Offshore structures under special loads including Fire resistance
Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

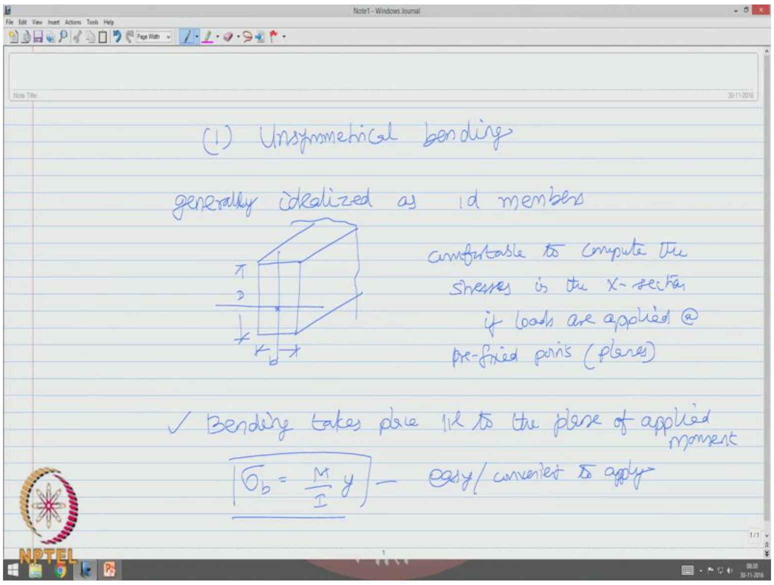
Module – 02
Advanced Structural Analyses
Lecture – 23
Unsymmetrical Bending – 1

Friends, welcome to the set of lectures under Module 2. In Module 1 we discussed about different structural action of offshore structures, we also discussed about conventional environmental loading that act on offshore structures. We understood how to estimate the environmental loads act on offshore structures, like wave load, win load, current load, etcetera using classical wave theories as well as power spectral density function or different spectrum.

We then paid attention in the first module to that of estimate of dynamic response of structural systems under special loads where we spoke about earthquake, loading ice loading, extreme waves, distinctly high sea waves, springing wave loads, ringing wave loads etcetera. We have understood how to model these special loads that act on offshore structures, what could be the return periods, what could be the probability of uncertainty associated with these kind of loads, and what could be the consequences cost by the special loads on offshore structures of different variety and structural form. For our understanding in case study we picked up complaint structural systems. Like for example, a tension leg platform and understood how the consequences of these special loads can affect significantly the degrees of freedom responses of these kinds of complaint structures under the special loads.

Now, interestingly we need to also shape up our understanding on the tools related to structural analysis when such members or subjected to special kinds of loads. So, in module 2 we will pay attention to some advance methods of structural analysis under special conditions so that we know how to analyze the systems or how to get the stresses and bending movement shear etcetera acting on the cross sections under these kind of special loads, but not necessarily the speciality arise from the kind of loads but they arise under different other circumstances which will discuss in this module.

(Refer Slide Time: 02:58)

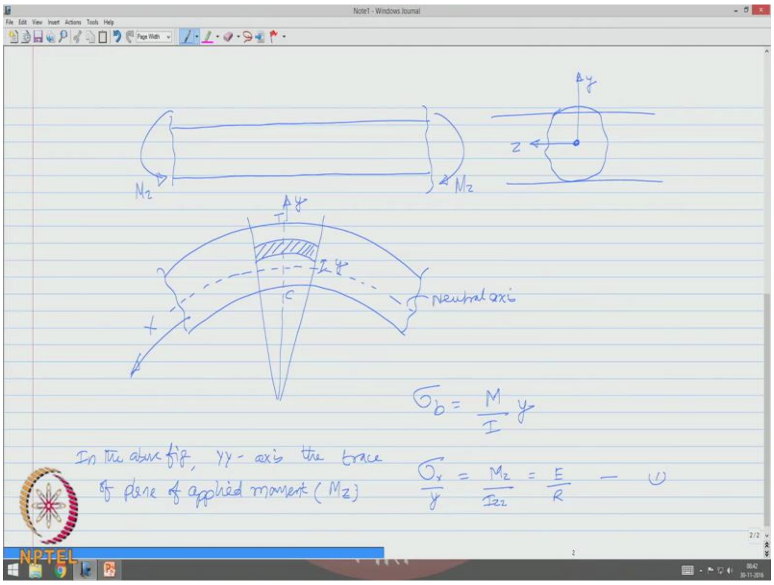


Under this module we will first talk about unsymmetrical bending. We know generally structural members, like beams, columns etcetera are generally idealized as one dimensional member. If you take a beam of rectangular cross section the length of the beam is so large compare to the cross sectional dimensions of the beam, so we idealize this as one dimensional member. Each member has generally preferably axis of symmetry or at least one axis of symmetry the intersection of which will be the centre of gravity the mass center of this body for any given cross section.

If the load is applied on this it is very comfortable and easy to compute the stresses. In the cross section if loads are applied at three fixed points or I should say planes generally so for in all structural analysis of a classical methods it is assume that bending takes place parallel to the plane of applied movement; if this statement is true then obtaining bending stress where classical theory of M by I into y from the flexural equation is easy and convenient to apply.

But in many cases this statement may not be true.

(Refer Slide Time: 05:55)

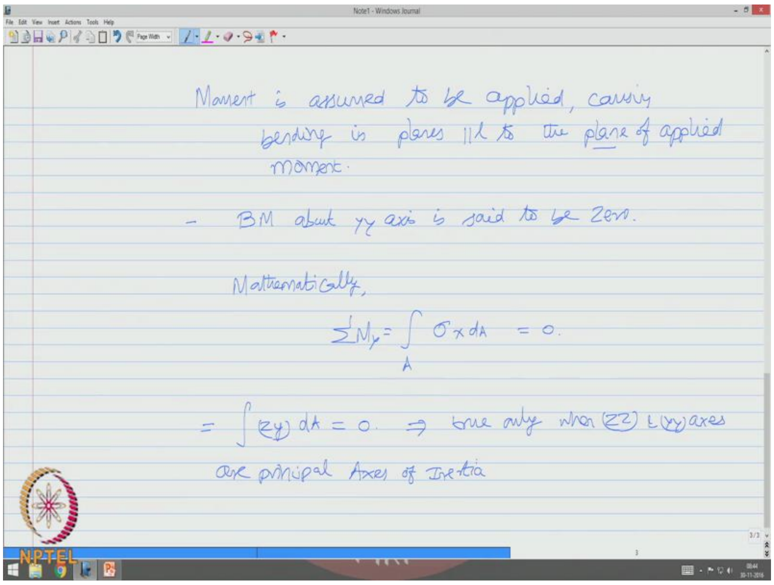


Let us take a beam subjected to a bending action of this which you make the beam to bend, this way which forms tension because elongation of fibbers on the top and compression because of contraction of fibbers at the bottom. If you cut a segment and take a layer at a distance y from the neutral axis, let say this my x x axis and this is my y y axis and the applied movement is Mz.

One can easily apply the conventional bending equation to find the bending stress caused by the movement Mz to be very specific they should then say sigma x by y can be Mz by IZZ which is also otherwise equal to E by R. So interestingly, it typical cross section of any form will have the z axis will have the y axis as marked here and subjected to Mz and that is the mass center of the body.

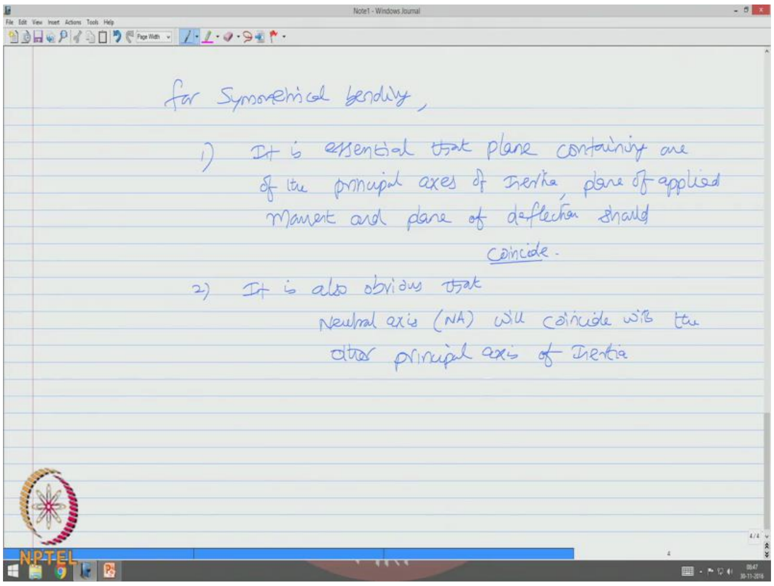
So obviously, one can now say that in the above figure y y axis actually represents the trace of plane of applied movement, whereas the applied movement is actually Mz.

(Refer Slide Time: 09:37)



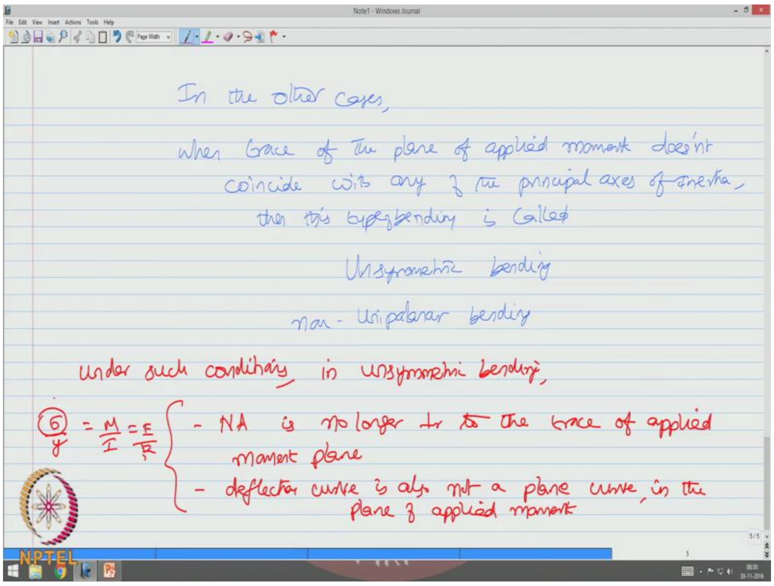
Therefore, movement is assumed to be applied causing, bending in planes parallel to the plane of applied movement. Under this condition we can then say bending movement about y y axis is said to be 0. Mathematically, algebraic sum of movements about y axis which can be given by sigma x dA, this is an said to be 0 which amounts to interestingly in equation which says integration of z by dA should be they said to 0, which will be true only when z z and y y axis are principal axes of inertia.

(Refer Slide Time: 11:49)



Therefore, for symmetrical bending we have the following observations: it is essential that the plane containing one of the principal axes of inertia, plane of applied movement and plane of deflection should coincide. Second, it is therefore obvious that neutral axis will coincide with the other principal axes of inertia. That is a standard case which generally happens in almost all members which are assembled to become a structural configuration.

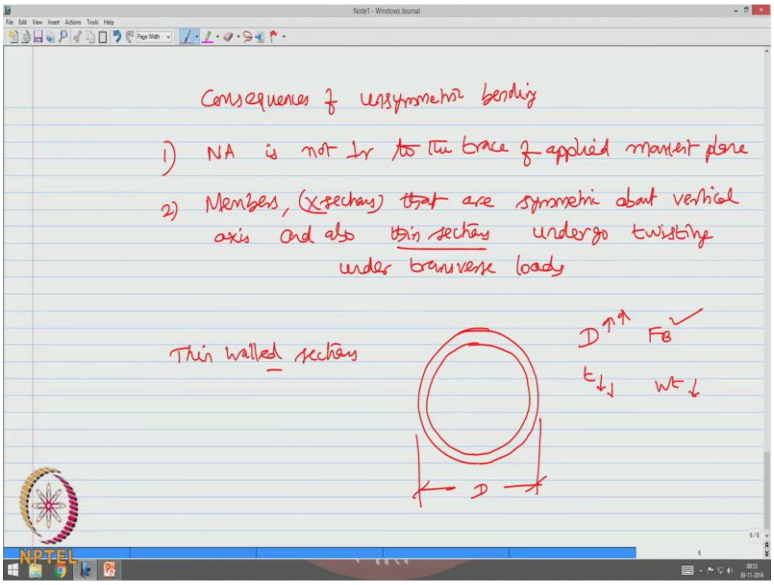
(Refer Slide Time: 13:54)



But in the case of unsymmetric bending; in the other cases, when trace of the plane of applied movement does not coincide with any of the principal axes of inertia, then this type of bending is called Unsymmetric bending. It is also other ways called as Uniplanar bending, in this case non uniplanar because is unsymmetrical. Therefore, under such conditions in unsymmetric bending neutral axis is no longer perpendicular to the trace of applied movement plane. Deflection curve is also not a plane curve in the plane of applied movement. So this complication will lead to non usage of standard theory to obtain the bending stress.

So, bending stress cannot be obtained by a straight forward equation of flexure as you see in this part. So, the complications will now come to estimate the bending stress, because neutral axis is no longer normal to the trace of applied movement and we all know the distance of extreme fibber is measured from the neutral axis normal to that. And movement should be on the trace of the applied plane and I should be respect to the axes of bending. So, all these complications will now add up to estimating the simple bending stress which cannot be now obtained with the standard equations of flexure.

(Refer Slide Time: 17:40)

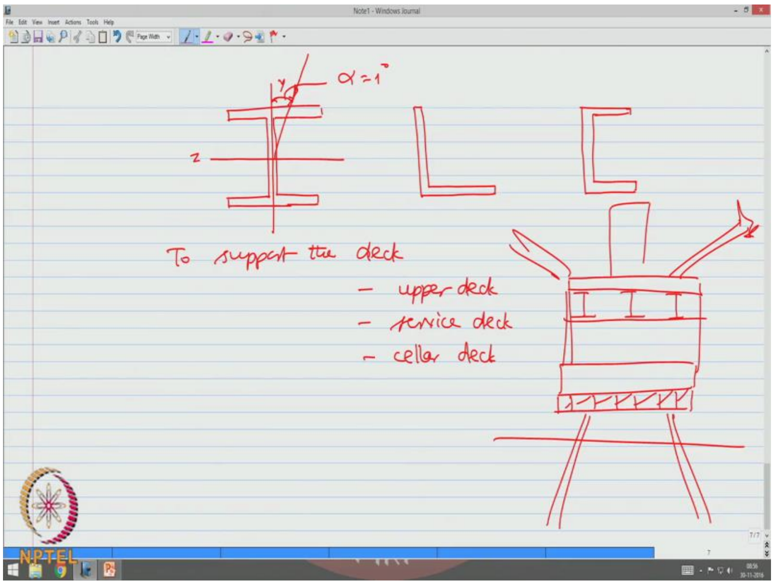


Let us ask what to be the consequence of unsymmetric bending. First, neutral axis is not perpendicular to the trace of applied movement plane. Members or cross sections which are not symmetric about vertical axis, and also thin sections undergo twisting under transverse loads.

Let see how this particular consequences or important for members in offshore structures. Interestingly, in offshore structures people use thin walled sections; the reason being we need a member may be circular in shape with large diameter to enable good buoyancy force less thickness to make it weight lesser so that installation, fabrication, construct ability, de-commissioning all becomes easy and simple.

So, obvious you are dealing with sections which are thin walled in nature biff the plane or bending does not coincide with of the applied movement, we always have un symmetrical bending cases. Therefore, estimating bending stresses in such cases under special loads as well as nominal loads will be tricky.

(Refer Slide Time: 20:46)

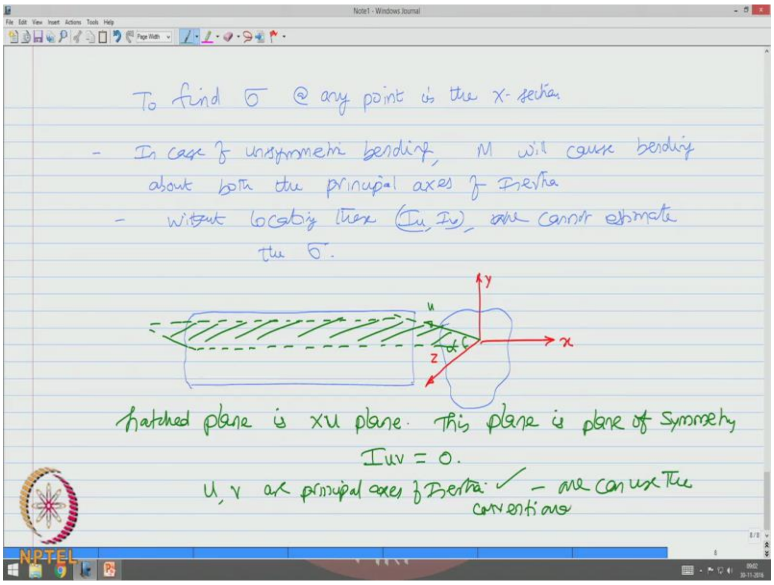


Let us take up examples of cross sections like I angles channels which have generally used to support the deck, the upper deck, the service deck, cellar deck. For example, deck may be a plate of some thickness you need to support the deck because they have largest pans, cross, beams, columns and another deck may be the deck is supported with a (Refer Time: 22:00) etcetera Where a platform may be constructed to have all arrangements like a flag boom like a (Refer Time: 22:13) etcetera.

So, when these members which any way have two axis symmetry, but if the loading is not applied of the plane of loading is not parallel to either one of them or it is applied with a small inclination to any one of the planes alpha may be even one degree the estimate a bending stresses or computing neutral axis position to estimate the bending stresses is not simple, it is complicated. We need to understand the analogy behind estimating the bending stresses by locating the principal axes of inertia and then locating the position of neutral axis and then measuring the distance of fibbers decoyed estimate stresses with respect to neutral axis along the principal axes of inertia to get the bending stresses.

It is also observed that even a small inclination of one degree of loading which can happen accidentally in a given construction practices in offshore structures can cause a good hike in the bending stress which can be as high as 25 to 30 percent. We will take an example and demonstrate this later. Now, let us try to understand systematically how to find the bending stress in case of unsymmetric bending.

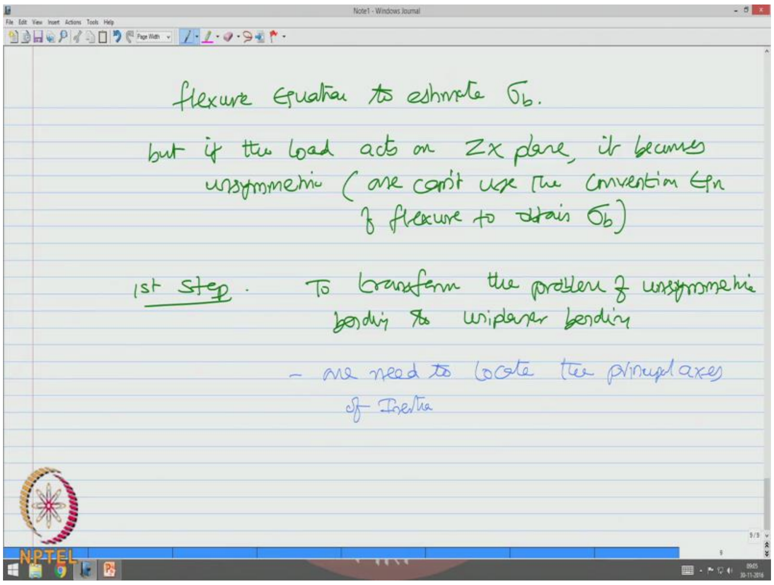
(Refer Slide Time: 24:00)



So, objective is to find the bending stress at any point in the cross section. We already know that in case of unsymmetric bending movement will cause bending about both of the axis of inertia. So, without locating these principal axes one cannot estimate the bending stresses.

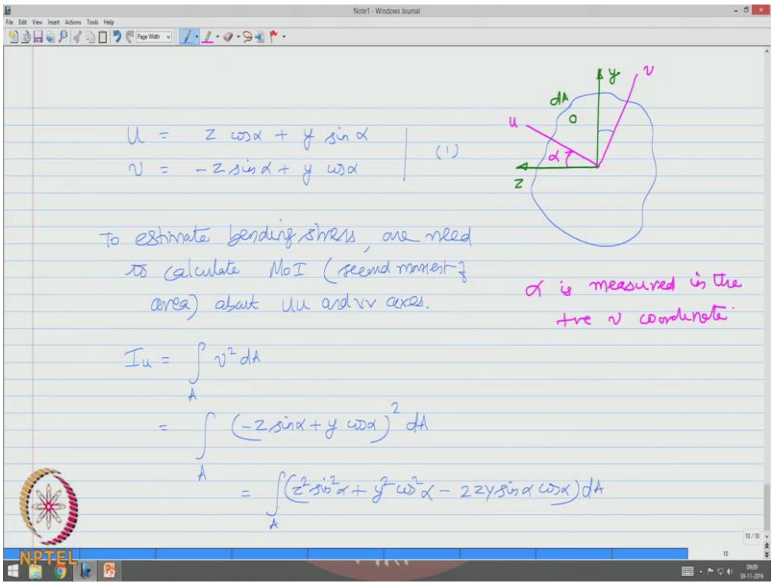
Let us take a beam whose cross section may be of initiate which has got axis z and y, and you will put z slightly incline let say this is my x axis. I have now a principle plane u which makes an angle alpha with z. Let us talk about this plane which is the x u plane. So, the hatched plane is x u plane this plane is plane of symmetry, for a simple reason the cross product of Iuv is said to 0; where u and v are principal axes of inertia. Similar able to locate these principal axes of inertia then one can use the conventional flexure formula to estimate the bending stress.

(Refer Slide Time: 28:11)



But, if the load acts on z x plane it becomes unsymmetric; one cannot use the conventional equation of flexure to obtain the bending stress; so what one should do in that case? The first step obviously will be to transform the problem of unsymetric bending to uniplanar bending. How to do this? To do this one need to locate the principal axes of inertia. So, the problem is now originated to a simple start saying for a given cross section try to first identify the principle axes of inertia.

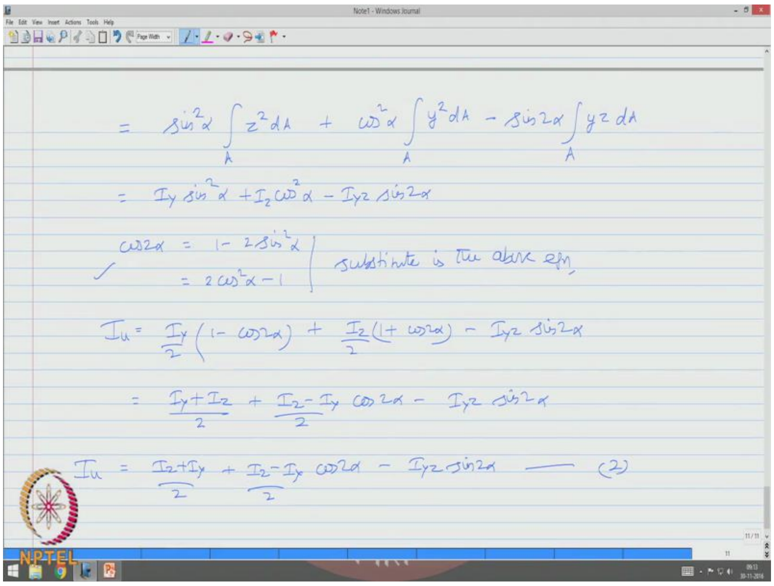
(Refer Slide Time: 30:36)



So, let us take a cross section of inertia, let us mark in axis as z and y. Let us take any point whose elemental area is dA. Now I want to locate two axis which is u and v at an angle alpha from z. So interestingly is measured in the positive v coordinate, that is the sin convention we have following; in that case u can be given by z cos alpha plus y sin alpha, because you can be then resolve as z cos alpha and y sin alpha. Whereas, v will be the opposite direction, so minus z sin alpha plus y cos alpha; because we know if this angle is alpha this is also the same angle, so y cos alpha and minus z sin alpha; we calls equation number 1.

To estimate the bending stresses at any point the cross section one need to calculate the movement of inertia which is the second movement of area about u u and v v axis. So, from the first principles Iu is integral over the entire area which is v square dA which can be said as integral for the entire area. V has taken from equation 1, so minus z sin alpha plus y cos alpha to the whole square of dA which can be integral z square sin square alpha plus y square cos square alpha minus 2 z y sin alpha cos alpha dA for the entire area A.

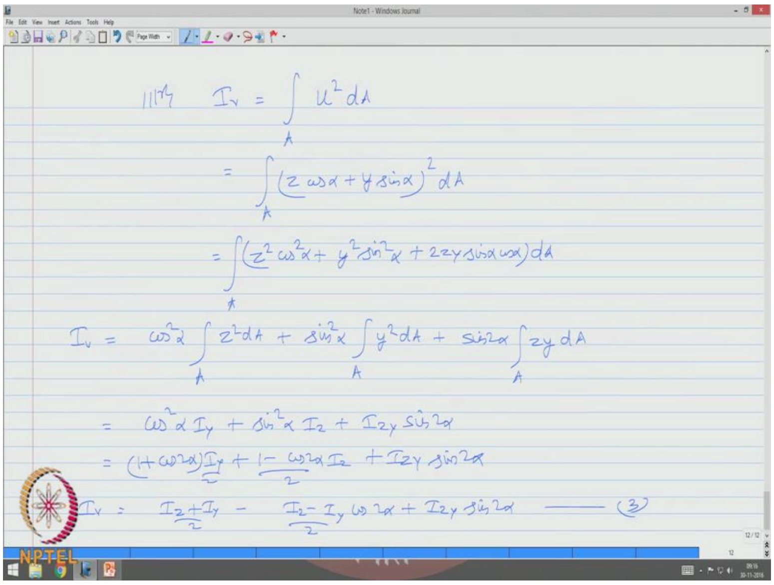
(Refer Slide Time: 34:28)



Which can be written as, sin square alpha take it off because there is not going to be a variable over dA integrate for the entire area z square dA plus cos square alpha entire area y square dA minus 2 sin alpha cos alpha is sin 2 alpha entire area y z dA. Z square can be now said as Iy sin square alpha. Please see this figure z is the horizontal axis, z square the second movement of area will give me y. So, z square will give me Iy plus cos square alpha Iz, because y square you give me Iz minus this is product amount of inertia which is sin 2 alpha.

We also know cos 2 alpha is 1 minus sin square alpha it is also equal to cos square alpha minus 1. So, let us substitute in the above equation. So we now say Iv sorry this Iu is equal to Iy by 2 1 minus cos 2 alpha plus Iz by 2 1 plus cos 2 alpha, I am substituting these equations here; minus Iyz sin 2 alpha; which now gives me Iy plus Iz by 2 plus Iz minus Iy by 2 cos 2 alpha minus Iyz sin 2 alpha. So, I can rewrite slightly Iz plus Iy by 2 plus Iz minus Iy by 2 cos 2 alpha minus Iyz sin 2 alpha which is my Iu, equation 2.

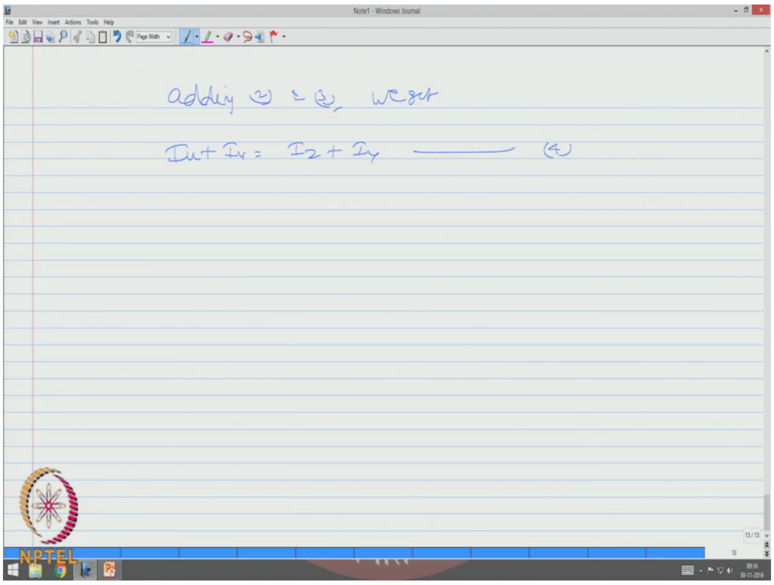
(Refer Slide Time: 38:04)



Similarly, Iv can be expressed as integral over entire area u square dA. Now u can be seen from this equation as z cos alpha plus y sin alpha. So integral A z cos alpha plus y sin alpha the whole square dA which can be z square cos square alpha plus y square sin square alpha plus 2 z y sin 2 alpha dA; which can be now separated as cos square alpha area z square dA sin square alpha area y square dA plus 2 z y sin alpha cos alpha plus sin 2 alpha area z y dA. We already know that this can give me Iy and the second integral can give me Iz and the third integral is Izy sin 2 alpha.

Substituting cos square and sin square in terms of cos 2 alpha we can now say 1 plus cos 2 alpha of Iy by 2 plus 1 minus cos 2 alpha Iz by 2 plus Izy sin 2 alpha which will now converged to Iz plus Iy by 2 minus Iz minus Iy by 2 cos 2 alpha plus Iz by sin 2 alpha which a call of equation number 3.

(Refer Slide Time: 40:46)



Interestingly, adding equation 2 and 3 we get Iu plus Iv is Iz plus Iy.

So friends, we will proceed further in the next lecture of estimating and locating the neutral axis and bending stresses with some examples. So, we have now realized and understood in this lecture that unsymmetric bending is an important application in any structural engineering problems; nevertheless it is important in members in offshore structures. Let us understand the analogy behind estimating bending stresses when they are subjected to unsymmetric bending.

Thank you.