Computer Methods of Analysis of Offshore Structures
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Module - 03
Lecture - 05
Stochastic Modelling (Part - 2)

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We are now continuing with the lectures on module 3, which is talking about stochastic processes. In the last lecture, we discussed about the details on estimating return period. We also explain how return period is a vital factor in stochastic process in general and random or stationary process in particular. In this lecture we will try to throw more light on; how do they do actually a stochastic modelling by taking an example.

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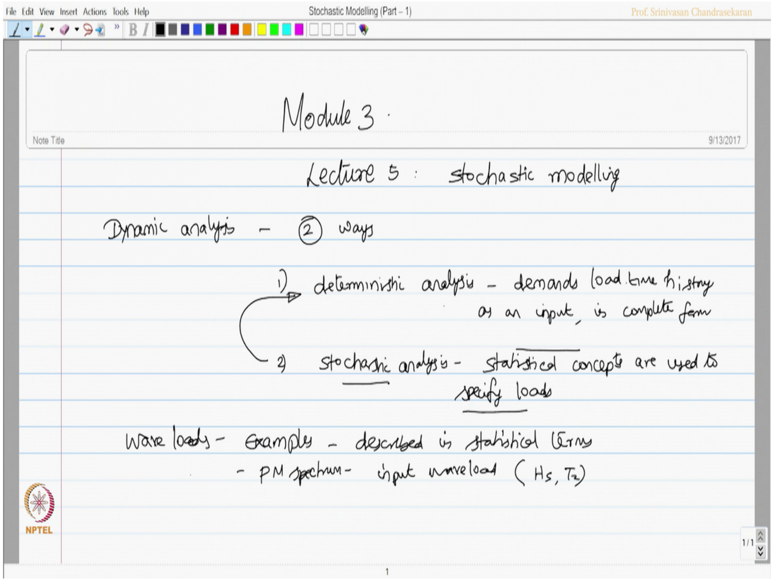

To reconnect we already said that dynamic analysis can be carried out by two ways; namely, a deterministic analysis which demands load time history as an input in complete form. Alternatively; one can also do stochastic analysis where statistical concepts are used to specify loads. Therefore, as we all agree the input loading which an offshore structure encounters is a random process which can be defined as a stochastic process under specific characteristics.

If you nearby to specify the loads, using a statistical concept; then I can do a stochastic analysis which can be an alternative to a detail deterministic analysis in form of estimating response when the input load is given which we already said they can be connected by a transfer function which forms or which follows a typical form or a proportionate form as that of a dynamic amplification factor.

Now, interestingly; wave loads or classical examples which can be described in statistical terms for example, if you look at the Pierson Moskowitz spectrum which is considered as one of the important input as wave loading. It deals essentially with two parameters significant wave height and zero mean crossing time period which are essentially a part of statistical analysis of a given data which is random in nature.

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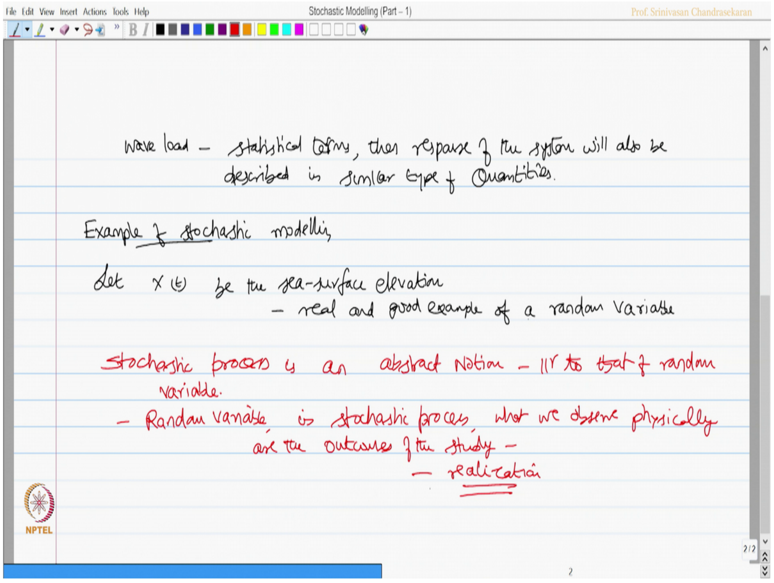

Once wave load can be expressed in statistical terms, then the response of the system will also be described in statistical terms. Let us take an example let X of t be the sea surface elevation which is actually your real and a good example of a random variable.

Now, let us make a statement; a stochastic process is an abstract notion which is more or less similar to that of random variable. Interestingly; similar to that of random variable in stochastic process, what we observe physically or the outcomes of the study we called this as realization. So, the term realization is physical outcome of that of the stochastic process which is in more or less a similar sense to the tough outcome of a typical random variable. Let us take an example. Let us say a real time history of sea surface elevation looks so complex let us try to plot a typical sea surface elevation.

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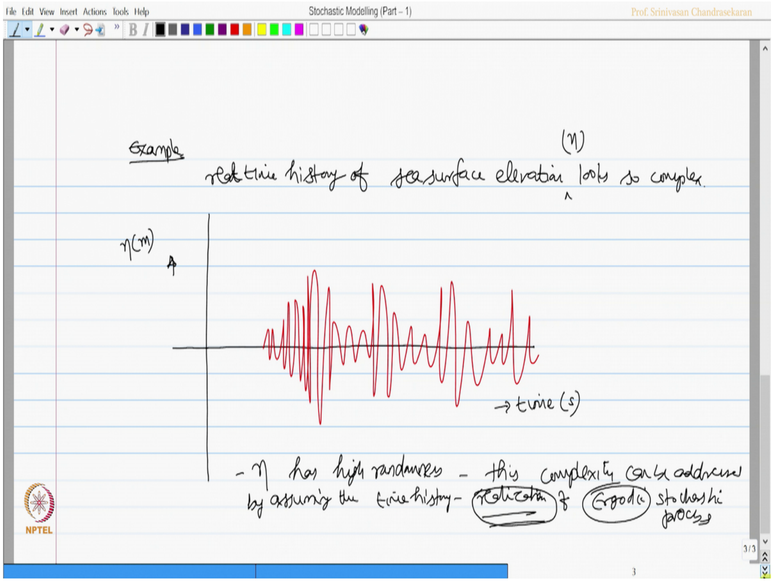

Let us say the x axis gives time in seconds and the y axis gives sea surface elevation eta in meters. Sea surface elevation is actually referred as eta in meters. So, the figure looks very complex. Sea surface elevation has high randomness. So, this can be overcome or this complexity can be addressed; by assuming the time history as a realization of Ergodic stochastic process. I think we clearly know what is ergodic? What is stochastic? Now. So, the complexity the time history can be addressed more or less in a simple form, by considering realization values of ergodic stochastic process.

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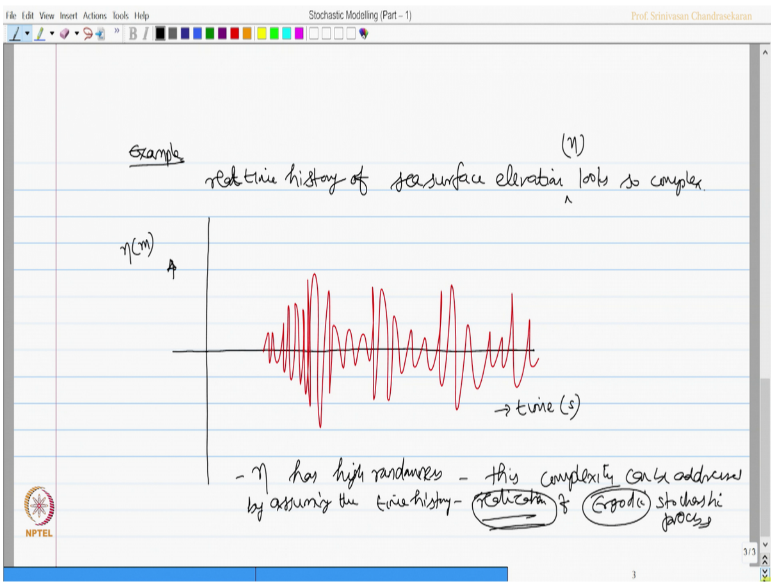

It has got an assumption, what is an assumption we make here? The assumption implies that; the statistical information about the process is in fact; contain in each single realization of the process, that is actually ergodicity; that is what we have discussed in the last lectures as well.

So, what is the second implication of this? The cumulative distribution function, we can refer this as CDF which is expressed as F of x t of x often ergodic process X of t is estimated by determining the relative amount of time that realization X of t of X of t assumes values lesser than or equal to the value X mathematically F of X of t X is limit

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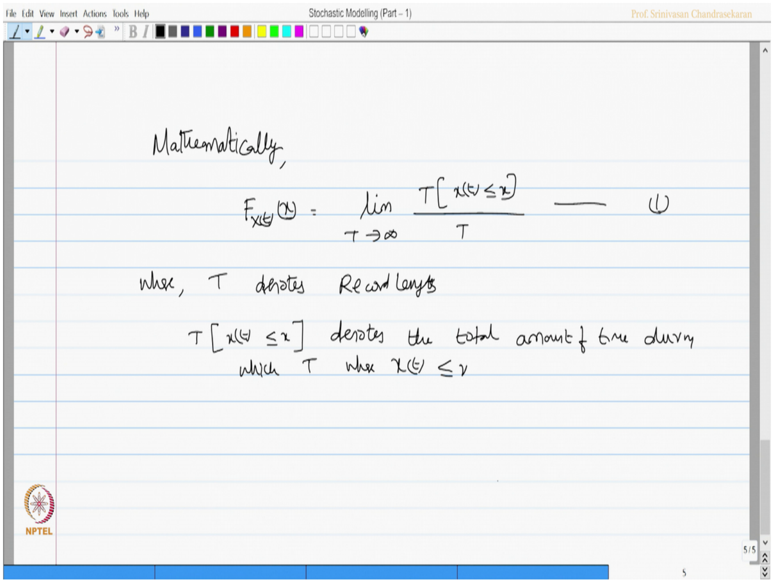

T tends to infinity T of x of t less than or equal to x of that of t equation number 1.

In this case T denotes record length. The term T of x of t less than equals x actually denotes the total amount of time during which T where x of t is less than or equal to x. So, x of t is a sample x is a value determined. So, x of t less than or equal to x is the realization of that sample over a record length T. So, that is how we express this mathematically.

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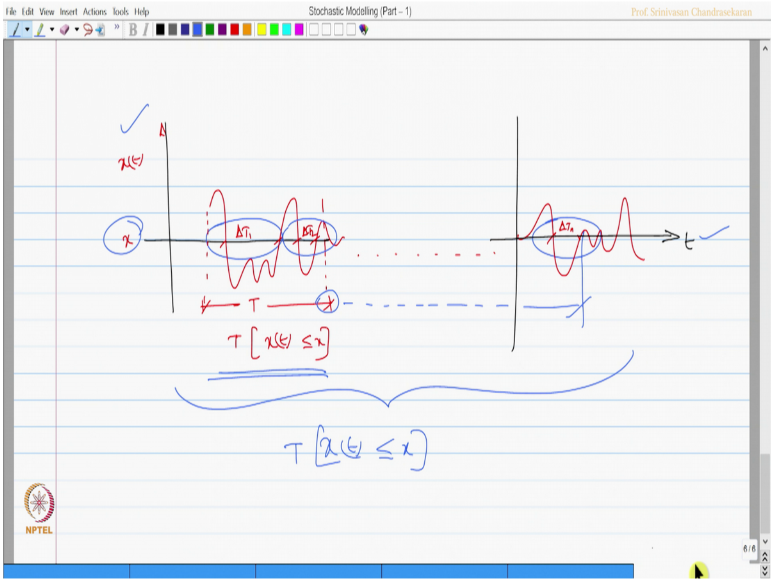

Graphically; if I say this is my plot and they assume that may record length is between these two points, this me record length T and of course, this is x of t and this value is actually x. So, what we are looking at is x of t lesser than or equal to x. So, it is actually this value. So, this is my delta T 1, because the x of t is less than or equal to x. Similarly this is again my delta T 2. So, these are all the time intervals from the recording T which satisfies the condition T the x of t less than 2 equal to x.

I continue this record, similarly with the same style which is extended over a time and the record continues. So, I can keep on estimating the values lesser than delta T n etcetera. So, this is the contribution, what I have marked here? What I have marked here? What I have marked here is the contribution fulfilling this expression over a record length of T. So, once I include this that my record length t will change to this. Maybe let us say it will here, now this becomes my capital T.

So, in a given value of x of t over time history; I can always fix up a known value x and try to find the realization of x of t less than or equal to x over the given record length T. So, that is what we are trying to do graphically.

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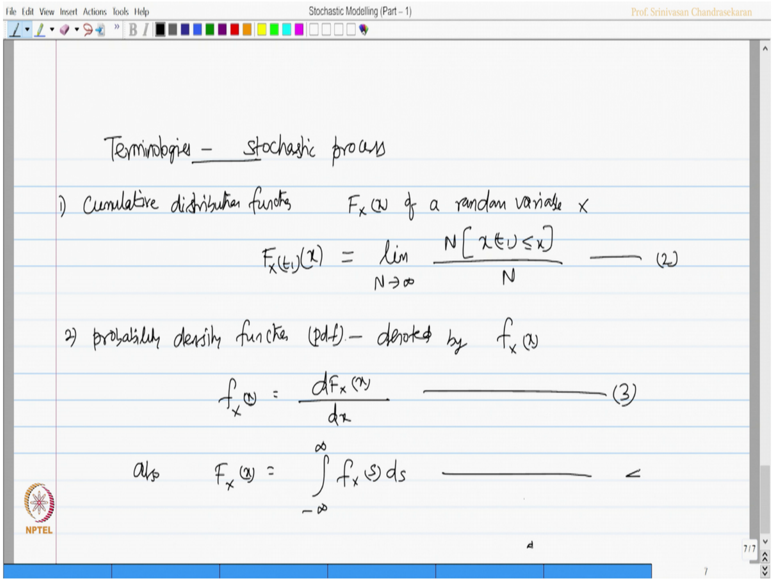

This what we explain mathematically in the previous case? This is what you explained mathematically in the previous case. Having said this; let us now look into few terminologies which are important in stochastic process. Let us talk about cumulative distribution function which is F of x of x of a random variable x F of x of t 1 of x is actually expressed as limit N tends to infinity N of x t 1 less than equal to x by N I call this equation number 2.

The next one is probability density function, which is p d f denoted by small f of x x and is given by small f of x x is d F x x by d x also F of x x s is minus infinity plus infinity f of x y s d s this statement is also true for stochastic processes.

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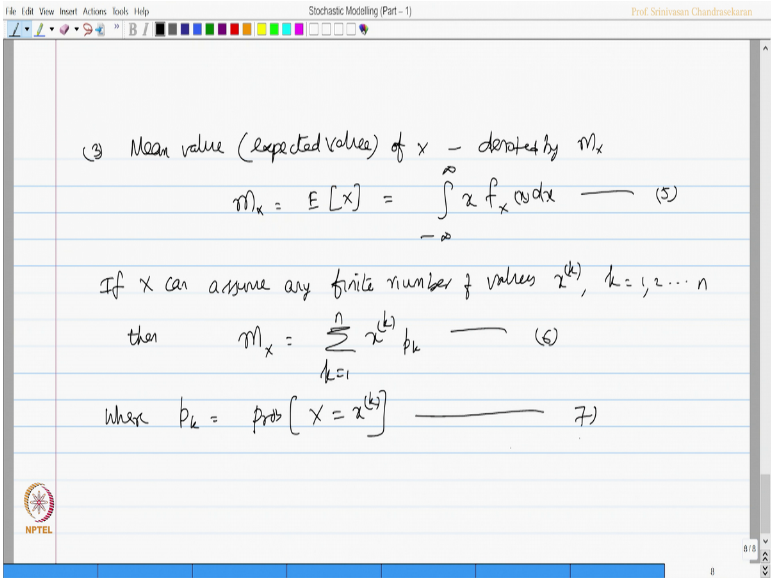

The mean expected value, what we call as mean value of x which is denoted by M x is given by expected value of x; which is actually equal to integration minus to plus infinity f of x x t x.

Now, if x can assume any finite number of values like for k equals 1, n then M x is given by k equals 1 to n x of k p k where p k is probability x equals x k.

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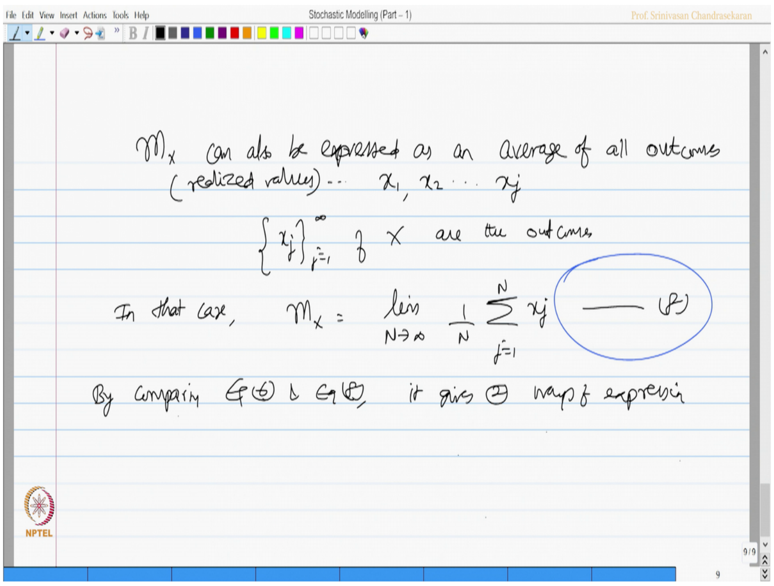

Interestingly, friends M x you can also be expressed as an average of all outcomes. So, let us insist the word realized values, because realization is an outcome of a given process all outcomes say for example, x 1, x 2, x j therefore, we can say the vector x j j equals 1 to infinity of x or the outcome. In that case the mean value is simply the limit N tends to infinity 1 by N of summation of j equals 1 to N x j which is a classical equation for finding the mean of a given data.

Interestingly friends, if you look at equation 6 and equation 8. So, if you look at by comparing, equation 6 and equation 8, it actually gives two ways of expressing the mean value is it not. Let us try to connect these two.