

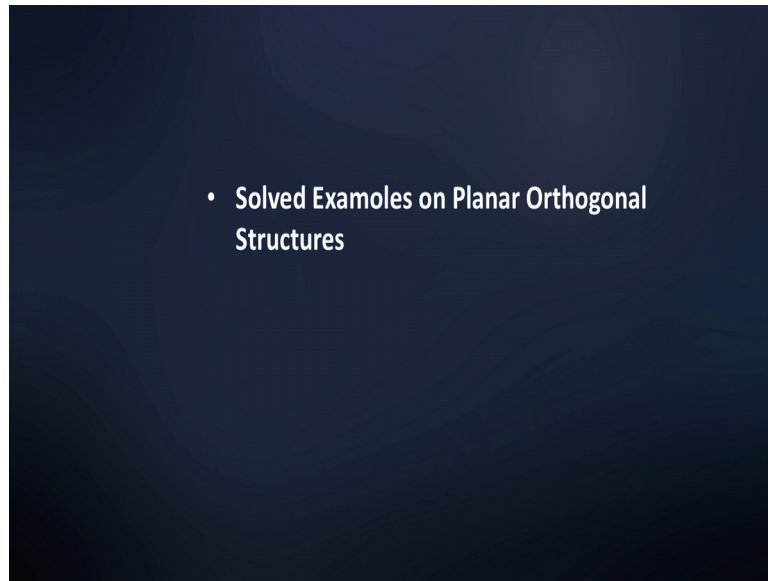
**Computer Methods of Analysis of Offshore Structures**  
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**Module - 01**

**Lecture - 15**

**Example problem - Planar non-orthogonal structure**

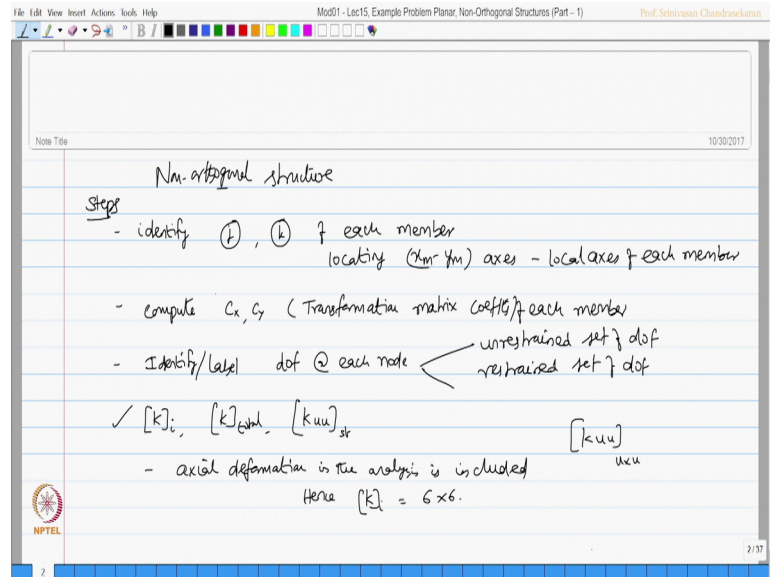
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Friends, welcome to the 15th lecture in module 1, where we are going to now solve couple of example problems on Planar Non-orthogonal Structures. We will also discuss the relevant computer codes with which the problems have been solved. We will explain the computer code steps and also the solutions obtained from the computer program.

Let us quickly **revise** the steps in solving a non-orthogonal structure.

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The steps involved could be the following for the given problem first identify the j end and k end of each member friends please recollect a very important statement identifying j end k end is actually locating the x m y m axes which we call local axes of each member. So, while doing; so, please understand that x m should be facing towards the positive length of the member and y m axes should be anticlockwise 90 degrees from the x m axes.

Locate the j end k end of each member after doing that let us compute the C x C y which are called the transformation matrix coefficients of each member also identify or label the degrees of freedom at each node first the unrestrained set of degrees of freedom then the restrained set of degrees of freedom we should do this.

So, then I will know what is the size of each matrix of the member what the size of the complete total matrix of the structure, I will also know size if the unrestrained stiffness matrix of the structure I will be able to do this. Now I can have a clue I think all of you will agree strongly with me. Since we are including the axial deformation in the analysis hence the size of stiffness matrix of each member will be 6 by 6 and the size of unrestrained stiffness matrix of the complete structure will be the unrestrained degrees of freedom.

So, we have to be very clear about the size of different sub matrices in the given stiffness matrix.

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Mod01 - Lec15, Example Problem Planar, Non-Orthogonal Structures (Part - 1) Prof. Srinivasan Chandrasekaran

Note Title 10/30/2017

- Estimate  $[k]_i$
- Identify the global labels of each member
- Assemble  $[K]_{\text{complete}}$  - Then plug-out  $[K_{uu}]$  submatrix from  $[K]_{\text{total}}$ .
- compute  $[FEM]_i$  in local axes.

$[K]_i = [T]_i^T [k]_i [T]_i$

$[FEM]_i = [T]_i [FEM]_i$

Diagram: A beam element of length  $l$ , modulus of elasticity  $E$ , moment of inertia  $I$ , and cross-sectional area  $A$ . Local axes  $x_m$  and  $y_m$  are shown. Global axes  $X_m$  and  $Y_m$  are shown. Displacements are labeled as  $\delta_t$ ,  $\delta_r$ ,  $\delta_s$ ,  $\delta_p$ , and  $\delta_q$ .

So, the next step is estimate the stiffness matrix of each member, we already have this equation with us derived in the last set of lectures; we know this is going to be a 6 by 6 matrix, we already have a fixed beam as our basic module, we know that this is going to be theta p theta q delta r delta s delta T and delta H where this is my x m axes this is my y m axes of the member the member has a length l I EI and A .

So, we already derived the 6 by 6 matrix of each member. So, estimate this for each member also identify the global labels of each member that is very important once we have done this, then K global of each member which will be T transpose k local T because transformation matrix of each member is now known to us, once I have this then assemble. So, let us do this for every member assemble k matrix global of the complete structure then plug out K bar uu sub matrix from K total K bar total.

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The screenshot shows a presentation slide with the following handwritten text:

- Estimate  $\{\bar{J}_i\}_{complete}$  . plug-out  $\{\bar{J}_i\}_u$
- $\{\bar{\delta}_i\} = [k_{uu}] \{\bar{J}_i\}_u$        $\{\bar{\delta}_r\} = \{\text{null vector}\}$
- $[\bar{M}]_i = [k]_i \{\bar{\delta}\}_i + \{FEM\}_i$

The slide also includes a title bar with 'Mod01 - Lec15, Example Problem Planar, Non-Orthogonal Structures (Part - 1)', a date '10/30/2017', and an NPTEL logo in the bottom left corner.

Then compute fixed end moments of i-th member in local axes, find fixed end moment of the i-th member in global axes which will be T transpose of the i-th member and fixed end moment of the i-th member; once I have this completed then estimate the joint load vector in global degrees of freedom for the complete structure from this plug out the joint load vector of the unrestrained degree from the global joint load vector.

Then use this equation delta bar global will be actually equal to  $K u u$  global inverse of  $J L u$  global, you get delta bar unrestrained friends it is very evident and you will understand and agree delta bar r of the system will be actually a null vector, it will be 0 restrained degrees of freedom after **obtaining** delta u; one can calculate the m bar of every member that is the global end moments and shear of every member from simply k bar of every member multiplied by delta bar of every member plus any fixed end moment of every member in global terms. So, we have to do this and identify the end moments **and** end shears.

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Example 1

$M_0 I = 0.0016 \text{ m}^4$   
 $E$  is constant  
 $A = 0.120 \text{ m}^2$

$\theta = \tan^{-1} \left( \frac{4}{2} \right) = 63.435^\circ$

member no	Ends	$l_i$	$\theta$	$C_x$	$C_y$	global labels
1	A B	$\sqrt{4^2+2^2} = 4.472 \text{ m}$	$+63.435^\circ$	0.447	0.894	(1, 2, 3, 4)
2	B C	4m	$0^\circ$	1	0	(1, 4, 2, 5, 3, 6)

Let us take an example and solve this problem let us take example 1; this is my structural system which is fixed here as well as fixed here let us call this end as A, B and C; let us mark the dimensions of this structure. So, this is 2 meters. And let us say this is 4 meters; let the height of the frame be 4 meters. Now interestingly, this is my global axes and for every member the local axes is going to be different.

Let say the I value the moment of inertia of each member is 0.0016; this member also has I this member also has I E is constant whatever may be the value; whatever maybe the material, it is constant through and through area of cross section is 0.120 meters square, this is meter 4, it is also constant. So, A and A once you have this let us say the system has a joint load which is ten kilo Newton applied on the structure, I think from simple geometry I can find is angle theta you know theta is actually tan inverse of 4 by 2 which is going to be 63.435 degrees.

By this logic this angle is also theta, now let us fix up the local axes of each member; this is my first member, this is my second member. So, let us say the local axes of this member has the j end here and the k end here and this becomes my x m and this becomes my y m similarly for the second member this becomes my j-th end k-th end and this becomes my x m axes and this becomes my y m axes. So, let us make a table and try to compute some information required for the calculation.

Let us say member number what are the ends that is the jth end and the kth end what is the length of the member what could be the value of theta and what could be the value of cos theta and sin theta C x and C y and what are the global labels of the member let us do this table for member number one member number one j end is that A; is it not; k end is that B for member number 2 j end is that B and this that C the length of the member AB is incline which will be square root of which will be square root of 4 square plus 2 square which will be actually equal to 4.472 meters.

The length BC is simply 4 meters. So, now, x m is oriented this way from the global axes x anticlockwise. So, positive; so, theta is going to be plus 63.435 degrees as far as the second member is concerned theta is 0 degrees; let us compute cos theta and sin; this is going to be 0.447 and this is 0.894 and this is 1, this is 0 cos 0 1 sin 0 0; let us now enter the global labels before that let us mark restrained and unrestrained degrees of freedom. So, let us mark unrestrained degrees in green. So, let us say there will be a rotation here which I marking as theta one there will be a horizontal displacement here which is a vertical; so delta 2 and delta 3.

These are the unrestrained degrees the restrained degrees are going to be let us say theta 4, delta 5 and delta 6 and theta 7, delta 8 and delta 9. So, now, let us mark the labels global. So, let us say j end k end for the member AB. So, I should say the labels are 7 and 1; these are the 2 moments 7 and 1, then the next is along the y axes. So, 9 and 2 the next is along the x axes. So, 8 and 3; how do you label this take a fixed beam; let us say this is x m this is y m; is it not; so 1, 2, then along y 3, 4, then 5 and 6. So, same style here 7 and 2, 7 and 1 are the moments the 9 and 2 or along y axes global and then 8 and 3 along x axes global.

Similarly, let us do it for the second member. So, you can do it easily; now 1 4 2 5 and 3 6; correct, it is done after we do this.

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Rotational Coeffts

$\overline{K}_{AB} = \frac{4EI}{L} = \frac{4E(0.0016)}{4.472}$

$\overline{K}_{BC} = \frac{4E(0.0016)}{4.0}$

$\frac{2EI}{L} = \frac{2E(0.0016)}{4.472}$

$\frac{2EI}{L} = \frac{2E(0.0016)}{4.0}$

$K_{AB}, K_{BC}$

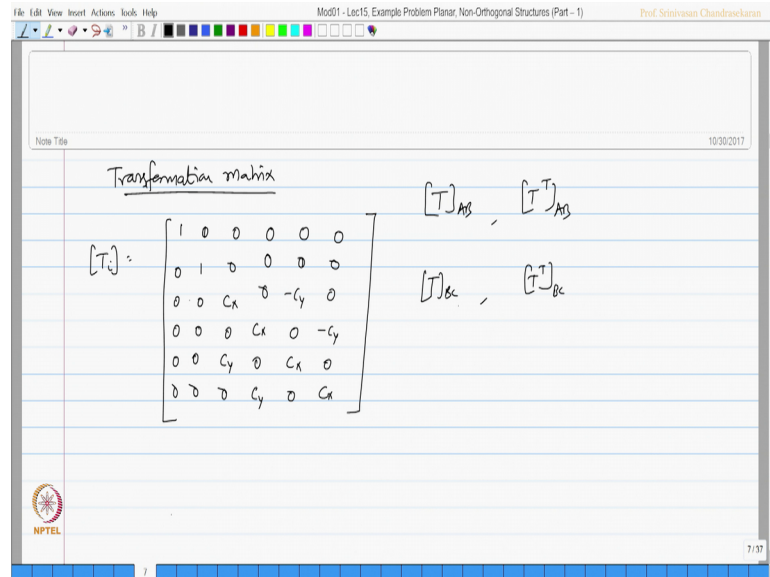
$\begin{bmatrix} p & q & r & s & t & h \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$

6x6

Let us compute the rotational coefficients because we need them for the stiffness matrix let us do it for the member AB you know it is going to be 4 EI by l which is going to be EI times 0.0016 by 4.472. Similarly 2 EI by l which is 2 times 0.0016 by 4.472; this is for the member AB, we can also do this for the member BC which is 4 times E 0.0016 by length of the member which is 4.0 and 2 EI by l which is 2 E 0.0016 by 4.0.

Once I have this values, I can now enter K AB, I can find the stiffness matrix local of AB and stiffness matrix local of BC from the standard equation of 6 by 6 which we already have which are meant for p q r s t and h similarly p q r s t and h, we have this matrix with us, we know this matrix; let us do it for K AB and K BC; similarly I can also now find the transformation matrix.

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Transformation matrix T of any member I is actually given by it is 6 by 6 matrix 1, then 1 and 0 0 C x 0 C y, 0 then 0 0 0 C x 0 C y, then 0 0 minus C y 0 C x 0, then 0 0 0 minus C y 0 C x.

Where C x and C y or available in the table, here you can see here we already have C x and C y for both the members. So, one can compute this transformation matrix for the member AB and for the member BC. So, there is no problem; is it not? I can also compute the transpose of this matrix for AB; the transpose of this matrix for BC that is also a very easy problem. So, let us do this. Now let us complete this after getting this.