Computer Methods of Analysis of Offshore Structures

Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Module – 01
Lecture – 05
Beam Element – 2 (Part – 2)

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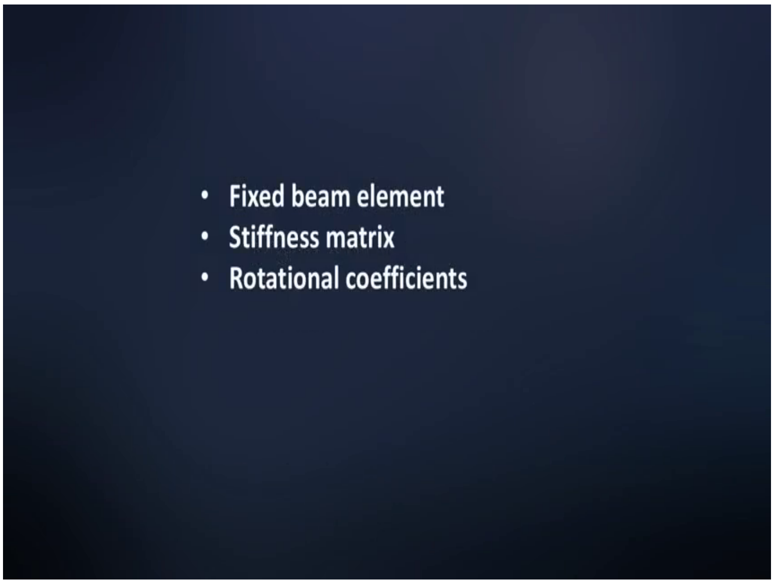

So, I will make use of this particular equation saying that flexibility and stiffness are cross to become identity matrix flexibility and stiffness matrix in the multiply, I get a identity matrix because one is inverse of the other.

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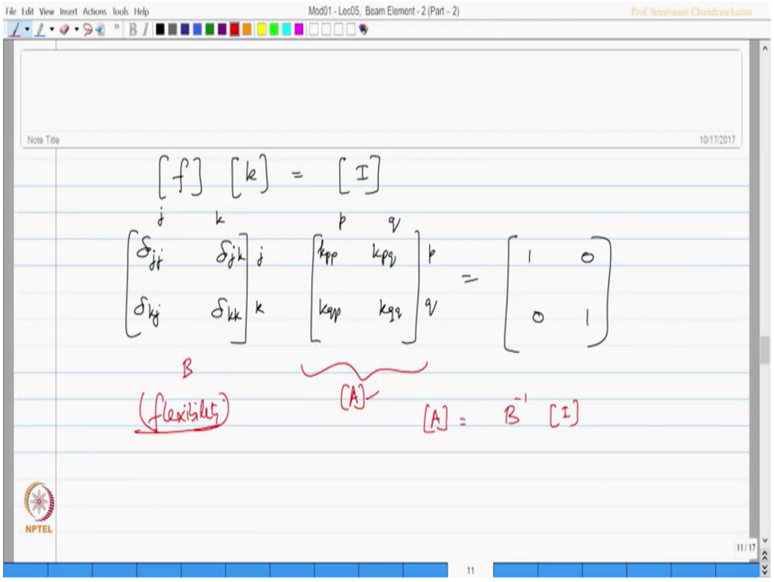

if I say delta j j delta j k delta k j and delta k k are the flexibility coefficients for j the and k th nodes; if I multiply this with p q and p q which are stiffness coefficients which is k p p, k p q, k q p and k q q, I will get an identity matrix which is 1, 0, 0, 1.

Friends please understand my job is to evaluate this matrix. So, this matrix if I call this as let us say a matrix I can evaluate A as B inverse of I. So, I am now going to evaluate the flexibility matrix or the flexibility coefficients invert that matrix to get my A matrix.

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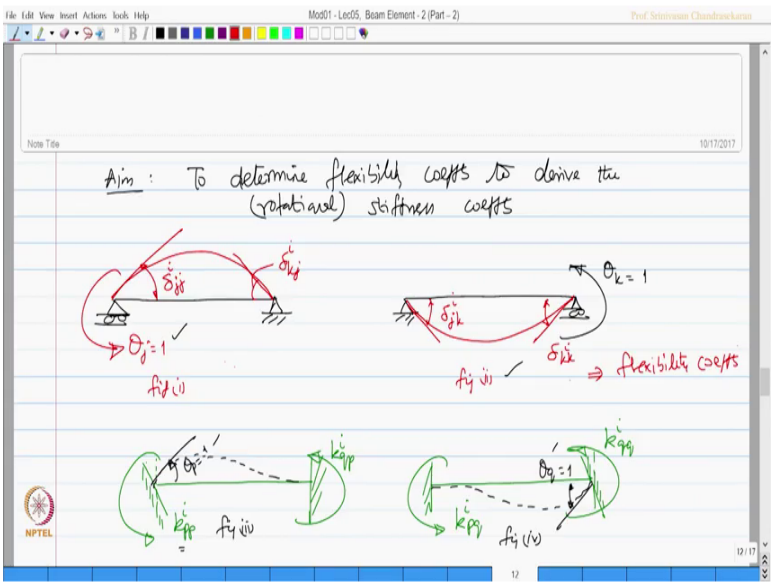

So, let us do that now. So, the job is to determine the flexibility coefficients to derive the rotational stiffness coefficients correct that is our task; now let us do that. So, let us take a beam which is simply supported; let us give unit rotation to this end.

Delta j j of the ith value. So, let us impose theta j is equal to 1 and I get delta j j and this value will be delta k j of the ith member, let us call this is figure one; let us draw the next figure; let me give unit rotation and the k th end now. So, the beam will deflect in this manner; let us say these are my rotational coefficients delta k k and delta j k, let us call this is figure 2 equivalently. Let us try to draw the beam with this end fixed, but this end rotated with k p p in post which will cause k q p for theta p as unity; I will call this as figure 3.

Similarly, let us say this end is fixed and this end is imposed by a rotation which is going to cause k q q which will cause in n movement k p q and that is going to cause rotation of theta q which is unity; I call this as figure 4 let us refer to these 4 figures and now understand; what we have done in this figures; figure one, we have give in rotation and the j th end figure 2 unit rotation the k th end figure 3 is rotational coefficients caused because of that rotation at j th end and k th end respectively.

So, now we can straight away say figure 1 and 2 shows the flexibility coefficients.

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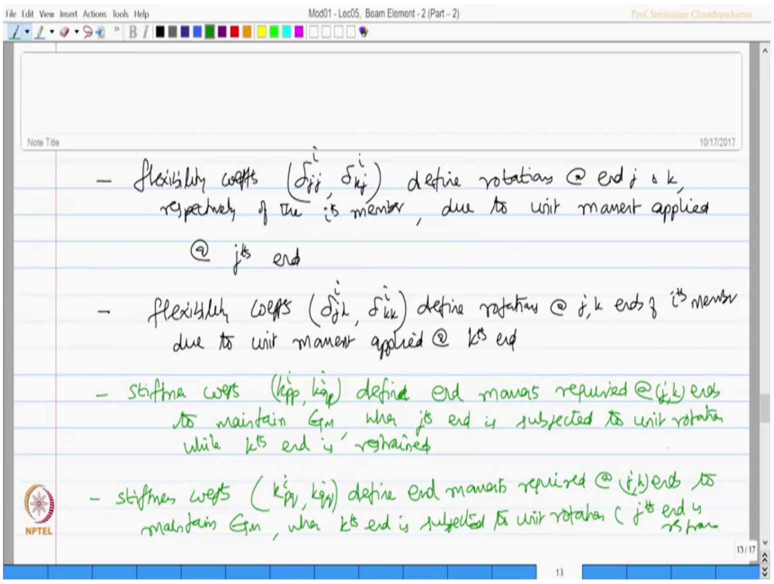

The flexibility coefficients delta j j and delta k j define rotations at the end j and k respectively of the ith member caused due to unit moment we are talking about flexibility. So, unit moment apply at j th end correct similarly flexibility coefficients delta j k and delta k k define rotations at j th and k th ends of the ith member due to unit moment apply at k th end you can see in this figure.

Unit moment applied at j th end and k th end will give you the flexibility coefficients respectively as you see in figures one and 2 similarly the stiffness coefficients k p p and k q p defined end moments that is required at j and k ends to maintain equilibrium when j th end is subjected to unit rotation while k th end is restrained you can see here this unit rotation, but k th end is no rotation restrained similarly stiffness coefficients k p q and k q q that is k p q and k q q of ith member.

Define end moments required at j and k ends to maintain equilibrium when k th end is subjected to unit rotation and j th end is restrained you can see in this figure j th end no moment no rotation k th end is subjected to unit rotation.

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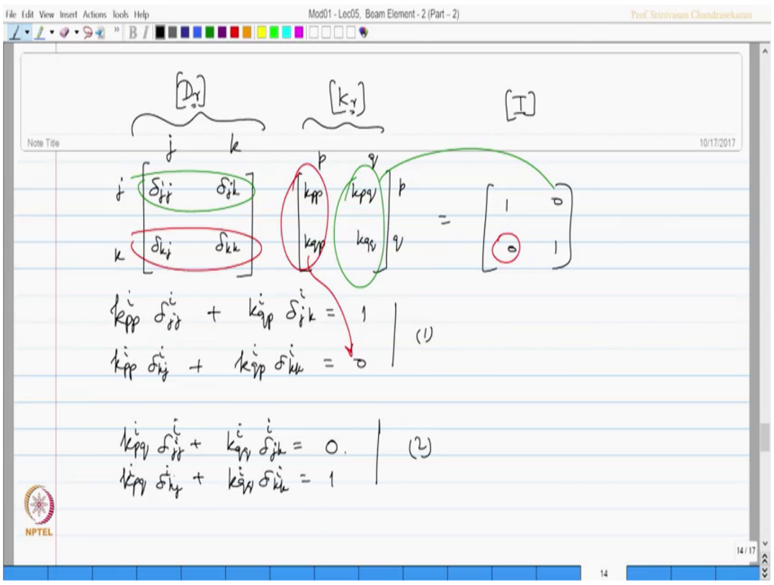

Having said this; let us now establish the fact that j k and j k. So, I should say the flexibility coefficients j j j k k j k k multiplied by the stiffness coefficients k p p, p q, q p. So, q q will give me identity matrix.

Let us expand this. So, kpp into j j plus k q p into j k is one and k p p delta k j plus k q p delta k k is 0 that is this row multiplied by this column is 0 kpp delta k j k q p delta k k is 0 this is one set of equation, we are similarly k p q delta j j plus k q q delta j k is 0 that is this by this is 0 than k p q delta k j plus k q q delta k k is one.

The second set of equation we have; now I call this matrix as flexibility matrix I call this matrix as stiffness matrix, I call this matrix as identity the subscript r stands for rotational degrees of freedom.

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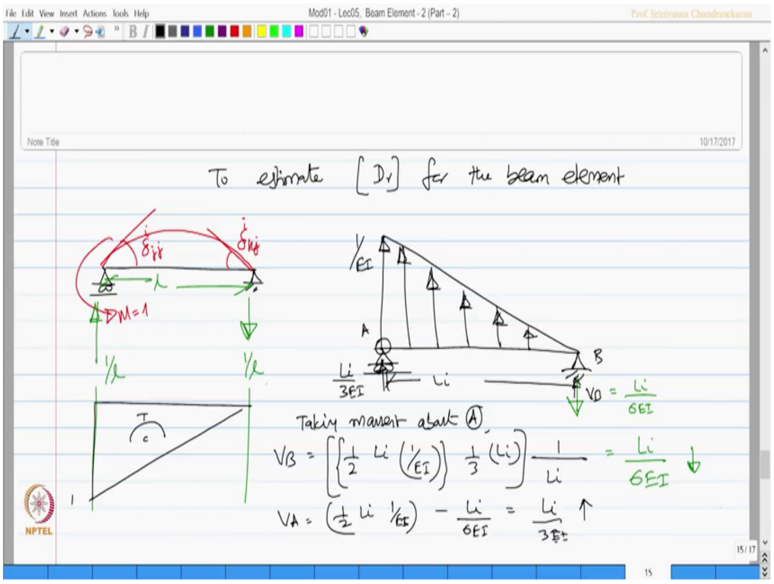

Now, my job is to estimate the matrix D r. So, now, to estimate the matrix D r for the beam element; so, once we get that will be able to find the k r. So, let us do that quickly. So, this is my beam, this is my deflected shape, I cause unit moment and this values going to be delta j j and this values going to be delta k j of the ith member.

So, if this is m, we all know that this is an anticlockwise moment which will be balanced by a clockwise couple which will be one by L and one by L where this is l. So, let us try to draw the bending moment diagram for this. So, this value is going to be 0 here and this going to be one which will have tension at the top and compression at the bottom. So, let us now replace this loading diagram with the conjugate beam, the conjugate beam will have a loading diagram as same drawing; now which will have an 1 by E I as the loading diagram.

So, this is going to be one by E I and this will remain as simply supported. So, I call this moment as A let us say the point a take moment about A. So, I call this as B therefore, the reaction is V B. So, I should say V B is straight away is half L i that is my distance and 1 by E I that is my height and the moment is going to be 1 third of L i; I have to divide this by L i again to get my V B where my V B will be in the reverse direction which will be actually equal to L i by 6 E I in this direction.

So, I mark that value here which will be L i by 6 here, therefore, V A will be actually half L i 1 by E I minus L i by 6 E I which will give me L i by 3 E I which is upward. So, this values going to be L i by 3 E I, similarly one can do for the other case.

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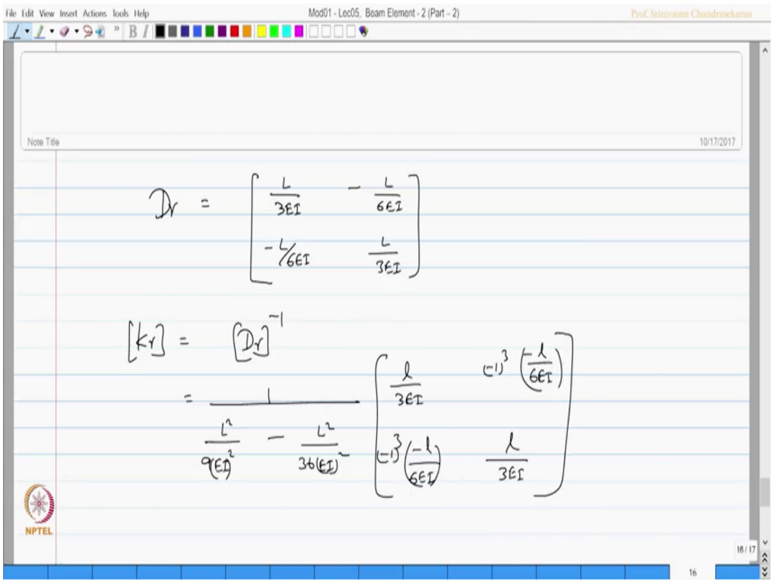

And see that this matrix is derived simply as L by 3 E I minus L by 6 E I minus L by 6 E I and L by 3 E I.

Therefore, k r will may actually D r inverse which will be actually equal to 1 by determinate of L square by 9 E I square minus L square by 36 E I square of L by 3 E I and L by 3 E I minus 1 cube of minus L by 6 E I minus 1 cube of minus L by 6 E I.

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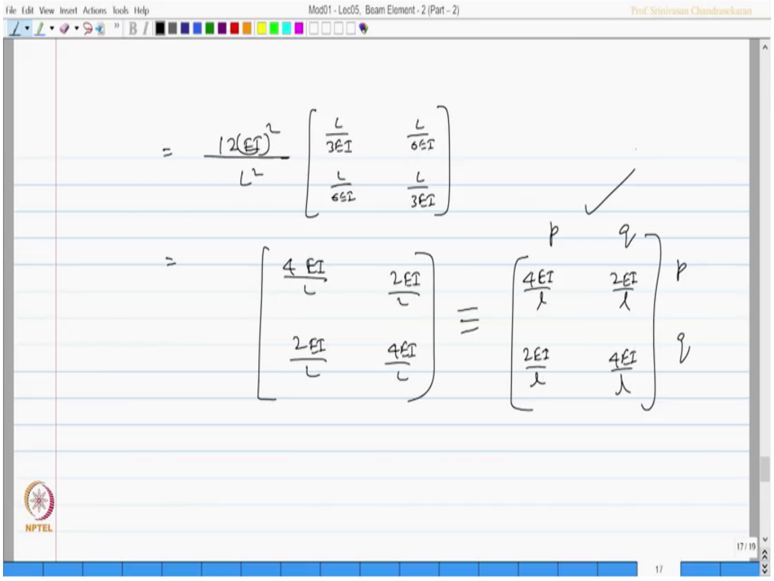

Which will amount to 12 E I square by L square of L by 3 E I; L by 3 E I; L by 6 E I; L by 6 E I which will be actually 4 E I by L. 2 E I by L. 2 E I by L and 4 E I by L.

So, now this is similar to p q; p q which is 4 E I by L 2, E I by L and 2 E I by L and 4 E I by L. So, friends, once we know this p q, the remaining matrix can be taken as seen from this equation. So, if we know this 4 remaining all can be computed.

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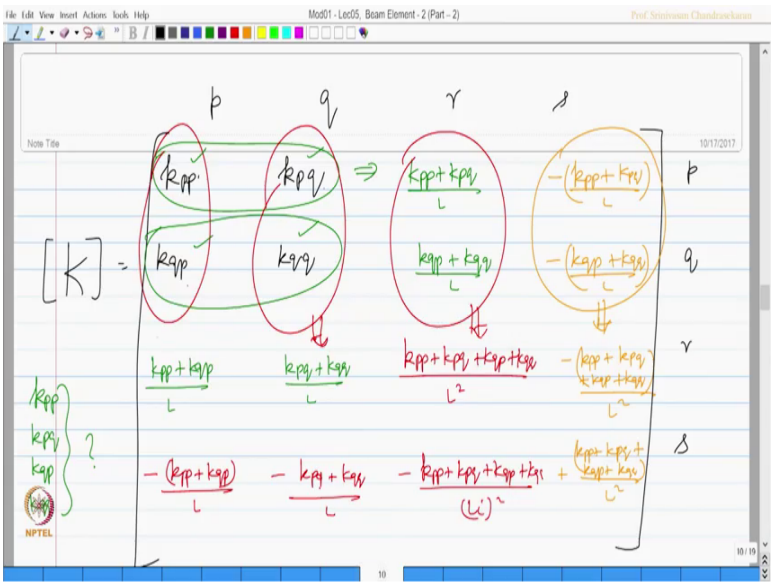

Now, I am writing the full matrix which is very easy to remember.

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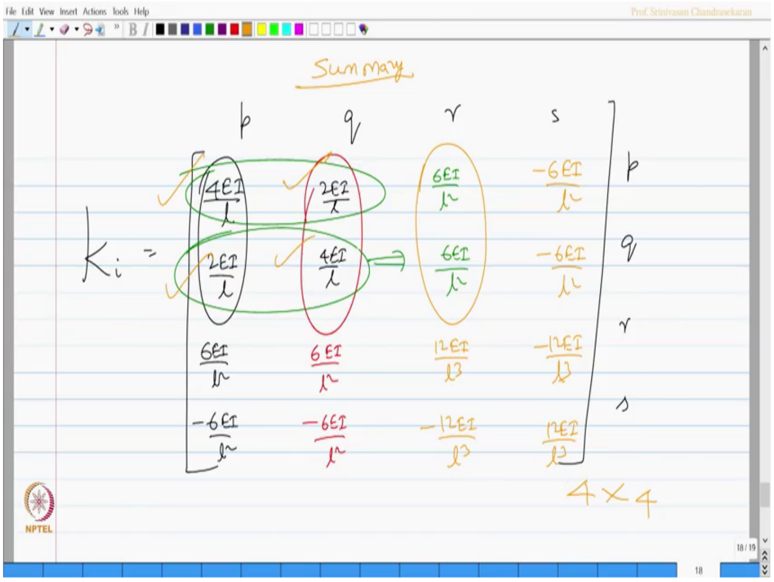

I am writing the full matrix of the ith member which is p, q, r, s which is p, q, r, s; the 4 values are 4 E I by L, 2 E I by L, 2 E I by L and 4 E I by L which we derive; just now I wrote that.

So, this will be; this value will be the some of these 2 by L again. So, 6 E I by L square; this is negative 6 E I by L square, this value will be some of these 2 by L again. So, 6 E I by L square and this is negative of that value 6 E I by L square, this value will be some of these 2 by L again. So, 6 E I L by square this will be some of these 2 which is 6 E I by L square. Now this value will be some of these 2 by L again. So, 12 E I by L q minus 12 E I by L q, once I get this the last column will be reverse of r column which is minus 6 E I by L square minus 6 E I by L square minus 12 E I by L q and plus 12 E I by L q.

So, friends as a summary we can see here. So, I can write this has a simple summary, we have for the beam element if I am able to compute these 4 coefficients, I can find the remaining 12 which is compromising 4 by 4 of rotational matrix of the ith beam which is got 16 coefficients which are derived based upon only the 4 coefficients of rotation.

Thank you very much.