

**Computer Methods of Analysis of Offshore Structures**  
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**Module - 01**  
**Lecture - 12**  
**Planar, Non-Orthogonal Structures 1 (Part - 2)**

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- Stiffness method
- Planar non-orthogonal structure
- Stiffness matrix including axial deformation for a beam element

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In case of non-orthogonal frames, orientation of the local axes ( $x_n - y_n$ ) may be such that this cannot be aligned or mapped to the same orientation of the reference axes ( $x - y$ ) for example, in figure members ①, ② have this problem.

Hence,  $[k_i]$  cannot be directly written with respect to the reference axes.

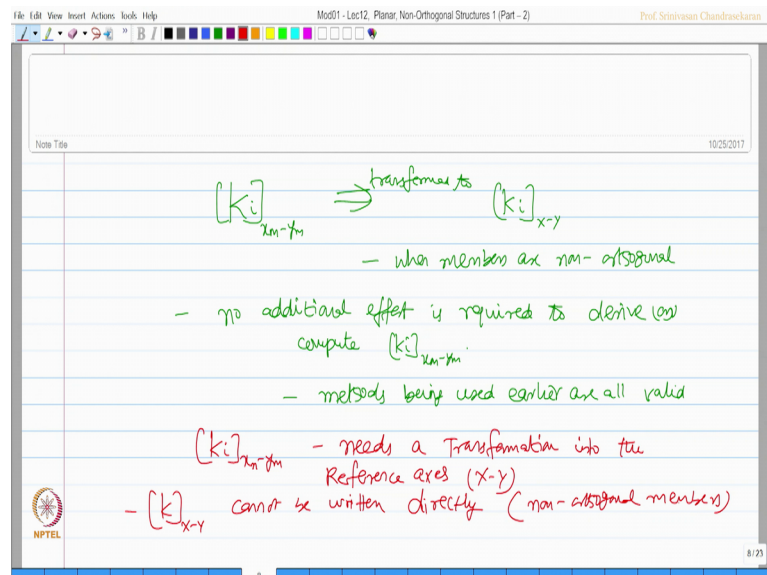
But can be transformed to the reference axes system.

$[k_i]$  - written for member (local) axes system will be valid - we need to transform this matrix.

Let us make a statement in case of non orthogonal frames orientation of the local axes that is  $x_m y_m$  may be such that this cannot be aligned or mapped to the same orientation of the reference axes that is  $x y$ , for example, in figure b, member 1 and 3 have this problem; is it not? In such cases, what do we do? What is required; hence the member stiffness matrix cannot be directly written with respect to the reference axes.

But there is a solution for this problem can be transformed to the reference axes system one good news is that  $K_i$  written for member axes or local axes system will be valid you do not have to change that only thing you have to do is we need to transform this matrix.

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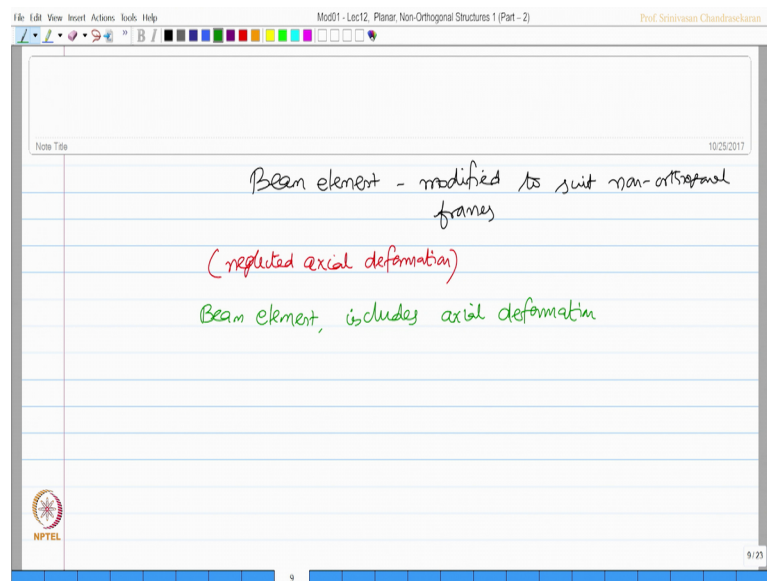
That is  $K_i$  written for  $x_m y_m$  frame need to be transformed to  $K_i$  of  $x y$  frame, we have to this; this is true when members are non orthogonal please understand this just not demand any additional effort to determine  $K_i$ . So, no additional effort is required to derive or compute  $K_i$  with  $x_m y_m$  frame its already known. So, whatever method, we have been using methods being used earlier are all valid, I have to only transform this that is all.

So, let me write  $K_i$   $x_m y_m$  needs a transformation into the reference axes  $x y$ . So, please understand  $K_i$  on  $x y$  frame cannot be written directly for non orthogonal members its very very important. So, what do you mean by non orthogonal members when the local axes of the member does not map aligned match exactly with the global axes or the reference axes they are called non orthogonal members. So, for these members the

stiffness matrix for the member cannot be written directly on the reference axes system, you can always write only on the local axes system and this should be transformed that is the problem here.

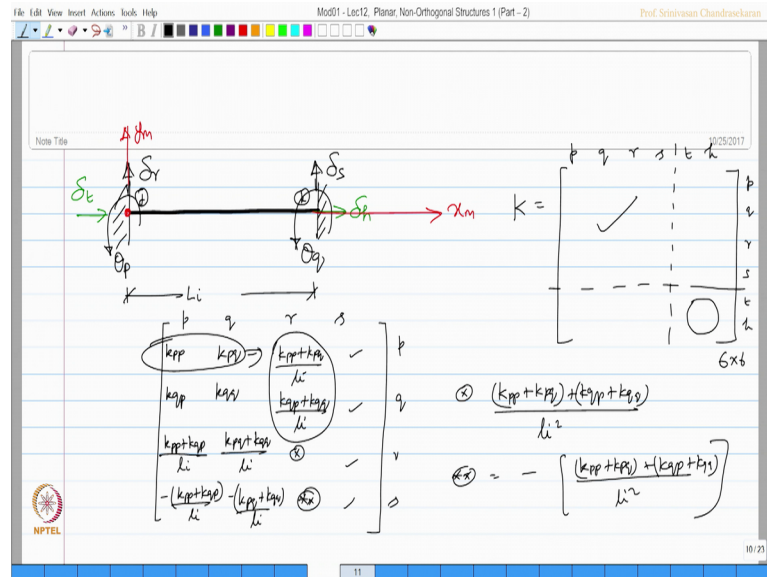
Now, let us take a simple beam element and see what additional data is required to use this element.

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In the non orthogonal frames, the question is now we have taking a beam element which is slightly modified in sense to suit to be a member of remember of non orthogonal frames, if you remember in the last set of derivation we actually neglected axial deformation; is it not. Now let us have a beam element, let us now have a beam element which includes axial deformation also. So, let us take a beam element.

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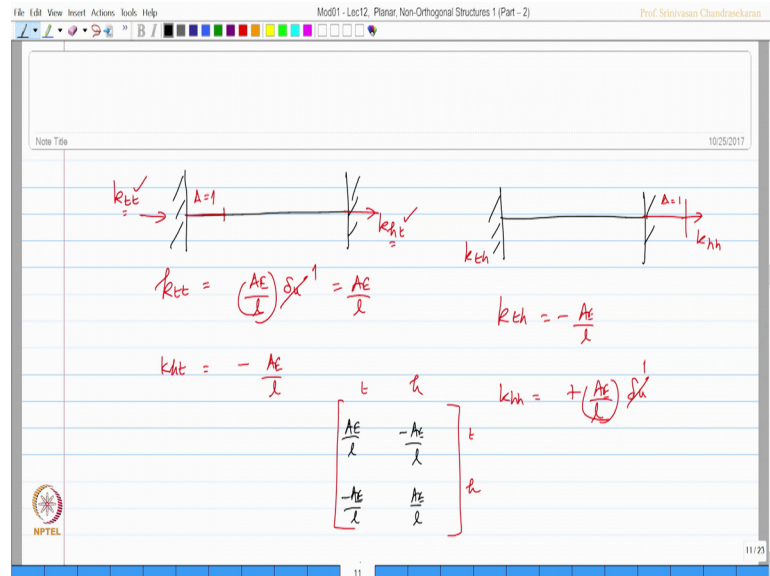


So, let me mark the beam element here which is fixed at both the ends is our basic model, let us mark the degrees of freedom, we already know this is theta p, this is theta q and this one is delta r, this delta s. Now let us introduce 2 more displacements axial at the j th end k th end of the member, we call this as delta t and this as delta h the member as got the length  $L_i$  of the member and this is my j th end, this is my k th end therefore, this will be my x m axes and this will be my y m axes and the origin is at j th end that is the origin, I do not have to explain you the rotational coefficients of stiffness matrix and the displacement coefficient at p, q, r and s.

So, now the stiffness matrix will be of size 6 by 6 is there are 6 degrees of freedom. So, it means p, q, r, s, t and h. Similarly p, q, r, s, t and h. So, we already know this matrix. So, let us write down that matrix let us write down only this sub matrix p, q, r, s, p, q, r, s. So, this is going to be  $k_{pp}$ ,  $k_{qp}$  and this is  $k_{qq}$  and this is  $k_{pq}$  rotational coefficients; once I get this, I can find out this value which will be simply  $k_{pp}$  plus  $k_{qp}$  by  $L_i$ , this will be minus of  $k_{pp}$  plus  $k_{qp}$  by  $L_i$  and this value will be  $k_{pq}$  plus  $k_{qq}$  by  $L_i$  and this is minus  $k_{pq}$  plus  $k_{qq}$  by  $L_i$  the third column will be  $k_{pp}$  plus  $k_{qp}$  by  $L_i$  that is this sum by  $L_i$  and this will be  $k_{pp}$  plus  $k_{qp}$  by  $L_i$  and this value, I am writing it here, I am just writing it here, there is no space. So, this value is going to be the sum of these 2 by  $L_i$ . So,  $k_{pp}$  plus  $k_{qp}$  plus  $k_{qp}$  plus  $k_{qq}$  by  $L_i$  square and this value will be simply minus of this value that is  $k_{pp}$  plus  $k_{qp}$  plus  $k_{qp}$  plus  $k_{qq}$  by  $L_i$  square where as the fourth column is actually the negative of the third column which can be filled up as it is. So,

this already we have derived there is no confusion of this; let us derive only this matrix separately.

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So, let us take a beam undergoing axial deformation by unity. So, now, this will be  $k_{tt}$  and this has to be  $k_{ht}$  that is force in the  $t$ th degree by giving unit displacement in the  $t$ th degree force in the  $h$ th degree by giving unit displacement in the  $t$ th degree. So,  $k_{tt}$  will be actually equal to  $AE$  by  $l$  of  $\Delta u$ , we know this is one therefore, this simply  $AE$  by  $l$  and  $k_{ht}$  will be negative of this value because this going to be opposite to  $k_{th}$ . So, minus similarly if I give unit displacement at this dimension, I will get  $k_{hh}$  and  $k_{th}$ . So,  $k_{th}$  or  $k_{hh}$  will be positive  $AE$  by  $l$  of  $\Delta u$  which is one this will be negative  $AE$  by  $l$ . So, I have a matrix now which is for  $t$   $h$ ;  $t$   $h$  only which will be  $AE$  by  $l$  minus  $AE$  by  $l$  minus  $AE$  by  $l$  and  $AE$  by  $l$ ; let us substitute this in the full stiffness matrix and write the full stiffness matrix of the structural system.

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Handwritten notes on a slide showing stiffness coefficients and a 6x6 stiffness matrix for a member. The slide is titled "Mod01 - Lec12, Planar, Non-Orthogonal Structures 1 (Part - 2)" and is by Prof. Srinivasan Chandrasekaran. The notes include:

$$k_{pp} = \frac{4EI}{l} \quad k_{qp} = \frac{2EI}{l}$$

$$k_{pq} = \frac{2EI}{l} \quad k_{qq} = \frac{4EI}{l}$$

The stiffness matrix  $[K]$  is shown as a 6x6 matrix with rows and columns labeled p, q, r, s, t, h:

$$[K] = \begin{bmatrix} \frac{4EI}{l} & \frac{2EI}{l} & \frac{6EI}{l^2} & -\frac{6EI}{l^2} & 0 & 0 \\ \frac{2EI}{l} & \frac{4EI}{l} & \frac{6EI}{l^2} & -\frac{6EI}{l^2} & 0 & 0 \\ \frac{6EI}{l^2} & \frac{6EI}{l^2} & \frac{12EI}{l^3} & -\frac{12EI}{l^3} & 0 & 0 \\ -\frac{6EI}{l^2} & -\frac{6EI}{l^2} & -\frac{12EI}{l^3} & \frac{12EI}{l^3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{AE}{l} & -\frac{AE}{l} \\ 0 & 0 & 0 & 0 & -\frac{AE}{l} & \frac{AE}{l} \end{bmatrix}$$

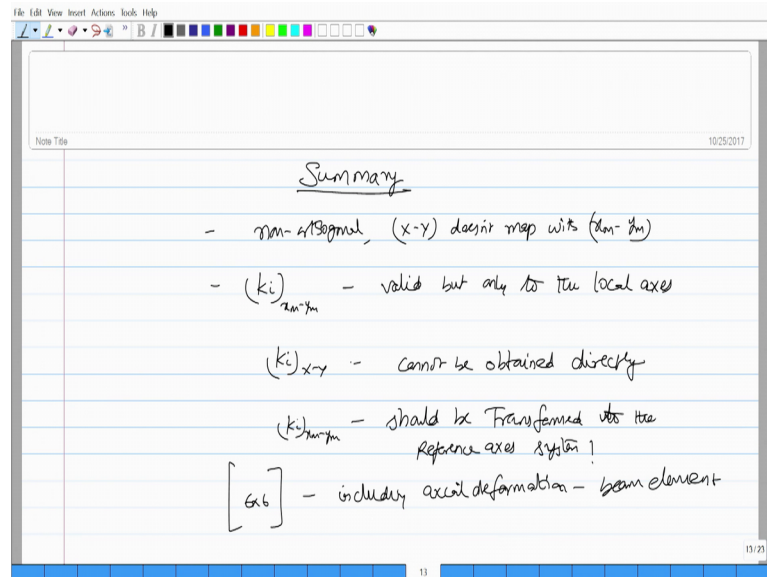
The matrix is labeled "6x6" and has a checkmark next to it. The NPTEL logo is visible in the bottom left corner.

And we also know the rotational coefficients that is  $k_{pp}$  is  $4EI$  by  $l$  and  $k_{pq}$  is  $2EI$  by  $l$  and similarly  $k_{qp}$  is also  $2EI$  by  $l$  and  $k_{qq}$  is  $4EI$  by  $l$ , we know these we have already derived them let us now write the full stiffness matrix for the member at the local axes.

Let us note the labels  $p, q, r, s, t$  and  $h$ . So,  $p, q, r, s, t$  and  $h$ . So, this is  $4EI$  by  $l$ , this is  $2EI$  by  $l$  this is going to be  $6EI$  by  $l^2$  square; this is minus  $6EI$  by  $l^2$  square and these 2 will be 0, this is going to be  $4EI$  by  $l$  this is  $2EI$  by  $l$  this is  $6EI$  by  $l^2$  square minus  $6EI$  by  $l^2$  square again 0. So, this going to be  $6EI$  by  $l^2$  square  $6EI$  by  $l^2$  square  $12EI$  by  $l^3$  cube minus  $12EI$  by  $l^3$  cube again, 0 is going to be minus  $6EI$  by  $l^2$  square minus  $6EI$  by  $l^2$  square minus  $12EI$  by  $l^3$  cube and  $12EI$  by  $l^3$  cube and this is 0; let us say these are all 0s and this will be  $AE$  by  $l$  minus  $AE$  by  $l$  minus  $AE$  by  $l$  and  $AE$  by  $l$  I get the full 6 by 6 matrix of the  $i$ th member at the local axes  $x_m, y_m$  I do not think any confusion in this specific case.

Now, the argument is if the local axes of the member does not orient with the global axes of the reference axes system I need to do the transformation. So, I have to derive the transformation matrix to transform this matrix to the reference axes system.

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So, friends, let us look at the summary we said when the members are non orthogonal the reference axes x y does not map with the local axes x m y m therefore,  $K_i$  which has been derived for x m y m is still valid, but only to the local axes  $k_i$  x y reference axes cannot be obtained or derived directly.

So,  $k_i$  x m y m should be transformed to the reference axes system. So, how to do this that we will see we have also derived the full stiffness matrix 6 by 6 including the axial deformation for a standard beam element which may be required in case of analysis of non orthogonal members.

Thank you very much.