Offshore structures under special loads including Fire resistance
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Module – 02
Advanced Structural Analyses
Lecture – 28
Shear Centre III

Friends, today we will extent the existing lecture on shear centre to do couple of more numerical examples and also solve the problem where the section is not symmetric about any one of the axis. So, this lecture is lecture 28, Shear Centre III in module 2; Title Advanced Structural Analyses under the NPTEL course of Offshore Structures under Special Loads Including Fire Resistance Design.

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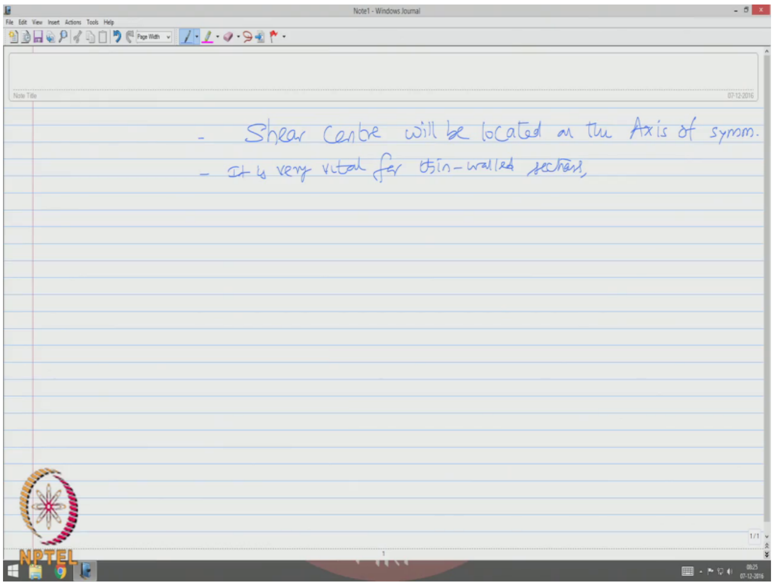

We already said that shear centre will be located on the axis of symmetric, if the section has at least one axis symmetric; shear centre will lie on that axis. If the vertical load is applied through this point; a section will not undergo twisting or portion. So, it is very vital for thin walled sections which are very common in offshore structures. In the last lecture we picked up an example, I retaliate that example straightly in extended the discussion further in this lecture.

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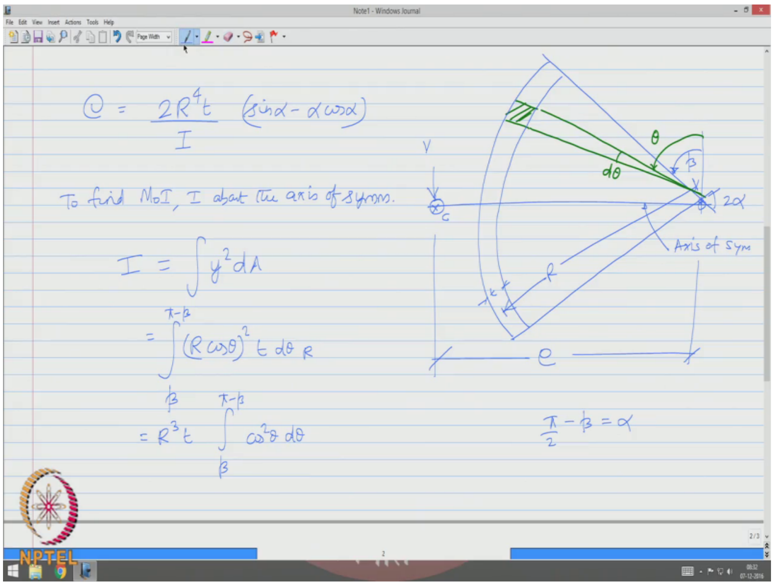

We picked up segment of a curve extending for angle; 2 alpha with radius R and sectional thickness the t, this be the axis of symmetry; shear centre lies on this axis c where the force applied to this will not cause a twisting and the distance of this point from the centre is the offsite of the shear centre which we call as e and we already derive that e is given by 2 R 4 t by I; sin alpha minus alpha cos alpha where to the vertical this angle is theta.

Now, let us discuss how to estimate the moment of inertia and the second moment of area of this section or this segment about the axis of symmetric, so to find the moment of inertia; I about the axis of symmetric. We know that I conventionally is given by this equation, we will cut the strip of segmental angle d theta at an angle theta from the vertical. So, now I can say integrate this R cos theta will be the distance second moment of area, t d theta will be area of strip and let this vary from beta to pi minus theta because this angle is beta. So, which gives me and the distance is R, so R cube t beta 2 power minus beta cos square theta; d theta.

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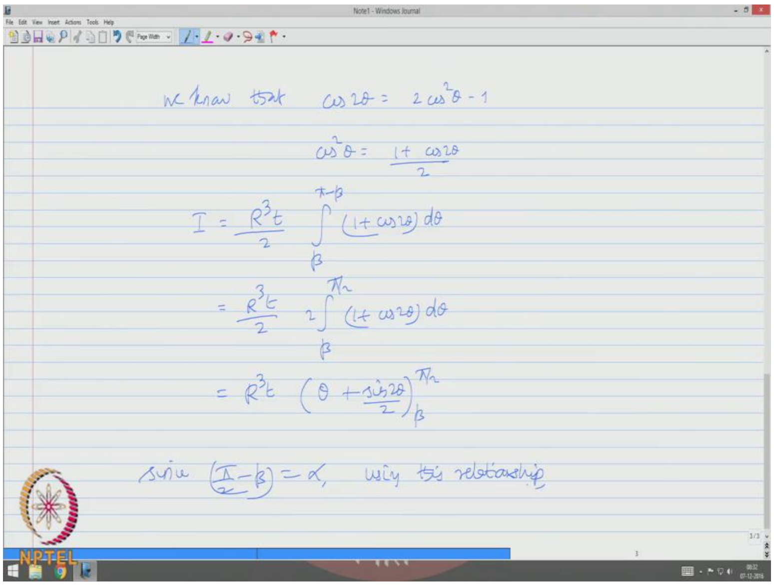

So, we know that cos 2 theta is 2 cos square theta minus alpha therefore, cos square theta this actually 1 plus cos 2 theta by 2. So, in the previous equation of cos 2 theta will substitute this as follows, so R cube t by 2, integral beta 2 pi minus theta 1 plus cos 2, theta d theta which simply says; now using the symmetry of integral they can now say beta to pi by 2 of twice interval there are two strips about symmetry.

So, now I can say R cube t theta plus sin 2 theta by 2 varying from beta to pi minus 2, if you look at the figure one can write that pi by 2 minus beta will be actually equal to alpha; this angle is beta. So, pi minus, pi 2 minus beta will be actually equal to alpha, so since pi by 2 minus beta is alpha.

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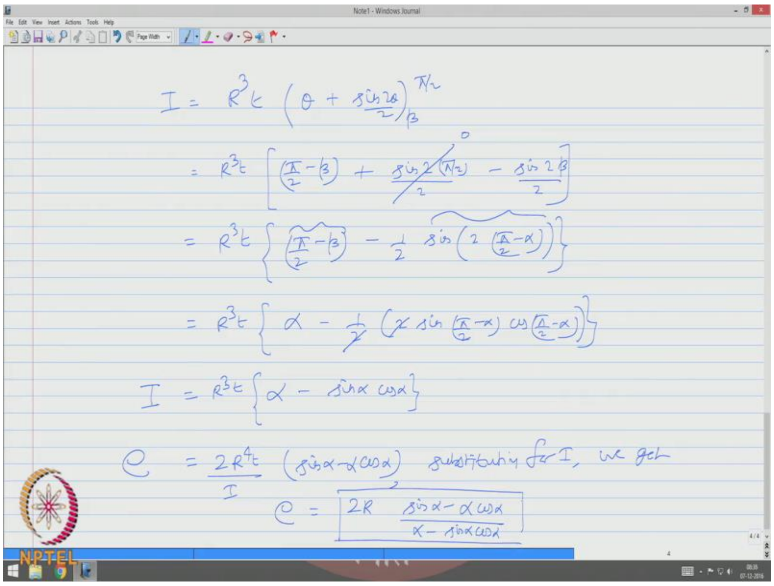

Using this relationship we can now say I is R cube t we earlier had theta plus sin 2 theta by two varying from beta to pi minus 2 means a R cube t pi by 2 minus beta that is upper and lower limits less sin 2 pi by 2, 2 minus sin 2 beta by 2 (Refer Time: 08:44) R cube t pi by 2 minus beta minus. So this term goes away minus half sin, again beta can be expressed as twice of pi by 2 minus alpha which can be R cube t, pi by 2 minus alpha beta is actually alpha minus half, so this term can be expressed as sin 2 alpha, so that can be said as twice sin alpha, cos alpha which can be R cube t, alpha minus sin alpha, cos alpha that is got you I. Therefore, e is actually equal to 2 R 4 t by I sin alpha minus cos alpha.

Now substituting for I we get e now equals 2 R sin alpha minus; there is an alpha here alpha cos alpha by alpha minus sin alpha; cos alpha that is my e.

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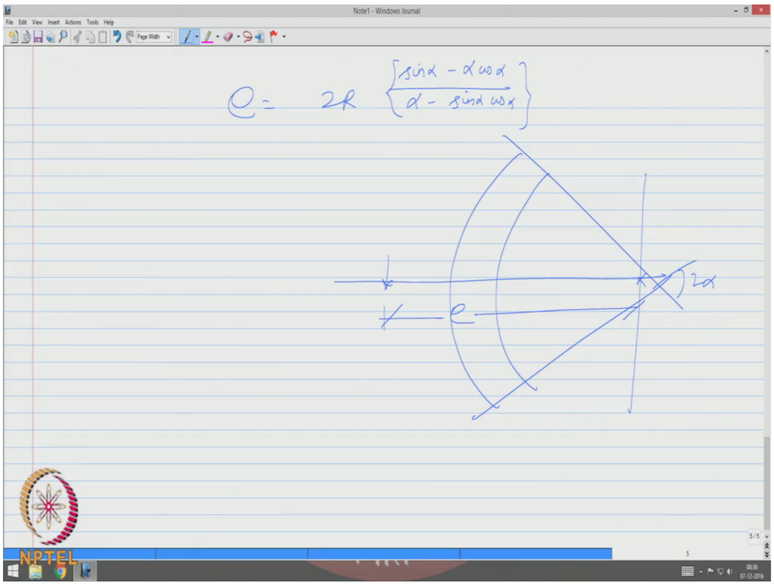

So, e is 2 R sin alpha minus alpha, cos alpha by alpha minus sin alpha, cos alpha. So, that is the distance of share centre for this figure on the axis of symmetry from this centre to the point which is e force angle extents 2 alpha at this centre.

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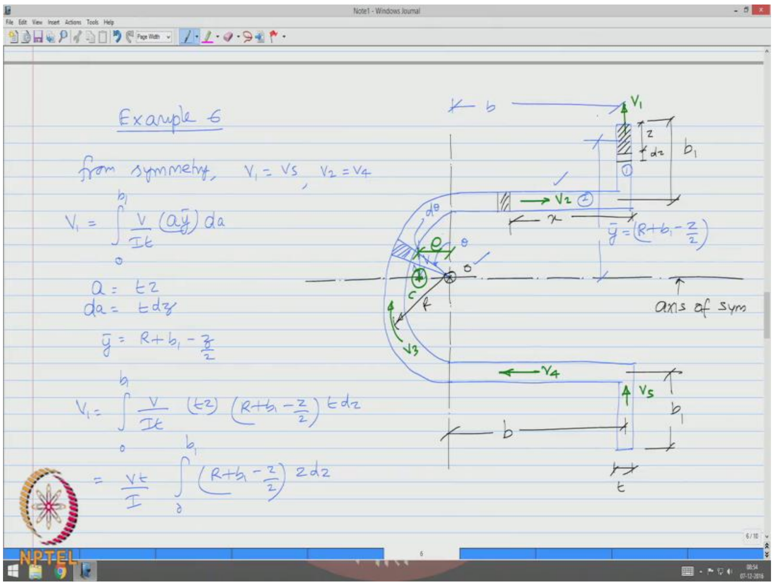

So will take up one more example and the example 6, with symmetric section in our section like this. So have a section like this, the dimension sections are; let us say, this is t and from this point, this distance is b to the centre and this of course remains the axis of symmetry of the section; therefore, shear centre line one this line this is o let us say the centre on this is R, the section is uniformed thickness t and this distance is b 1 of course this is also b 1. If look at the shear flow diagram is a, this is V 1, this is V 2 and this becomes V 3 and this is V 4 and this is V 5. So let us say the shear centre located somewhere here, this my shear centre and I call this distance from the centre as e. Now we know from symmetry V 1 is equal V 5 and V 2 equals V 4. Let us find V 1; to do V 1 will take a section of thickness d z whose dimension is z.

So, V 1 will be integral; 0 to b 1 V by I t; a y bar; d a in this case a is going to be t into z therefore, da is t dz and y bar the distance of the fiber from the axis of the symmetry. Actually this distance I am looking at this distance, this is from here and here that is y bar, for this case which is going to be let us say R plus V 1 minus z by 2 therefore, V 1 integral 0, b 1; V by I t, t z or plus V 1 minus z by 2, which gives me V t by I integral R plus b 1 minus z by 2 of z; dz.

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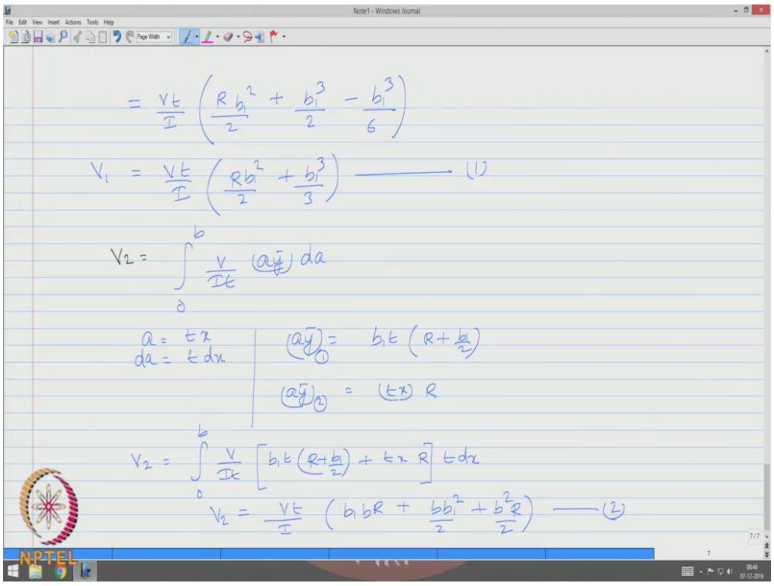

Which gives me V t by I, R b 1 square by 2 plus b 1 cube by 2 minus b 1 cube by 6 which will tell me V 1 as V t by I; R b 1 square by 2 plus b 1 cube by 3, equation number 1. Let us say I want to find V 2, so I take a section (Refer Time: 18:13) x from here or you want to find now the second moment of area of this whole strip about this part. So, V 2 is now equal to integral 0 to b because that is a limit of this section 0 to be V by I t uniform thickness a y bar, a is tx, da is t dx; that is called the strip. Now for the external (Refer Time: 19:13); a y bar can be straight away, b 1 into t that is for this (Refer Time: 19:20) let us call this (Refer Time: 19:22) as 1; for (Refer Time: 19:24) 1; b 1 into t, R plus b 1 by 2 is the y bar. So, now a y bar of piece 2; I call this is peace 2 will be t x into R.

So, now let us find V 2 0 to b V by I t, b 1 t; R plus b 1 by 2 plus tx into R of t dx which gets me V t by I, b 1; bR plus, b b 1 square by 2 plus b square R by 2 (Refer Time: 20:50) equation number 2.

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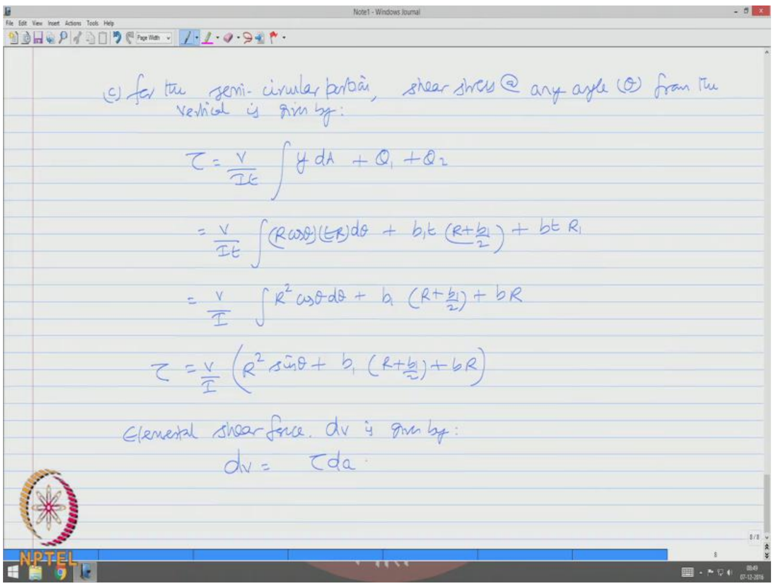

Now, we want to find for the semi circular portion; shear stress at any angle theta from the vertical is given by; draw the figure let us say we want to take. So, this angle I say is theta and of course, this small angle is the theta; taw is given by V by I t, integral y da plus piece 1 plus piece 2 which will be V by I t, R cos theta t into R, d theta that is for the semi circular strip plus b 1 into t that is piece 1, R plus b 1 by 2, plus b into t into R 1, which can be now said as V by I integral; R square cos theta, d theta plus b 1, R plus b 1 by 2 plus bR, which will be V by I R square sin theta plus b 1 of R plus b 1 by 2; plus b alpha that is my tau.

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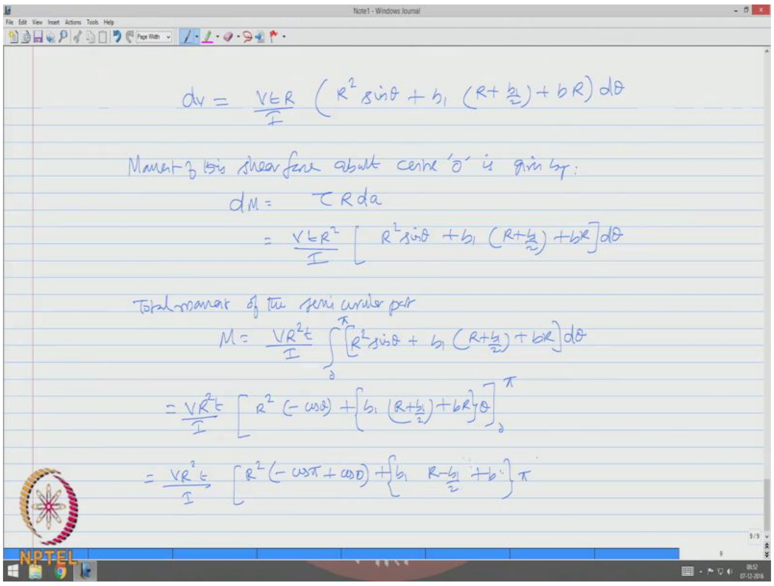

So the elemental shear force d V is given by dv is tau da, which is Vt R by I R square, sin theta plus b 1 of R plus b 1 by 2 plus bR, d theta.

Now, we want to take moment of this force, so moment of this shear force about centre o; is given by the taking moment about this point, centre o is given by dm is equal to tau R da where there is distance, we are now going to be V t R square by I, R square sin theta plus b 1, R plus b 1 by 2 plus bR, d theta. Now the total moment of the semi circular portion will give me M as V; R square t by I, integral 0 to pi, R square, sin theta plus b 1 of R b 1 by 2 plus bR of d theta which gives me V R square t by I, R square minus cos theta plus b 1 of R plus b 1 by 2 plus bR of theta varying 0 to pi by substituting the upper and lower limits, we will get R square minus cos pi plus cos 0 plus b 1 of R plus b 1 by 2 plus bR of pi.

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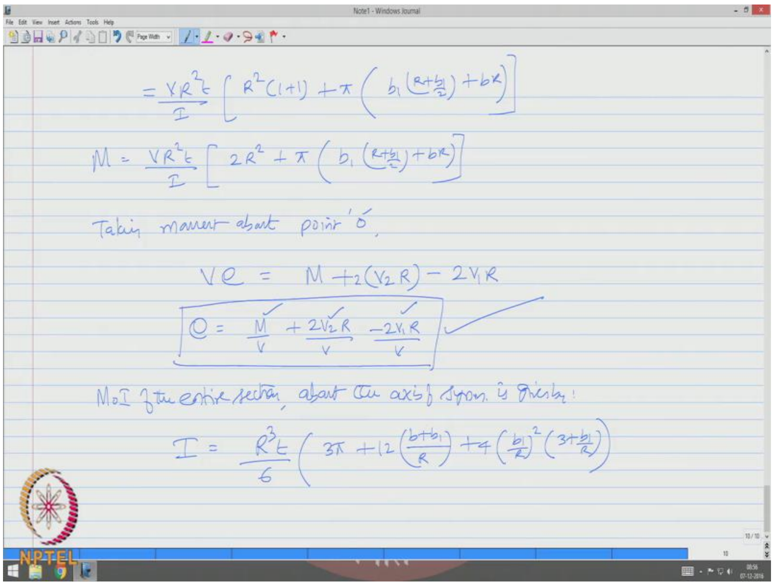

Which leads to bR square t by I, R square 1 plus 1, plus pi times of b 1; R plus b 1 by 2 plus bR. Therefore, m will be V R square t by I 2 R square plus pi times of b 1; R plus b 1 by 2 plus bR. Now taking moment of all these forces about m, about the shear centre or let us say about on the shear centre; let us take moment about the point o, we know look at this figure; the force V; you going to act here and this is distance e. So, it is going to be V into e which is this M plus in our V 1 and the distance is b (Refer Time: 28:16) but that is anti clock wise, similarly for V 2; the distance is R.

So, M plus V 2 into R 2 because V 2 and V 4 are symmetric, so 2 minus 2 V 1 into R, so e is now going to be M by V plus 2 v, two R by V minus 2; V 1; R by V. So, interestingly we have equation for V 1, V 2 and M; they can find very easily. Now the moment of inertia of the entire section about the axis of symmetry is given; I is going to be I would request you to do it for the first principles and understand how this can be arrived; 3 pi plus 12 times of b plus, b 1 by R plus 4 times of b 1 by R whole square into 3 plus b 1 by R, so that is moment of inertia and this one distance of shear centre.

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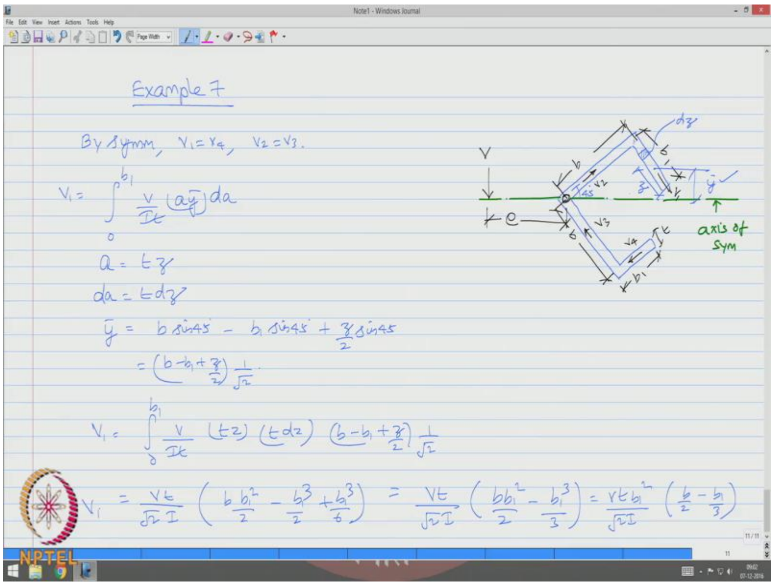

So friends we are able to derived the distance of shear centre or locate the point of shear centre, from the cg of a given section which has at least one axis of the symmetry. We will take upon one more example, the section has one axis symmetry let us draw the dimension of section, so this distance is b 1 and this is distance is b and this also b and this is also b. The section has uniformed thickness; this t, so let us say my load is going to act here and let us mark the shear centre distance from this point as e, let us marks the shear flow.

So, shear flow this is going to be V 1, this going to be V 2, V 3 and V 4, so by symmetry we know V 1 is equal to V 4 and V 2 is equal to V 3. I want to find V 1, I will take a section let us say this is my section what I am cutting and this distance from here is z and this thickness is dz. So integral 0 to b 1; V by I t; a y bar da, so a is tz, da is tdz; y bar is this centre; from the symmetry which is going to be this angle is 45 degrees. So, b sin 45 minus b 1 sin 45, so b sin 45 will be this vertical distance, this distance and V 1 will be subtract thus plus z by 2 sin 45 will give me y bar which is b minus b 1 plus z by 2 of 1 by root 2.

So, therefore V 1 is 0 to b 1 V by I t; tz into tdz; b minus b 1, plus z by 2; 1 by root 2 which will be V t by root 2; I bb1 square by 2 minus b 1 cube by 2 plus b 1 cube by 6; it tells me V 1 is V t by root 2; I bb1 square by 2 minus b 1 cube by 3 which can be simplified as V t; b 1 square by root 2 I; b by 2 minus b 1 by 3 that is my V 1.

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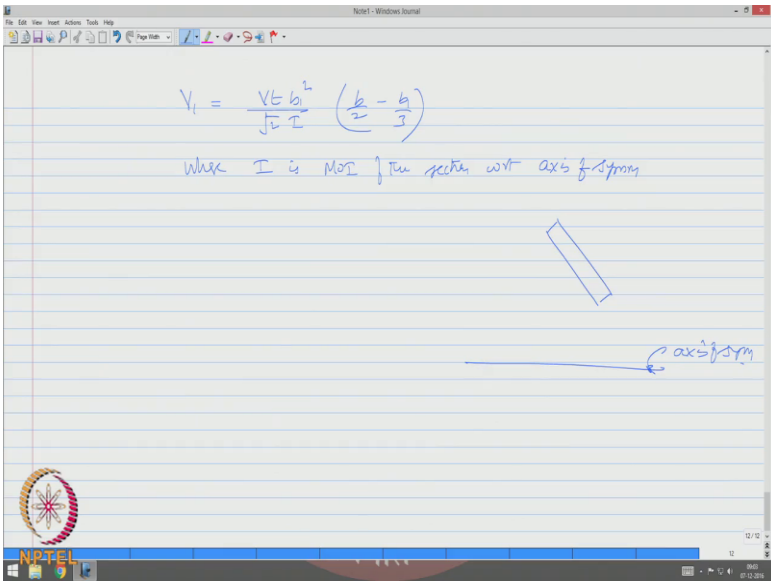

One can get in the similar fashion V 2, if you look at this experience for the V 1; it is V t, b 1 square by root 2 I b by 2 by b 1 by 3 where I is the moment of inertia of the complete section (Refer Time: 36:23) with respect to axis of symmetry.

Now, this member inclined and axis of symmetry here I need to find the moment of inertia of section; of this piece; this specter is place. So, it is slight tricky I will live this is as an homework to you for the day; do this, we will try to find do this in the next lecture and then extend this principle for sections with no axis of symmetry as well.

Thank you very much.