

**Computer Methods of Analysis of Offshore Structures**  
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**Module - 01**  
**Lecture - 09**  
**Example Problem 2 (Part - 2)**

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- Stiffness method
- Single bay single storey frame

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The image shows a handwritten matrix  $[K]_{AB} = EI$  for a single bay single storey frame. The matrix is a 4x4 global stiffness matrix with nodes 1, 2, 3, and 4 labeled at the top and right. The matrix is:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ \begin{matrix} 1.333 & 0.667 & 0.333 & -0.333 \\ 0.667 & 1.333 & 0.333 & -0.333 \\ 0.333 & 0.333 & 0.111 & -0.111 \\ -0.333 & -0.333 & -0.111 & 0.111 \end{matrix} & \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \end{bmatrix}$$

Handwritten annotations include circles around the diagonal elements (1.333, 1.333, 0.111, 0.111) and a note  $k_{11} = 1.333$  to the right of the matrix. The NPTEL logo is visible in the bottom left corner.

Once we have these values let us try to enter K AB matrix which is stiffness matrix for the member AB which I am entering here. The labels are as you see from here for AB the labels are 1 3 9 4. So, I should say the labels are 1, 3, 9, 4 which I am entering. And just now for the K AB the rotation of the constants are these. So, let us say 1.333, 0.667, then 0.333 which you already derived so I do not think we have to explain this again you can verify the previous lecture and try to fill up this matrix easily without any confusion.

So, let us fill up this matrix from the rotational coefficients. So, I do not think there is an ambiguity in this let us once again explain for the benefit of the users. Adding these two divide by 1 that is 6 meter will give you this. This value is negative of this coefficient. Similarly, adding these two by 1 will give you this; the fourth value is a negative of the third value. Similarly, adding these 2 by 1 will give you this. Adding these 2 by 1 will give you this. Adding these 2 by 1 again will give you this; this value will be negative of this.

The fourth column is negative of the third column as you see here. Let us write down for K BC; the next member which is again EI.

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$$K_{AB} = EI \begin{bmatrix} 1 & 0.5 & 0.375 & -0.375 \\ 0.5 & 1 & 0.375 & -0.375 \\ 0.375 & 0.375 & 0.188 & -0.188 \\ -0.375 & -0.375 & -0.188 & 0.188 \end{bmatrix}$$

Let us write down the labels for this. If you look at this BC has labels 1, 2, 5, 8. So, I should say 1, 2, 5, and 8.

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$$\{\Delta\} = \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \delta_7 \\ \delta_8 \\ \delta_9 \end{Bmatrix}$$

$$\bar{A}B : \frac{4EI}{l} = \frac{4(EI)l}{6} = 1.333EI$$

$$\frac{2EI}{l} = \frac{2EI(l)}{6} = 0.667EI$$

$$BC : \frac{4EI}{l} = \frac{4EI(l)}{4} = EI$$

$$\frac{2EI}{l} = \frac{2(EI)l}{4} = 0.5EI$$

$$CD : \frac{4EI}{l} = \frac{4EI(l)}{6} = 1.333EI$$

$$\frac{2EI}{l} = \frac{2EI(l)}{6} = 0.667EI$$

Similarly for BC 4 EI by l is 1 and 2 EI by l is 0.5; so 1.5 by l which is 0.375 and minus 0.375. Say this is 1, and 0.5, 1.5 by l, 0.375 minus 0.375 again 1.5 by l 0.375, 0.375. 0.375 plus 0.375 by l which is 0.188 minus 0.188. The fourth column is the opposite of the third column by sign. So, let us fill up this. So, K BC is also completed.

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$$[K]_9 = EI \begin{bmatrix} \textcircled{2} & \textcircled{3} & \textcircled{9} & \textcircled{4} & & & & & \\ & 1.333 & 0.667 & 0.333 & -0.333 & & & & \\ & 0.667 & 1.333 & 0.333 & -0.333 & & & & \\ & 0.333 & 0.333 & 0.111 & -0.111 & & & & \\ & -0.333 & -0.333 & -0.111 & 0.111 & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix}$$

Let us go for K CD which is EI let us have the labels of KCD.

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Example 2

$[k]_{ABC\bar{D}}$  = size  $9 \times 9$

$\{\theta_1, \theta_2\}$  unrestrained d.o.f

$\{\theta_3, \delta_4, \delta_5, \theta_6, \delta_7, \delta_8, \delta_9\}$  - restrained d.o.f

$\begin{matrix} u \\ k_{xx} \\ (2 \times 2) \end{matrix}$	$\begin{matrix} y \\ k_{yy} \\ (2 \times 2) \end{matrix}$	$u$	Member	$j^{\text{th}}$ end	$k^{\text{th}}$ end	d.o.f labels
$\begin{matrix} (k_{xx}) \\ (7 \times 7) \end{matrix}$	$\begin{matrix} (k_{yy}) \\ (7 \times 7) \end{matrix}$	$y$	$\bar{B}A$	B	A	$(\theta_1, \theta_3, \delta_7, \delta_8)$
			$\bar{B}C$	B	C	$(\theta_1, \theta_5, \delta_5, \delta_6)$
			$\bar{C}D$	C	D	$(\theta_2, \theta_6, \delta_9, \delta_7)$

If you look at this CD as labels 2 6 9 7; so 2, 6, 9 and 7; 2, 6, 9 and 7 and we already know  $4EI$  by 1 is 1.33 and  $2EI$  by 1 is 0.667; so 1.33 0.667 addition of these 2 by 1 again, so 0.333 minus 0.333. Similarly 1.333 0.667 addition of this by 1 again so 0.333, and minus 0.333; these two 1.333 plus 0.67 by 1 again which is again 0.333 again 333: so 666 by 1 0.111 minus 0.111.

The fourth column is opposite of third minus 0.333 minus 0.333 minus 0.111 and plus 0.111. So, we have no issues we have got the stiffness matrix for  $K_{AB}$ ,  $K_{BC}$ , and  $K_{CD}$ .

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$$[K]_{uu} = EI \begin{bmatrix} 1.333 & 0.5 \\ 0.5 & 1.333 \\ 2.333 & 2.333 \end{bmatrix}$$

$$[K]_{uu} = EI \begin{bmatrix} 2.333 & 0.5 \\ 0.5 & 2.333 \end{bmatrix}$$

$$[K]_{uu}^{-1} = \frac{1}{5.193 EI} \begin{bmatrix} 2.333 & -0.5 \\ -0.5 & 2.333 \end{bmatrix}$$

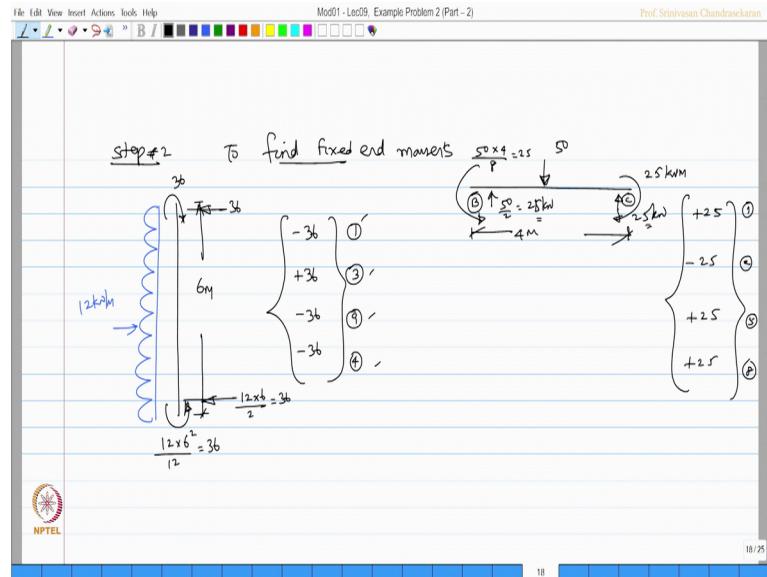
So, now I want to form. So, let us talk about  $K_{uu}$  matrix which will be 2 by 2 because the understand degrees of freedom are only 2 by 2 for this problem. Let us estimate this value from the known matrices of a b c.

So, let us say 1 1 I want to see. So, let us look into all. So, there is easy way of doing this; let us see how this can be done without any error. Carefully see here 1 is present in AB BC, 2 is present in BC and CD. So, for first row let us straight look into both these matrices, because 1 is present in both so let us look into K AB. First row first column 1.33 I am writing it here  $k_{11}$  1.33 first row  $k_{11}$ . So, let us enter that value here 1.33.

Then let us look into the next matrix which is 1 2. This is 1 1 again 1 value here let us take that value plus 1.0 which becomes 2.333. Similarly 1 2: 1 2 is available in K BC 1 2 is this value so let us say 0.5. Similarly for 2: let us go back to this matrix 2 1 is 0.5 you do not have to go to K AB this is nothing of 2 order here no; no 2 number is here. So, 2 1 is 0.5; so 2 1 is 0.5. Similarly it let us go to 2 2 which is going to be 1; 2 2 you also have one more 2 2 in the next matrix which is this.

So, the total will be 1 plus 1.333 which is 1 plus 1.333 which is 2.333 absolutely symmetric and square matrix. So,  $K_{uu}$  matrix is EI times of 2.333 0.5, 0.5 and 2.333. Let us find  $K_{uu}$  inverse which will be 1 by 5.193 EI. You can easily estimated from the first principles which will be again 2.333 2.333, minus 0.5 minus 0.5.

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The next step is to find the fixed end moments caused because of the loads on each span of the member. Let us do one by one. Let us say span AB subjected to a uniform distributed load of 12 kilo Newton per meter and this distance is 6 meters; is or not. So, this will cause a moment as I am indicating here. So, this value will be 12 into 1 square by 12 which will be 36, this will also be 36, and this reaction will be 12 into 6 by 2 which will also 36 and 36.

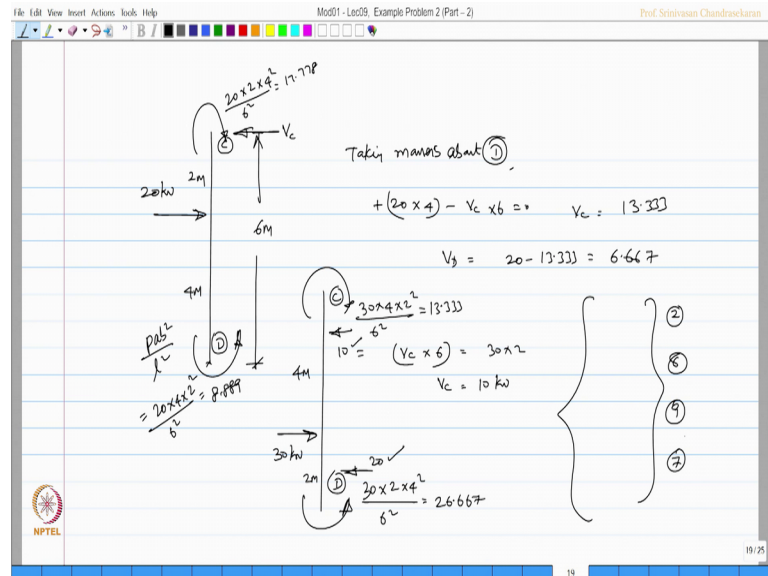
So, now I can write the vector which will be of degree is 1, 3, 9, and 4 these are degrees you can see here, member AB as degrees 1, 3, 9, and 4. So, 1, 3, 9, and 4. For 1 1 is marked here two is marked here and so on. So, 1 is here 1 will be here. So, this is anticlockwise is positive therefore this is going to be minus 36, this is 3 which is anticlockwise, so plus 36 and 9 and 4 both are minus 36 because they are towards left.

Similarly let us do it for the beam span BC this is B and this is C central concentrated load 50 which will have moments of this type and ensures of this type, the span is 4 meters. So, this is going to be  $wl^2$  square by  $pl$  by 8. So, 4 by 8 which is going to be 25 this is also going to be 25 kilo Newton meter, and this reaction is also going to be 50 by 2 which is 25 kilo Newton; this is also 25 kilo Newton.

So, if I write the vector for the span BC the labels will be 1, 2, 5 and 8, because you know there is a label of BC 1, 2, 5, and 8. So, 1, 2, 5, and 8 the values will be

anticlockwise of plus 25 this is clockwise of minus 25 and the reaction ensures is upward positive so plus 25 and this also positive plus 25.

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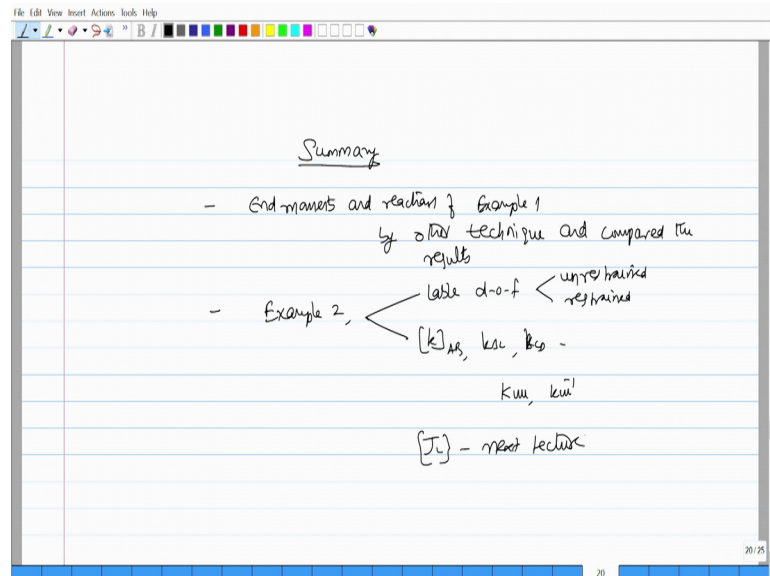
Let us do it for the third case which is CD. CD is got two loads. So, this is C and this is D end this is separated by a distance 6 meters that is a span of the beam, ok.

It has got 1 load which is 20 kilo Newton. So, this is going to be 2 meters this is 4 meters. So, this will generate the moment P AB square by 1 square which will be 20 into 4 into 2 square by 6 square which gives me 8.889 and this value will be 20 into 2 into 4 square by 6 square which will be 17.778. And taking moment towards the point D this reaction will be let us call is V C. So, taking moments about D 20 into 4 plus minus V C into 6 will be 2, so that gives me V C as 13.333 and V D will be 20 minus 13.333 which will be 6.667.

Similarly, let us do it for the other load. There is again C n D n the other load is of intensity 30 kilo Newton this is 2 meters and this is 4 meter. So, this value will be 30 into 2 into 4 square by 6 square which makes 26.667. And this value will be 30 into 4 into 2 square by 6 square which makes it as 13.333 and the reactions this reaction and this reaction will be 10 and 20 which can be said as V C into 6 is equal to 30 into 2 which V C will be ten kilo Newton's which we got and the total is 30 therefore these 20.

Now, let me write down the fixed end moments of the member CD for the labels 2, 6, 9, and 7 that is the label of CD; 2, 6, 9, and 7: 2, 6, 9, and 7. So, I should say to summarize this we have to add this let us make a summary and continue this problem in the next lecture.

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So, in this lecture we estimated the end moments and reactions of example 1 by other technique and compare the answers; we did this. And we found the results are exactly same. The second conclusion we can make is we are working on example 2. We are able to locate and label the degrees of freedom both unrestrained and restrained. We are able to find the stiffness matrix for all the three members K BC, CD. And then we also found out K uu, K uu inverse. We are in the process of estimating the joint load which will discuss in the next lecture.

Thank you.