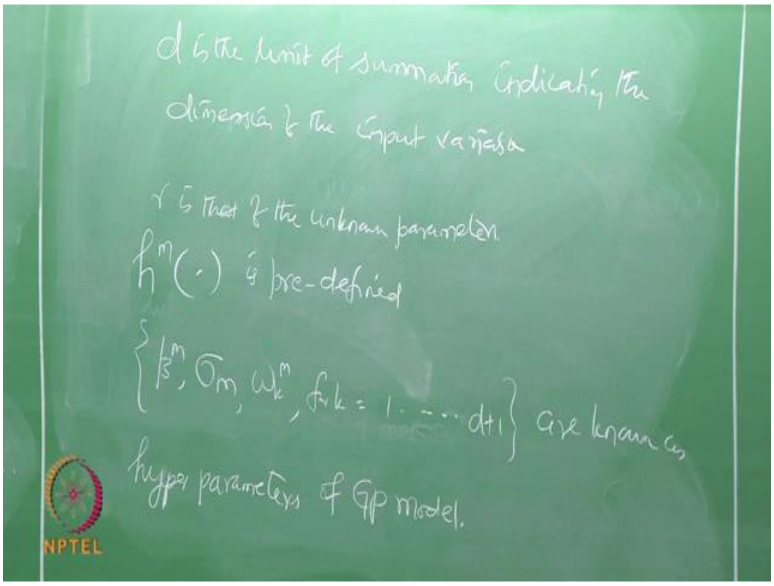

Where d is the limit of summation indicating the dimension of the input variable; r is that of the unknown parameters; h m is predefined for a given problem; beta m, sigma m, w km for k equals 1 to d plus 1 are known as hyper parameters. These hyper parameters of the Gaussian process model.

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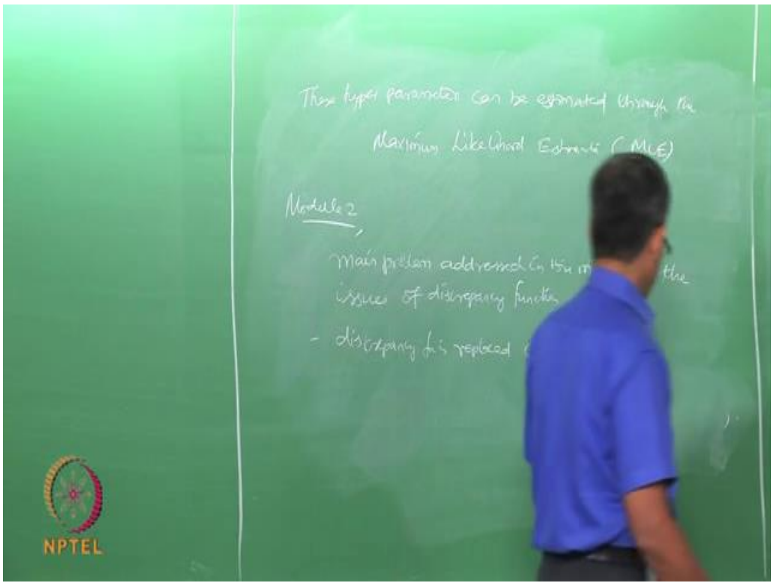

These hyper parameters can be estimated through the maximum likelihood estimation. This is a very recommended generalized technique which can be used as one of the powerful module in the modular Bayesian approach. In module two of the modular Bayesian approach this deals with the Gaussian process modeling the main problem addressed in this module is the issues of discrepancy function. In this module, discrepancy function is replaced with the Gaussian process model; whereas, in the earlier case the Gaussian process model replace the numeric model. So, the discrepancy function is replaced using Gaussian process model.

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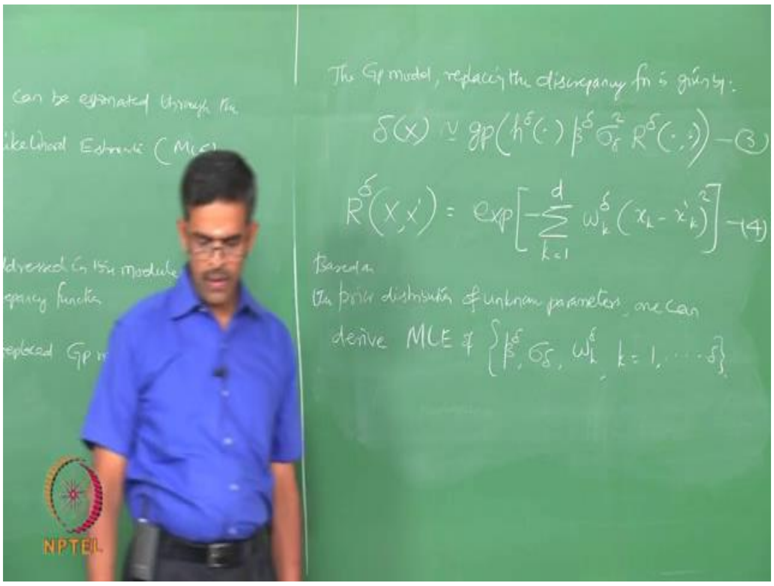

Now the Gaussian process model which replaces the discrepancy function is given by the discrepancy function is called as del X which is approximated to the Gaussian process model of h delta of the function beta delta sigma delta square R delta of this, let me call this as equation 3. The R delta is given by the exponential of summation of k equals 1 to d w k delta of x k minus x dash k square which is equation number 4, where the meaning applied as same as in the earlier case. The prior distribution of unknown parameters based on the data obtained from the numeric and experimental studies one can derive the maximum likelihood function. So, we can say based on the prior distribution of unknown parameters one can derive the maximum likelihood estimates of beta delta, sigma delta, w k delta where k is varying from 1 to delta.

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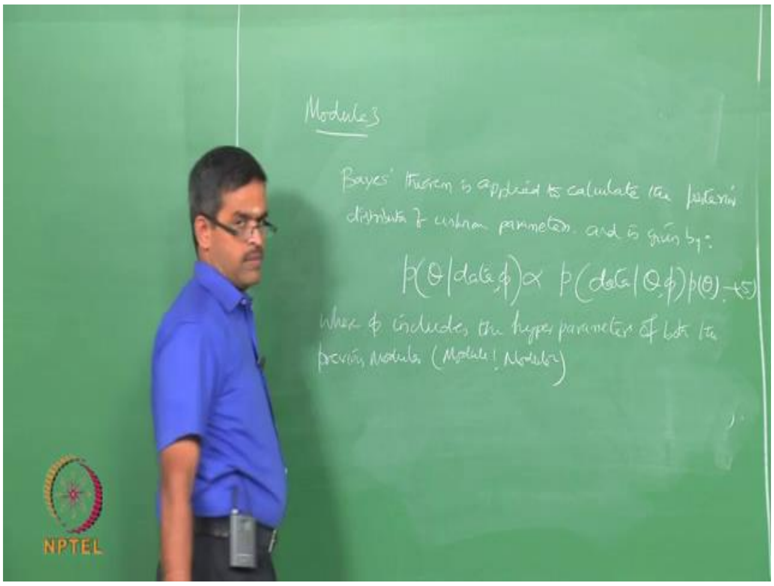

In case of module three of the Modular Bayesian approach, base theorem is applied to calculate the posterior distribution of unknown parameters. The base theorem as applied to the posterior distribution of unknown parameter is given by probability of theta with the data available on phi is proportional to probability of data given theta and phi with probability of theta, let us call equation number 5. Where in the above equation phi includes the prediction of experimental includes the hyper parameters of both the previous modules that is module 1.

In module 4, it deals with prediction of experimental response and discrepancy function together all the four modules of Modular Bayesian approach is considered to be one of the effective tool to address auto model their propagation of uncertainty. Alternatively, one can also look at the literature for a full Bayesian approach. The earlier what we saw was a Modular Bayesian approach; alternatively full Bayesian approach can also be used to model the propagation of non-linearity.

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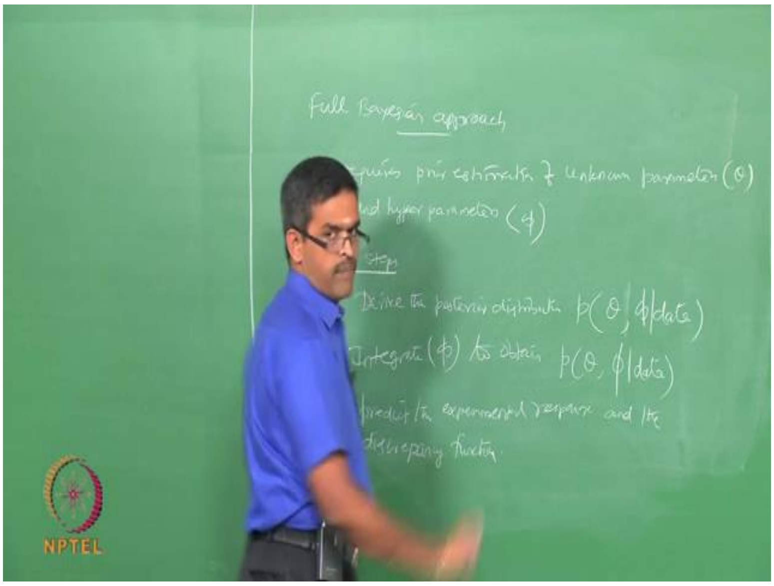

According to this, this method requires a prior estimation of unknown parameters this method requires prior estimation of unknown parameters, in our case let us say theta, and hyper parameters in our case phi. So, following steps are important in case of full Bayesian approach derive the posterior distribution first, which is probability of theta phi given data followed by which integrate the hyper parameter to obtain the probability phi given data proceed to predict the experimental response and the discrepancy function.

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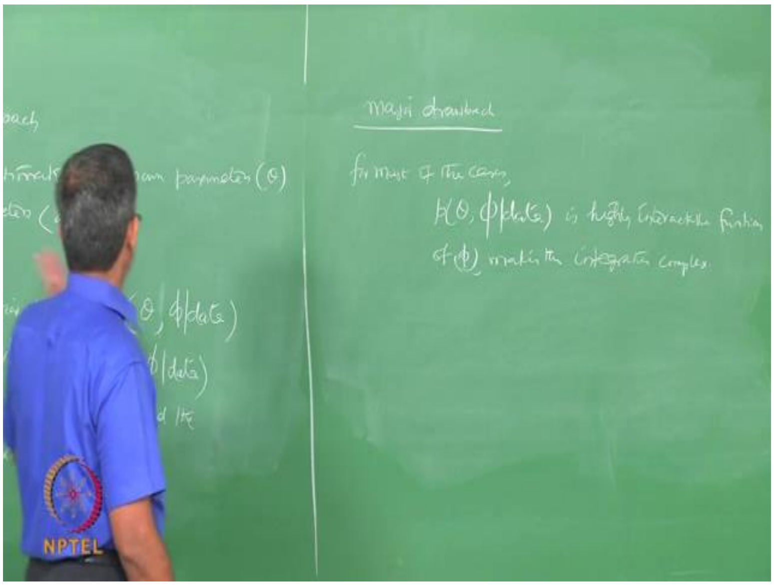

This method of course, has a major drawback. For most of the cases, the probability p theta phi data is highly intractable function of phi making the integration a complex procedure. So, it is rather very difficult to obtain or to follow the fully Bayesian approach, therefore literature suggested Modular Bayesian approach which we discussed few minutes back.

So, ladies and gentlemen, in this lecture, we try to introduce you the probabilistic tools. We mainly focused on probabilistic and non-probabilistic approaches to model the propagation of uncertainty. Of course, we have established the fact in the previous lectures that uncertainties govern the reliability tools; and reliability is a positive phase of looking at the failure of the system; whereas, risk is more or less a prescriptive tool, which results in recommendations of rules and governs. We close the lecture at this moment; we will continue with the discussions on probabilistic tools and plausible reasoning in the next lecture. If you have any doubts for any clarification, please do write the NPTEL portal site and also parallely read all the references listed in the website of the course.

Thank you.