Computer Methods of Analysis of Offshore Structures
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Module - 01
Lecture - 18
Planar truss system

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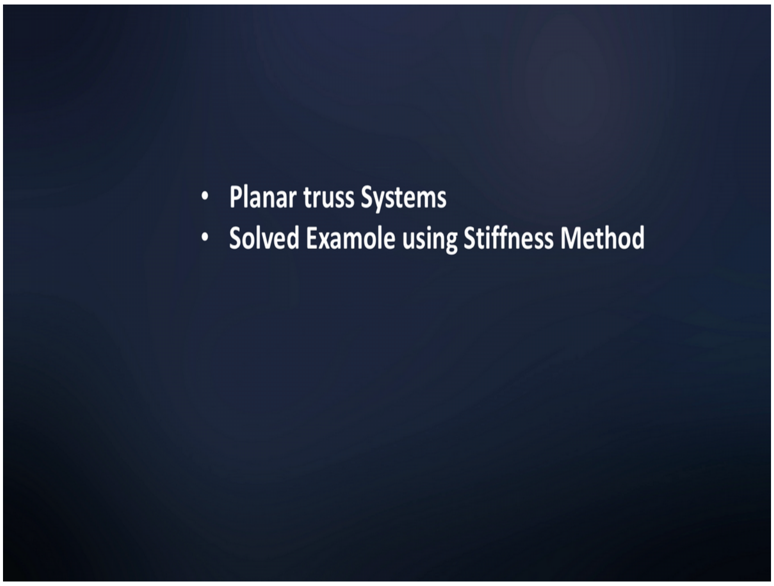

Friends let us continue with the 18th lecture in module 1. So far we have discussed about framed structures we just got bending elements, both orthogonal and non orthogonal members in the given planer system. In this two lectures we will talk about how to solve the planar truss system using stiffness method. You know truss members have orientations which are non orthogonal. Therefore, there is no specialty about orthogonal and non orthogonal analysis as for as truss members are concerned; usually we all agree that truss members do have diagonal members in a truss system therefore, non orthogonal members are very common in truss system.

So, we will talk about planar truss system, which also contain non orthogonal members in a general analysis.

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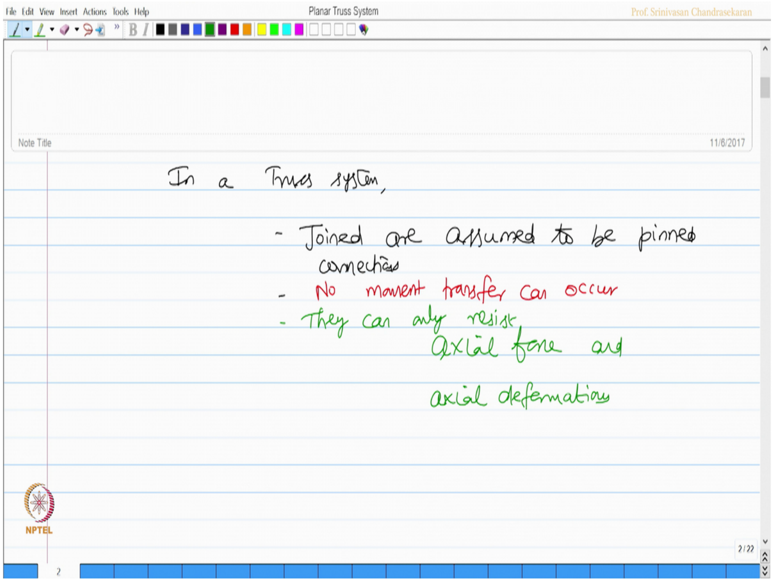

In a truss system, joints are assumed to be pinned, it means no moment transfer can occur therefore; they can only resist axial force and axial deformations.

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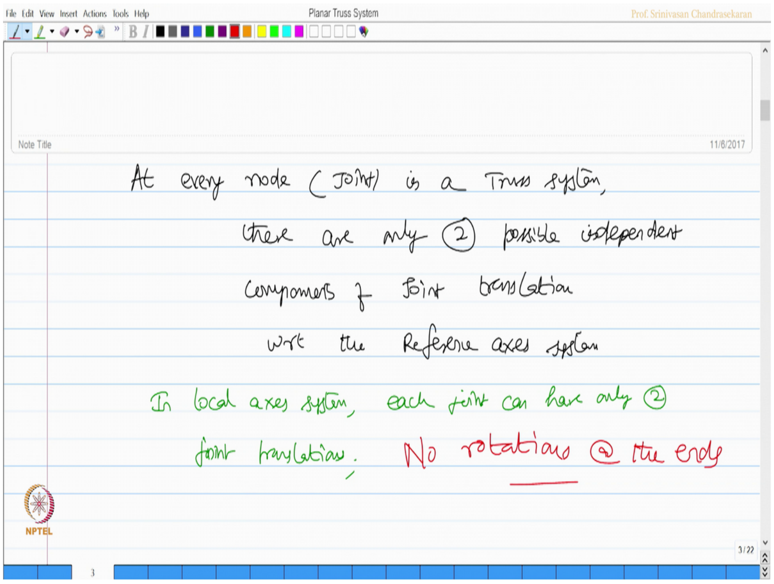

So, therefore, at every node or let us say joint in a truss system, there are only two possible independent displacements of joint translation with reference to the reference axes; similarly in local axes system each joint can have only two joint translations.

So, no rotations at the ends; having said this let us try to draw the transformation between the global to the local axes, say this is the truss member inclined to an arbitrary value theta.

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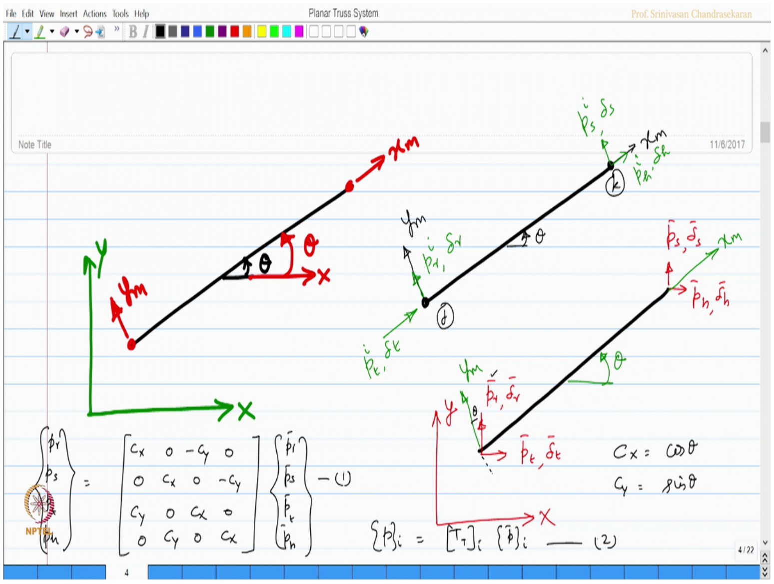

There is a style of marking this theta because this theta should be always measured with reference to the global axes. It means what is the inclination of the local axes x m with reference to the global axes x. So, the angle of theta is with reference to the capital X to small x that is theta.

Let us say we have a member which is inclined arbitrarily, which has two joints or two nodes let us name these as j and k as usual and let us this angle be measured from the global x to the local x as theta. So, this is my x m axis this is going to be my y m axis which is local coordinate system and we all know that there will be independent translations, happening along x and along y we know we will name this as P r of the i-th member and the displacement delta r similarly at disjoint p s of the i-th member and displacement this s similarly in the axial direction it is p t of the i-th member and displacement delta t and this is p h of i-th member and delta h.

This convention is similar to what we have discussed in the beam elements, also accept that I have removed the end rotations theta p and theta q that is all I have kept r s t h as usual let us compare this with another axes system, but in this case I am going to mark the degrees of freedom related to the global axes. So, this is going to be my along y which is p r bar delta r bar, we know that bar represents the global responses.

Similarly, in this case is going to be p r s delta bar s similarly along x this is going to be p bar t delta bar t and along x in the k-th node will be p bar h delta bar h. So, this represents my reference axes system x and y and this represents my local axes system x m and y m and theta is measured from the global towards local in the anticlockwise direction. They also say that c x is cos theta and c y represents sin theta. So, now, what I want is to convert the local responses with respect to the reference axes responses which I called as transformation.

So, I can write the transformation vector here very easily which will be let us say p r p s p t and p h which is connected to the global responses, which is p r bar, p s bar, p t bar and p h bar using a transformation matrix. So, let us quickly see how do we transform this you know if I resolve p r bar we resolve p r bar along y m; obviously, this angle is theta this is also theta. So, I can say this going to be p r will be p r bar cos theta and if we resolve this p s down it will be minus sin theta. So, I can write it as a transformation like this similarly in the other end I can say this is going to be c x and minus c y.

Similarly, I can always find p t that is along x axis there is the transformation which can be C y 0 C 6 0 which will have contribution from r and s similarly the other end will have contribution from t and h; so C y 0 C x. So, now, I can write this as p vector is equation number one now I can write a new equation saying p vector in local for i-th member will be transformation matrix for the truss of i-th member multiplied by p bar of i-th member equation number 2.

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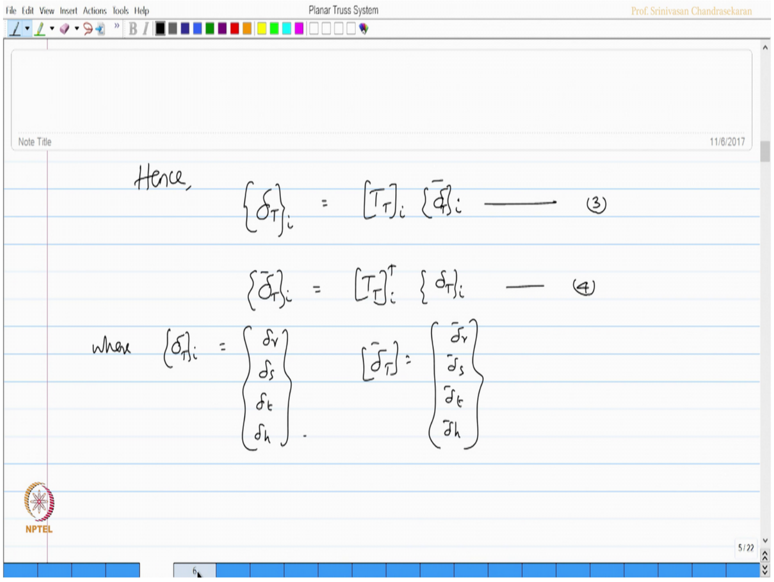

Therefore all relationships like the displacement of the truss member of the i-th member will be again connected to the transformation matrix of the i-th member with the top the responses of the i-th member in the reference axes system, I call this equation number 3.

Further if you want to find the responses of the truss member in reference axes system delta bar, this can be simply given by T transpose of the truss member with that of the local axes responses which is 4. Where delta transpose in this local axes will be simply delta r, delta s, delta T and delta h whereas, delta bar in the global reference axes system will be simply delta bar, delta s, delta t and delta h respectively, now since we have derived the stiffness coefficients for the member of a beam element.

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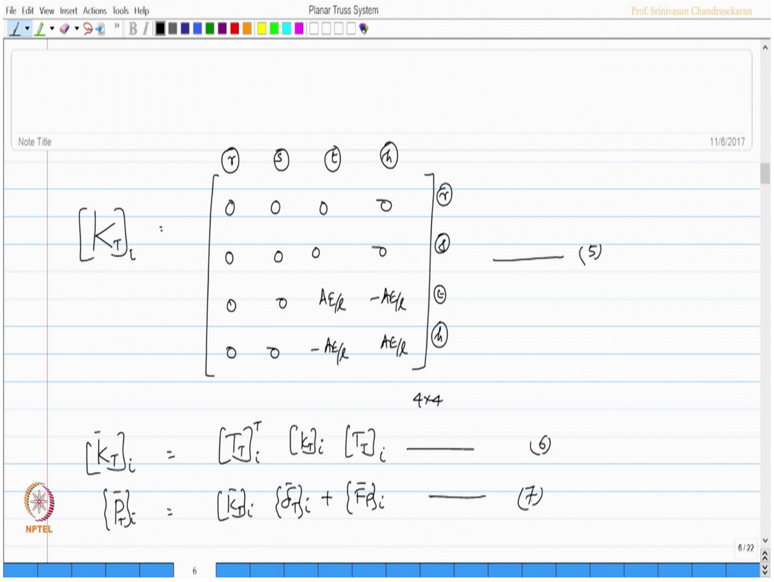

Now, we can do the same thing by just transforming it for the axial response member, I can say now the stiffness matrix of the truss member of an i-th element can be simply given by a 4 by 4 matrix, which will have r s t and h as rows and columns and you know this will be 0 and this will be AE by l minus AE by l minus AE by l and AE by l. This is standard stiffness matrix for a truss member without n rotations if we really want to define the global stiffness matrix of this with reference to the reference x system the i-th member can simply say use the transformation matrix transpose u is the local matrix of this and then again multiply this with the transformation matrix of the i-th member to get this equation 5 I can call this is equation six.

Now, I can also write the responses of the truss member in reference axes system will be simply k bar of the truss member multiplied by delta bar of the truss member plus if there are any end reactions of the truss member.

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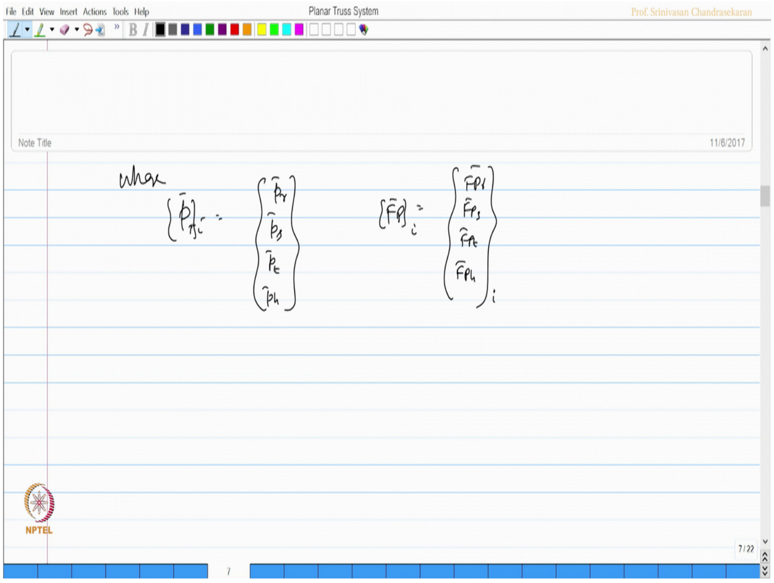

Where p bar of the truss member vector is simply p bar r p bar s, p bar t and p bar h and f p bar of the truss member will be end reactions in r degree of freedom, in s degree of freedom in t degree of freedom and h degree of freedom respectively of the i-th member ok.

So, it is very simple the planar truss problem looks much simpler than the beam element problem, let us take an example on apply this to a problem and solve the problem using stiffness method.

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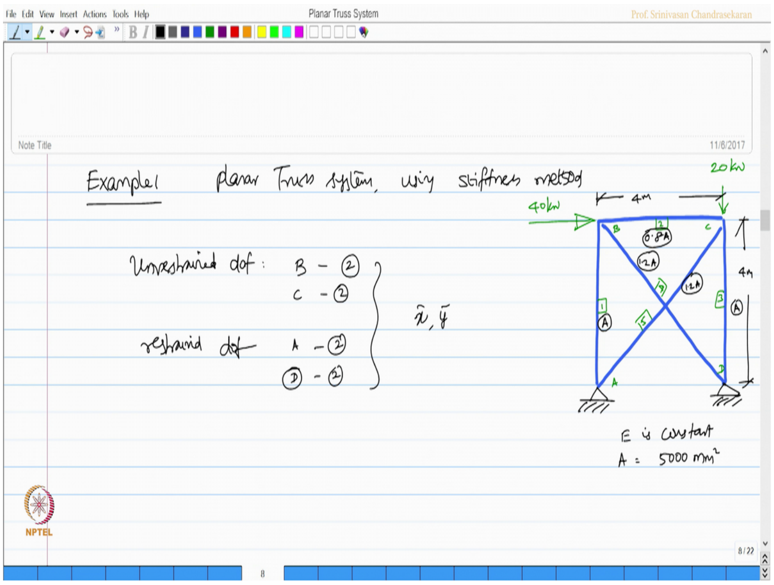

We have also give you the computer code will use the computer code rather to solve this problem. So, we are now going to solve example 1 of a planar truss system using stiffness method. So, let us draw this problem here, the supports are this way indicated the loads are indicated here, it is going a 40 kilo Newton load are played at this node and the 20 kilo Newton load are played at this node.

Let us name the loads as A B C and D that is name the members as 1, 2, 3, 4, 5. So, there are 5 members in this truss system let us make a small table which are required now let us say the area of cross section of this numbers let us say E is constant what are may be the value for the material, this member has got a as the value this number has point a a as the value and this number has A as the value and these two members has 1.2A as the value where A is actually 5000 mm square.

So, this becomes 4000 for the member BC, 5000 for both the members AB and CD for the members 4 and 5 that is member BD and AC this is about 1.2 times of 5 which is 6 thousand. So, A is known E is known, now let us mark the length the dimensions this is about four meters this is also 4 meters. Now let us mark the unrestrained and restrained degrees of freedom, let us draw the figure slightly in the bigger size let us see how many un restrain degrees of freedom this stress has the displacement degrees you know joint B has t 2 joint C has 2.

These are unrestrained and the restrained degrees of freedom joint A has 2 and joint D has 2 these are nothing, but displacements along x and along y is not let us mark them. So, say this is my truss system ok.

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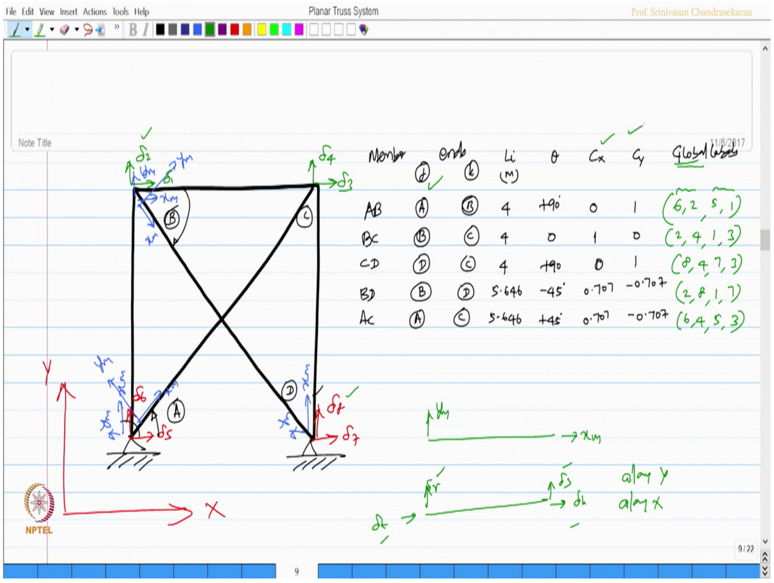

Let us mark the unrestrained degrees this is going to be delta 1 and delta 2 and to be delta 3 and delta 4 then the restrained degrees delta 5 delta 6, 7 and delta 8 in the reference system is this, this is my global x and global y now I want to mark the local x and y of every member let us do that in blue color

So, this member will have x m and y m, here this member will have x m x m and y m here, the fourth member will have x m and y m here and the fifth member will have x m and y m here. So, let us make a small table the table is very interesting and easy you say the member let us say the ends where is my j-th end, where is my k-th end for the member what is the length of the member in meet us what is the angle theta of the member therefore, what is my c x and c y and what are my global labels let us do that.

So, for the member AB the joint A and B are a j end and k end respectively this is 4 meters theta is plus 90, you have to measure x m angle with the reference axes. So, this value you know this is 90 degree therefore, cos and sin can be worked out I will fix the global level slightly later now let us do this BC, this is for B and C 4meters again, but this angle is 0 a for this is one and this is 0 similarly for the member CD this is D and C you know the origin is here this is again 4 meters again plus 90.

So, 0 and 1, for the member BD original set B because this is A this is B this joint is C this joint is D. So, BD is or not the length is root of root 2 into 4 meters that is 5.646 now the angle, you have to measure this angle with reference to this x x axis which is going to be minus 45 its anticlockwise is positive is not. So, this going to be 0.707 this is minus 0.707; for the member AC it is at a and c length is 5.676 meters.

If you look at this angle, this angle is anticlockwise 45; so plus 45. So, it is going to be 0.707 and as points 0.707. Let us look at the global levels please compare a standard truss element a standard truss element, which will have x m and y m the degrees of freedom are r s t and h is not? So, first two labels are along y, the second two labels are along x correct. So, now, let us look at the member AB. So, first two labels are along y global y. So, 6 and 2 were the labels 6 and 2 the next two labels are along x. So, it is 5 m 1.

Similarly, for the member BC you can write now 2413 that is along y and then along x similarly for the member CD the D is here, the 8473. The first two refers degrees of freedom starting from the j-th node, the second two refers degrees of freedom again starting from j-th node correct let us for the member BD the origin is here therefore, along y 2 and 8 p very careful we are looking for global level; so 2 and 8 and then along x 1 and 7 correct.

So, for AC origin is here along global y 6 and 4; so 6 and 4 then along global x 5 and 3. I do not think there is any doubt here still you can understand very clearly there will no difficulty in this correct once we mark this based upon the values of c x and c y once I know the transformation matrix, I can easily find out transformation matrix for each member because I know C x and C y I can now find each member transformation matrix very easily. One can also find KAB using this relationship; you know K local is going to be from equation 5.

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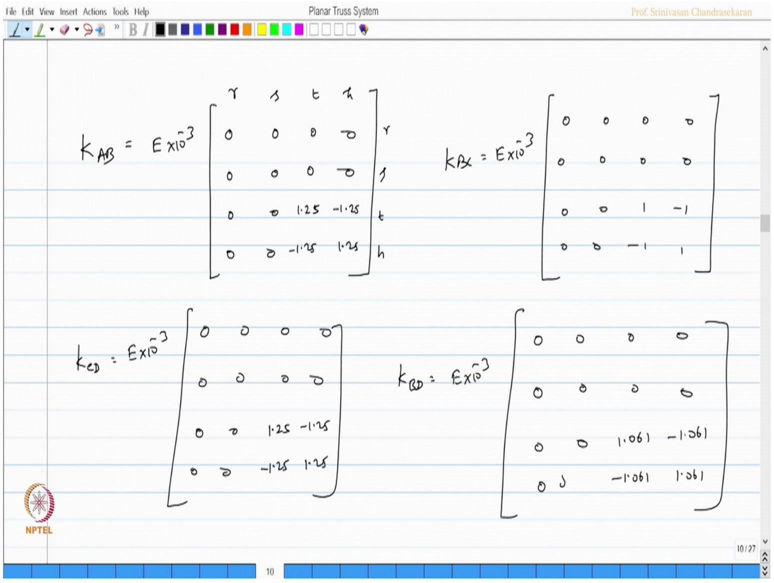

So, use question 5 AE and l are known therefore, I can easily find K AB which will be E into 10 power minus 3 for the local labels r s t and h will be 0 0 0 0, 4 zeros here 2 zeros here these also r s t and h 1.25, minus 1.25 minus 1.25 and 1.25 the K AB. Let us do this K BC which will be E into 10 power minus 3 again similarly I can do this for local K CD which will be E 10 power minus 3 and so on is not?

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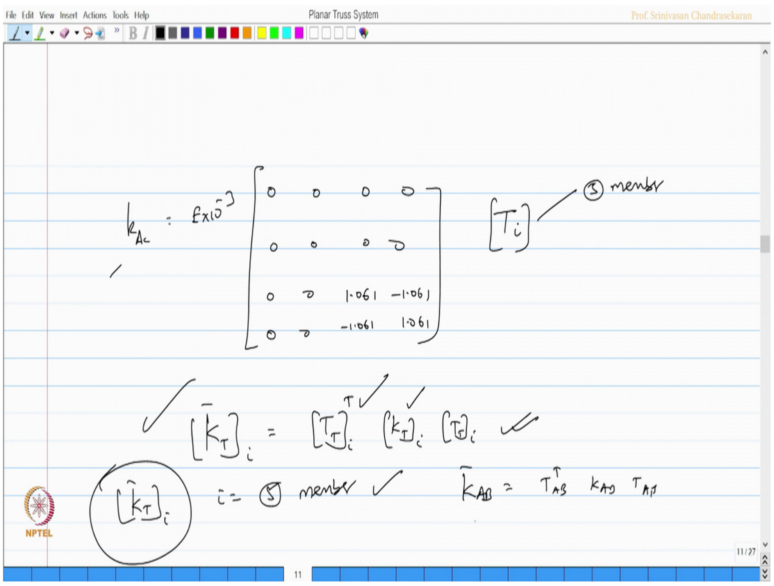

I can do it for KBD which is E 10 power minus 3, I can do it for KAC which is E 10 power minus 3. So, now, friends we have the stiffness matrices for all the 5 members you can see here for the member a b, b c, c d, d b, d and a c we have all the 5 members we also have the transformation matrix for all the members for a b for all the 5 members we have the transformation matrix because we have the relationship of c x and c y. Now I can always find K global of the truss member of any member, which is given by this relationship T transpose K local and T all the member.

So, I use this expression and find K global of all the members. So, now, I get k global of the truss member, for all the 5 members because I have the transformation matrix of all the 5 members I have local stiffness matrix of all the 5 members I can use this relationship and compute this. Now what we have with us says the K global matrix of all the 5 members which I am now writing here.

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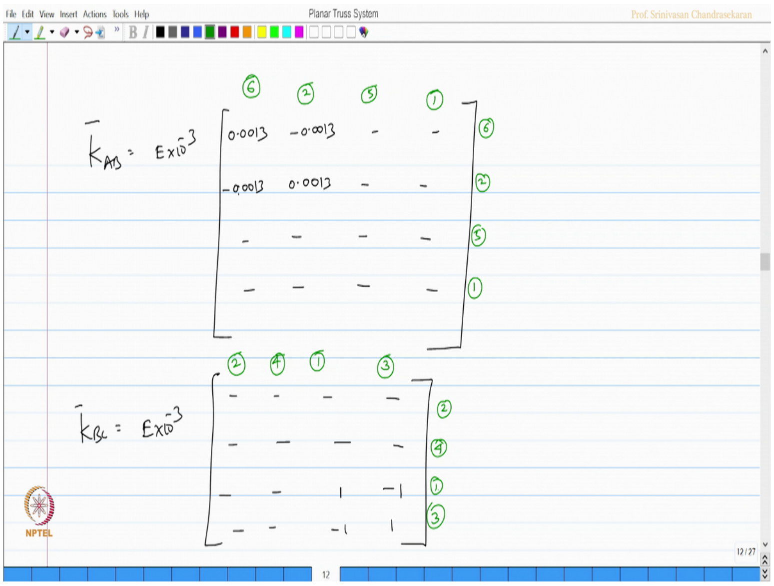

So, I should say K bar AB which is E into 10 power minus 3 by using this relationship K bar AB will be T transpose of AB multiplied by K AB then T of K AB is not? We can easily get this K AB will be given by 0.0013 minus 0.0013 0 0 that is K AB let us find K BC.

Now, let us marl the degrees of freedom label here; we already know K AB as a degree of freedom 6251. So, let us mark the labels here because we need this labels 6251, 3251 we need this labels to assemble this stiffness matrix later now K BC which will be E 10 power minus 3 now the labels are in this case 2413 they can also find K CD.

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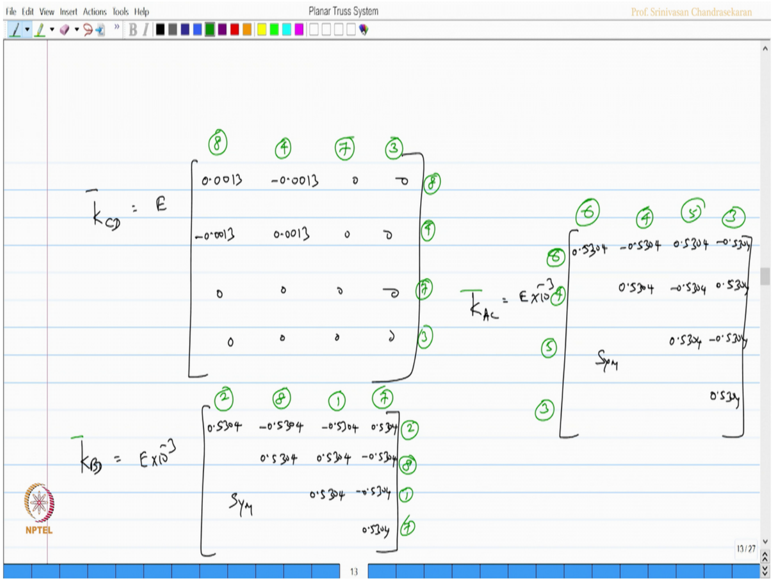

Which will be E K AB has no multiplier simply E this also has no multiplier simply it is E end to this.

E times of 0.0013 0.0013 0 0; now or the labels for K CD are 8473. Let us do this for K BD with a diagonal member which will have a multiplier of this value 0.5304, now the labels are going to be 281 and 7 similarly I can do this for K AC. So, the labels are going to be 6 4 5 and 3. So, we have all the K bar for all the members, we need assemble this and apply the equation and solve the problem which will do the next lecture.

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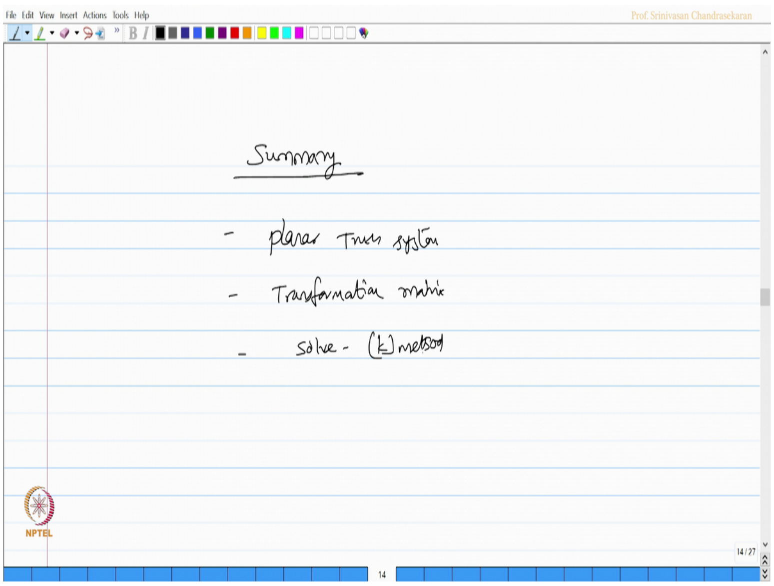

So, in this lecture we understand that how to derive the stiffness matrix for plane truss system, how to derive the transformation matrix for each member within attempted solve a problem using stiffness method. We will continue the solution the next lecture.

Thank you very much.