Offshore structures under special loads including Fire resistance

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Module – 02
Advanced Structural Analyses
Lecture – 34
Curved Beam-V

Friends, let us continue to discuss on the topic Curved Beams. In this lecture we will solve couple of numeric examples to understand how the Winkler Bach equation can be applied to estimate the stresses at the extreme fibres in the intrados and extrados of a curved beam with large initial curvature.

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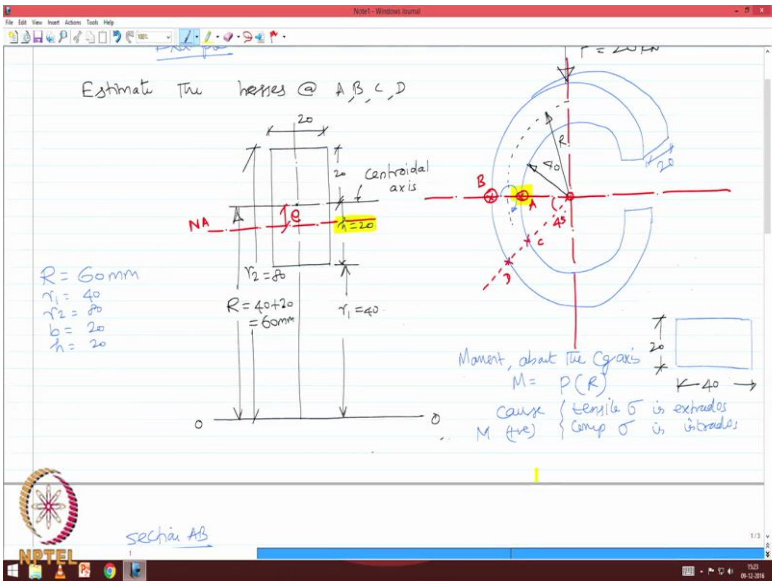

We will take up an example which can be a section like this, let us say this becomes the centre; we will look at the cross section, it is a typical rectangle of dimensions 40 and 20 and this radius being 40, it is subjected to a force P which is 20 kilo Newton.

We need to estimate the stresses at A and B and also at another section which is said 45 degree C and D. So, the question asked is estimate the stresses at A, B, C, D. In all these problems; it is interesting to first know certain geometric parameters which are important to estimate the stresses. So let us say this is my cross section, this is my plane o o, this dimension is 20, this is my centroidal axis which is at the radius R, one can see the radius can be now estimated as 40 plus 20; 60 millimetres and this dimension is 20, this dimension is also 20 and we know this is to be called as r 1; which is 40 and this dimension is called as r 2 which is 80, the neutral axis will be shifted within x and c e, which we need to estimate.

Let us write down certain parameters, so R is 60, r 1 is 40, r 2 is 80, b is 20 and h is 20. It means the section is about 20 millimetre this way, it is very interesting that moment about the centroidal axis will be actually equal to P into R which will create a moment which is going to cause tensile stresses in the extrados and compressive stresses in the intrados; therefore, M in this case is positive.

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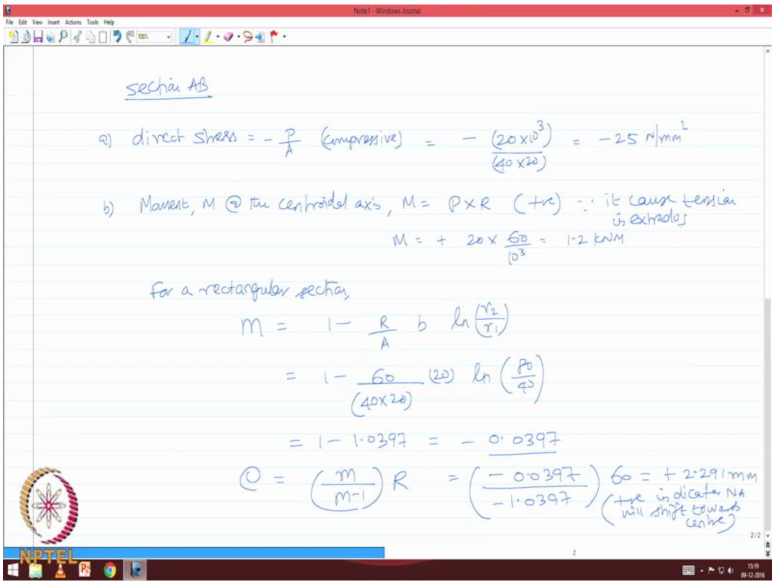

Let us talk about the calculations for the section AB; section AB is located as shown in the figure here. Let us say the direct stress will be P by A; which will be compressive, which will be minus 20; 10 power 3 Newton by cross sectional area 40 into 20 which gives me minus 25 Newton per mm square.

Moment, M at the centroidal line is actually equal to P into R which will be positive because it causes tension in extrados. So, moment will be plus 20 into 60 by 1000 which is 1.2 kilo Newton meter. We know that for a rectangular section; M is given by 1 minus R by A b natural algorithms of r 2 by r 1, let us substitute for this values; 1 minus 60 by 40 into 20 and b is 20; natural algorithm of 80 by 40, which is 1 minus 1.097 minus 0.0397 which is a sectional property required to calculate the (Refer Time: 09:11) the (Refer Time: 09:12) e is M by M minus 1 into R. So minus 397 minus 1.0397 multiplied by 60 which is plus

2.291 millimetre; which indicates that positive indicates neutral axis will shift towards the centre of curvature.

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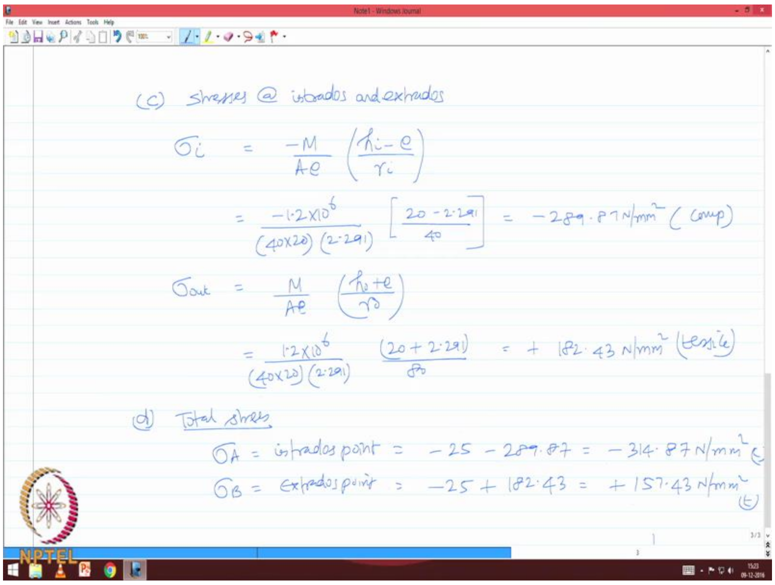

Let us now calculate the stresses at intrados and extrados, the extreme fibres, the general expression sigma i intrados is minus M by A e; h i minus e by r i which is minus 1.2; 10 power 6 by 40 into 20 into e is 2.291 which we just now calculated multiplied by 20 minus 2.291, you see h is 20 by 40. We are looking for the point A which is intrados, so 2.291 which is actually equal to minus 289.87 Newton per mm square, negative indicates it is compressor. Sigma extrados will be M by A E; h o plus e by r o; which is 1.2; 10 power 6 by 40 into 20 into e which is 2.291; 20 plus 2.291 by or o which is r 2; which is 80, which gives me plus 182.43 Newton per mm square, positive indicates this tensile.

Now the total stress, let us say at A which is an intrados point is the direct stress which is compressive and the stress due to the moment which is also compressive, which gives me minus 314.87 Newton per mm square. Stress at the point B; which is extrados point the extreme fibre on the outside, this is the direct stress and the stress due to moment is tensile, so the next stress is 157.47 Newton per mm square, this is compressive and this is tensile. We need to also work out the stress at one more section C D, which is located at 45 degrees; let us go for section C D.

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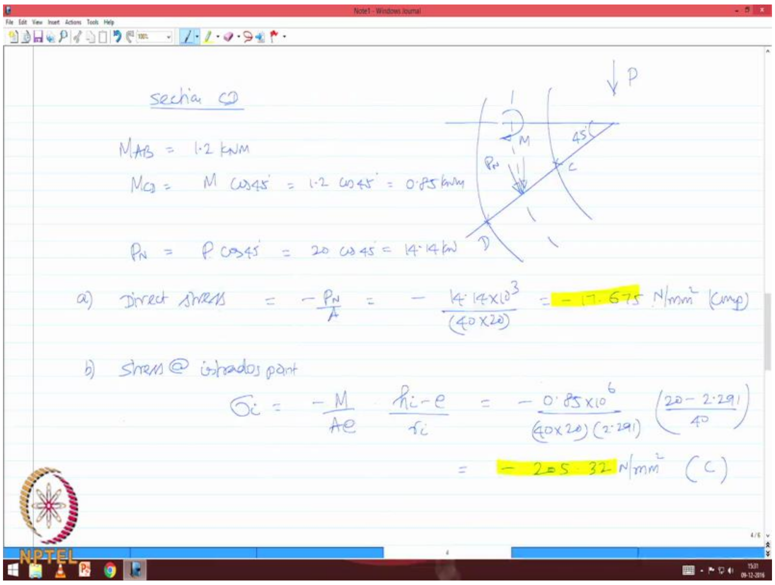

So now, section C D is located at 45 degree, the load is applied here, the r is known to me, the moment here is known. So, M along A B is known to me which is 1.2 kilo Newton meter, therefore M along the line C D will be this moment of cos 45 which is 1.2 cos 45 which will be 0.85 kilo Newton meter. Now I also want to force normal to this which I say as P normal, I have this force which is P therefore, P normal will be again P cos 45 which will be 20 cos 45 which is 14.14 kilo Newton.

Now, let us work out the direct stress which is minus P by A which is minus 14.14 into 10 power 3 Newton divided by A which is 40 into 20, I get minus 17.675 Newton per mm square as compressive. Stress at intrados point is given by sigma i is minus M by A e; h i minus e by r i, which is now equal to minus 0.85; 10 power 6 by 40 into 20 into e which is 2.291 and h i is 20 minus 2.291 by r i which is 40 which gives me a value of minus 205.32 Newton per mm square compressive.

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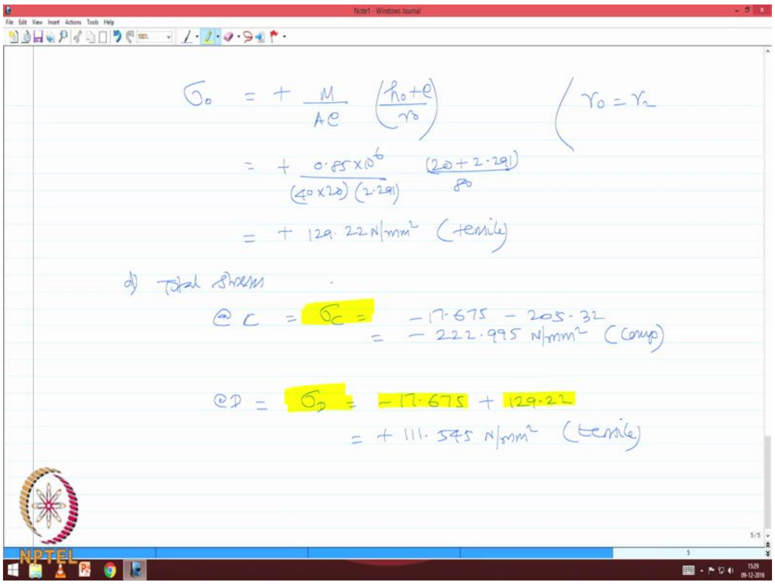

Sigma at the extrados point is given by plus M by A e; h o plus e by r o; r o is as same as r 2, h is now positive 0.85; 10 power 6 by 40 into 20 into 2.291; 20 plus 2.291 by 80, which gives me plus 129.22 which is tensile. Therefore, the total stresses at C which is sigma C is minus 17.675 there is a direct stress compressive minus 205.32; which we already have from here, which is negative 22.995 which is compressive. At point t which is sigma d; which is extrados the direct stress is same, but the stress due to moment is 129.22 which is plus 11.545, which is tensile.

So, friends you have seen the equation derived in the last lectures, one can estimate the stresses at intrados and extrados points for various applications including the direct stress and the stress due to the moment. One can also easily conclude what is the nature of stresses, one based upon the sign convention, two based upon the nature of the moment caused on the curved beam.

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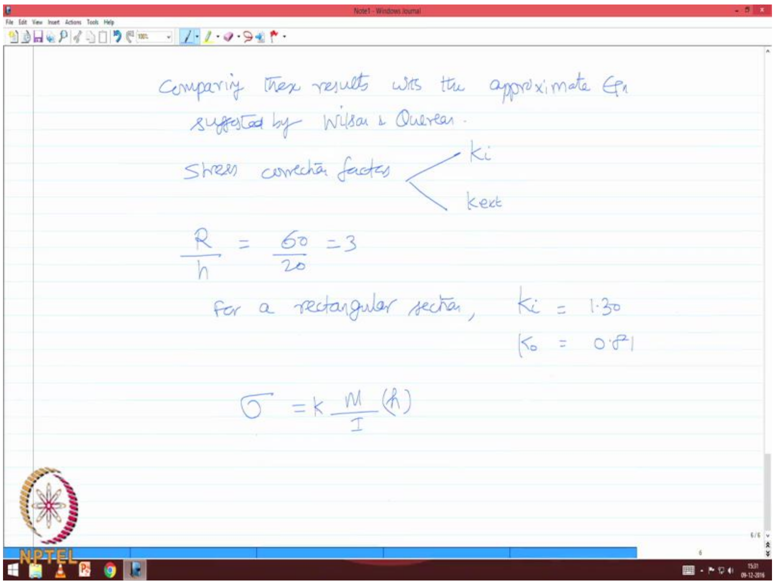

Interestingly, let us compare these results with the approximate equation suggested by Wilson and Querean. Wilson and Querean gave stress correction factors for both intrados and extrados, to know this we need to compute R by h value, for our problem this is 60 by 20; which is 3.

To look at the table back for a rectangular section; k i, the stress correction factor for intrados is 1.3, which I gave you in the last lecture and for outer; it is 0.81. Now the general stress can be simply M by i into h that is a distance of the fibre into the correction factor.

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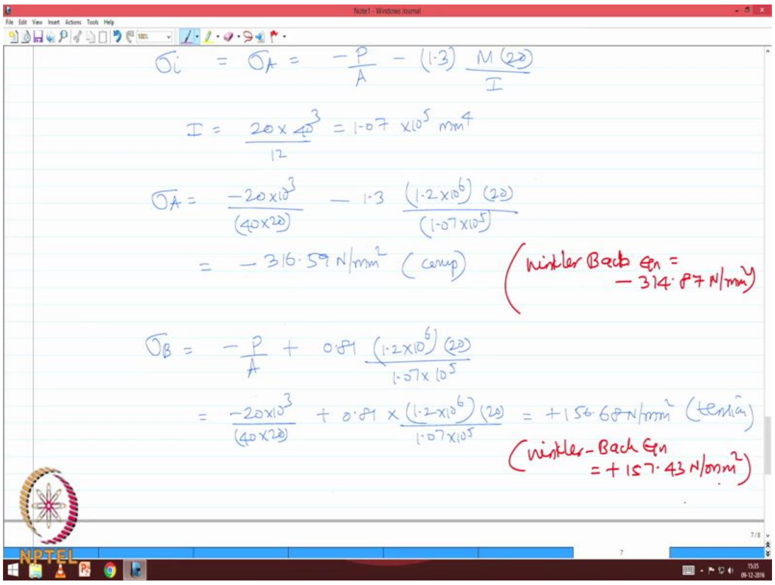

So, if you really want to compute sigma i, then I should say this is going to be equal to stress at A; which is due to the direct stress, due to the moment; 1.3 times of M into 20 is the h value divided by I. Let us compute the I value for this section, which is 20 into 40 cube by 12; which is 1.07; 10 power 5.

So, therefore let us compute sigma AI this is intrados which is minus 20; 10 power 3 by 40 into 20, there is a direct stress compression minus 1.3 intrados compression. So, 1.2; 10 power 6 that is the movement and 20 is a distance of the intrados fibre from the centroidal axis, divided by 1.07; 10 power 5; the moment of inertia which will get this value as minus 316.59, which is compression of the point A. Friends, let us compare this with the value what we got for sigma A; which is 314. So, I can compare the actual value what we got from Winkler Bach equation is minus 314.87 Newton per mm square.

Similarly, let us do it for sigma B, which is minus P by A plus 0.81 into 1.2; 10 power 6, the distance of extrados 5 by 20 mm by i value which is 10 power 5; which is minus 20; 10 power 3 by 40 into 20 plus 0.81 times of 1.2; 10 power 6 into 20 by 1.07; 10 power 5, which comes to plus 156.68 plus indicate it is tensile. When you compare this with the results of sigma B, they can very clearly see there is 157. So, Winkler Bach gave with the value as plus 157.43, which has very close to what we have from the approximate solution as well. So, this is how one numerical can be easily solved using the equations what we derive in the last set of lectures.

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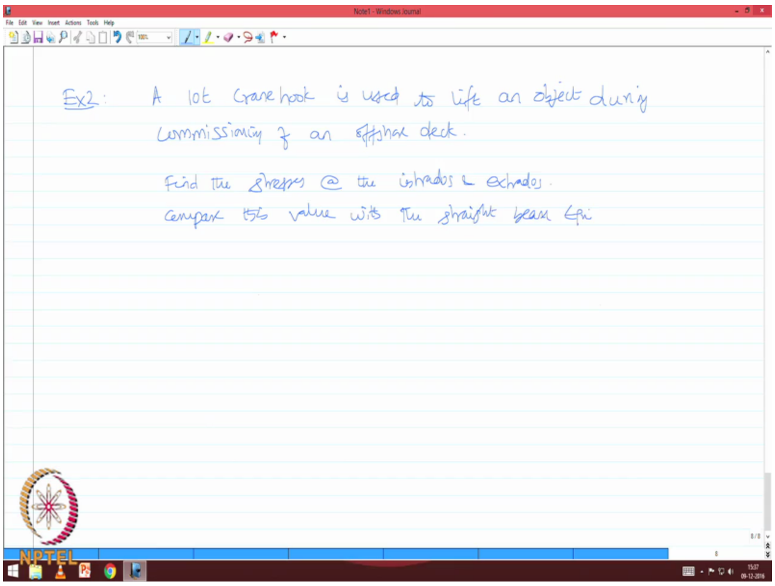

We will take up one more example, which is slightly interesting in applicable to an offshore structure. Let us say example 2: A 10 ton crane hook is used to lift an object during commissioning of an offshore deck. Find the stresses at the intrados and extrados. Compare this value with the straight beam equation, so that is a problem given.

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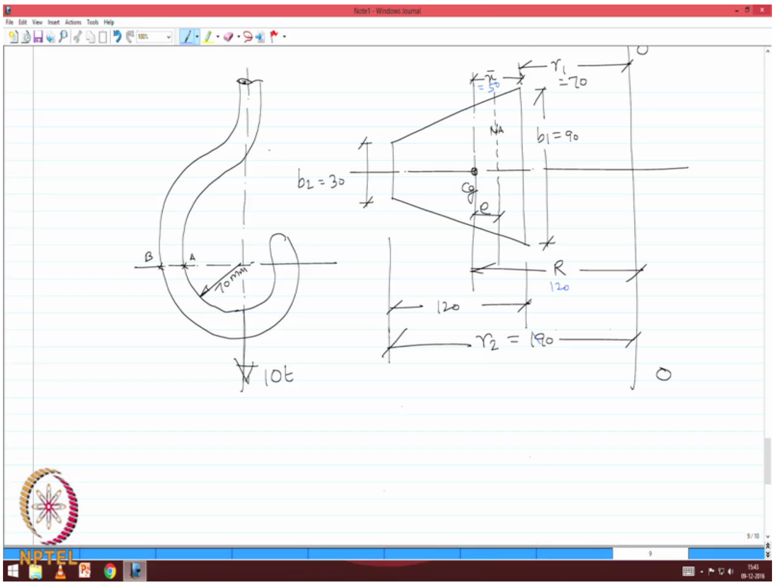

Let us say the problem is interesting because the section is actually a crane hook, which has the geometric centre as indicated and this radius is 70 millimetre. We want to compute the stresses at two points A and B when this is a subjected to a load of 10 tons.

The cross section of this looks like this; as a trapezium section, so this becomes my o o axis now, I need to locate the c g; this becomes my c g. This dimensions are given as 120; this is b 1 as per our normal cloture which as 90; this dimension is actually b 2; which is 30, but say this is r 1, which is actually equal to 70 and this is r 2, which actually equal to 190; this is my r which you need to calculate because I have to first compute this distance c g from the base. The neutral axis will have an (Refer Time: 29:05) e, this is my neutral axis which need to be computer for this problem. So, the first step is to locate the neutral axis that is to find x bar; that is a first step.

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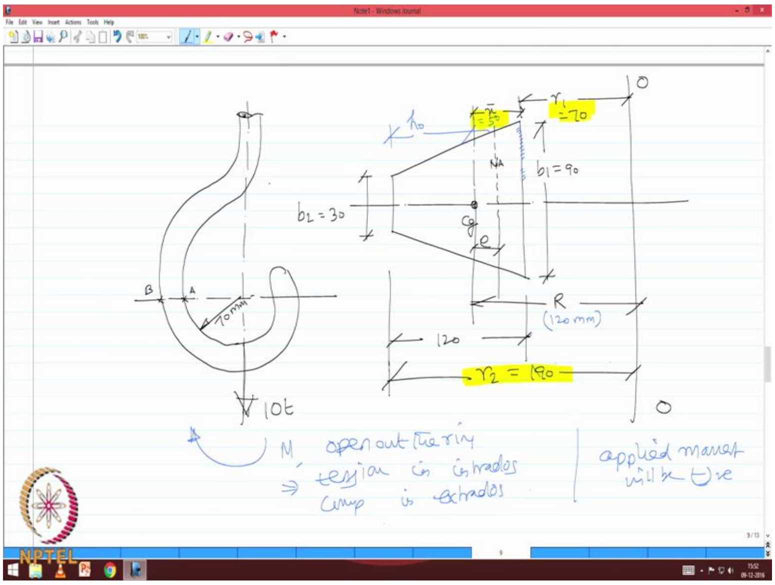

So, x bar can be computed as sigma A x bar by sigma A first principles. So, this is my section, this is 90, this is 30 and this is what I call as x bar, let us say divide this into different pieces, say this is piece 1, piece 2 and piece 3. So, x bar will be now equal to half base height because this height is 120 into one-third of 120; that is in x bar of the piece number 1. There are two such pieces; piece number 1 and 3 has the same dimension plus the rectangle which is 30 by 120 whose centroid is 60; the whole divided by the complete area which is half h; A plus B, which comes to be 144; 10 power 3 plus 216, 10 power 3 by 7200 which gives me x bar as 50 millimetre.

Let us write down the value here, this value is 50. Therefore, r will be now 120 millimetres.

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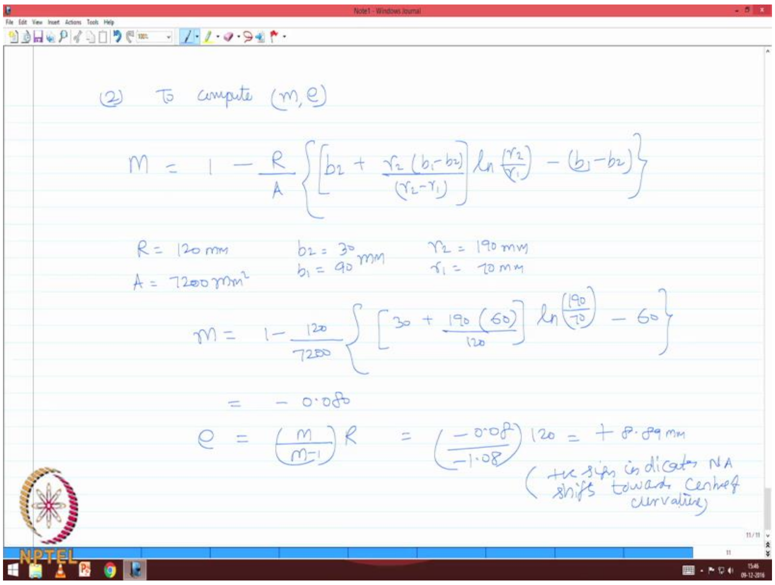

Now, the second step is going to be to compute M and e for this cross section. We know M is given by the expression what we derived in the last lecture; 1 minus R by A; b 2 plus r 2; b 1 minus b 2 by r 2 minus r 1 of natural algorithm of r 2 by r 1 minus b 1 minus b 2, that is the equation what we already derived in the last lecture for a trapezoidal section. Let us substitute for our problem R is 120, A is area 7200 mm square, b 2 is 30 and b 1 is 90 millimetres, r 2 is 190 and r 1 is 70 millimetres.

Substituting M is 1 minus 120 by 7200; 30 plus 190; b 1 minus b 2; that is 60 divided by r 2 minus r 1; that is 120, natural algorithm of r 2, by r 1; 190 by 70 minus 60 that is b 1 minus b 2. So, this gives me M value as minus 0.080, therefore e as we know is M by M minus 1 into R. So, minus 0.08 minus 1.08 into R which is 120, which indicates a positive value of 8.89 millimetres, which indicates that positive sign indicates neutral axis shifts towards centre of curvature.

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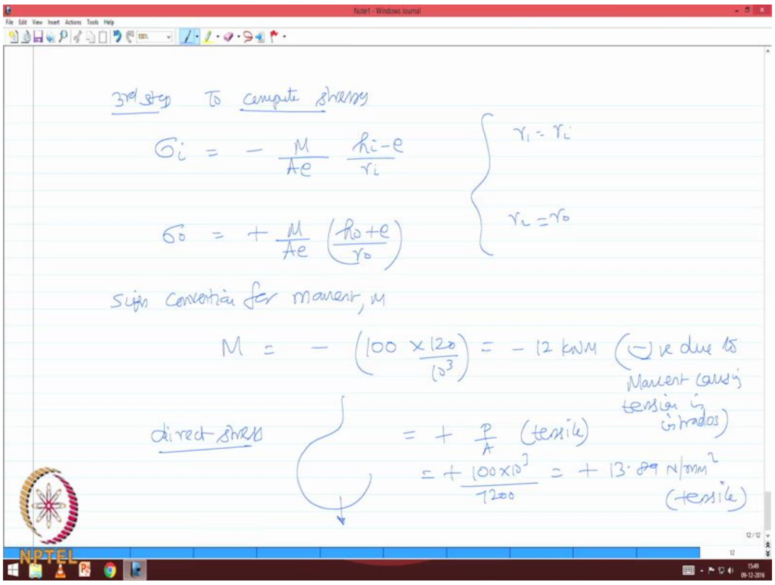

The third step is to compute stresses, we know stress at intrados is given by minus M by A e; h i minus e by r i and stress at extrados is given by plus M by A e; h o plus e by r o, r i is nothing but r 1 and r o is nothing, but r 2. Interestingly the sign convention for moment we already know that, in this case look at this figure carefully this will create a moment of this nature which will try to open out the ring. It means this will create tension in intrados and compression in extrados, but our general convention is moment should cos tension in the extrados. Therefore, applied movement now will be considered negative, so applied moment is negative; so M is going to be minus 100 kilo Newton into R in meters.

So, which is going to be minus 12 kilo Newton meter; negative due to moment causing tension in intrados. We can also find the direct stress, look at the figure I have a hook and it is pulling therefore, this is going to cos tension; the direct stress will be now positive P by A positive because it is going to be tensile, which is going to be plus 100; 10 power 3 by area of the trapezium which is 7200, which is going to be plus 13.89 Newton per millimetre square which is tensile.

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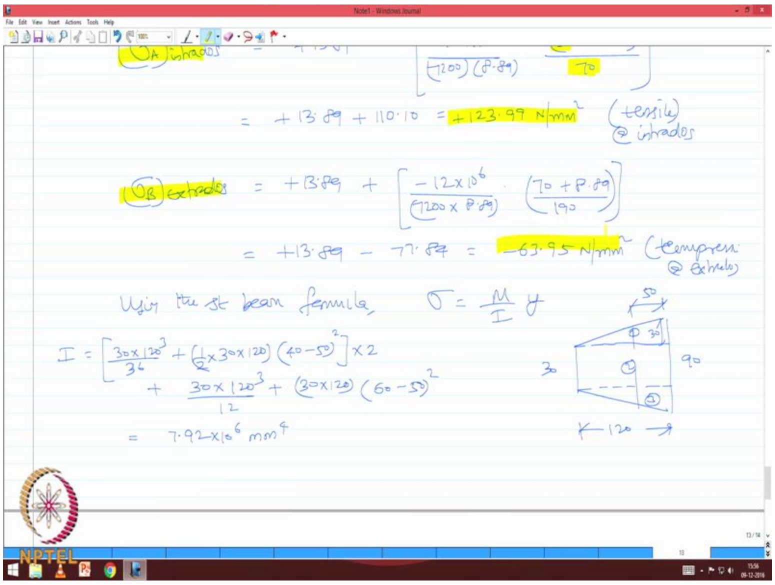

So, let us try to find the stress at intrados which will be plus 13.89 minus; my M is negative therefore, minus area into e; 8.89 then 50 that is distance of intrados minus e; 8.89 divided by r 1; which is 70.

Can please see this figure, r 1 is 70 and this surface is intrados and the distance of that is 50. Let us say you have taken 50 here and 70 here in the equation. So now this value will be equal to plus 13.89 minus, minus – plus, plus 110.10 which gives me 123.99 Newton per mm square positive sign indicates it is tensile at the intrados. Let us talk about the extrados, which is going to be plus 13.89 plus minus 12; 10 power 6 by 7200 into 8.89; h o is 70 plus e 8.89 divided by 190.

We can look at this figure; r 2 is 190 and h o is this distance, that is distance of the external as fibre from here this is h o, which is 70 (Refer Time: 40:12) 120 minus 50. So, substituting we get plus 13.89 minus 77.84 which is minus 63.95 Newton per mm square which indicates, it is compressive at extrados. If I use the straight beam formula which is simply M by I into y, let us compare this to do this, I must compute I value; let us say this is my section is 90, 30, this is 120 and the c g is at 50. I divide this into two parts; 1, 2 and 3; I is second moment of area let us compute this. So, 30 is this distance into 120 cube by 12 by 36 that is for the triangle plus A k square, area is half into 30 into 120, k is going to be 40 minus 50 in the whole square of two pieces plus for a rectangle B D cube by 12 plus A k square 30 into 120 is the area k is going to be 60 that is 120 by 2 minus 50; the whole square. So, I get this value as 7.92; 10 power 6 mm 4.

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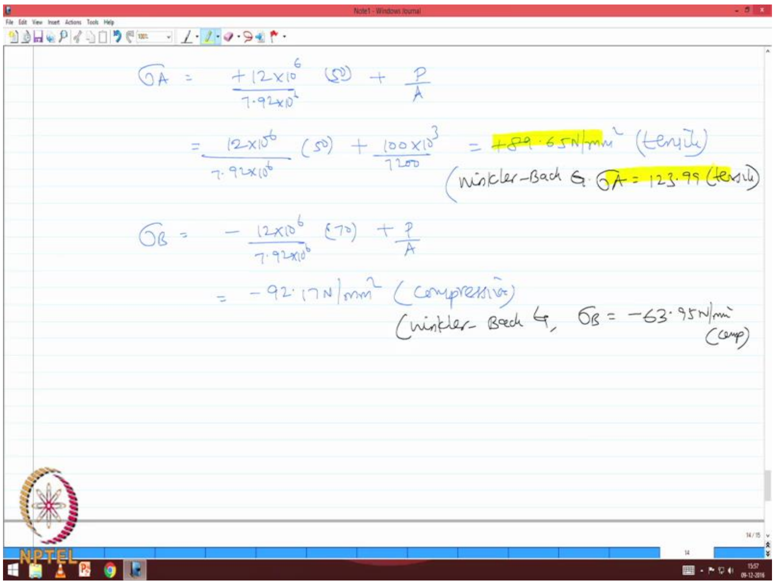

Let us use sigma A; M by I, so plus sigma A, the moment is causing at the point A, tensile stresses 7.92; 10 power 6 into 50 plus P by A because P by A tensile stress all the time, which is going to be 12; 10 power 6 by 7.92; 10 power 6 into 50 plus 100 into 10 power 3 by 7200 which gives me plus 89.65 Newton per mm square tensile and in the extrados, it is going to be compression into 70 plus P by A which gives me minus 92.17 Newton per mm square compressive.

If we compare this values, what we have from the actual value, so the Winkler Bach equation gave me sigma A as 123.99 tensile. We can see here that is sigma A, similarly sigma B is minus 63.95, so Winkler Bach equation gave me sigma B as minus 63.95 Newton mm square compressive. So, there is a significant variation between the straight beam equation and the Winkler Bach, if we do not use an appropriate equation for curved beams to find out the stresses in the extreme fibres in intrados and extrados, the design can go wrong.

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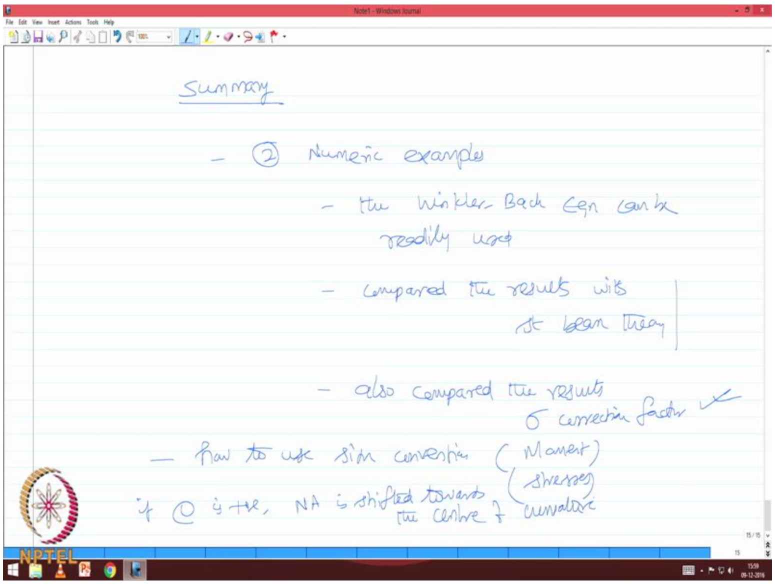

Summary, we did two numerical examples, showed how the Winkler Bach equation can be readily used, we have also compared the results with straight beam theory the difference is significant. We also compared the results with stress correction factors suggested by the researches, the results were close to Winkler Bach. So, in these examples we learnt how to use the sign convention for two things; one for the moment, two for the stresses. We have also learnt, if e is positive it means neutral axis is shifted towards the centre of curvature.

So, friends hope that with these five lectures on curved beams, which we have discussed on beams with small initial curvature and large initial curvature, one will be able to estimate the stresses for any kind of a curved beam with different cross sections whose M value and e value at derived and now they are known to you.

Thank you.