Computer Methods of Analysis of Offshore Structures

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Module - 01
Lecture - 20
3d Structures : Analysis by Stiffness method (Part – 2)

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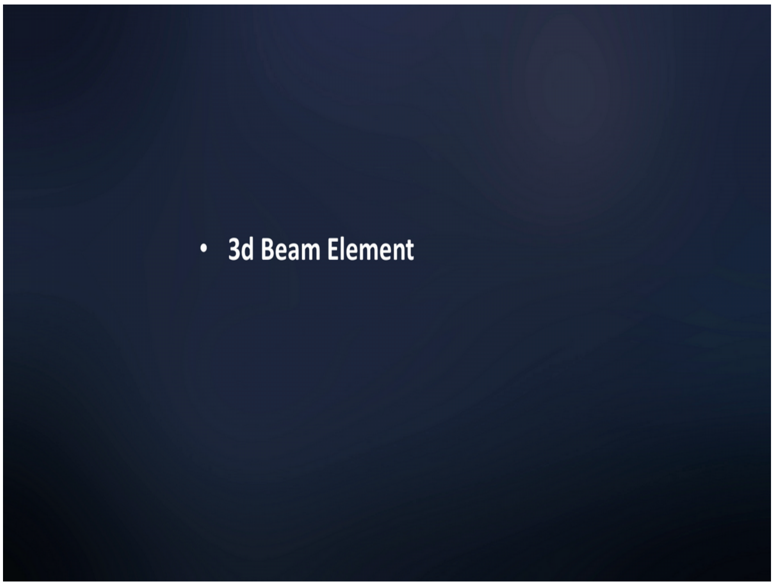

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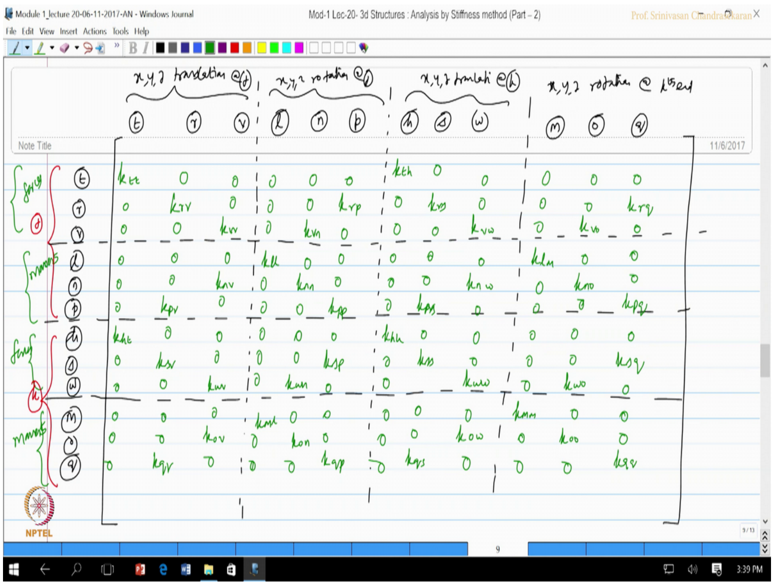

Let us, now write down the coefficients contribute in this matrix. Let us write down the labels: t r v, l n p, h s w, then m o q that is an order of writing this.

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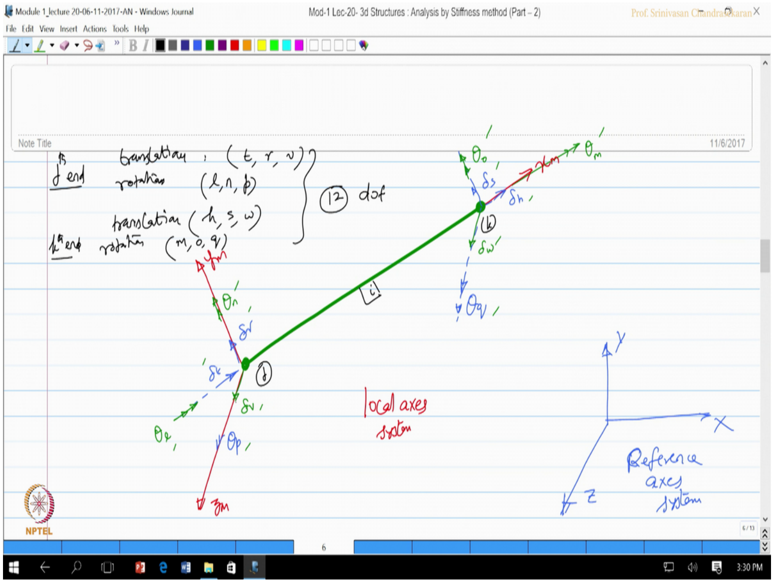

Actually, we will look at them carefully t r v are actually translations and l and p are rotations at j-th end. So, let us carefully write it here.

These are x y z translations at j-th end, these are x y z rotations at j-th end. Naturally this will be h s and w then m o and q will be x y z translations at k-th end this is x y z rotations at k-th end correct. Let us also mark the labels here this is t r v, then l n p, h s w, m o and q.

So, now these are happening at the j-th end, these are happening at the k-th end, and these are forces, and these are moments, Similarly these are forces, these are moments. Let us fill up these values. When you give unit displacement along t only t and h are influence you see here.

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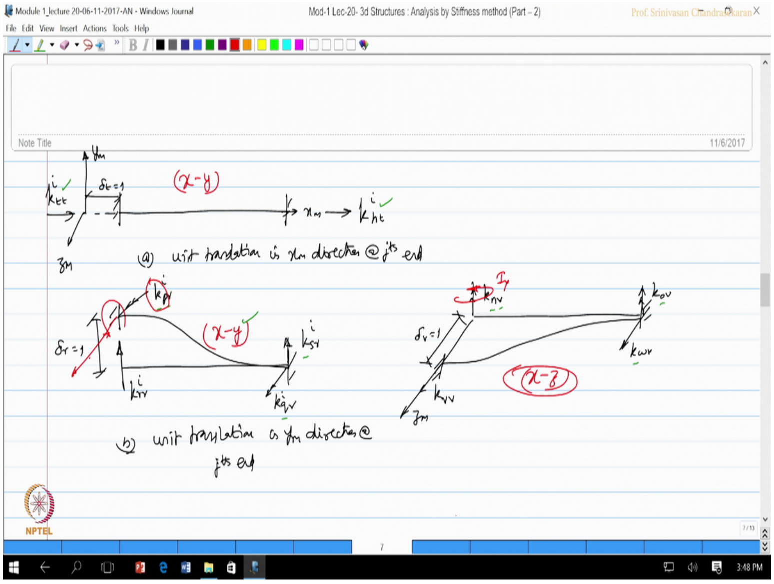

Only t and h are influenced. So k tt then k ht remaining all are zeros. Similarly, when I do it for h at the k-th end you see it will be k th and k hh remaining all will be zeros. When you give unit displacement along r along r; r is along y. So, you get moment about so you are doing on x y plane. So, r p r p s and q are invoked so let us do that when you give along r r p s and q are invoked remaining all are 0.

Similarly, at the other end when you give along s you know r s p and q are invoked remaining all are 0. When you give unit displacement along v you know let us see here along v vn o and w are invoked. So, when you give along v vn so k vv vn o and w are invoked remaining all are 0. Similarly, contemporary at k-th end will be w. So, k vw k nw k ww and k ow remaining all are 0, so 0 0 1 more 0 and 1 0 0 0 ww zeros and 0 ok.

Now, let us give moments about l when I give moment about l l and m are invoked. So, l and m are invoked, so l and m are invoked remaining all are 0. Similarly, at m l and m are invoked remaining all are 0 so 0 0 0 and 0 0. Now you will give unit rotation about n v n o and w are invoked.

So, v n o and w v n o and w are invoked remaining all are 0. Similarly contemporary o will be; k vo k no k wo and k o o the last 1 is rotation about p. So, p q r as will be involved. So, k rp k p p s k sp and k pq remaining all are 0. Similarly q, so k rq k pq k sq and k qq remaining all are 0 so on.

So, friends I do not think you have any difficulty in entering this matrix for a member identifying the corresponding degrees of freedom influenced by giving unit displacement along x y z and unit rotation about x y z both at the j-th end and k-th end respectively.

Now, our job is to fill up these values of coefficients from the known principle quantities of beam element, we have to form the company stiffness matrix of a 3d beam element which we will do now. Let us enter the same matrix again, now I am saying this is k matrix of i-th member. Let us enter the labels t r v l n p h s w and m o q this is center at the values, t r v l n p h s and w m o and q ok.

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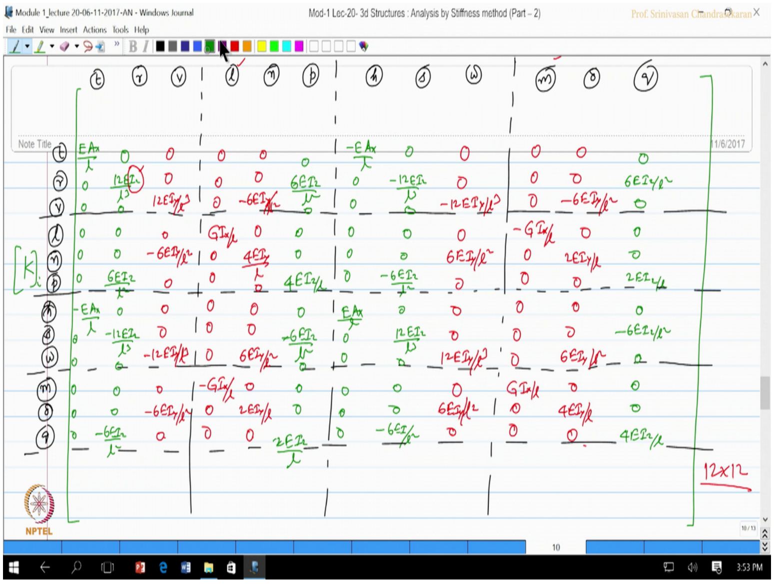

Now, you know by giving unit displacement along x, we will get this value which is known to us which is a e by l. So, E Ax by l and t and h will be effected so minus E Ax by l, similarly by giving along h this will be minus E Ax by l and plus E Ax by l remaining all are 0, I have not enter the 0 values. So, let us enter them.

Next 1 is giving unit displacement along r along the y axis we know comparing the beam element the stiffness matrix can be entered in simple as r p. So, r p s and q will be invoked r and s are reactions p and q are moments. Therefore, in simple terms this will be 6 E I z by l square q will be again minus 6 E I z by l square the reactions will be 12 E I z by l cube and minus 12 E I z by l cube, 0 0 0 0 0 0 0 0. Let us, do this for s also in s you know 3 will be negative only along x will be positive. So, this is positive, remaining all will be negative, remaining all are zeros by just convention known to us, then let us give unit rotation about z axis we have only know this value. So, that is going to cause invoke p q r s.

So, p r s and q only this will be there we all know that is going to be 4 E I z by l. So, 4 E I z by l and q is going to be 2 E I z by l which we already known for a 2 dimensional main member I am entering them in green and the reactions are going to be along r is positive. So, 6 E I z by l square along s it will be negative 6 E I z by l square remaining all will be 0; so 0 and 0 0 0 and 1 0 and another 0 below and 0 0. Similarly, we can do it for q also this is going to be 6 E I z by l square, then at p this will be 2 E Iz by l where, q is going to be 4 E I z by l and for s it is going to be negative minus 6 E Iz by l square.

So, reactions are l square and moments are real remaining all are 0 2 0 0 2 zeros, then member 0 0 0 0 then value. So, we have entered the 6 columns which are conventional for a simple beam element which we already know, Let us do the remaining with red color which is new for the 3 dimensional member. So, let us give unit displacement along v and we will get now let us try to understand how do we get E I z here how do we get E I z here.

So, look at the bending plane when I give unit displacement the rotation contribution comes about the z axis. So, k p comes about the z axis that is why reducing I z here that is why I z everywhere. Let, us now go to the unit displacement along z axis which is v, when I give unit displacement along z axis which is v, it is x z plane. So, now when I give displacement the rotation is going to happen about y axis. So I should use I y ok.

So, be careful we should use I y now. So, now this is going to be 0 0 12 E I y by l cube 0, the again v means n. So minus 3, 6 E I y by l square then zeros then w will be invoked. So, minus 12 e I y by l cube then minus 6 E I y by l square. So, v then correspondingly it is going to w is it not v and correspondingly w, so let us fill up this column now.

So, w column which will have the same values, but with the different sign convention I y l cube 0 6 E I y by l square, then 3 zeros, then 12 E I y by l cube then 1 0 6 E Iy by l square and 0. Now, let us give unit rotation about x axis that is what we are going to do about x axis you are rotating it about x axis. So, you get torsional moment ok.

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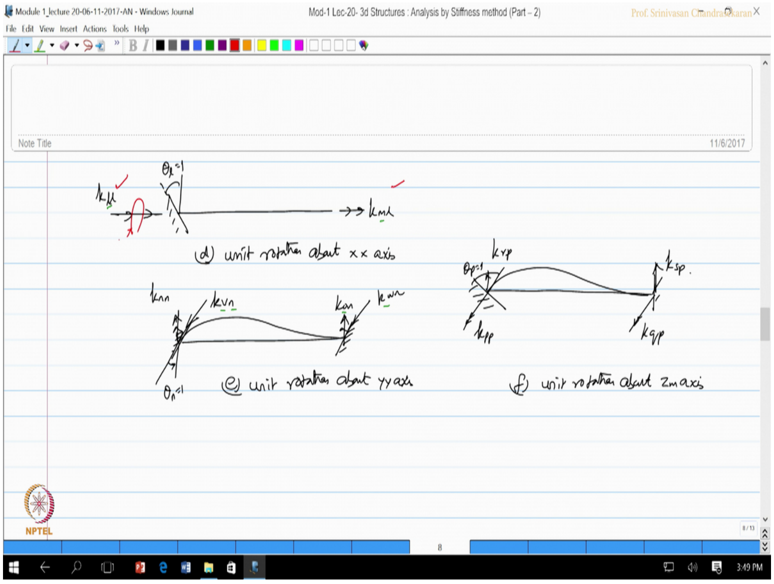

So, I should use now only we can see here only l and m are invoked only l and m are invoked. So, l will be again, so these 3 are zeros this will be G Ix by l in till m all will be zeros is going to be minus G Ix by l, similarly with m l and m will be invoked. So, this will be 0 minus G Ix by l remaining all are zeros and m it is going to be G Ix by l positive remaining all are 0.

Let us, now talk about the last column which is rotation about n. So, when I give that I am going to invoke moment about I y moment about I y. So, I should say if I talk about n the degrees of freedom are n v o and w, so n v o and w. So, let us talk about n v, so t r 0 v is present, so it is going to be minus 6 E I y by 12 by l square then this is 0 4 E I y by l next is 0 these 2 are zeros, w will have 6 E I y by l square and 0 and o o will have 2 E I y by l.

Let us the last column o, which will be again these 2 are again zeros. So, this will be minus 6 E Iy by l square. So, 0 going to be 2 E I y by l 0, then this is going to be 6 E Iy by l square 0 and 4 E Iy by l 0. So friends, we have now all the coefficients of a 12 by 12 matrix which qualifies for the member stiffness of a beam element which is arbitrarily oriented in space it is going to make the space element.

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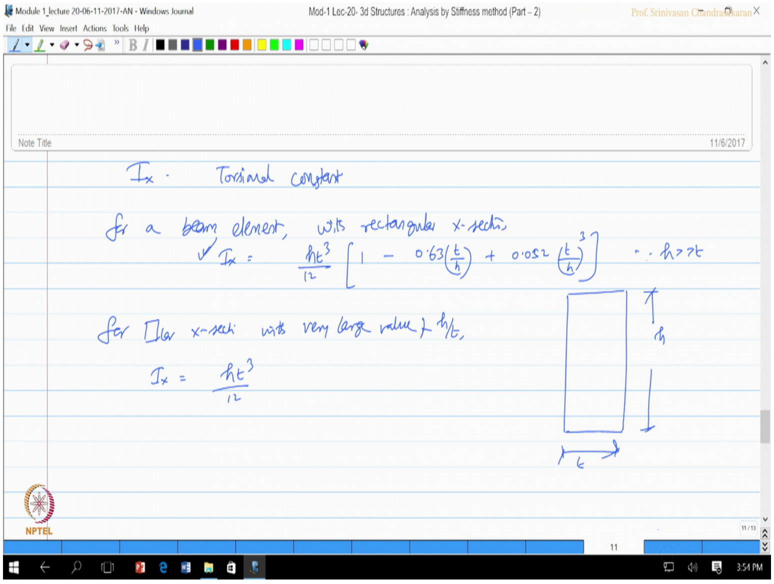

Now, let us see how to estimate this I x, this I x I x actually is called torsional constant, now for the beam element with rectangular cross section Ix can be given by ht cube by 12 1 minus 0.63 t by h plus 0.052 t by h cube. So this is approximate where, h is very large compare to t. So, I have a rectangular cross section this is my h and this is my t, if h is very large compare to t then I can use this approximate equation to compute the torsional constant.

For rectangular cross section with very large value of h over t Ix can be approximated as simply ht cube by 12 where this ratio will go very low.

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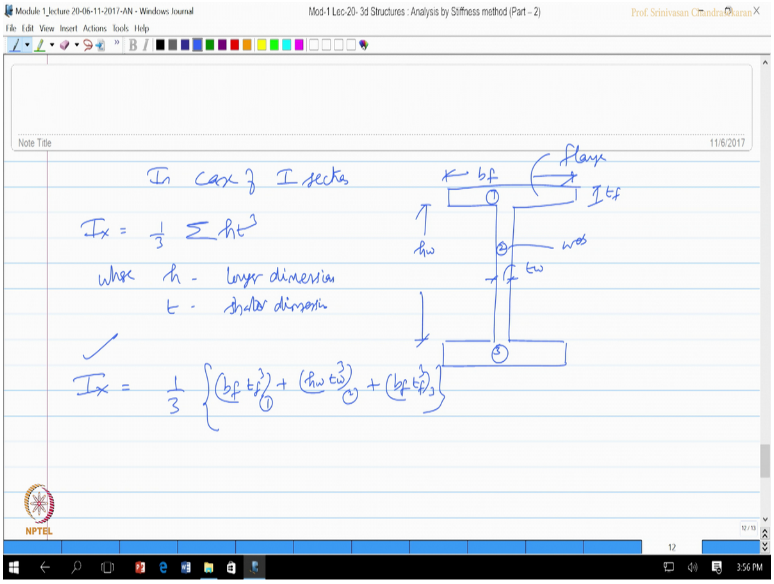

In case of I sections as shown, let us say this is tf this is bf breadth of the flange, this is tw and this is hw. So, this is the web this is the flange Ix can be now the torsional constant is one-third of summation of ht cube where, h is the longer dimension of rectangle and t is the shorter dimension of rectangle.

Having said this, now I can find the torsional constant for this I section as 1 by 3 of h is the longer 1 let us say I call this as member 1, member 2 and member three, for member 1 it is going to be bf tf cube this is for member 1, then for member 2 the longer 1 is hw tw cube member two, plus again for member 3 it is bf tf cube is for member 3. So, I can find the torsional constant approximately, using this relationship.

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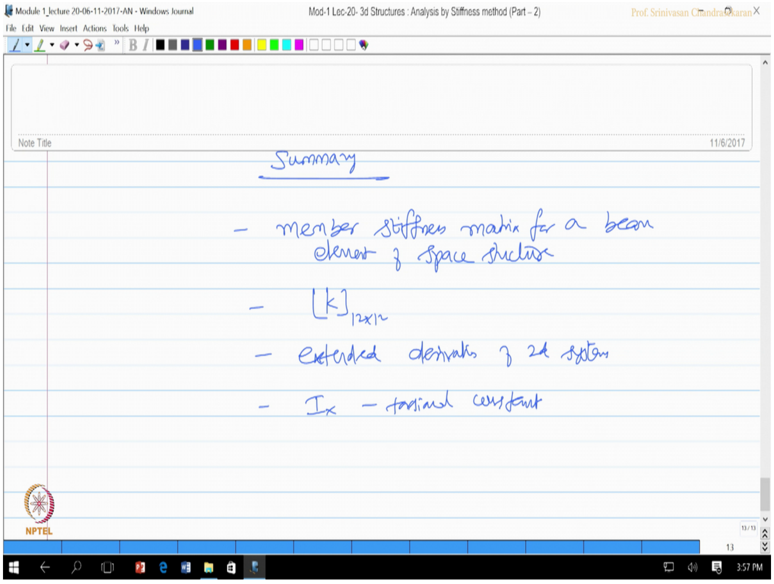

So, now for the 3 dimensional member aligned in space, we know how to compute the member stiffness matrix for a beam element of space structure. So, we know that this k it is going to be 12 by 12, which is actually an extended derivation of 2d system. We also explained you how to compute the torsional constant using approximate relationship for these members.

Friends, we will continue with discussion in next lecture and try to see how we will transform these member stiffness matrices to global stiffness matrix with reference to the reference or global axes system. Then we will assemble these to form the k total of the entire space structure to analyze the structure.

Thank you very much.